DE SITTER DIAGRAMMAR AND THE RESUMMATION OF TIME

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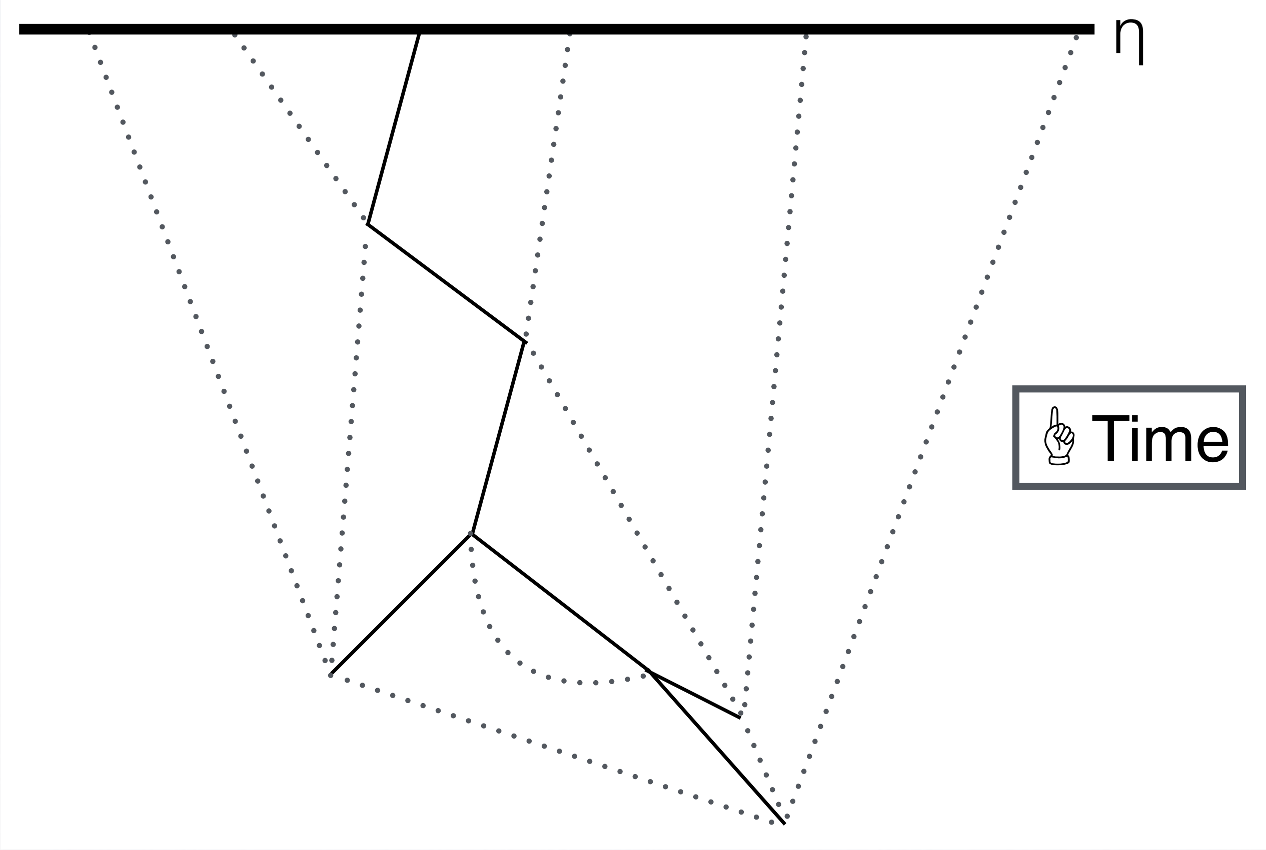
w/ Raman Sundrum

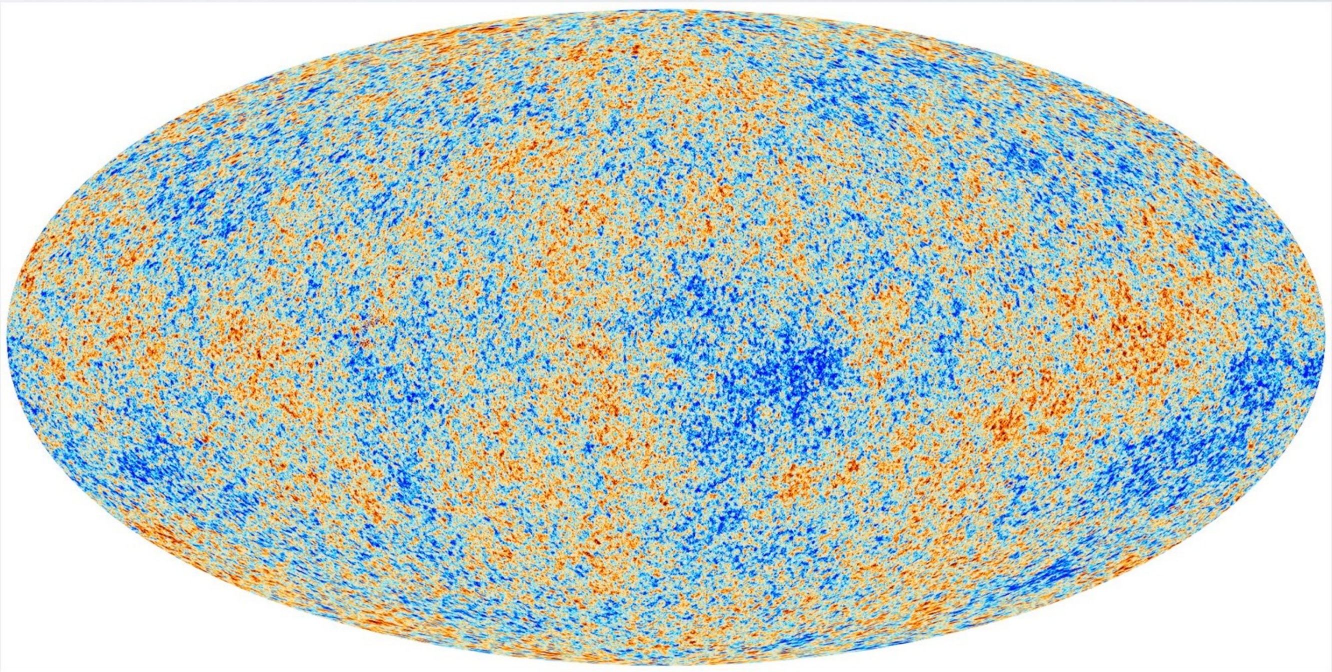


Newton 1665 Seminar, SISSA
6/16/20

1912.09502
2007.xxxxx

DE SITTER AFFECTS US

- **Cosmological inflation:** Nearly **De Sitter** evolution
- Current era of **cosmological acceleration**, **De Sitter**-like
- **Eternal inflation**, asymptotic **De Sitter**, possible resolution to **cosmological constant**.



Temperature anisotropy map from Planck satellite

$$<\varphi_k \varphi_{-k}> \sim 1/k^{3+(1-n_s)}$$

$$n_s = 0.968 \pm 0.006 \text{ (PLANCK)}$$

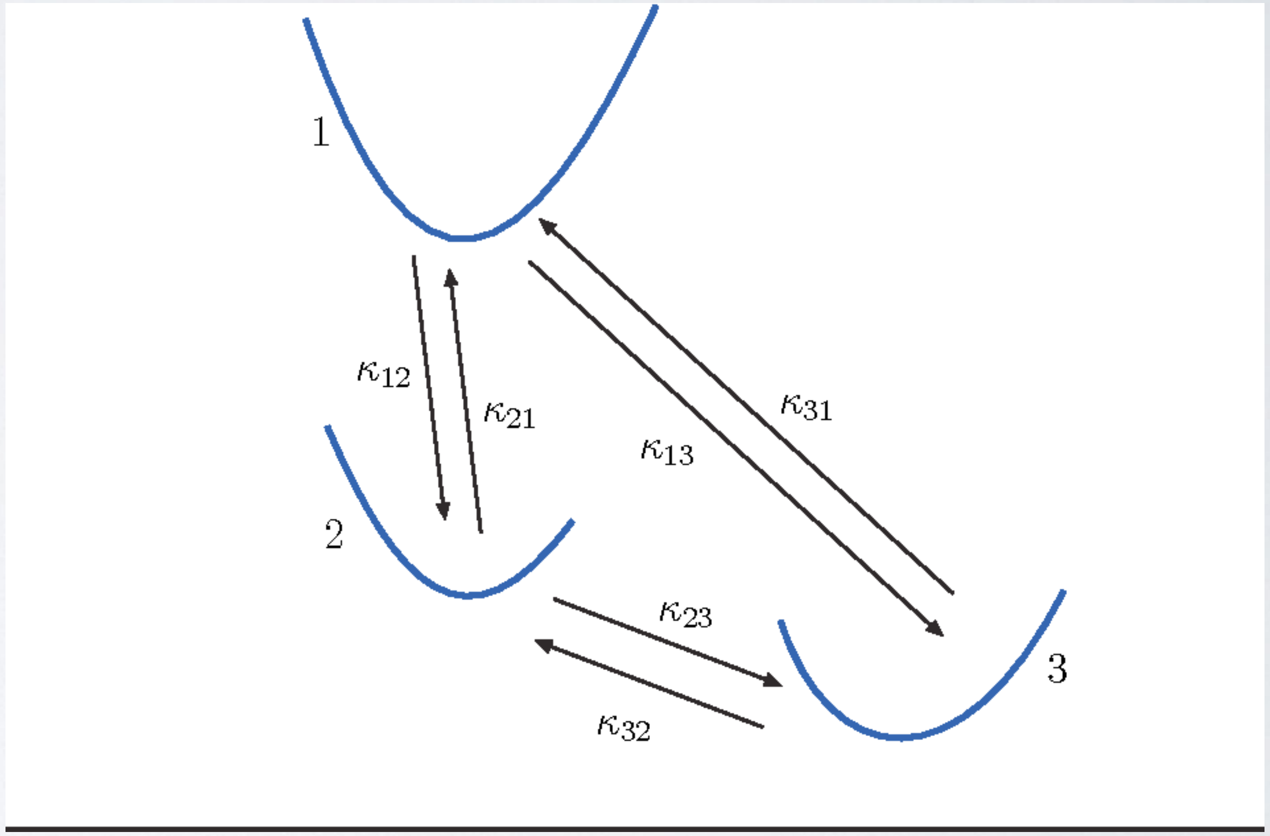
BABY LANDSCAPE

- Eternal Inflation & the Landscape offer a solution to the CC problem

- 20+ years after dark energy discovery, can we do controlled calculations?

