Predictions in Inflationary Cosmology from Quantum Gravity with Purely Virtual Quanta

Marco Piva

NICPB, Tallinn

July 6^{th} 2020

Overview of the theory

- Purely virtual quantum (fakeon):

  a degree of freedom that mediates interactions and circulate inside loops but cannot appear as external state in physical processes.
  [D. Anselmi, JHEP 1706 (2017) 086, D. Anselmi and MP, JHEP 1706 (2017) 066]

Quantum gravity with fakeons

- Degrees of freedom of the theory:

  massless spin-2 $h_{\mu\nu}$, (graviton) massive scalar ϕ, (inflaton) massive spin-2 $\chi_{\mu\nu}$. (fakeon)

- Parameters:

$$M_{\rm Pl}, \qquad \Lambda_C, \qquad m_\phi, \qquad m_\chi.$$

- Physical content in cosmology:

  scalar perturbations, tensor perturbations, no vectors or additional tensors/scalars.

Predictions in inflationary cosmology

D. Anselmi, E. Bianchi and MP, arXiv:2005.10293

- Amplitudes and spectral indices in de Sitter and quasi de Sitter.

$A_{\mathcal{R}}$	A_T	r	$n_{\mathcal{R}}-1$	n_T
$\frac{m_\phi^2 N^2}{3\pi M_{\rm Pl}^2}$	$\frac{4m_\phi^2}{\pi M_{\rm Pl}^2}\frac{2m_\chi^2}{m_\phi^2+2m_\chi^2}$	$\frac{12}{N^2}\frac{2m_\chi^2}{m_\phi^2+2m_\chi^2}$	$-\frac{2}{N}$	$-\frac{3}{2N^2}\frac{2m_\chi^2}{m_\phi^2+2m_\chi^2}$

Black: Starobinsky inflation.

- A consistency condition gives a bound on the fakeon mass

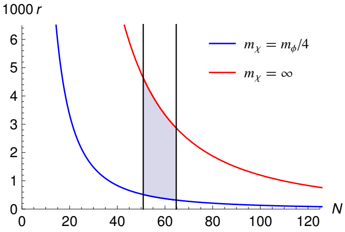
$$m_\chi > m_\phi/4$$

The bound restricts the possible values for the tensor-to-scalar ratio r

$$\frac{1}{9} \lesssim \frac{N^2}{12} r \lesssim 1.$$

$$N = 60 \qquad 0.4 \lesssim 1000r \lesssim 3,$$

$$-0.4 \lesssim 1000 n_T \lesssim -0.05.$$



N in the range $n_{\mathcal{R}} = 0.9649 \pm 0.0042$ at 68% CL.

Unitarity

$$\mathrm{SS}^\dagger = 1, \qquad \mathrm{S} = 1 + i\mathrm{T}.$$

- Unitarity equation (optical theorem)

$$2\mathrm{Im}\mathrm{T} = \mathrm{T}\mathrm{T}^\dagger.$$

- Pseudo-unitarity equation (cutting equations)

$$2\mathrm{Im}\mathrm{T} = \mathrm{T}\mathrm{H}\mathrm{T}^\dagger,$$

  in general

$$\mathrm{H} = \mathrm{diag}(\ldots, 1, \ldots, 1, -1, \ldots, -1, \ldots).$$

- To have unitarity

$$\mathrm{H} = \mathbb{1} \qquad \text{or} \qquad \mathrm{H}|_\mathrm{V} = \mathbb{1} \quad + \quad \textbf{consistent projection}$$

- Known consistent projections:
  - Faddeev-Popov method;
  - Fakeon prescription.

$$\mathrm{H} = \mathrm{diag}(\ldots, 1, \ldots, 1, 0, \ldots, 0, \ldots).$$

Note: Having a decay width is not enough to consistently project away.

Reformlation of Lee-Wick models

[D. Anselmi and MP JHEP 06 (2017) 066.]

Consider a class of HD theories with

$$D_{\rm LW}(k,m,M) = \frac{1}{(k^2 - m^2 + i\epsilon)P_n(k^2/M^2)},$$

where $P_n(z)$ is a polynomial with deg n and complex conjugated pairs of zeros.

- Lee-Wick formulation is incomplete and leads to inconsistencies (CLOP included).
- Formulate the theory in Euclidean spacetime and **nonanalytically Wick rotate**.

  Deformation of integration domains before Wick rotation
  or use the **average continuation**.

  This operation gives a consistent projection and the theory is unitary.
  [D. Anselmi and MP, PRD 96 (2017) 045009], [D. Anselmi, JHEP 02 (2018) 141.]

