

Dark energy after gravitational wave observations

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with Paolo Creminelli 1710.05877,
+ Matthew Lewandowski and Giovanni Tambalo, 1809.03484
+ Vicharit Yingcharoenrat, 1906.07015, 1910.14035

22 June 2020
Online “Newton 1665” seminars

Motivations

- General relativity is very well tested on Solar System scales
- Its validity is extrapolated on larger scales

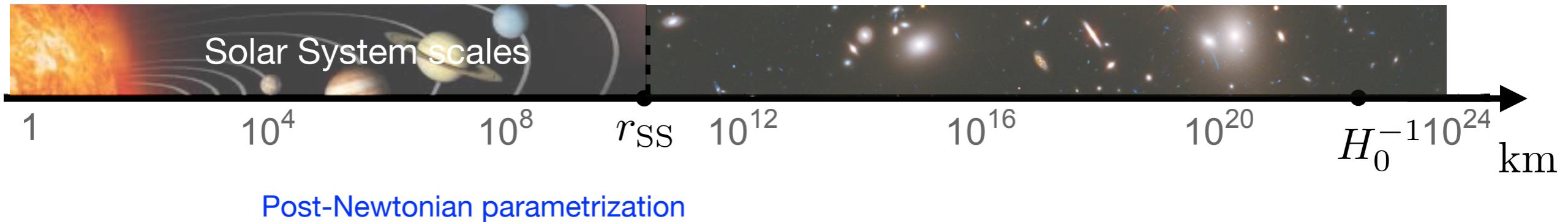


Table 4: Current limits on the PPN parameters.

Parameter	Effect	Limit	Remarks
$\gamma - 1$	time delay	2.3×10^{-5}	Cassini tracking
	light deflection	2×10^{-4}	VLBI
$\beta - 1$	perihelion shift	8×10^{-5}	$J_{2\odot} = (2.2 \pm 0.1) \times 10^{-7}$
	Nordtvedt effect	2.3×10^{-4}	$\eta_N = 4\beta - \gamma - 3$ assumed
ξ	spin precession	4×10^{-9}	millisecond pulsars
α_1	orbital polarization	10^{-4}	Lunar laser ranging
		4×10^{-5}	PSR J1738+0333
α_2	spin precession	2×10^{-9}	millisecond pulsars
α_3	pulsar acceleration	4×10^{-20}	pulsar \dot{P} statistics
ζ_1	—	2×10^{-2}	combined PPN bounds
ζ_2	binary acceleration	4×10^{-5}	\ddot{P}_p for PSR 1913+16
ζ_3	Newton's 3rd law	10^{-8}	lunar acceleration
ζ_4	—	—	not independent [see Eq. (73)]

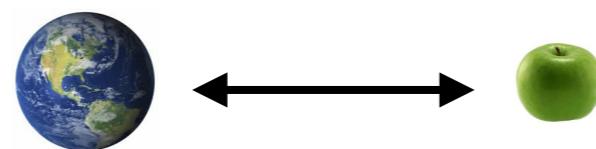
Motivations

- On large scales, constraints are weaker
- No measured deviations from Λ CDM (caveat H₀ tension), but dark sector not well understood