- Our final result is probability distribution over field strengths observed in different Hubble patches at late times

$$P[\{\phi_k\}, t] = |\Psi[\{\phi_k\}, t]|^2$$



Allowing backreaction of scalar onto geometry gives probability distribution over different Cosmological Constants

IR DIVERGENCE

- Let's compute the free theory scalar **two-point function**

$$\langle \phi_k \phi_{-k} \rangle \sim \frac{H^2(1+k^2\eta^2)}{k^3}$$

$$ds^2 = dt^2 - e^{2Ht}\vec{dx}^2 = \frac{1}{(H\eta)^2}(d\eta^2 - \vec{dx}^2)$$

$$\eta = -\frac{e^{-Ht}}{H}$$

- What if we go to position space?

$$\langle \phi(x)\phi(y) \rangle \sim \int \frac{dk}{k} \sim \log(k_{UV}) - \log(k_{IR})$$

- Regulate IR to compute in presence of IR pathology

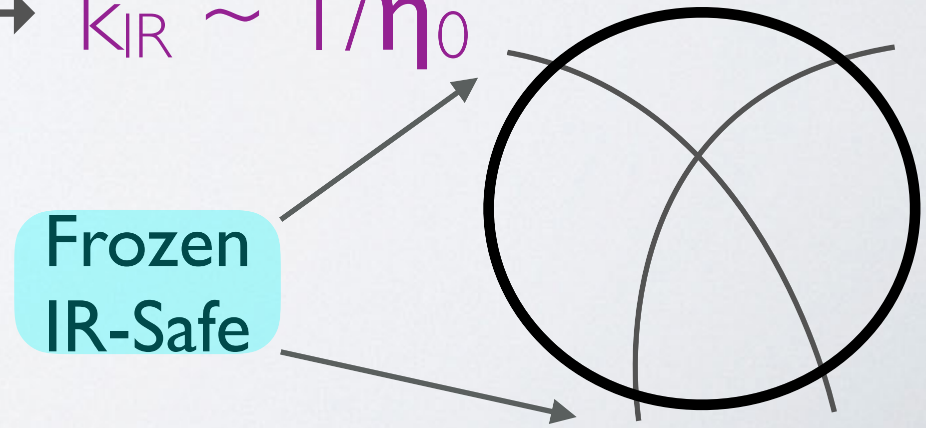
POSITION SPACE

- Finite result:

$$\langle\phi(x)\phi(y)\rangle = H^2\left[\frac{\eta\,\eta'}{\Delta\eta^2 - r^2} + \log\left[k_{\rm IR}^4(\Delta\eta^2 - r^2)^2\right]\right]$$

- But every propagator comes with **log[k_{IR}]**

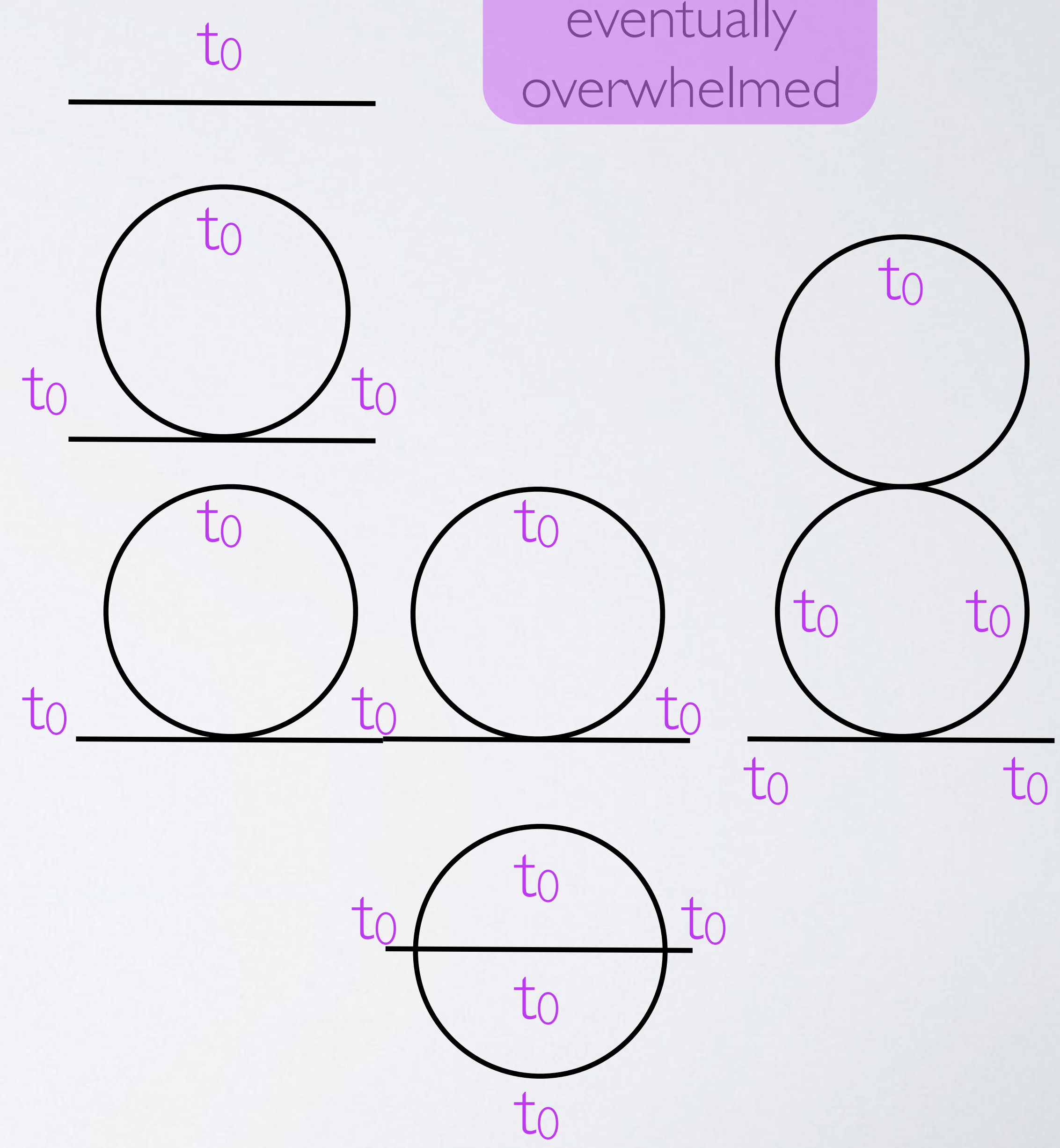
- Cutoff at horizon-scale at initial time → $k_{IR} \sim 1/\eta_0$



PERTURBATION THEORY BREAKS

- $\log[k_{IR}] \sim t_0$
- **At late times** theory becomes **nonperturbative**.
- How bad is it?
- How to regain control?

No matter how small **λ**, eventually overwhelmed



IN-IN FORMALISM

- Let's compute (meta)observable

$$\langle \mathrm{BD}' | e^{iH_{int}\eta} \bar{T}\, \phi(x)^n \, T e^{-iH_{int}\eta} | \mathrm{BD}' \rangle$$

- Expectation value, not S-matrix

Exercise for the audience: Prove this equality

$$\langle \phi(t,x)^n \rangle = \sum_{V=0}^{\infty} (-i)^V \int_{t_0}^{t} dt_V \ldots \int_{t_0}^{t_3} dt_2 \int_{t_0}^{t_2} dt_1$$
$$\times \left\langle \left[\left[\ldots \left[\phi_I(t,x)^n, H_I(t_V) \right] \ldots, H_I(t_2) \right], H_I(t_1) \right] \right\rangle$$

- Commutators, time-ordering enforce **manifest causality**.