- Superrenormalizable theories of quantum gravity

$$S_{\rm QG}(g) = -\frac{1}{2\kappa^2}\int \sqrt{-g}\left[R + 2\Lambda_C + R_{\mu\nu}P_n(-\nabla^2/M^2)R^{\mu\nu} + RQ_n(-\nabla^2/M^2)R\right]$$

  Still a problem of uniquness...

Prescription for the propagator $\frac{\pm 1}{k^2 - m^2}$

Standard particle : $$\frac{1}{k^2 - m^2 + i\epsilon}.$$

Ghost : $$\frac{-1}{k^2 - m^2 + i\epsilon}.$$

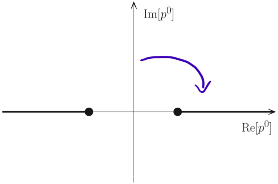
Fake particle (fakeon):

[D. Anselmi JHEP 06 (2017) 086.]

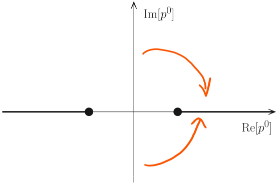
$$\pm \frac{k^2 - m^2}{(k^2 - m^2)^2 + \mathcal{E}^4}.$$

+ integration domain deformations

Prescription for the amplitude $\mathcal{A}(p)$



$$\mathcal{A}_+(p) = \mathcal{A}(p + i\epsilon)$$



$$\mathcal{A}_{\rm AV}(p) = \frac{1}{2}\left[\mathcal{A}_+(p) + \mathcal{A}_-(p)\right].$$

A theory of particles and fakeons is unitary.

D. Anselmi, JHEP 02 (2018) 141.

$D(k^2, m^2, \mathcal{E}) = \frac{1}{(k^2-m^2)^2+\mathcal{E}^4}$.

Minkowski models

$$i\mathcal{M}(p) = c\int_{\mathbb{R}} \frac{\mathrm{d}k^0}{2\pi}\int_{\mathbb{R}^3}\frac{\mathrm{d}^3k}{(2\pi)^3}D(p-k, m_1, \mathcal{E})D(k, m_2, \mathcal{E}).$$

Not unitary, nonlocal and non-Hermitean divergences.

[U. G. Aglietti and D. Anselmi, Eur. Phys. J. C 77 (2017) 84.]

Lee-Wick models

$$i\mathcal{M}(p) = c\int_{\mathbb{LW}} \frac{\mathrm{d}k^0}{2\pi}\int_{\mathbb{R}^3}\frac{\mathrm{d}^3k}{(2\pi)^3}D(p-k, m_1, \mathcal{E})D(k, m_2, \mathcal{E}).$$

Incomplete prescription, inconsistencies in diagrams, violates Lorentz invariance.
(Lee and Wick, Nakanishi, Anselmi and MP)

Fakeon models

$$i\mathcal{M}(p) = c\int_{\mathbb{LW}} \frac{\mathrm{d}k^0}{2\pi}\int_{\mathcal{D}_3}\frac{\mathrm{d}^3k}{(2\pi)^3}D(p-k, m_1, \mathcal{E})D(k, m_2, \mathcal{E}).$$

Unitary, unambiguous and Lorentz invariant.

[D. Anselmi and MP, JHEP 1706 (2017) 066.]

The fakeon propagator is not the Cauchy principal value.

$$\lim_{\mathcal{E}\to 0}\frac{k^2-m^2}{(k^2-m^2)^2+\mathcal{E}^4} = \mathcal{P}\frac{1}{k^2-m^2} = \lim_{\epsilon\to 0^+}\frac{1}{2}\left(\frac{1}{k^2-m^2+i\epsilon}+\frac{1}{k^2-m^2-i\epsilon}\right).$$

[D. Anselmi J. High Energ. Phys. 03 (2020) 142.]

The theory of quantum gravity and fakeons

$$S_{\mathrm{QG}}(g) = -\frac{M_{\mathrm{Pl}}^2}{16\pi}\int\sqrt{-g}\left[R + \frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - \frac{1}{6m_\phi^2}R^2\right]$$

Properties:

- Unitarity
  [D. Anselmi and MP, PRD 96 (2017) 045009, D. Anselmi, JHEP 02 (2018) 141].
- Renormalizability (once Λ_C is reinstated).
- Violation of microcausality
  [D. Anselmi and MP, JHEP 11 (2018) 21].
- No violation of macrocausality
  [D. Anselmi and A. Marino, Class. Quantum Grav. 37 (2020) 095003].