DES '18 $|\mu - 1| < 8 \times 10^{-2}$

$$|\Sigma - 1| < 4 \times 10^{-1}$$

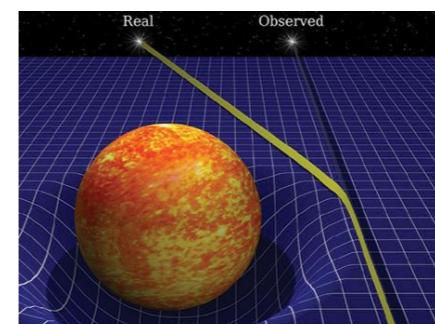


fifth force

$$dt^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)d\vec{x}^2$$

$$\nabla^2\Phi = 4\pi G \mu \delta\rho_m$$

$$\nabla^2(\Phi + \Psi) = 8\pi G \Sigma \delta\rho_m$$



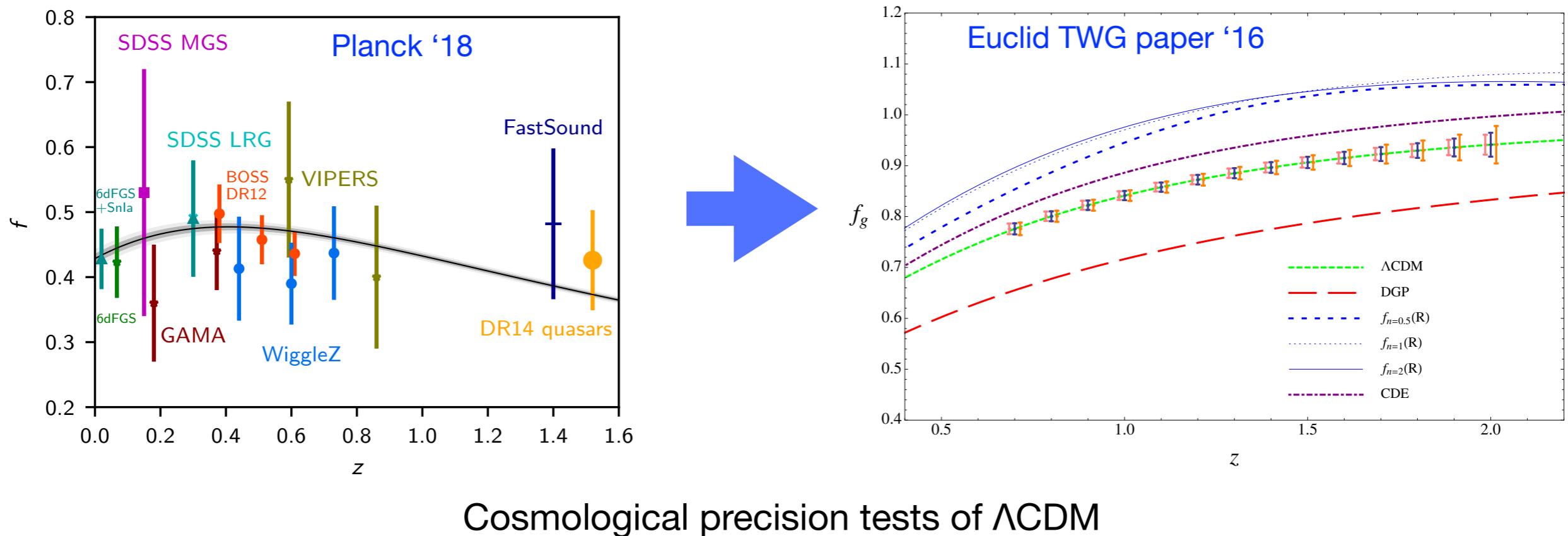
anomalous light
bending

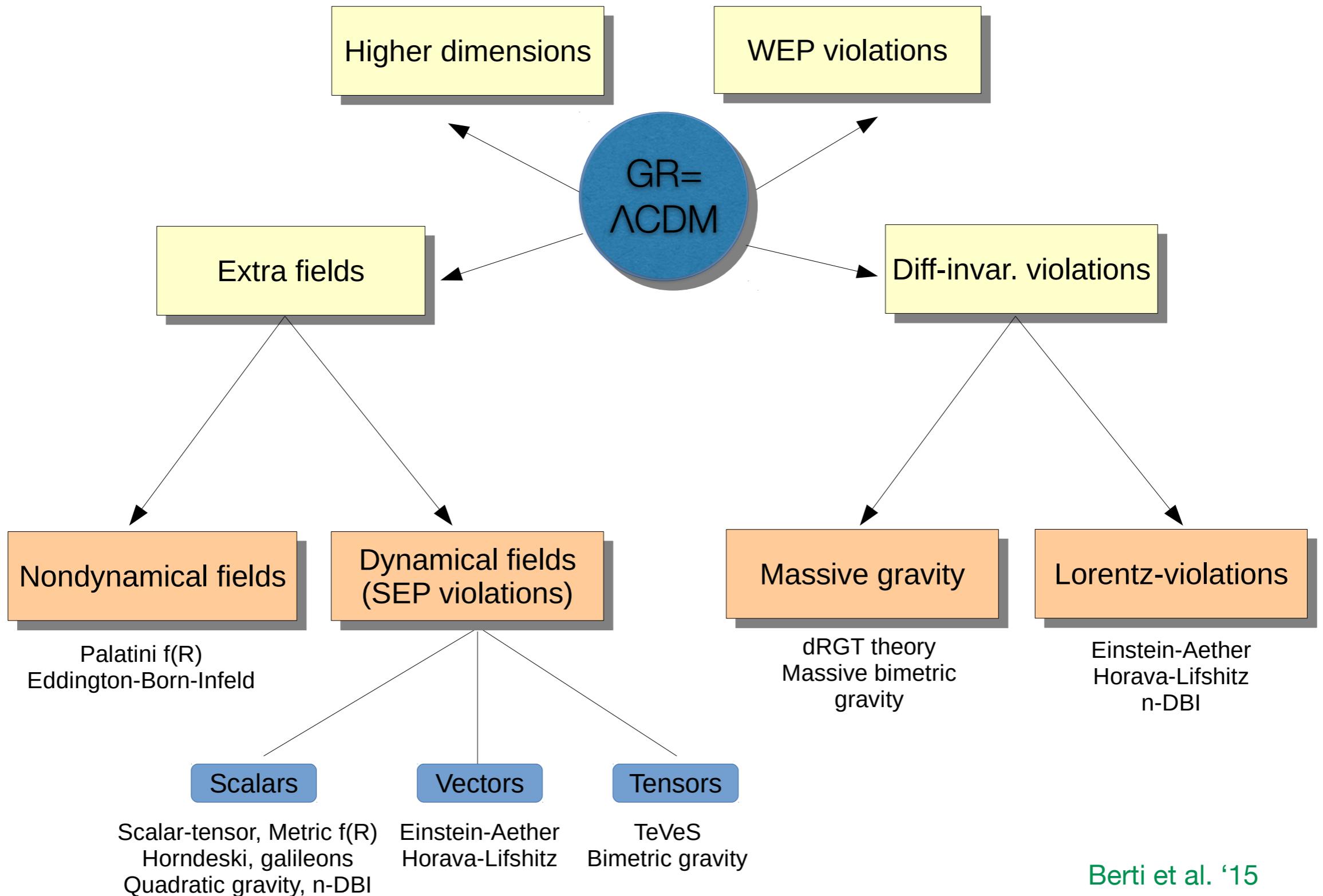
Motivations

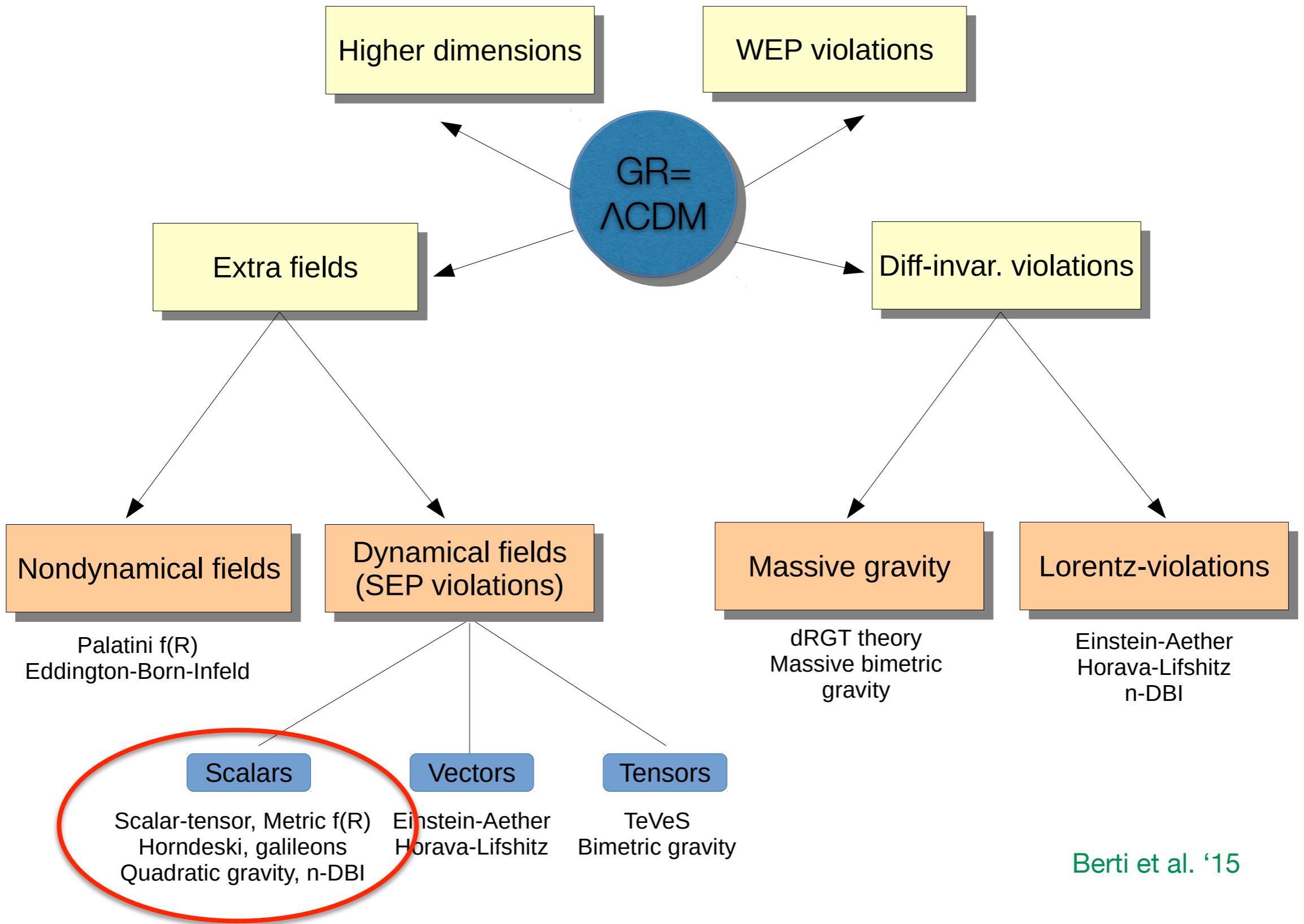
- On large scales, constraints are weaker
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- Is Λ CDM the ultimate model or the simplest approximation given the current precision of data?
- Is there new physics on very large scales?

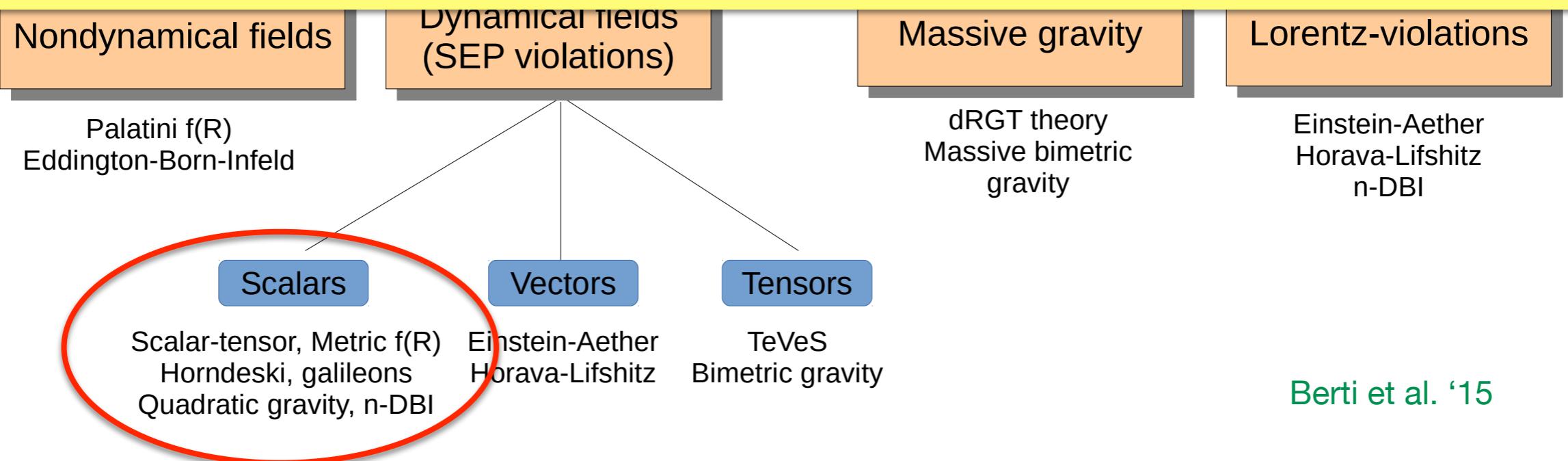






GW observations

- severely constrain cosmological modifications of gravity
- dramatically reduce the parameter space of scalar-tensor gravity (self-accelerating and screening) and the discovery-potential of new physics in LSS surveys



Berti et al. '15

Generalized scalar-tensor theories

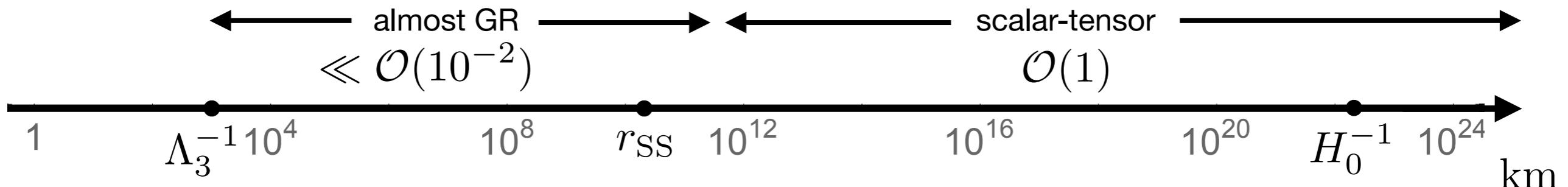
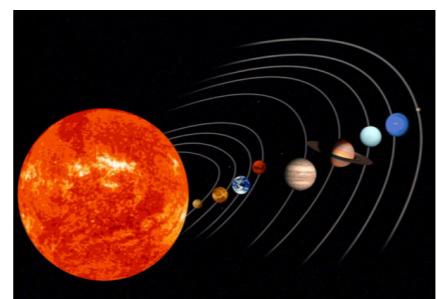
$$\begin{aligned}\mathcal{L} = & G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\square\phi & \square\phi \equiv \phi_{;\mu}^{\mu} & X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} \\ & - 2G_{4,X}(\phi, X)\left[(\square\phi)^2 - (\phi_{;\mu\nu})^2\right] \\ & + G_5(\phi, X)G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)\left[(\square\phi)^3 - 3\square\phi(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3\right] \\ & - F_4(\phi, X)\epsilon^{\mu\nu\rho}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu}\phi_{;\nu'}\phi_{;\rho}\phi_{;\rho'} \\ & - F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu}\phi_{;\nu'}\phi_{;\rho}\phi_{;\rho'}\phi_{;\sigma}\phi_{;\sigma'}\end{aligned}$$

Generalized scalar-tensor theories

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& + G_5(\phi, X)G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)\left[(\square\phi)^3 - 3\square\phi(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3\right] \\
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\end{aligned}$$