Reformulation in Weinberg: hep-th/0506236

IN-IN MANIFEST CAUSALITY

- Graphical argument makes heavy use of **Weinberg's "nested commutator" reorganization** of in-in, but there is an objection to it:

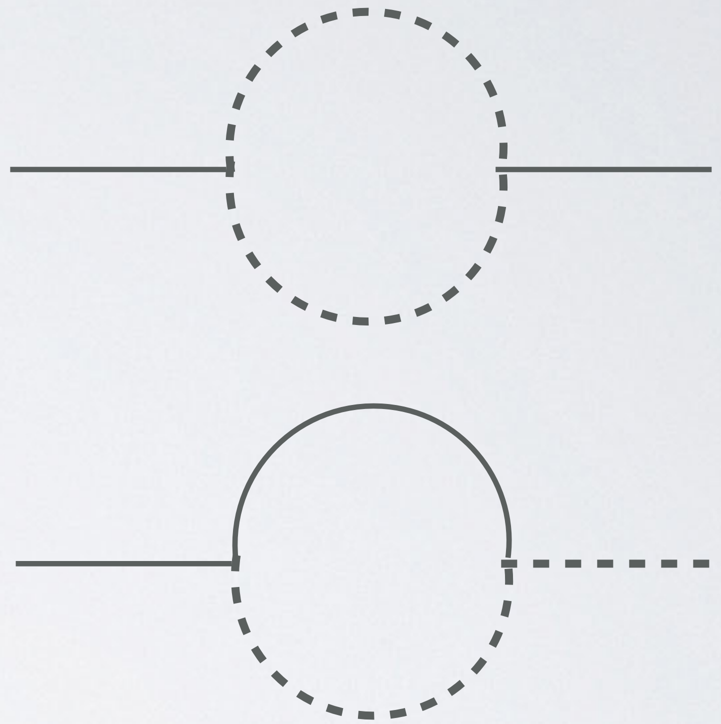
$$\langle\Omega|\phi_{\text{Heis.}}(t,x)^n|\Omega\rangle\Big|_{\text{IR}-\text{reg.}} \propto \langle\text{BD}'|\left[\bar{T}\exp\left(i\int_{t_0(1+i\epsilon)}^{t} H_I(t')dt'\right)\right] \times \phi_I(t,x)^n\left[T\exp\left(-i\int_{t_0(1-i\epsilon)}^{t} H_I(t'')dt''\right)\right]|\text{BD}'\rangle$$

- The typical **iε prescription breaks unitarity** and the symmetry between bra and ket evolution

- However, there is **an equivalent ε-deformation, which is explicitly unitary, and preserves the manifest causality** of Weinberg, $\varphi \rightarrow \varphi e^{\varepsilon t}$ [cf. Kaya: 1810.12324 and MB & Sundrum: 2007.xxxxx]

DE SITTER DIAGRAMMAR

- Two types of propagators:
  - $G_R = \theta(\eta'-\eta)[\varphi(\eta'),\varphi(\eta)] \sim k^0$
  - $G_+ = <\{\varphi(\eta'),\varphi(\eta)\}> \sim 1/k^3$
- Leading contribution **minimizes commutators**
- Causality→
  - >0 commutator per vertex
  - >0 commutator with ext. field



Solid: Commutator (One per vertex)
Dashed: G_+

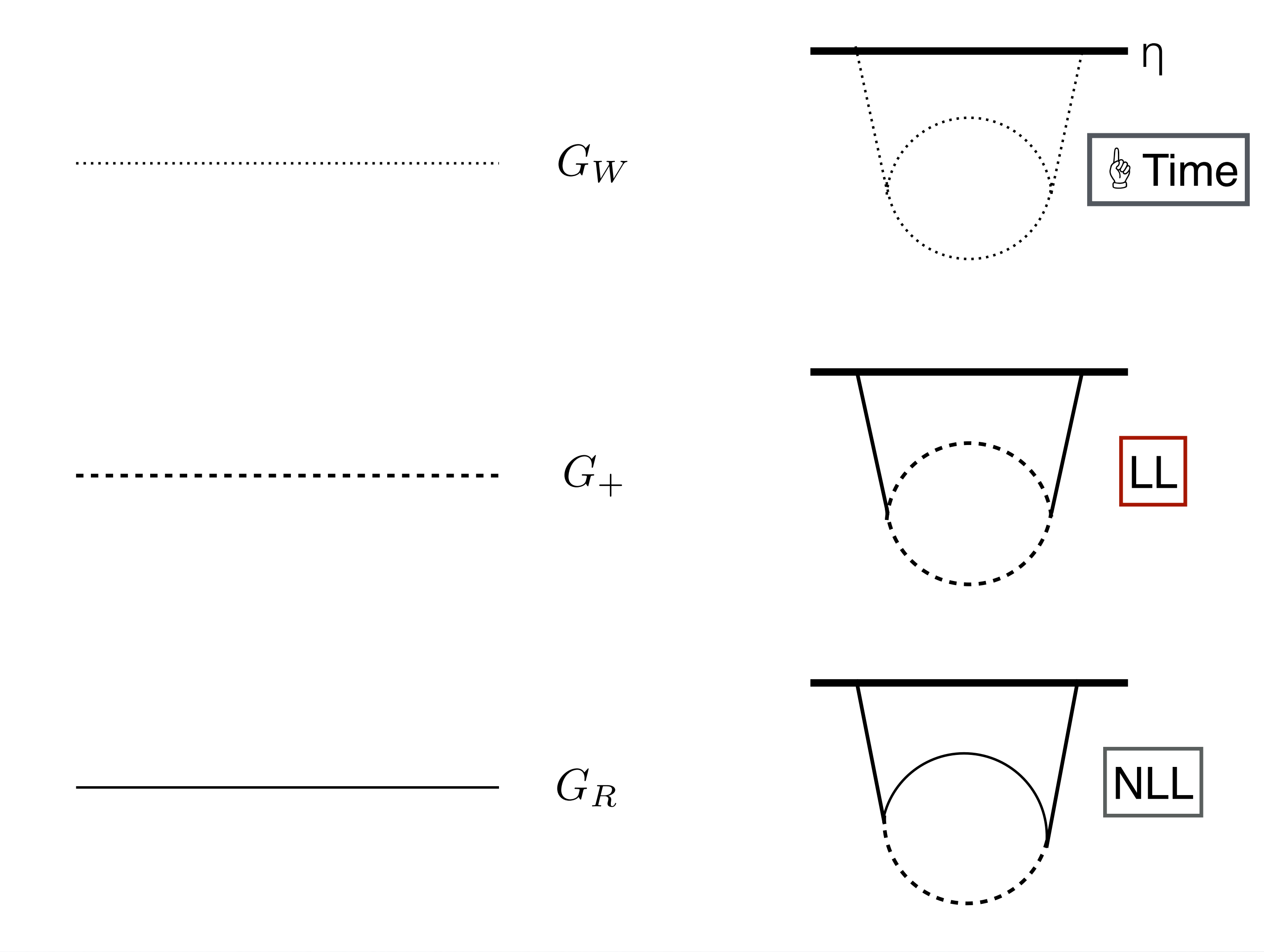
G_+→ log in position space
Integrand ~ $\mathbf{\log[k_{IR}]^{P-V}}$

LEADING LOG

Every diagram topology **decomposed in G_+ and G_R,** then **power count**

$G_W = (G_+ + G_R)/2$

$G_W = \langle\varphi(\eta')\varphi(\eta)\rangle \sim 1/k^3$



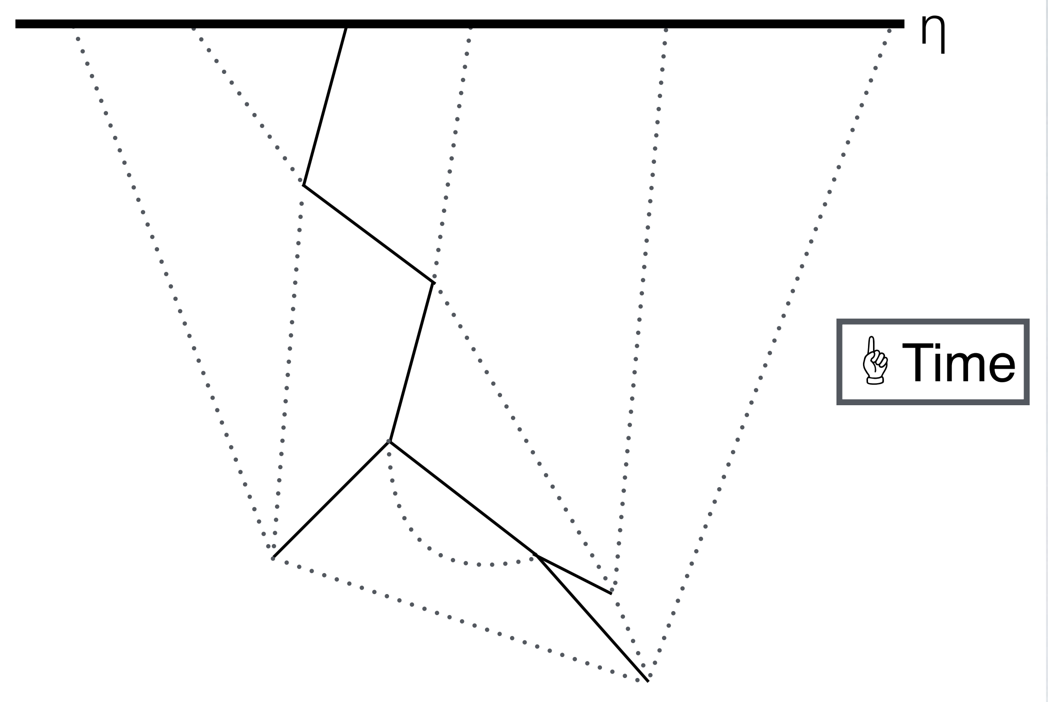
In-In Formalism of Musso hep-th/0611258

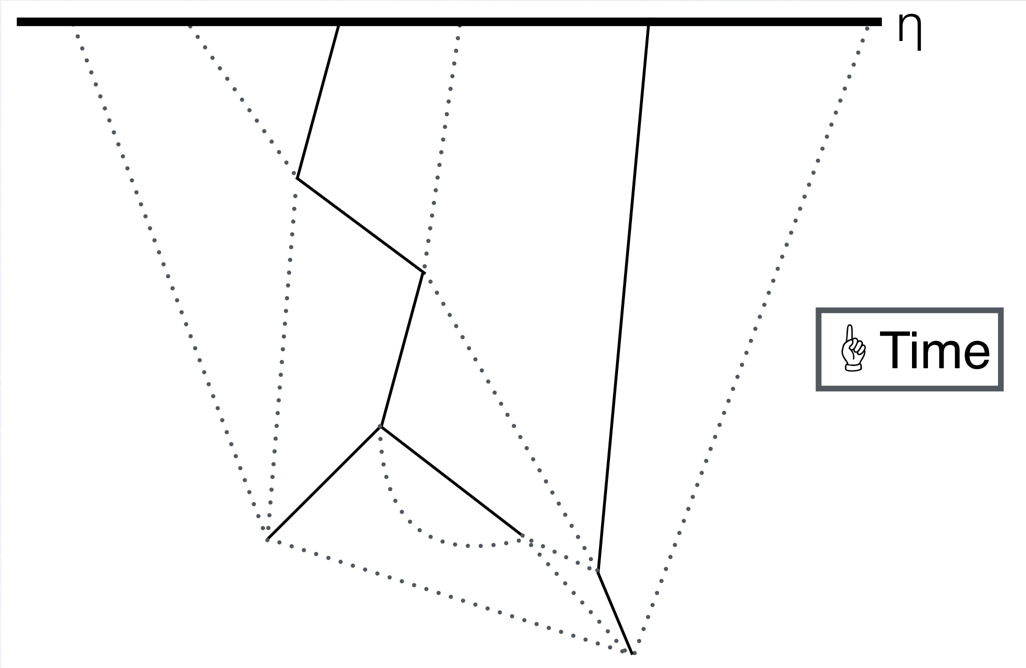
LOOPS FROM TREES

- Leading-Log budget is **V retarded propagators**
- Must **touch every vertex and correlation point, and connect to later time.**