The theory of quantum gravity and fakeons

D. Anselmi and MP, JHEP 11 (2018) 021.

Equivalent action:
auxiliary fields ϕ, $\chi_{\mu\nu}$ + Weyl transformation + field redefinitions.

$$S(g,\phi,\chi,\Phi) = S_{\rm EH}(g) + S_\chi(g,\chi) + S_\phi(g+2\chi,\phi) + S_m(ge^{\kappa\phi}+2\chi e^{\kappa\phi},\Phi).$$

$S_{\rm EH}$ = Einstein-Hilbert, S_χ = $(-)$Pauli-Fierz + interactions, S_m = Standard Model,

$$S_\phi(g,\phi) = \frac{3\zeta}{4}\int\sqrt{-g}\left[\nabla_\mu\phi\nabla^\mu\phi - \frac{m_\phi^2}{\kappa^2}(1-e^{\kappa\phi})^2\right].$$

Violation of microcausality

Resumming the self energies the corrected χ propagator at the peak m_χ is

$$\langle \chi_{\mu\nu}(p)\chi_{\rho\sigma}(-p)\rangle_{s\sim\bar{m}_\chi^2} = -\frac{i\kappa^2}{\zeta}\frac{Z_\chi}{s-\bar{m}_\chi^2+i\bar{m}_\chi\Gamma_\chi}\Pi^{(2)}_{\mu\nu\rho\sigma}(p,s),$$

$$\Gamma_\chi = -\frac{m_\chi^3}{M_{\rm Pl}^2}C, \qquad C = \frac{N_s+6N_f+12N_v}{120}, \qquad \Gamma_\chi < 0.$$

Breit-Wigner distribution

$$\frac{i}{E-m+i\frac{\Gamma}{2}} \qquad \longrightarrow \qquad \mathrm{sgn}(t)\theta(\Gamma t)\exp\left(-imt-\frac{\Gamma t}{2}\right).$$

$$\text{Duration} \sim 1/|\Gamma_\chi|$$

$$\text{If } m_\chi \sim 10^{12}\text{GeV then } 1/|\Gamma_\chi| \sim 4\cdot 10^{-20}\text{s}$$

For time intervals of the order $1/|\Gamma_\chi|$ causality loses meaning.

Projected action and classicalization

[D. Anselmi, Class. and Quantum Grav. 36 (2019) 065010.]

- At classical level the fakeon prescription is applied on the Green function

$$\mathcal{A}_{\mathrm{AV}}(p) = \frac{1}{2}\left[\mathcal{A}_{+}(p) + \mathcal{A}_{-}(p)\right] \quad \Rightarrow \quad G_{\mathrm{f}} = \frac{1}{2}\left[G_{\mathrm{ret}} + G_{\mathrm{adv}}\right]$$

$$\ddot{V} + \Omega^2 V = \left(\frac{\mathrm{d}^2}{\mathrm{d}t^2} + \Omega^2\right) V = F(t) \qquad \Rightarrow \qquad V(t) = (G_{\mathrm{f}} * F)(t).$$

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2} + \Omega^2\right) G_{\mathrm{f}}(t,t') = \delta(t-t').$$

- In flat spacetime $\Omega = \mathrm{const.}$

$$G_{\mathrm{f}}(t,t') = \frac{\sin(\Omega|t-t'|)}{2\Omega}.$$

- In de Sitter spacetime $\Omega = \Omega(t)$.

$$G_{\mathrm{f}}(t,t') = \frac{i\pi\,\mathrm{sgn}(t-t')}{4H\sinh\left(n_\chi \pi\right)}\left[J_{in_\chi}(\check{k})J_{-in_\chi}(\check{k}') - J_{in_\chi}(\check{k}')J_{-in_\chi}(\check{k})\right], \qquad \check{k}^{(\prime)} = \frac{k}{a(t^{(\prime)})H}.$$

- Projected classical action

$$S(U,V) \qquad \longrightarrow \qquad S^{\mathrm{prj}}(U) = S(U,V(U)).$$

Cosmological perturbations

Classical background: FLRW $\quad g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \mathrm{d}t^2 - a(t)^2(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)$

$$\varepsilon = -\frac{\dot{H}}{H^2}, \qquad \eta = 2\varepsilon - \frac{\dot{\varepsilon}}{2H\varepsilon}.$$

$$S(u) = \frac{1}{2}\int \mathrm{d}t\ a(t)^3 \left[f(t)\dot{u}^2 - h(t)\ddot{u}^2 - g(t)u^2\right], \qquad u \equiv u_{\mathbf{k}}(t)$$

$$\Downarrow$$

$$S'(U,V) = \frac{1}{2}\int \mathrm{d}t\ Z\left(\dot{U}^2 - \omega^2 U^2 - \dot{V}^2 + \Omega^2 V^2 + 2\sigma UV\right).$$