Self-acceleration and **screening**: large classical scalar field nonlinearities

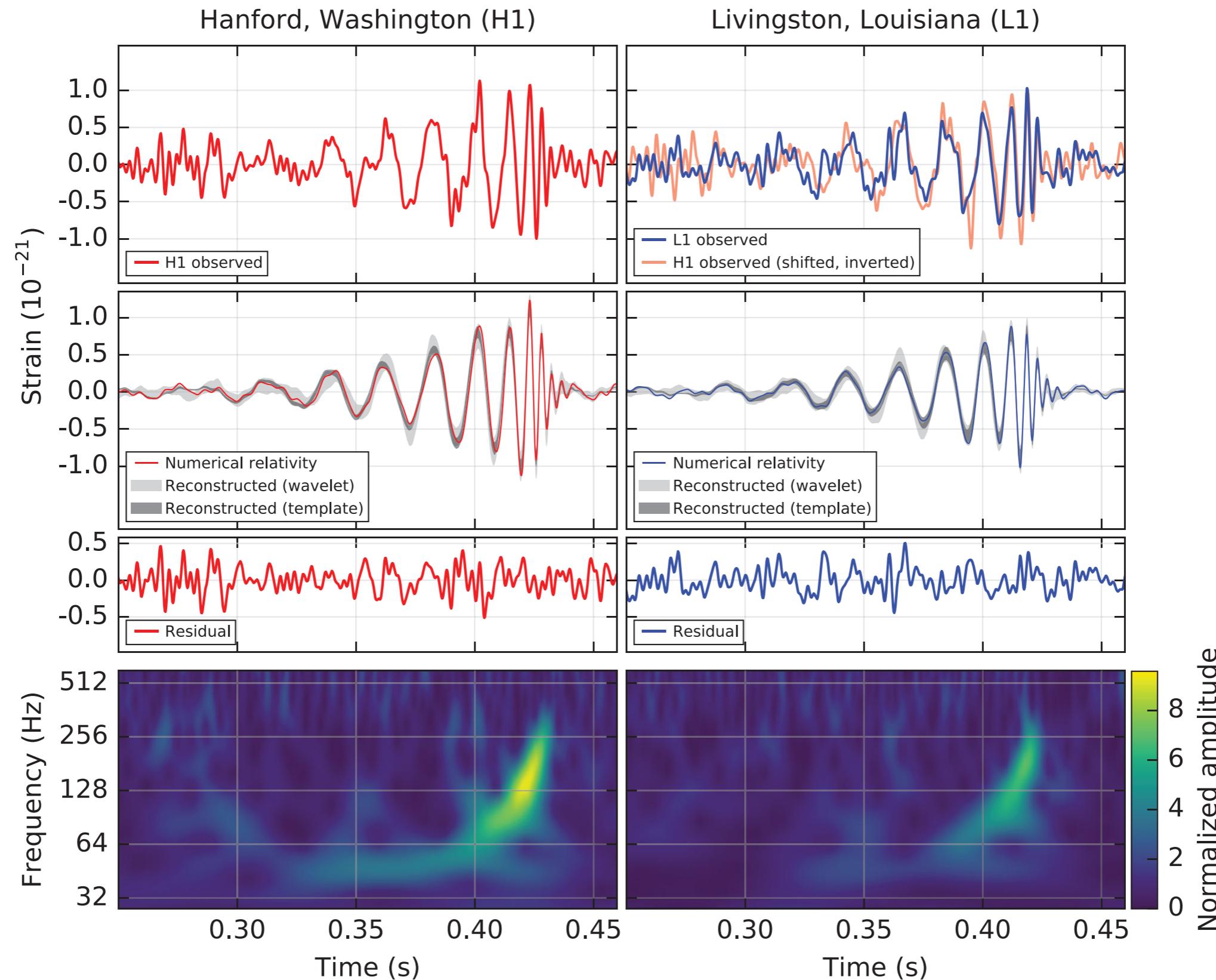
$$\begin{aligned}
\Lambda_3 &\equiv (H_0^2 M_{\text{Pl}})^{1/3} \\
&\sim (1000 \text{ km})^{-1}
\end{aligned}$$



GW150914: Gravitational Waves

Abbott et al. '16

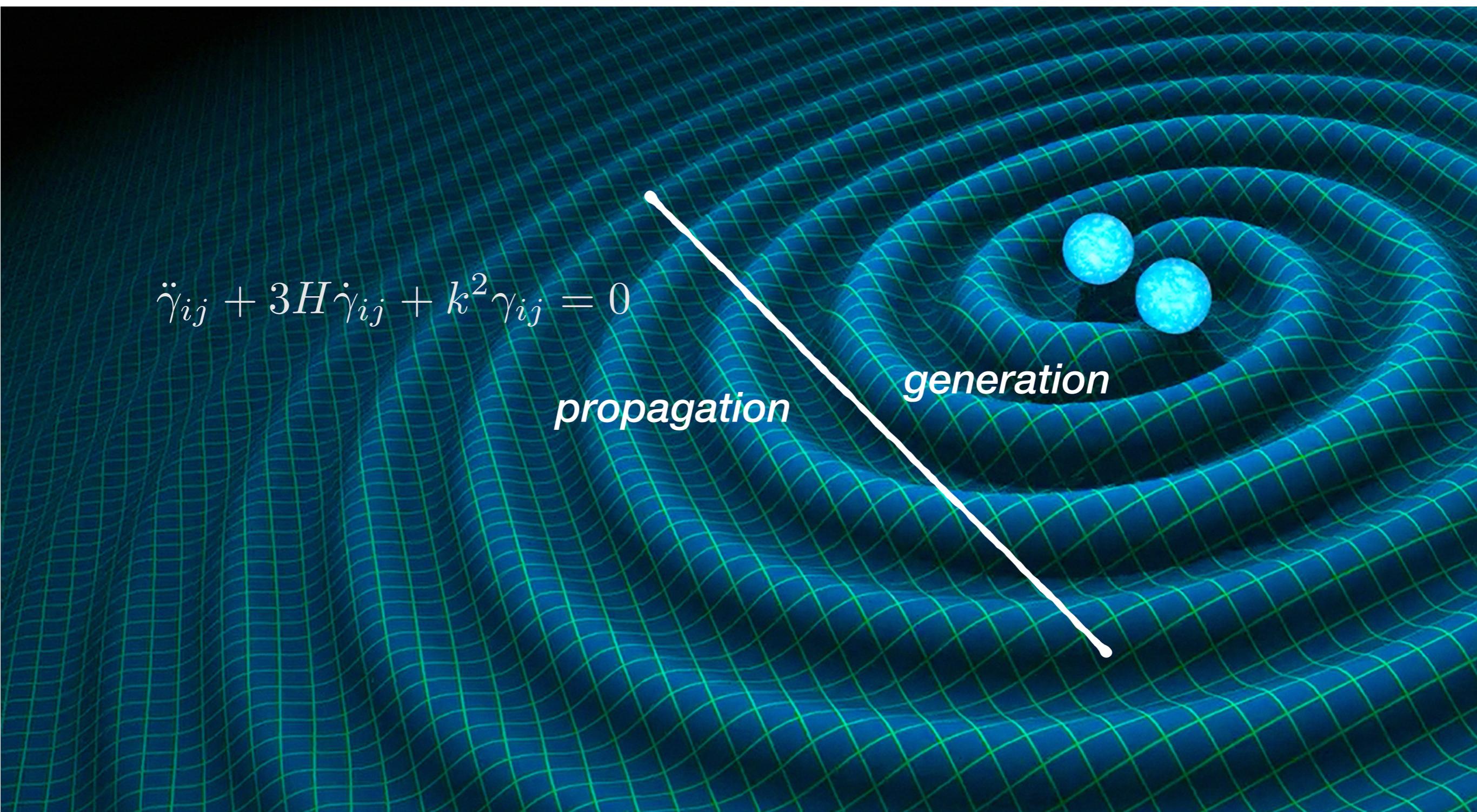
first detection: 09/14, 2015



Gravitational wave equation

Gravitational wave equation:

$$ds^2 = -dt^2 + a^2(t) [\delta_{ij} + \gamma_{ij}] d\vec{x}^i d\vec{x}^j , \quad \gamma_{ii} = 0 = \partial_i \gamma_{ij} , \quad H = \dot{a}/a$$



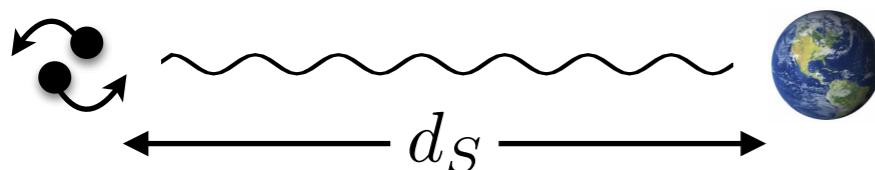
Modified gravitational wave propagation

Modified gravity spontaneously breaks Lorentz Invariance. Acts like a medium, where gravitons are **absorbed** and **dispersed**. Effects accumulate on long time-scale.

$$\ddot{\gamma}_{ij} + [(3 + \alpha_M)H + \Gamma(k)] \dot{\gamma}_{ij} + [c_T^2 k^2 + f(k)] \gamma_{ij} = 0$$

$$\frac{\Gamma(k)}{\omega} \lesssim \frac{1}{d_S \omega}$$

$$\frac{f(k)}{\omega^2} \lesssim \frac{1}{d_S \omega} \sim 10^{-18} \times \frac{2\pi \times 100 \text{ Hz}}{\omega} \frac{40 \text{ Mpc}}{d_S}$$