$$\left\langle \left[\left[\cdots\left[\phi_I(t,x)^n, H_I(t_V)\right], \ldots\right], H_I(t_2)\right], H_I(t_1)\right]\right\rangle \neq 0$$

$$\left\langle \left[\left[\cdots\left[\phi_I(t,x)^n, H_I(t_V)\right], \ldots\right], H_I(t_2)\right], H_I(t_1)\right]\right\rangle = 0$$





"De Sitter superhorizon modes are semiclassical"

- **G_R subgraphs are all trees!**
  - Each tree touches one and only one correlation point

FROM INTEGRAND TO INTEGRAL

- Soft expansion:

$$G_{+\,\mathrm{soft}} = \frac{H^2}{k^3}\,\theta(k - k_{\mathrm{IR}}),$$

$$G_{R\,\mathrm{soft}} = \theta(\eta' - \eta)\frac{i\,H^2}{3}(\eta^3 - \eta'^3)$$

$$G_{R\,\mathrm{soft}} \approx \theta(\eta' - \eta)\frac{i\,H^2}{3}\eta^3$$

Dominant contribution for strongly-ordered times, |η| >> |η'|

- **Loop momenta** are **G_+** momenta

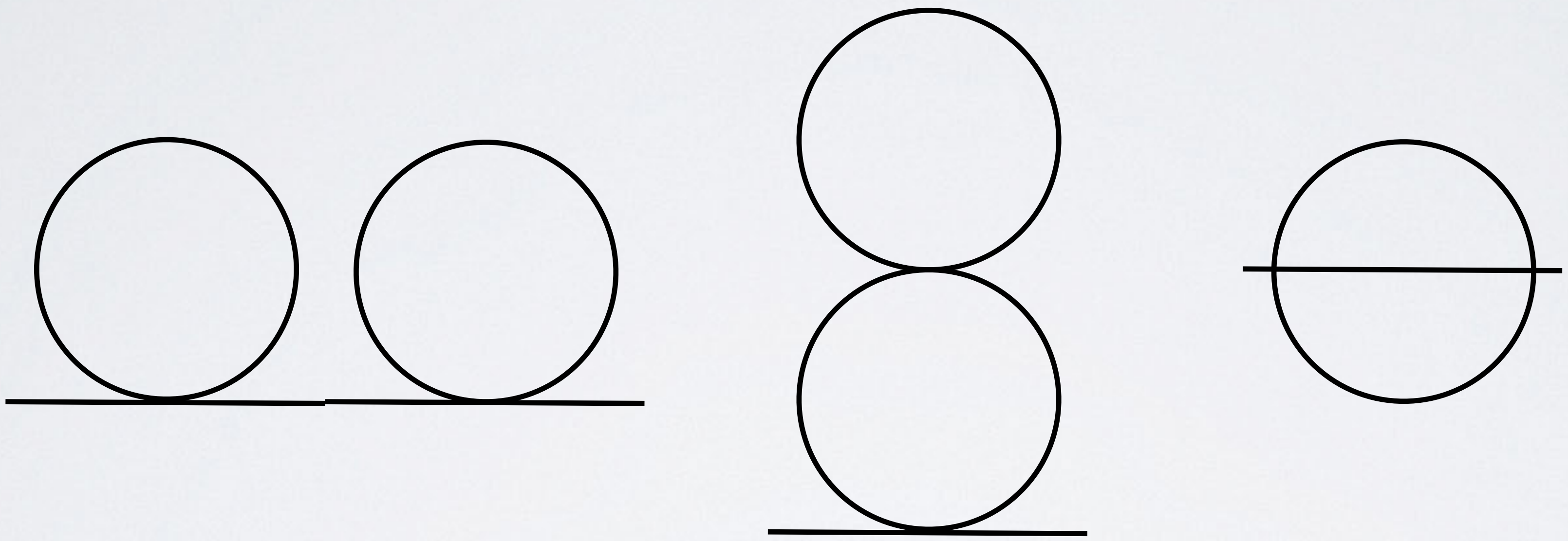
- We get **one log(k_{IR}) per propagator**:

P-V G_+ propagators
$$\int_{k_{\mathrm{IR}}} \frac{d^3k}{k^3} \sim \log(k_{\mathrm{IR}})$$

V G_R propagators
$$\int_{1/k_{\mathrm{IR}}}^{\eta'} \frac{d\eta}{(H\eta^4)}\eta^3 \sim \log(k_{\mathrm{IR}})$$

t^P

- All Feynman diagram topologies contribute to leading-log.

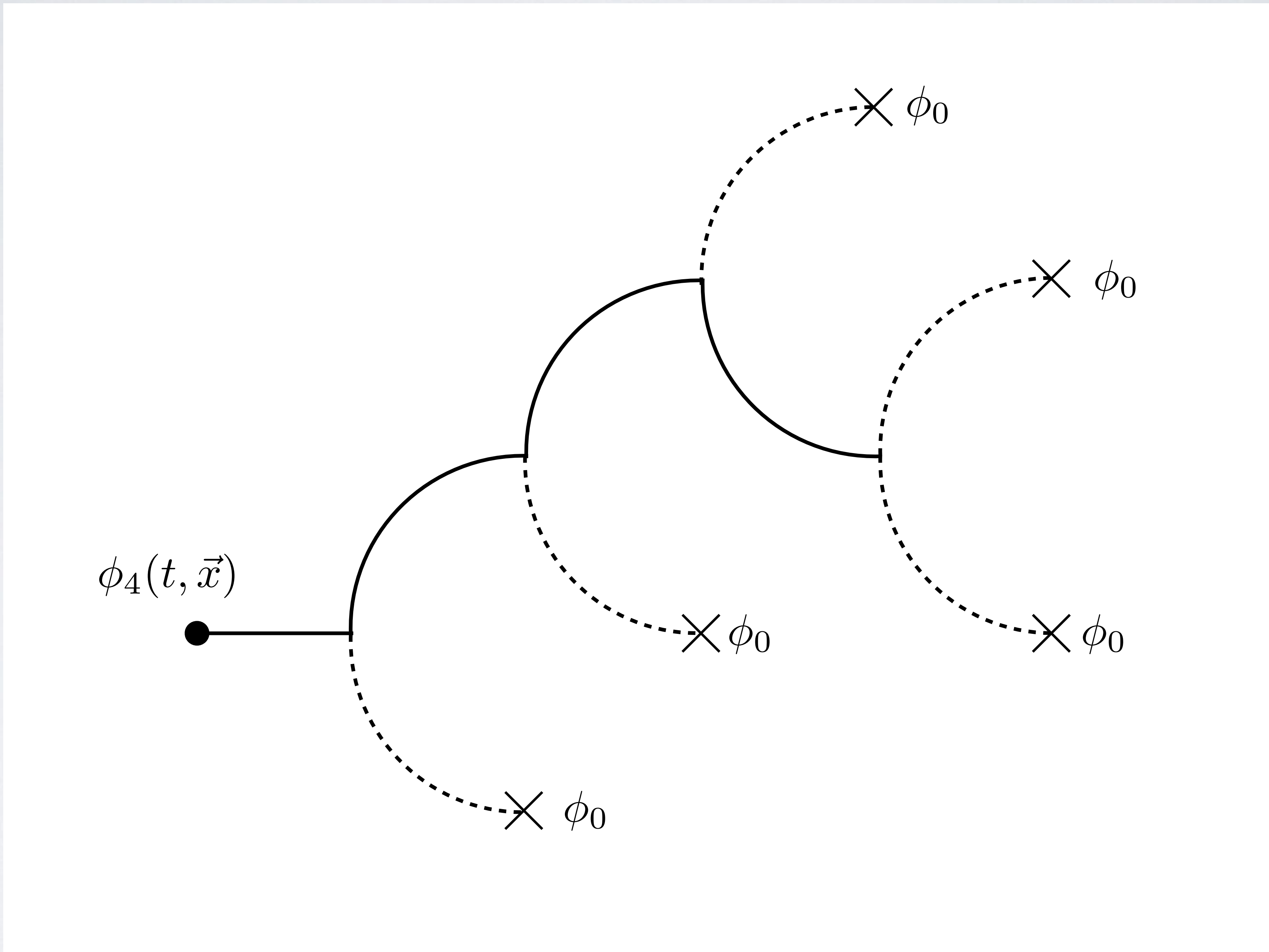


$\sim t^5$

- Only dimensionful scale to balance k_{IR} is correlation time:

$$\log(k_{IR}\eta)^P \sim t^P$$

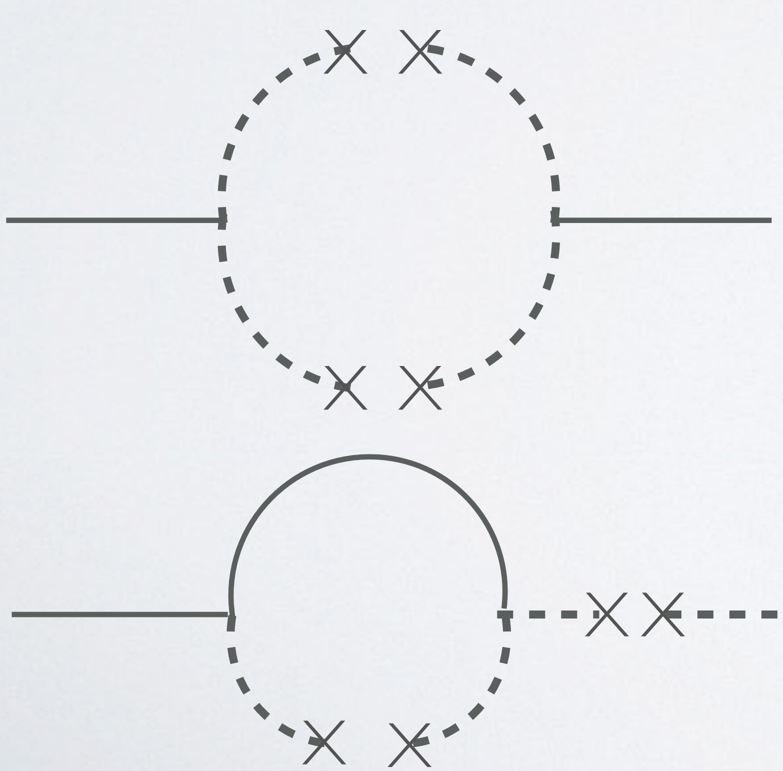
CLASSICAL PERTURBATION THEORY

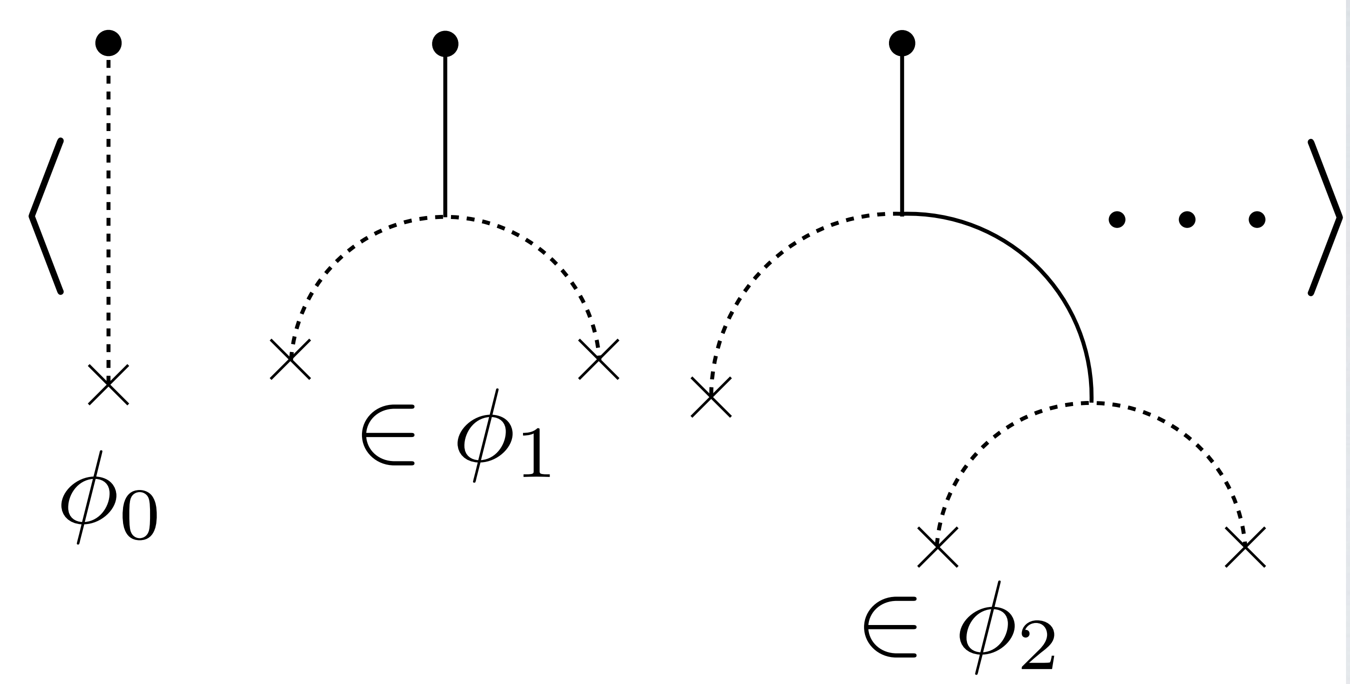


Classical Field Theories can be perturbatively solved
as convolutions of G_R propagators with the free-field solution, φ_0

SEWING CLASSICAL TREES

First do **classical perturbation theory on every field**. Then **sew together classical solutions with G_+** = $\langle \varphi_0 \varphi_0 \rangle$





FIRST ORDERNESS

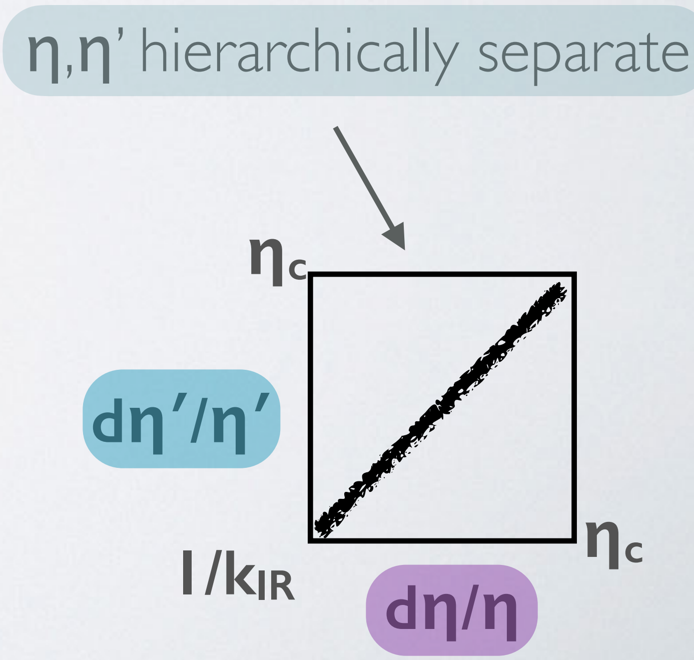
- IR of De Sitter satisfies first order equation of motion for weak potential

$$\ddot{\phi} + 3H\dot{\phi} + \frac{k^2}{a^2}\phi + V'(\phi) = 0$$
$$\Rightarrow 3H\dot{\phi} + V'(\phi) = 0$$

- Free theory retarded propagator

$$G_R(\eta, \eta'; k) \approx \theta(\eta' - \eta)\frac{iH^2}{3}(\eta^3 - {\eta'}^3) \approx \theta(\eta' - \eta)\frac{iH^2}{3}\eta^3$$

- Retarded propagator for $\dot{\phi} = 0$ in DS



CURING MASSLESS DE SITTER

- **φ** in our correlation functions given by retarded trees convolved with **φ**$_0$
- **φ** satisfies inhomogeneous equation of motion

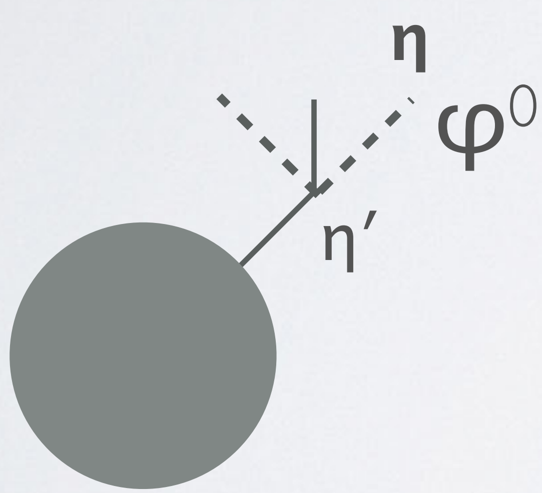
$$\dot{\phi} = -\frac{1}{3H}V'(\phi) + \dot{\phi}^0$$

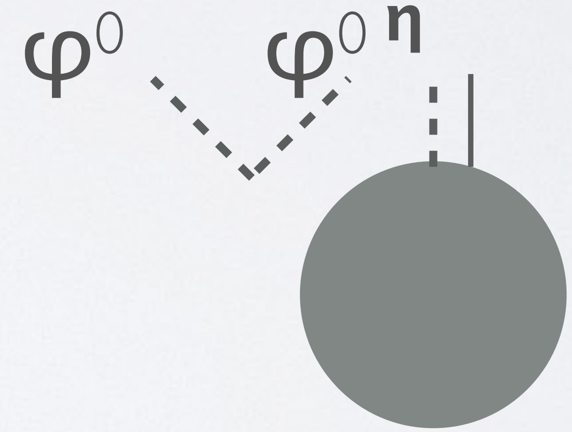
- Source term accounts for time-dependence in the quantum fluctuations

CORRELATOR UPDATE EQUATION

- Differentiate our (meta)observable

$$\langle \dot{\phi^n} \rangle = \langle n\,\phi^{n-1}\dot{\phi} \rangle = -\frac{1}{3H}\langle n\,\phi^{n-1}\,V'(\phi)\rangle + \langle n\,\phi^{n-1}\dot{\phi}^0\rangle$$





$$(H\eta)\partial_\eta G_{R\,\mathrm{soft}} \sim (H\eta)\partial_\eta \frac{H^2}{k^3} = 0$$

$$(H\eta)\partial_\eta G_{R\,\mathrm{coincident}} \sim (H\eta)\partial_\eta \log(k_{\mathrm{IR}}\eta) = H^3$$

CONSISTENCY CHECK

$$\langle \dot{\phi^n} \rangle = -\frac{1}{3H}\langle n\,\phi^{n-1}\,V'(\phi)\rangle + \frac{H^3}{8\pi^2}\langle \phi^{n-2}\rangle$$

- Does the power counting we supposed to derive equation emerge from solving it?