Procedure for tree level computations:

i) Solve the EOM for V by means of the fakeon Green function;

ii) Insert the solution back in S';

iii) Quantize the new action with the standard methods.

$$S_w^{\mathrm{prj}} = \frac{1}{2}\int \mathrm{d}\tau \left[w'^2 - \bar{k}^2 w^2 + \frac{w^2}{\tau^2}\left(\nu_{\mathrm{t}}^2 - \frac{1}{4}\right)\right], \qquad \bar{k} = k\Big(1 + \mathcal{O}(\varepsilon)\Big).$$

Consistency condition

The fakeon eom's can always be turned into the form

$$\ddot{V} + m(t)^2 V = \mathcal{O}(\sqrt{\varepsilon}), \qquad m^2(t) = H^2 n_\chi^2 + \frac{k^2}{a^2}.$$

Imposing a no-tachyon condition on $m(t)^2$ in the flat-space limit

$$m(t)^2\big|_{k/(aH)\to 0} > 0. \qquad \Rightarrow \qquad n_\chi \in \mathbb{R}.$$

It can be seen also from the Green function

$$G_{\mathrm{f}}(t,t') \underset{k\to 0}{\longrightarrow} \frac{1}{2Hn_\chi} \sin\left(Hn_\chi|t-t'|\right)$$

All perturbations give the same bound

$$\boxed{m_\chi > \frac{m_\phi}{4}.}$$

Results: Amplitudes and spectral indices

D. Anselmi, E. Bianchi and MP, arXiv:2005.10293

- Leading order

$A_{\mathcal{R}}$	A_T	r	$n_{\mathcal{R}}-1$	n_T
$\frac{m_\phi^2 N^2}{3\pi M_{\rm Pl}^2}$	$\frac{4m_\phi^2}{\pi M_{\rm Pl}^2}\frac{2m_\chi^2}{m_\phi^2+2m_\chi^2}$	$\frac{12}{N^2}\frac{2m_\chi^2}{m_\phi^2+2m_\chi^2}$	$-\frac{2}{N}$	$-\frac{3}{2N^2}\frac{2m_\chi^2}{m_\phi^2+2m_\chi^2}$

Black: Starobinsky inflation.

- At this order

$$r \simeq -8n_T.$$

- Higher order corrections

$$A_{\mathcal{R}} = \frac{GN^2m_\phi^2}{3\pi}\left(1-\frac{\ln N}{6N}+\mathcal{O}\left(\frac{1}{N}\right)\right).$$

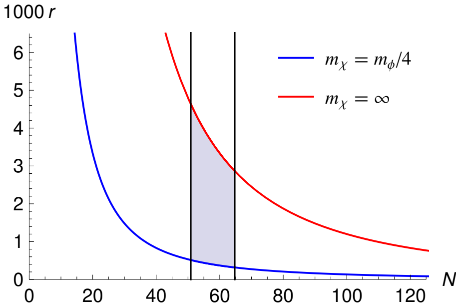
$$A_T = \frac{8G}{\pi}\frac{m_\chi^2 m_\phi^2}{m_\phi^2+2m_\chi^2}\left(1-\frac{3m_\chi^2}{N(m_\phi^2+2m_\chi^2)}\left(1+\frac{\ln N}{12N}\right)+\mathcal{O}\left(\frac{1}{N^2}\right)\right).$$

With $N=60$, the first correction to A_T is between 0.3% and 2.5%.

Results: ratio

- From Planck 2018, $n_{\mathcal{R}} = 0.9649 \pm 0.0042$ at 68% CL.
- From the predictions, $n_{\mathcal{R}} = 1 - 2/N$.
- From the bound $m_\chi > m_\phi/4$,

$$\frac{1}{9} \lesssim \frac{N^2}{12} r \lesssim 1.$$



$$N = 60 \qquad 0.4 \lesssim 1000r \lesssim 3, \qquad -0.4 \lesssim 1000n_T \lesssim -0.05.$$

Conclusions

New predictions from quantum gravity with purely virtual quanta

- Purely virtual quanta provide a local, unitary and renormalizable theory of quantum gravity.
- The theory is essentially unique.
- Computational power (Feynman diagrams).
- Predictive.
- Falsifiable.
  - Amplitudes and spectral indices of scalar and tensor perturbations.
  - Once new cosmological data will be available, m_ϕ and m_χ will be fixed and other predictions will be stringent tests of the theory.