See e.g. Yunes, Yagi, Pretorius '16; Abbott et al. '17

Modified gravitational wave propagation

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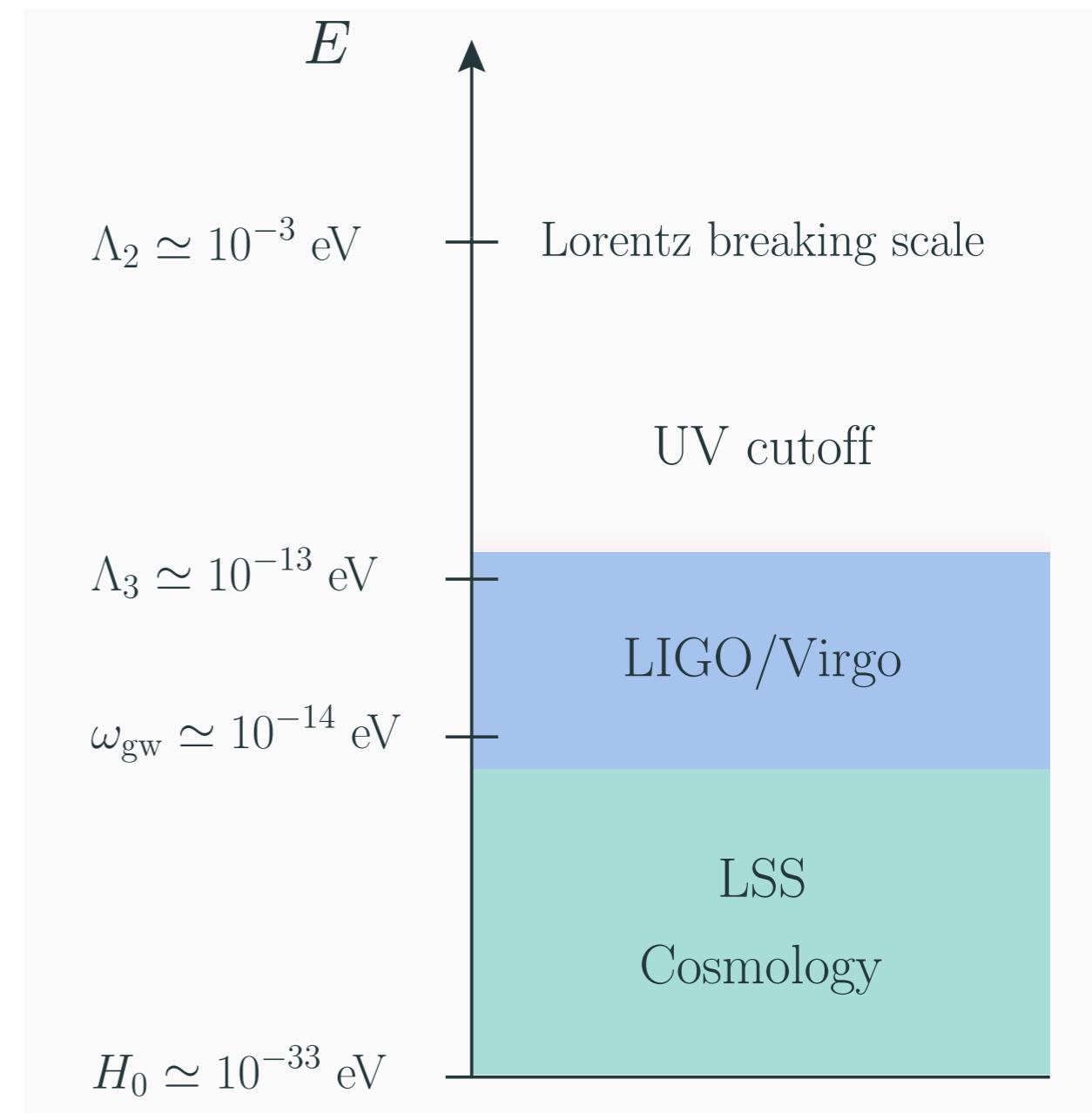


Modifications in the wave equation are related to
modifications of gravity in the LSS:

$$\mu = \mu(\dots), \quad \Sigma = \Sigma(\dots)$$

$$\nabla^2 \Phi = 4\pi G \mu \delta \rho_m$$

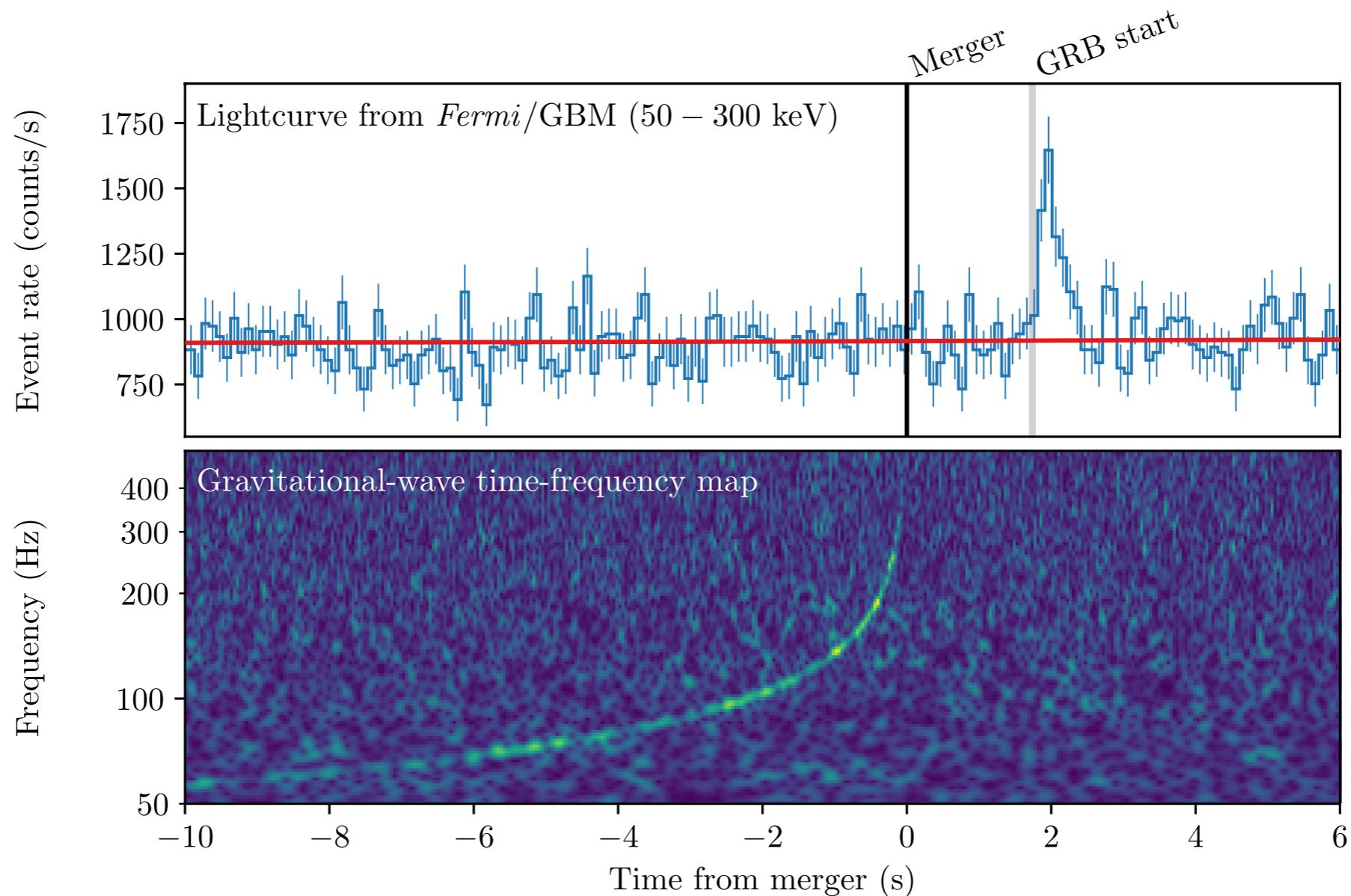
$$\nabla^2(\Phi + \Psi) = 4\pi G \Sigma \delta \rho_m$$



GW170817: neutron star merger



Multi-messenger observation



$$-3 \times 10^{-15} \leq \frac{c_T - c}{c} \leq 7 \times 10^{-16}$$

$C_T=1$ implications

$$\begin{aligned}\mathcal{L} = & G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\square\phi & \square\phi \equiv \phi_{;\mu}^{;\mu} & X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} \\ & - 2G_{4,X}(\phi, X)\left[(\square\phi)^2 - (\phi_{;\mu\nu})^2\right] \\ & + G_5(\phi, X)G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)\left[(\square\phi)^3 - 3\square\phi(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3\right] \\ & - F_4(\phi, X)\epsilon^{\mu\nu\rho}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu}\phi_{;\nu'}\phi_{;\rho}\phi_{;\rho'} \\ & - F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu}\phi_{;\nu'}\phi_{;\rho}\phi_{;\rho'}\phi_{;\sigma}\phi_{;\sigma'}\end{aligned}$$

$c_T=1$ implications

$$\begin{aligned}
 & \dot{\gamma}_{ij}^2 - (\partial_k \gamma_{ij})^2 \\
 \mathcal{L} = & G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\square\phi \\
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 \end{aligned}$$

$\square\phi \equiv \phi_{;\mu}^{\mu} \quad X \equiv g^{\mu\nu}\phi_{;\mu}\phi_{;\nu}$

$$c_T^2 - 1 \propto -2G_{4,X} - G_{5,\phi} - (H\dot{\phi} - \ddot{\phi})G_{5,X} + X F_4 - 3 H X \dot{\phi} F_5$$

$c_T=1$ implications

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$$c_T^2 - 1 \propto -2G_{4,X} - G_{5,\phi} - (H\dot{\phi} - \ddot{\phi})G_{5,X} + XF_4 - 3HX\dot{\phi}F_5$$

Most general theory compatible with $c_T=1$: $G_5 = F_5 = 0$, $XF_4 = 2G_{4,X}$

Creminelli, FV '17; Sakstein, Jain '17 ; Ezquiaga, Zumalacarregui '17 ; Baker+ '17

$$\delta c_T \sim (\Lambda_3/\Lambda_2)^4 \sim 10^{-40} \ll 10^{-15} \quad \text{c}_T=1 \text{ tuning is stable}$$

Can we rule out more?

$$\begin{aligned}\mathcal{L} = & G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\square\phi \\ & - 2G_{4,X}(\phi, X)\left[(\square\phi)^2 - (\phi_{;\mu\nu})^2\right] \\ & - F_4(\phi, X)\epsilon^{\mu\nu\rho}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu}\phi_{;\nu'}\phi_{;\rho}\phi_{;\rho'}\end{aligned}$$