YES!

$$\Big\langle\, \phi^{2n}(t,\vec{x})\,\Big\rangle_{\rm VEV} = (2n-1)!!\,\left(\frac{H^2}{4\pi^2}\ln a\right)^n \left\{1 - \frac{n}{2}(n+1)\,\frac{\lambda}{36\pi^2}\ln^2 a + \frac{n}{280}\Big(35n^3+170n^2+225n+74\Big)\Big[\frac{\lambda}{36\pi^2}\ln^2 a\Big]^2 + \dots\right\}$$

From gr-qc/0505115

FOKKER-PLANCK EQUATION

- With equations for arbitrary n-point function, we confirm the following ansatz for generating function, which is **Starobinsky's famous Fokker-Planck equation** for De Sitter

$$\dot{p}(\phi,t) = \frac{1}{3H}\partial_\phi[V'(\phi)\,p(\phi,t)] + \frac{H^3}{8\pi^2}\partial_\phi^2\,p(\phi,t)$$

- While generic solution is difficult, we can straightforwardly get late-time behavior

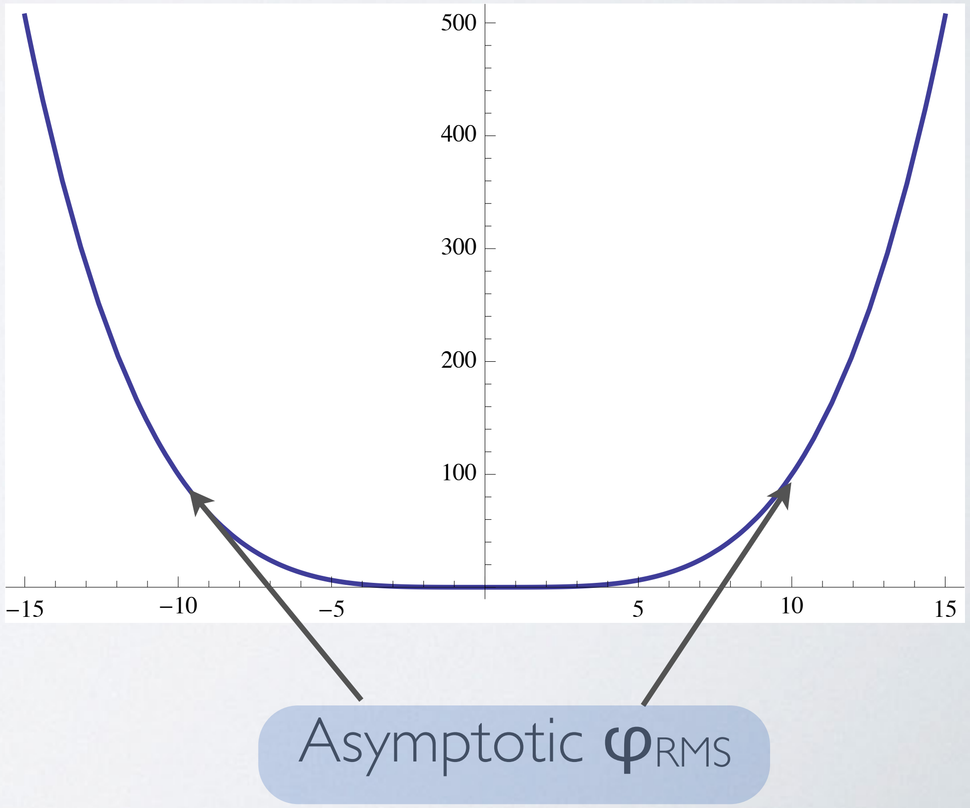
$$\langle\phi^n\rangle = \int d\phi\,p(\phi,t)\,\phi^n$$

$$p(\phi,t) = Ne^{-\frac{8\pi^2}{3H^4}V} + \sum_{n=1}^{\infty}\Phi_n(\phi)e^{-\Gamma_n t}$$

LATE TIME LIMIT

- Fokker-Planck solution has zero eigenvalue with all others positive. Solution is nonperturbative.
- At very late times, any dependence on initial conditions is washed out. We flow to distribution dictated by interaction alone.
- Correlators stay finite $\langle \phi^2 \rangle_{t\to\infty} \sim \frac{H^2}{\sqrt{\lambda}}$

$$V(\phi) \sim H^4$$



Unresummed theory breaks down before $t \sim \lambda^{-1/2}$

With LL resummation, still have control

FUTURE DIRECTIONS

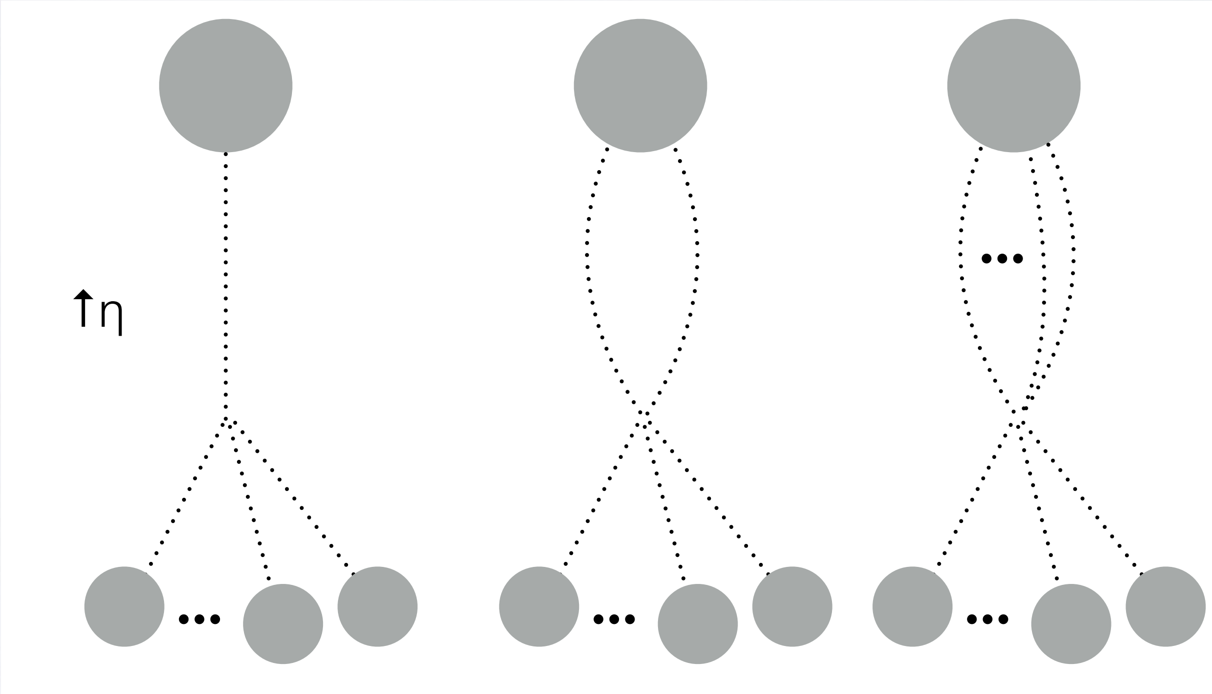
- We have recovered behavior reminiscent of the **parton shower**,* is this a hint of strong dynamics in dual?
  - Both DS and PS have leading **Markovian description**
  - Probabilities flow in both (fixed point in DS) (Fokker-Planck vs. DGLAP)
  - Factorization in both (jets in QCD vs. Hubble patches)
- Resummation at NLL? Quantum corrections in DS will **shift density-matrix off the diagonal**. Possible lesson for/from merging fixed-order Matrix Elements and Parton Shower in QCD?
- Given our novel diagrammatic analysis for scalars, can we tackle similar **open question for gravitons**, does their IR divergence destabilize DS?
- **Toy landscape** at late times (distribution over φ). Warmup problem for **eternal inflation**

See also
1911.00022: Senatore & Gorbenko

CLOSING LOOPHOLES

- For **general number of retarded propagators**, $N_R > V$

$$\langle \phi(\eta, 0)^n \rangle|_{\lambda^V} \sim \lambda^V \log(k_{\mathrm{IR}}\, \eta)^{V+P-N_R}$$



- **Expanding in kη just brings compensating powers of η_{earliest}.** No way to get ahead of soft, strongly-ordered case.

Momentum cutoff determined by earliest momentum that vertex touches.
If N_i propagators terminate at η_i, then it is earliest vertex for N_{i-1}.