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$$XF_4 = 2G_{4,X}$$

- **Beyond Horndeski:** $\alpha_H \equiv -\frac{X^2 F_4}{G_4}$

- **Braiding:** $\alpha_B = \frac{\dot{\phi} X G_{3,X}}{H G_4}$

Forecasted constraints from the large-scale structure

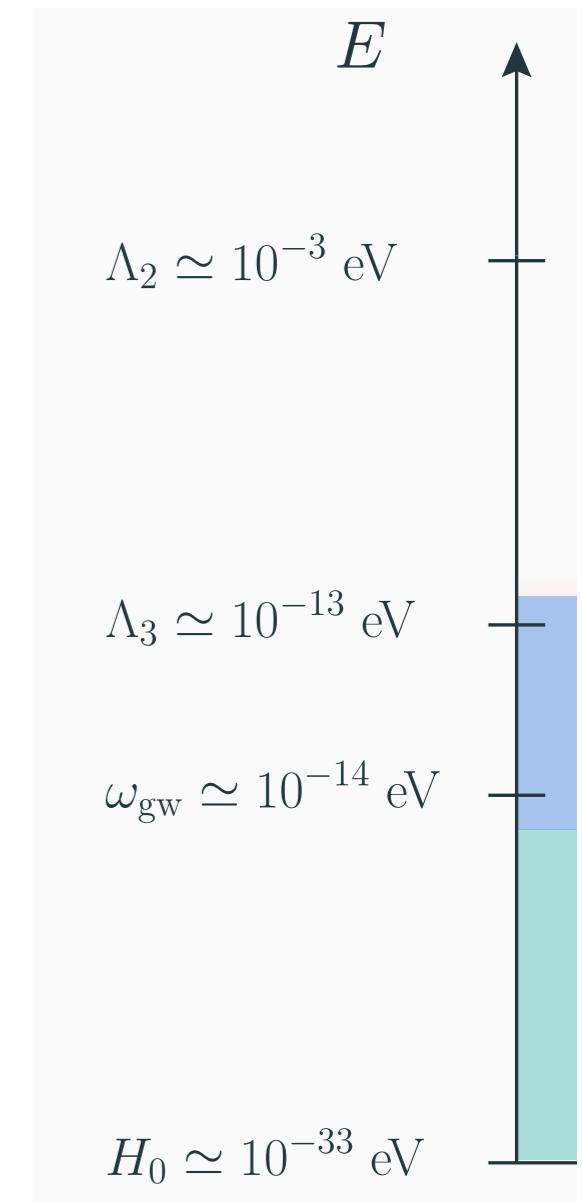
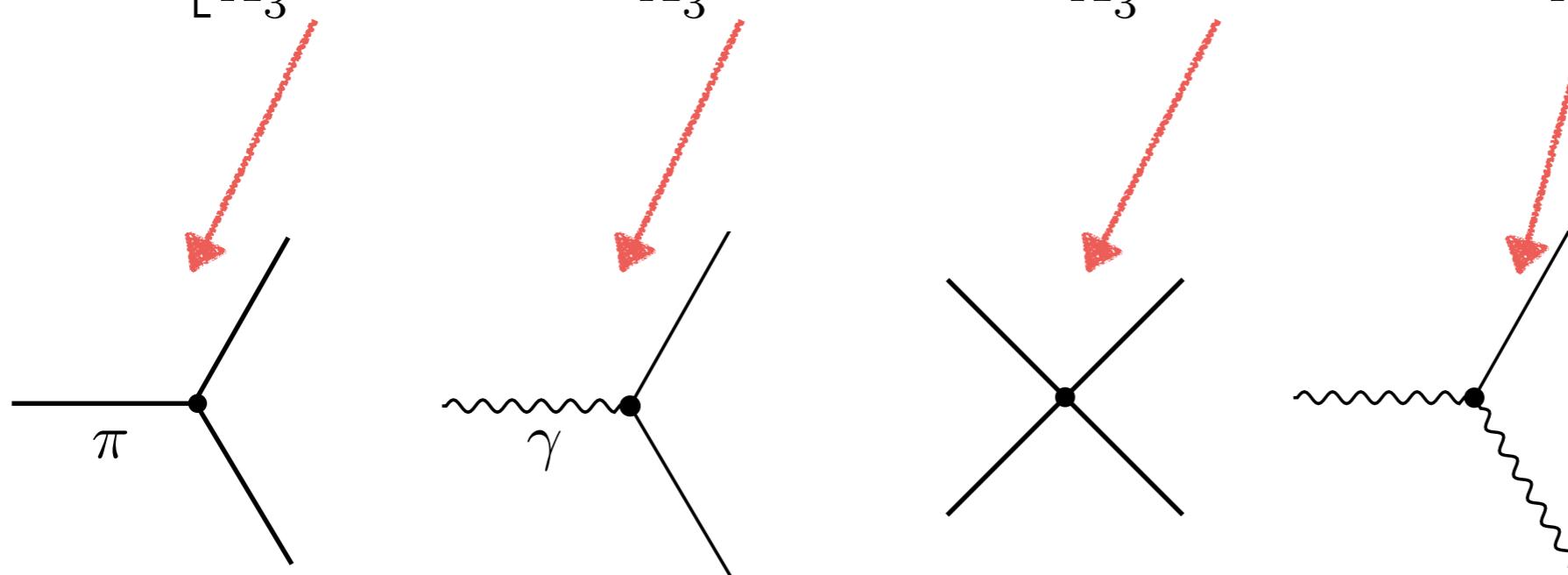
$$|\alpha_H| \lesssim 10^{-2}$$

$$|\alpha_B| \lesssim 10^{-2}$$

Expanded action for α_H

$$\mathcal{L} = \frac{1}{2} (\dot{\pi}^2 - c_s^2 (\partial_k \pi)^2) + \frac{1}{4} ((\dot{\gamma}_{ij})^2 - (\partial_k \gamma_{ij})^2) \quad \pi \equiv \delta\phi/\dot{\phi}_0$$

$$+ \alpha_H \left[\frac{1}{\Lambda_3^3} \partial^2 \pi (\partial \pi)^2 + \frac{1}{\Lambda_3^3} \ddot{\gamma}_{ij} \partial_i \pi \partial_j \pi + \frac{1}{\Lambda_3^6} (\square \pi)^2 (\partial \pi)^2 + \frac{1}{\Lambda_2^2} \dot{\pi} \dot{\gamma}_{ij}^2 \right]$$

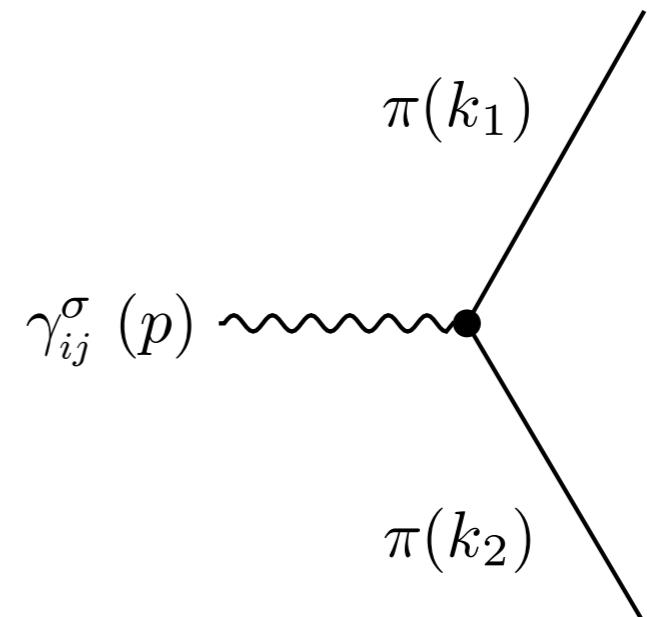


Graviton decay into dark energy

Creminelli, Lewandowski, Tambalo, FV '18

$$\mathcal{L}_{\gamma\pi\pi} = \frac{\alpha_H}{\Lambda_3^3} \ddot{\gamma}_{ij}^c \partial_i \pi_c \partial_j \pi_c$$

$$\Lambda_3 \equiv (M_{\text{Pl}} H_0^2)^{1/3}$$

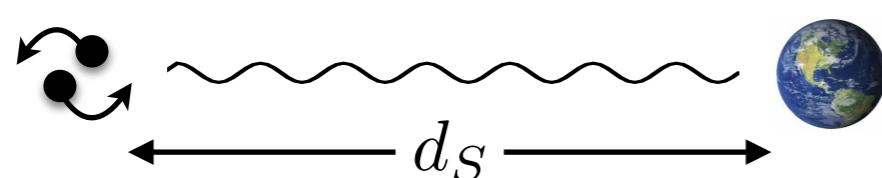


Beyond Horndeski interactions imply GW decay into scalar fluctuations π .
 Analogous to light absorption into a material

Decay allowed for $c_s < 1$ (c_s = sound speed of π fluctuations; assume $c_T=1$)

$$\Gamma \simeq \left(\frac{\alpha_H}{\Lambda_3^3} \right)^2 \frac{\omega_{\text{gw}}^7 (1 - c_s^2)^2}{c_s^7} \quad \text{decay rate}$$

$$d_S \Gamma < 1 \quad \Rightarrow \quad \alpha_H < 10^{-10}$$



irrelevant for LSS observations $\alpha_H \lesssim 10^{-2}$
 (unless $c_s=1$ with great precision)