SCALAR QFT IN DE SITTER

- In general, we work in time-momentum space

$$S = \int d\eta\, d^3k\ \frac{1}{2(H\eta)^2}\left((\partial_\eta \phi)^2 - k^2\phi^2\right) - \frac{1}{(H\eta)^4} V(\phi)$$

- The **ground state is more subtle** in DS than Minkowski
  - No global timelike Killing vector (Energy will **Redshift**)
  - Spacetime expansion crates particles (relative to Minkowski vacuum)
- Standard choice is **Bunch-Davies** (uniquely DS-invariant, Mink.-like in UV)

BUNCH-DAVIES VACUUM

- Ground state? Which classical solution multiplies annihilation operator?

$$\phi(\eta, \vec{k}) = H\eta^{3/2}\left[a_k H_\nu^{(2)}(k\eta) + b_k H_\nu^{(1)}(k\eta)\right]$$

- Standard choice is Bunch-Davies vacuum (b_k=0):*
  - DS invariant, a,b, k-independent
  - Minimizes energy as t→-∞
  - Coefficient becomes positive frequency as t→-∞

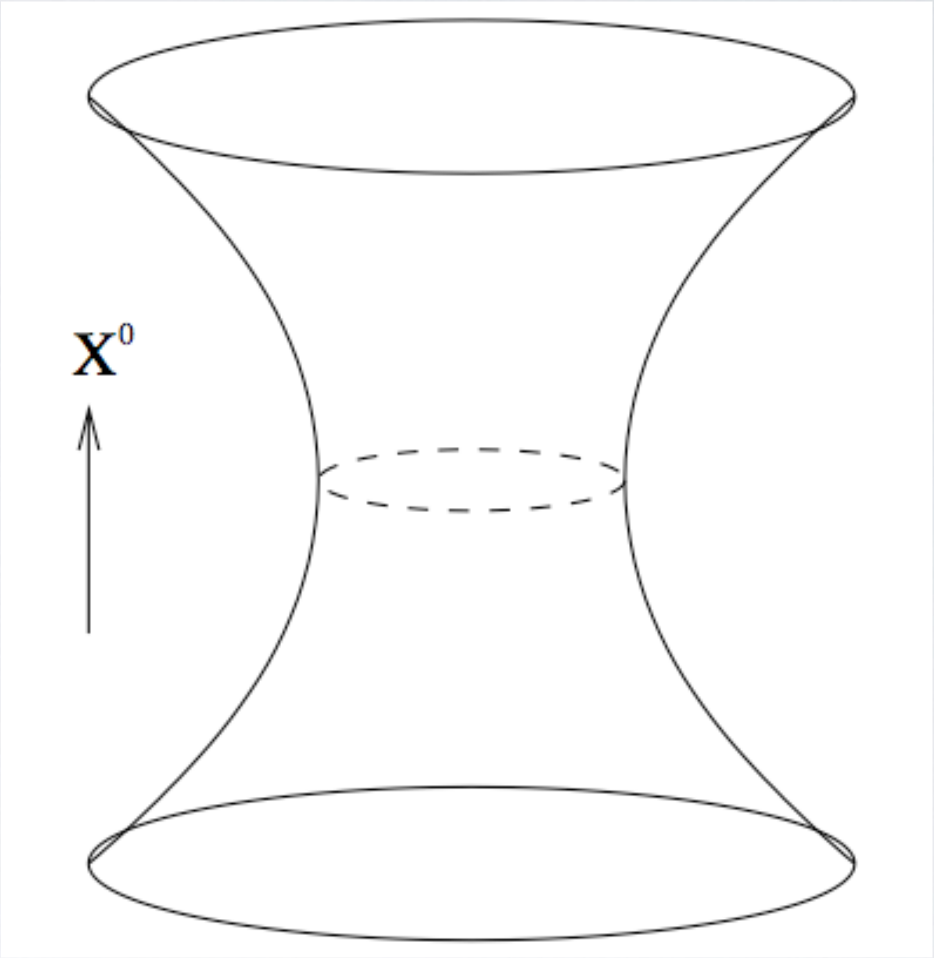
* Alternate DS invariant states, **α-vacua**, are likely pathological (non-local, acausal), see Lowe, Holman

LIGHTNING DE SITTER OVERVIEW

- Global DS as an embedded hyperboloid
- Patches of DS (inflation-like) → FRW and conformal metrics:

$$ds^2 = dt^2 - e^{2Ht}\vec{dx}^2$$
$$= \frac{1}{(H\eta)^2}(d\eta^2 - \vec{dx}^2)$$
$$\eta = -\frac{e^{-Ht}}{H}$$

$a \equiv e^{Ht}$

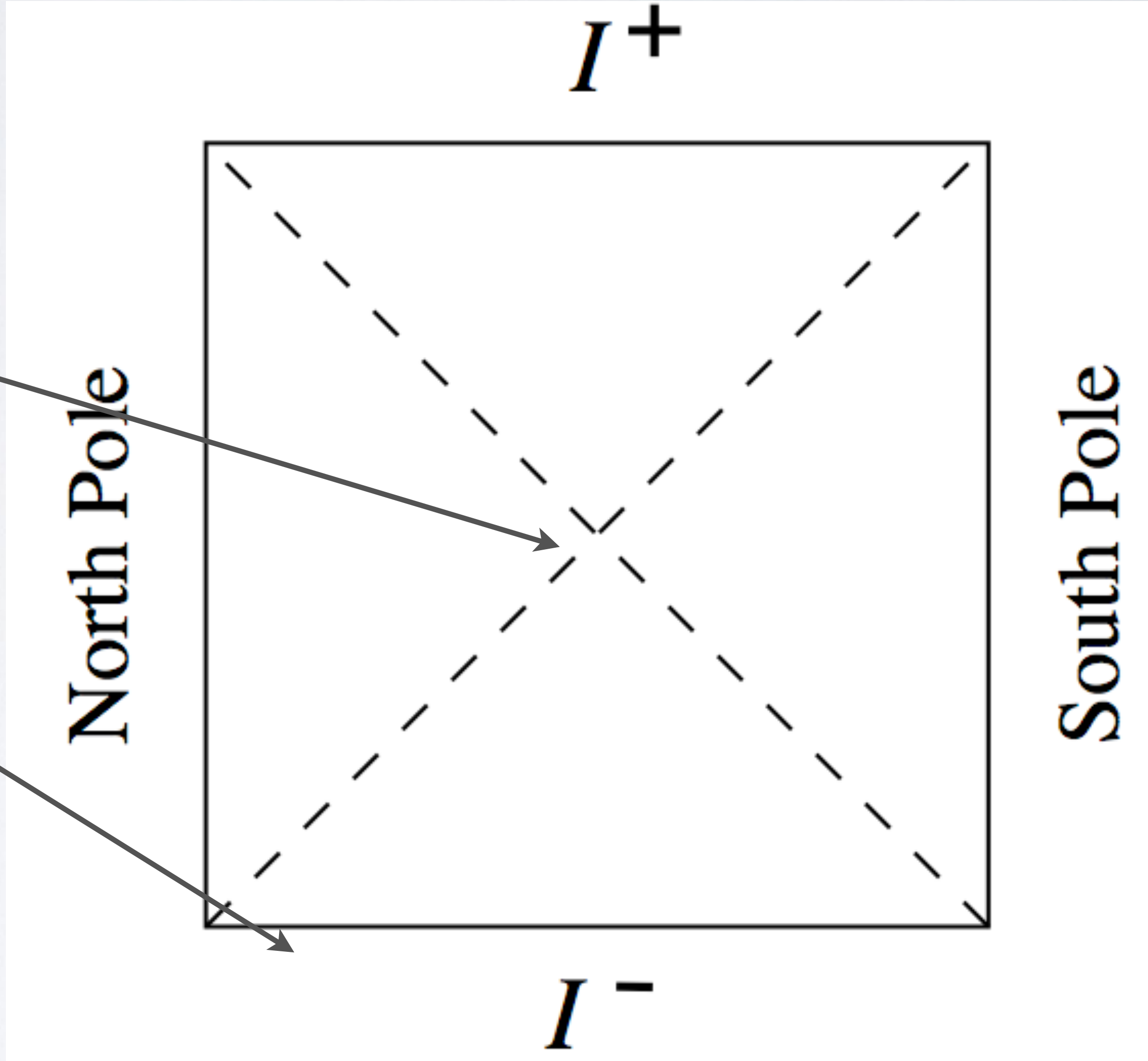


$$\frac{1}{H^2} = X_0^2 - X_1^2 - \ldots - X_d^2$$

$$x_{\text{phys}} = e^{Ht} x_{\text{com}} \qquad k_{\text{phys}} = e^{-Ht} k_{\text{com}}$$

FORMAL DE SITTER

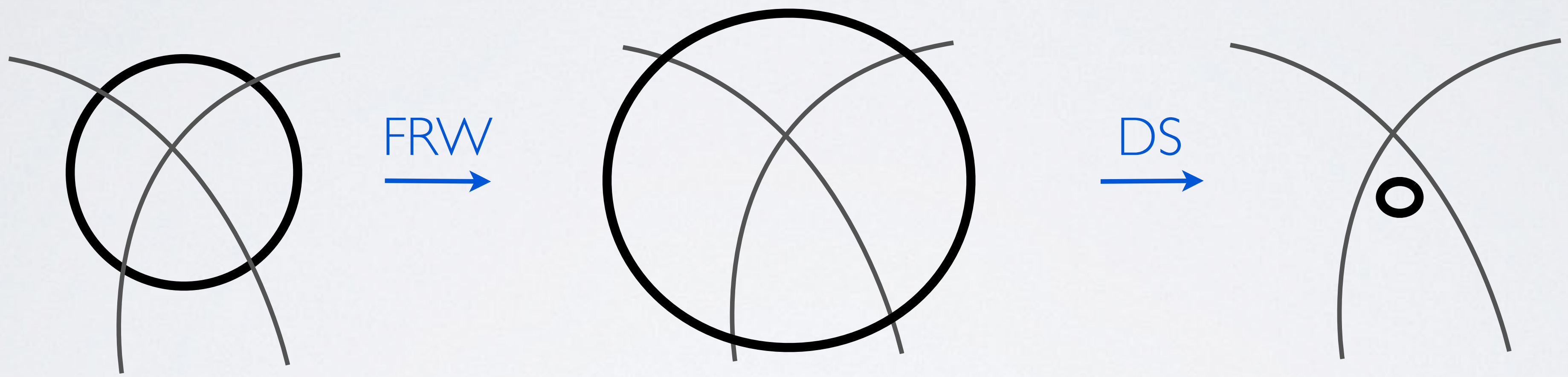
- QFT in De Sitter raises **conceptual questions** (infrared, late times) [think QED in the 40s and 50s]
- What are **microstates** and the **holographic dual**?
- Open questions with gravitons: (Polyakov, Rajaraman, Senatore, Anninos, Freedman, Tsamis, Woodard) Do they **destabilize DS** or **relax CC**?
- Starobinsky (1986): "Stochastic Inflation" to understand scalars. Can we derive it clearly?



Penrose diagram of DS
Past and future infinities are spacelike
Observer only has causal contact in triangle

CARTOON HISTORY OF AN IR-SAFE UNIVERSE

- **Regulate** by starting De Sitter at a finite time.



- Modes that never get inside comoving horizon are frozen *and safe* → **Comoving IR cutoff**

LEADING IR DEPENDENCE

- Schematic contribution to correlator **at V^{th} order**

$$\langle \phi(\eta,0)^n\rangle\Big|_{\lambda^V} \sim \lambda^V \int_{1/k_{\rm IR}}^{\eta} \frac{d\eta^{(V)}}{(H\eta^{(V)})^4}\cdots\int_{1/k_{\rm IR}}^{\eta^{(2)}}\frac{d\eta^{(1)}}{(H\eta^{(1)})^4}\int_{k_{\rm IR}}\frac{d^3k_1}{(2\pi)^3}\cdots\int_{k_{\rm IR}}\frac{d^3k_{P\text{–}V}}{(2\pi)^3}$$

$$\times \prod_{n=1}^{P\text{–}V} G_{+\,\rm soft}(k_n)\prod_{m=1}^{V} G_{R\,\rm soft}\left(\eta^{(k_m)},\eta^{(l_m)}\right)$$

G_+

G_R

- Tracking $k_{\rm IR}$ dependence, find it in lower integration limits

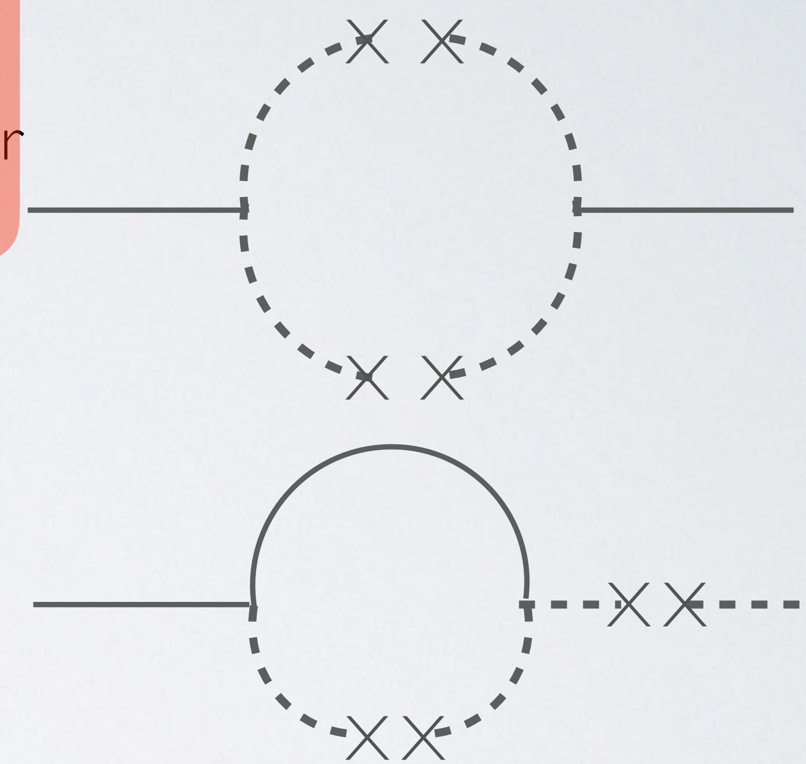
TO CURE LATE-TIME DIVERGENCE?

- Can iteratively solve **classical** equations of motion and correct action

Inhomogeneous free theory solution accounts for Quantum Noise

$$\partial_\eta^2\phi_1 - \frac{2}{\eta}\partial_\eta\phi_1 + k^2\phi = -\frac{1}{(H\eta)^2}V'(\phi_0)$$

$$\phi_1(x) = -\int \frac{d^4y}{(H\eta)^2} G_R(x,y)V'(\phi_0)$$



Draw all possible trees that touch every vertex and at least one external point

Pair φ_0 with $\langle\{\varphi(\eta'),\varphi(\eta)\}\rangle$

- All dangling φ_0 sewn up in free-theory two-point functions, G_+

- $\varphi(\varphi_0)$ in is the same function with and without quantum noise in φ_0

PERSISTENT FIRST ORDERNESS

- Perturbation theory corrects $d\varphi/dt = 0$

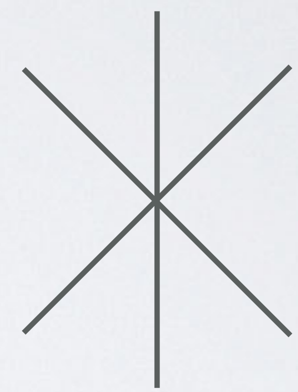
- We solve to all orders,

$$\dot{\phi} = -\frac{1}{3H}V'(\phi) + \dot{\phi}_0$$

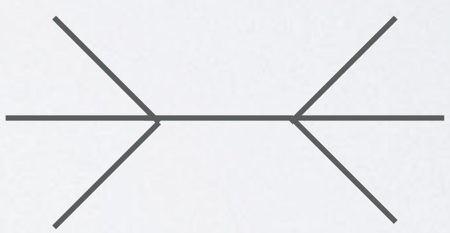
Quantum Noise

- Can we revive acceleration?

$$\ddot{\phi} \equiv \partial_t(\dot{\phi}) = \partial_t\left[-\frac{1}{3H}V'(\phi)\right] = \frac{1}{9H^2}V'(\phi)V''(\phi)$$



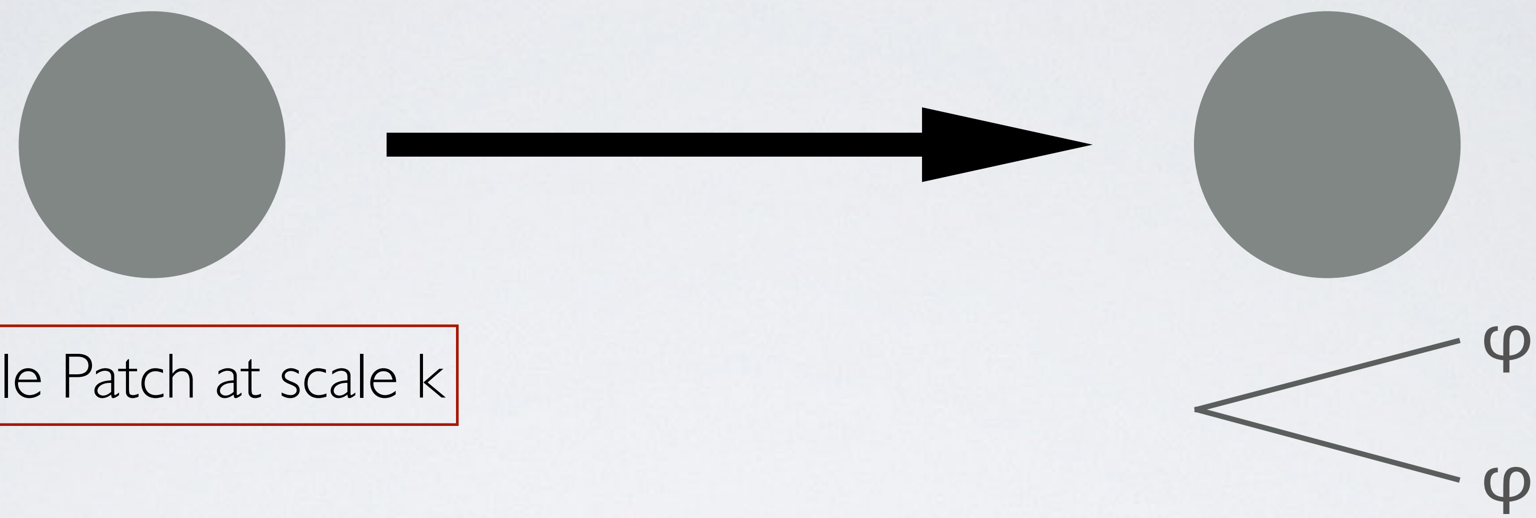
$\sim\lambda^2 t^6$



$\sim\lambda^2 t^7$

Effective interaction $\lambda^2\varphi^{2n-2}$,
Always beaten by original by a log

PARTON SHOWER UNIVERSE



Hubble Patch at scale k

Hubble Patch at scale k+dk
(Must account for quantum fluctuation)

$$|\psi[\phi, k + dk]|^2 = \int \mathcal{D}\phi^0 \, \mathcal{D}\sigma \; P[\sigma] \, |\psi[\phi^0, k]|^2 \, \delta[\phi - \phi^0 - \Delta\phi_{\text{classical}}(\phi^0) - \sigma]$$

"PDF"

"Splitting Function"

What are implications for?

- Holography
- IR stability of gravitons
- Emergence of timelike dimensions
- Classical Universe from Quantum fluctuations