Coherent decay

Creminelli, Tambalo, FV, Yingcharoenrat, '19

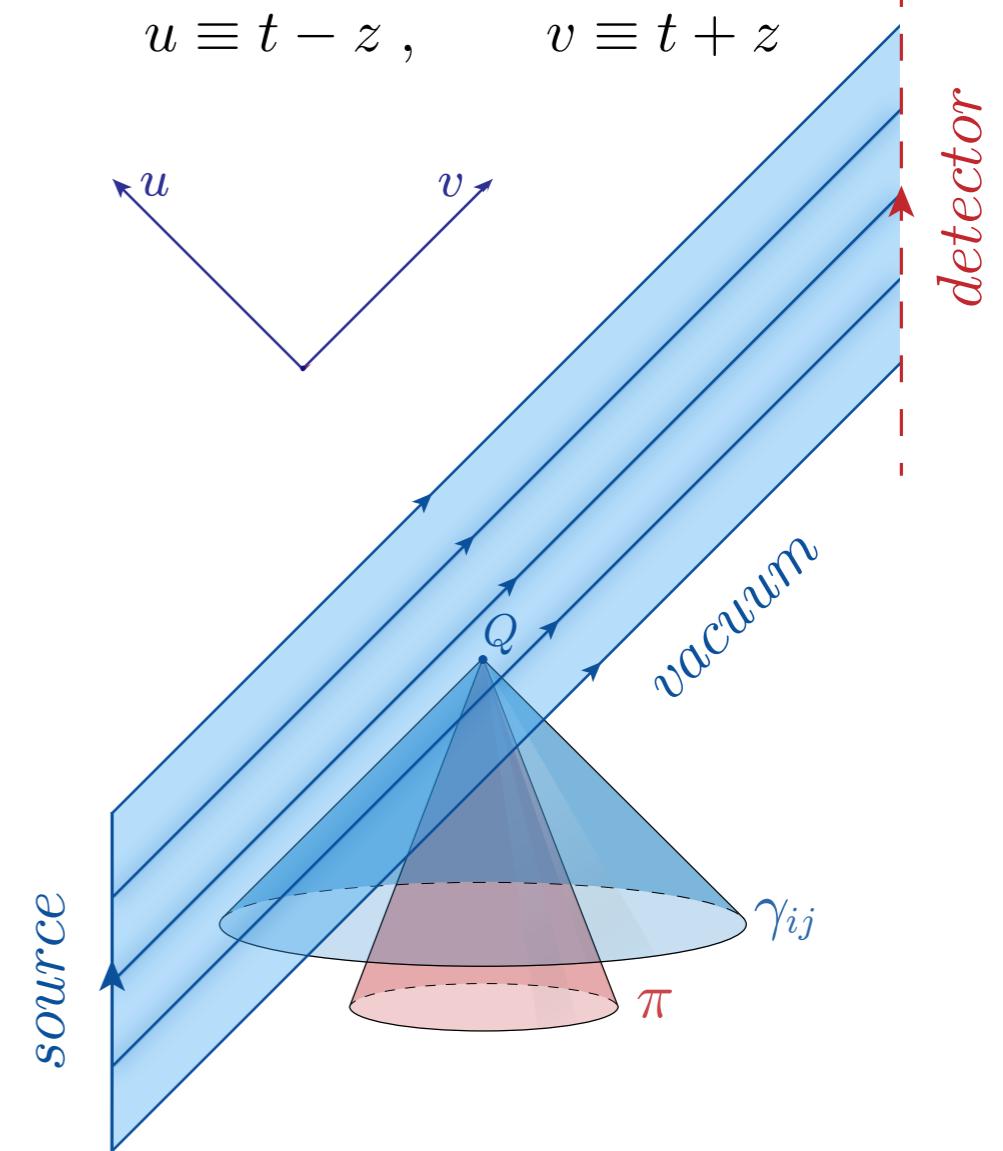
Decay enhanced by the large occupation number of the GWs \sim preheating

Classical wave:

$$\gamma_{ij} = M_{\text{Pl}} h_0^+ \cos(\omega u) \epsilon_{ij}^+, \quad \beta = \frac{|\alpha_H|}{\alpha c_s^2} \left(\frac{\omega}{H} \right)^2 h_0^+$$

Oscillator with changing frequency:

$$\ddot{\pi} - c_s^2 [\nabla^2 + \beta \cos(\omega u) \epsilon_{ij}^+ \partial_i \partial_j] \pi = 0$$



Coherent decay

Creminelli, Tambalo, FV, Yingcharoenrat, '19

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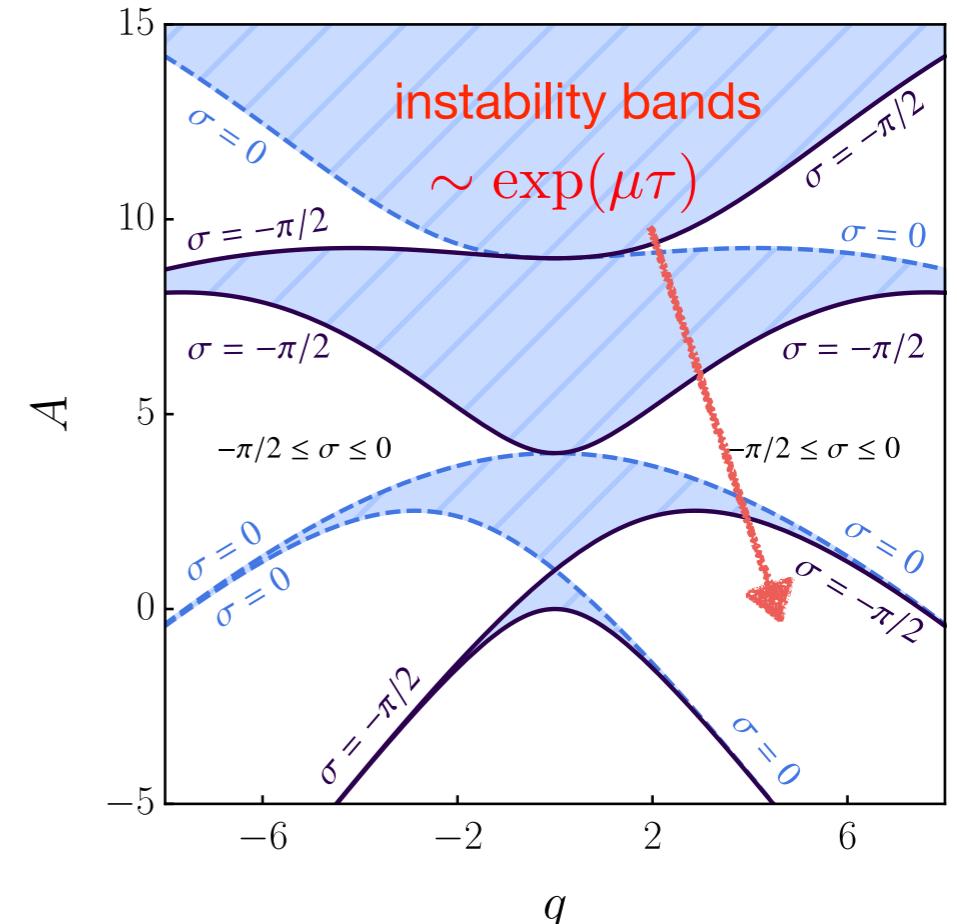
Oscillator with changing frequency:

$$\ddot{\pi} - c_s^2 [\nabla^2 + \beta \cos(\omega u) \epsilon_{ij}^+ \partial_i \partial_j] \pi = 0$$

Each Fourier mode satisfies a Mathieu equation
 \Rightarrow parametric resonance.

$$\frac{d^2 \pi_{\vec{k}}}{d\tau^2} + (A_{\vec{k}} - 2q_{\vec{k}} \cos(2\tau)) \pi_{\vec{k}} = 0$$

Resonant modes grow exponentially: $\pi_{\vec{k}} \sim e^{\mu_{\vec{k}} \tau}$

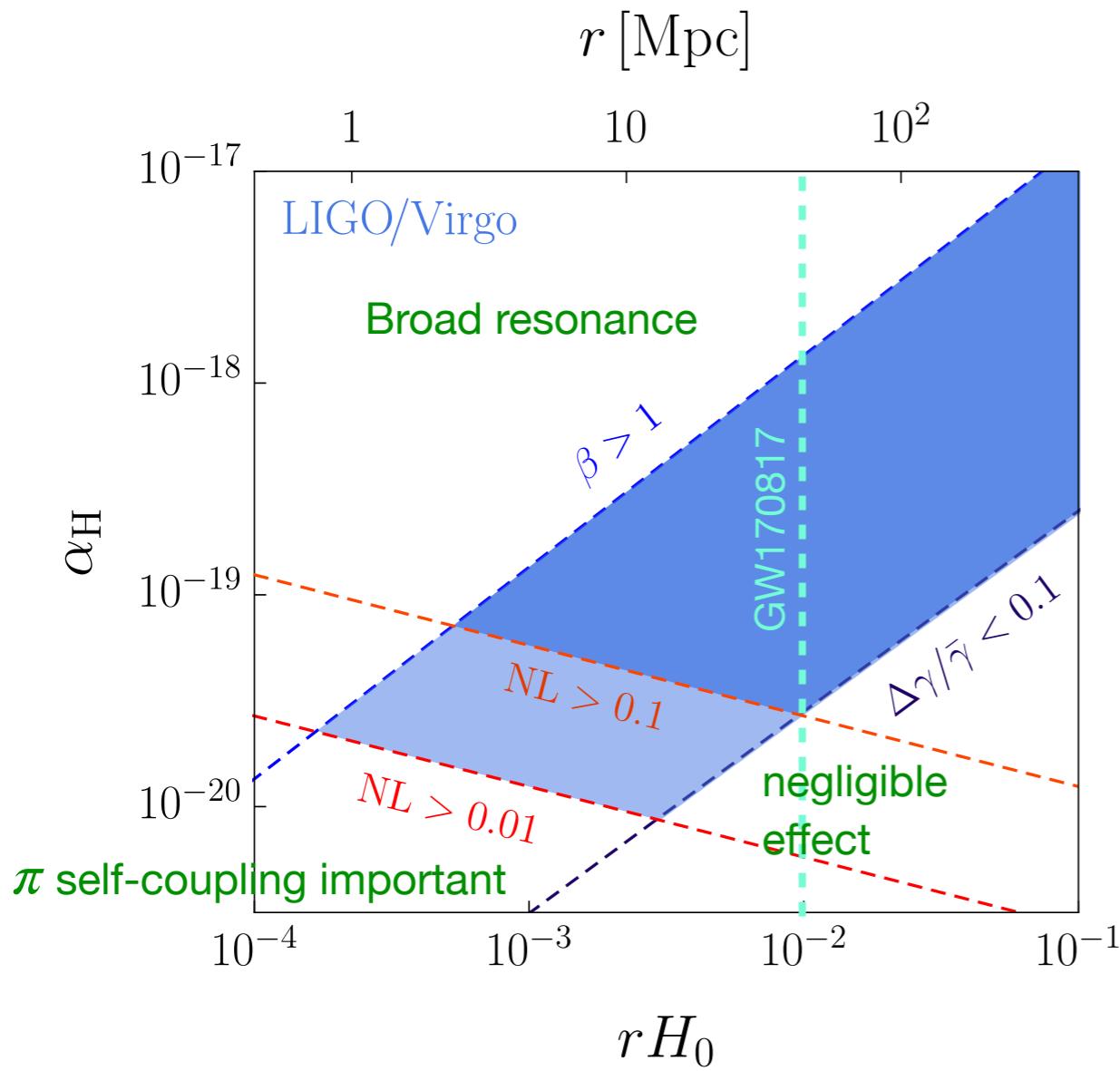


Narrow resonance $\beta \ll 1$: $\mu \sim \beta/4 \Rightarrow \rho_{\pi} \propto e^{\beta \omega u / 4} \Rightarrow \Delta \gamma_{ij} \propto v \gamma_0 e^{\beta \omega u / 4} \epsilon_{ij}^+$

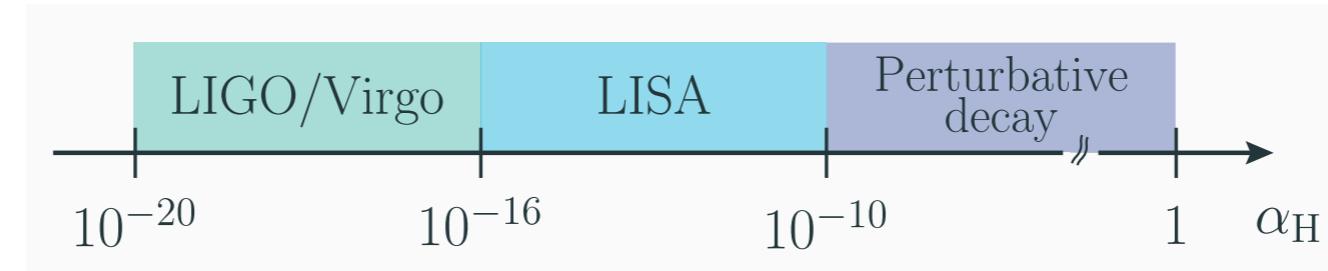
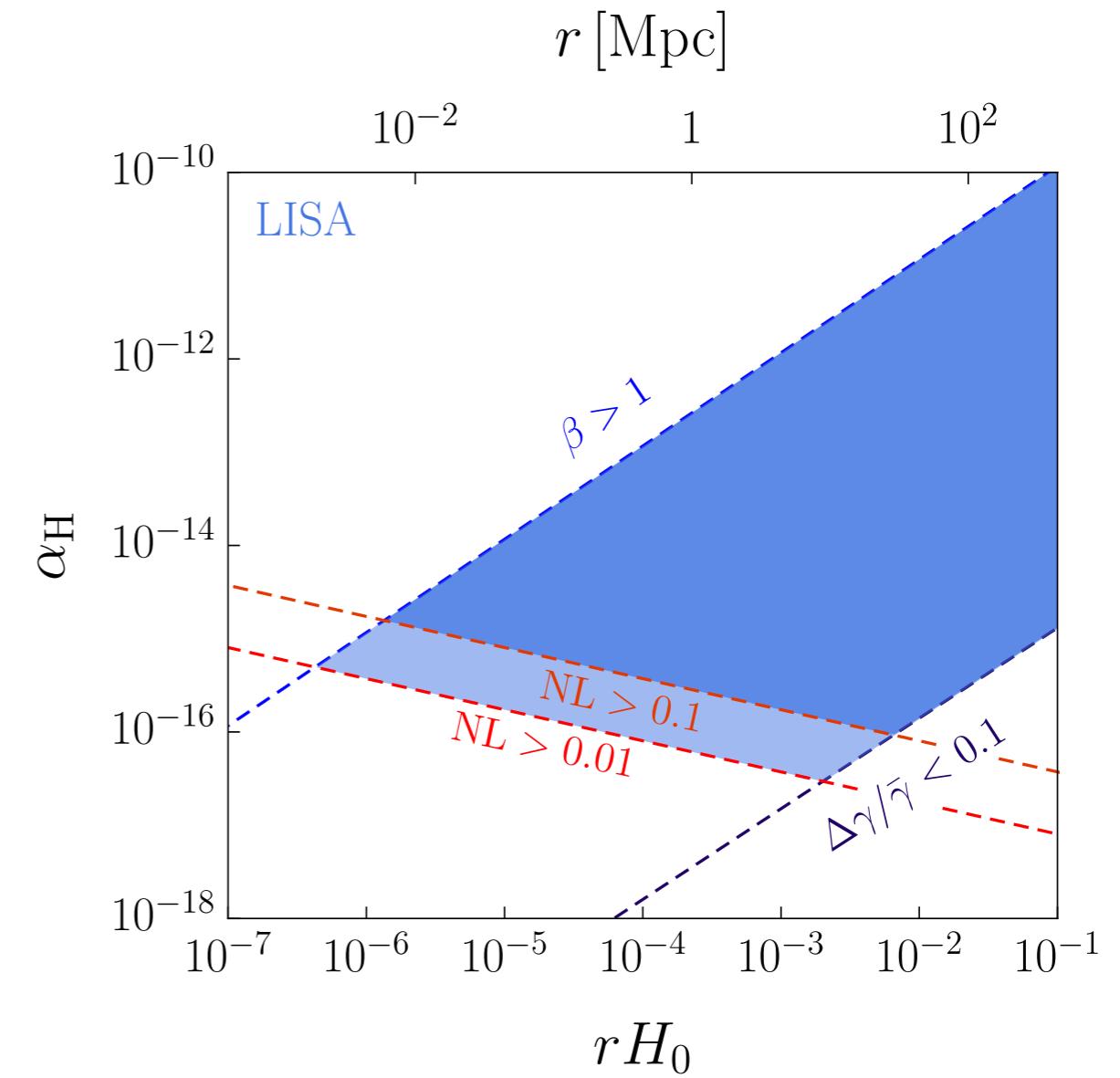
Same direction and polarization. Same frequency + higher harmonics (precursors)

GW modification

$$f = 30 \text{ Hz}, \quad M_c = 1.2M_\odot$$



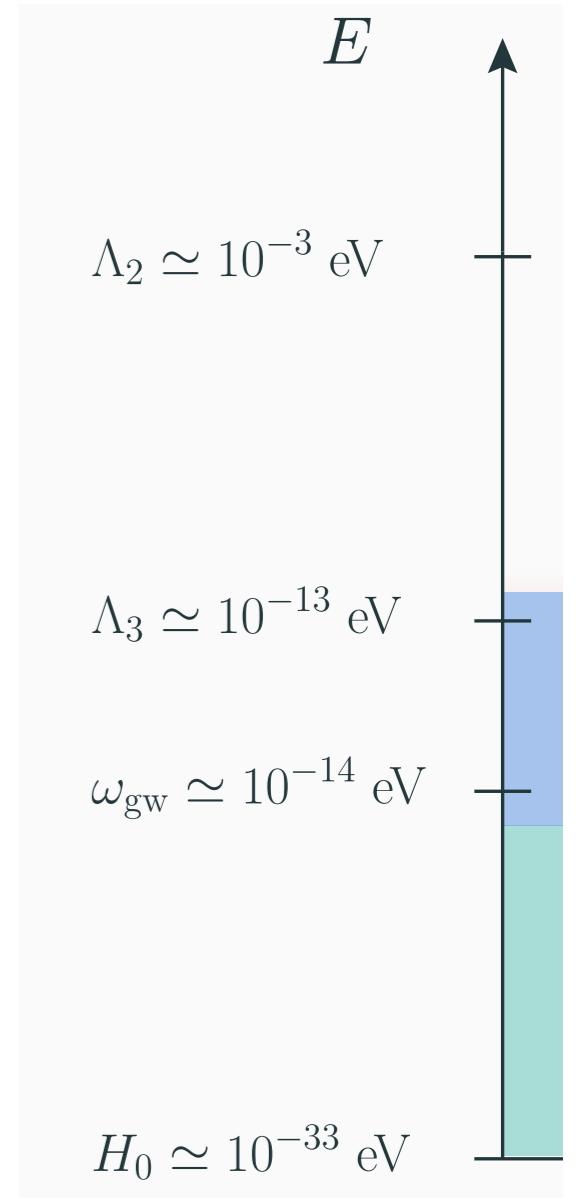
$$f = 10^{-2} \text{ Hz}, \quad M_c = 30M_\odot$$



Expanded action for α_H

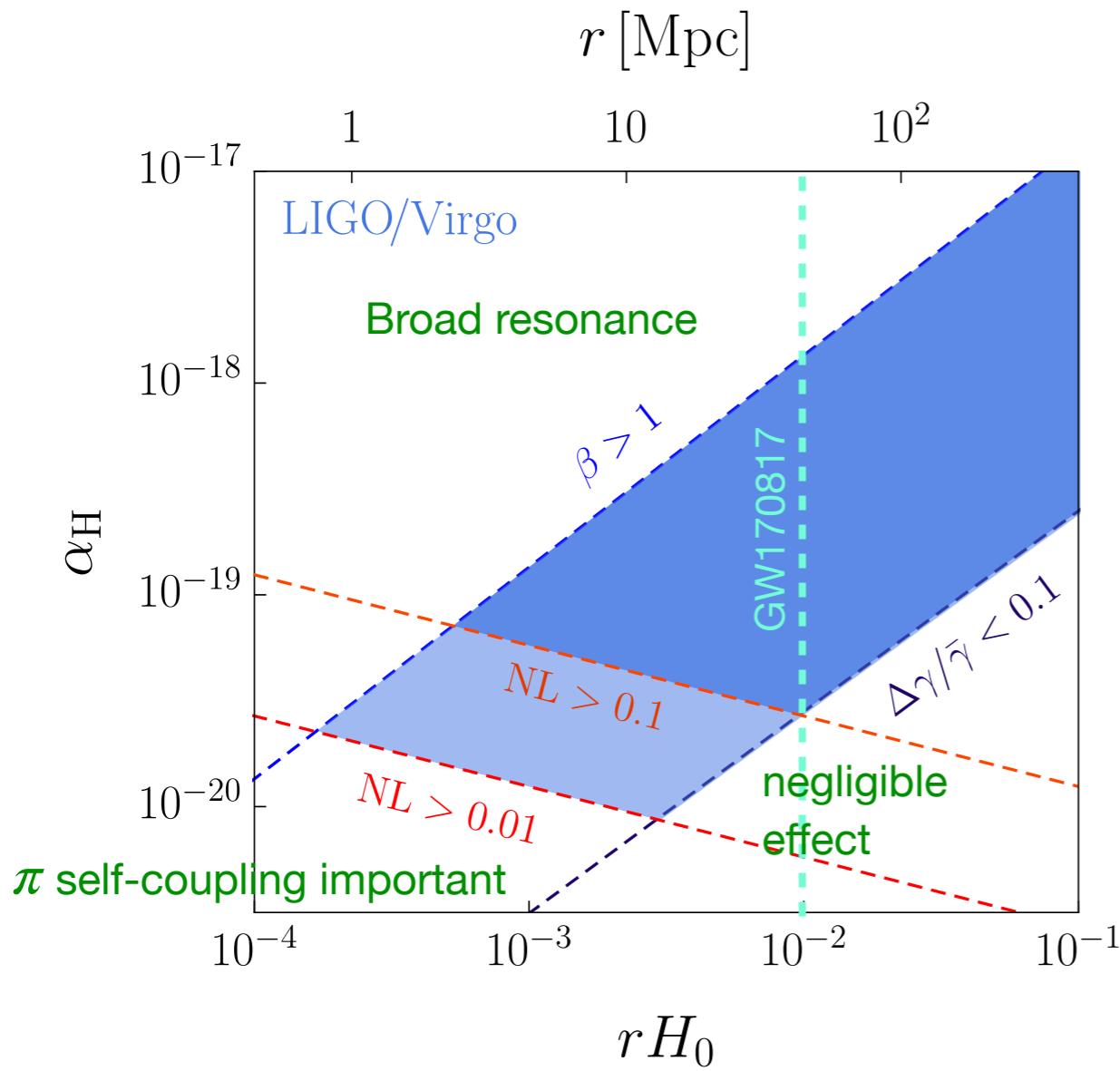
$$\mathcal{L} = \frac{1}{2} (\dot{\pi}^2 - c_s^2 (\partial_k \pi)^2) + \frac{1}{4} ((\dot{\gamma}_{ij})^2 - (\partial_k \gamma_{ij})^2) \quad \alpha_H \equiv -\frac{X^2 F_4}{G_4}$$

$$+ \alpha_H \left[\frac{1}{\Lambda_3^3} \partial^2 \pi (\partial \pi)^2 + \frac{1}{\Lambda_3^3} \ddot{\gamma}_{ij} \partial_i \pi \partial_j \pi + \frac{1}{\Lambda_3^6} (\square \pi)^2 (\partial \pi)^2 + \frac{1}{\Lambda_2^2} \dot{\pi} \dot{\gamma}_{ij}^2 \right]$$

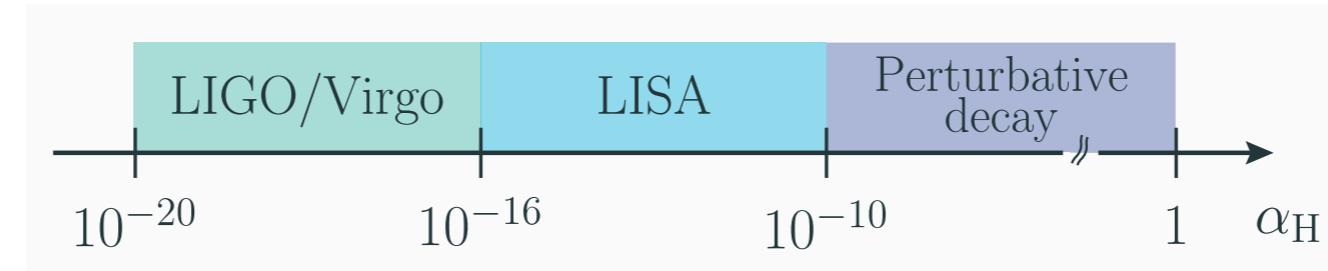
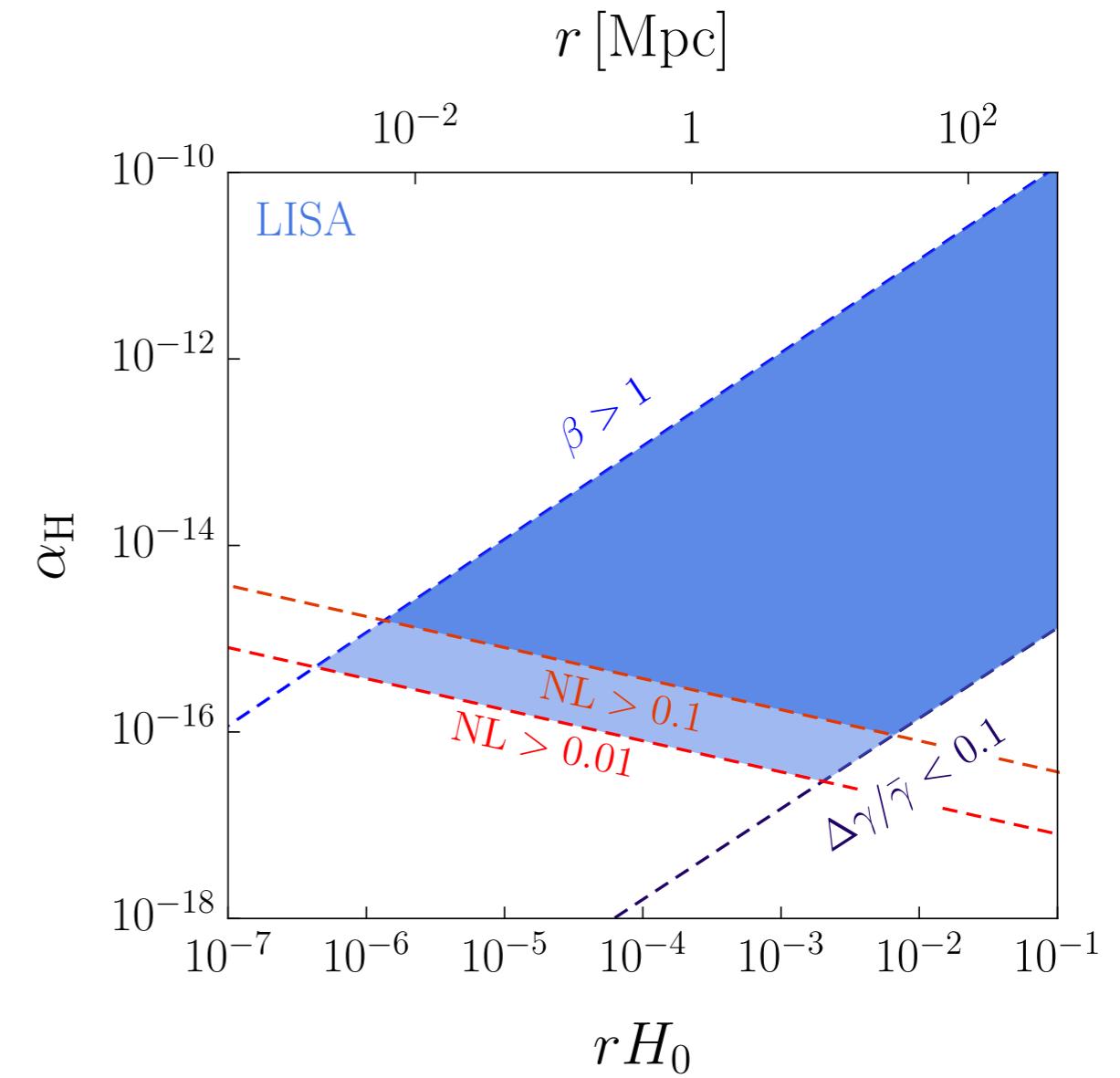


GW modification

$$f = 30 \text{ Hz}, \quad M_c = 1.2M_\odot$$



$$f = 10^{-2} \text{ Hz}, \quad M_c = 30M_\odot$$



Theory after no decay

$$\mathcal{L} = G_4(\phi, X)R + G_2(\phi, X) + \boxed{G_3(\phi, X)\square\phi}$$

$$- 2G_{4,X}(\phi, X) \left[(\square\phi)^2 - (\phi_{,\mu\nu})^2 \right]$$

$$- F_4(\phi, X) \epsilon^{\mu\nu\rho}_{\sigma} \epsilon^{\mu'\nu'\rho'}_{\sigma} \phi_{;\mu} \phi_{;\mu'} \phi_{;\nu} \phi_{;\nu'} \phi_{;\rho} \phi_{;\rho'}$$

$$\square\phi \equiv \phi_{;\mu}^{;\mu} \quad X \equiv g^{\mu\nu} \phi_{;\mu} \phi_{;\nu}$$

$$XF_4 = 2G_{4,X}$$

- **Beyond Horndeski:** $\alpha_H \equiv \frac{X^2 F_4}{G_4}$

- **Braiding:** $\alpha_B = \frac{\dot{\phi} X G_{3,X}}{H G_4}$

Expanded action for α_B

$$\mathcal{L} = \frac{1}{2} (\dot{\pi}^2 - c_s^2 (\partial_k \pi)^2) + \frac{1}{4} ((\dot{\gamma}_{ij})^2 - (\partial_k \gamma_{ij})^2)$$

$$+ \alpha_B \left[\frac{1}{\Lambda_3^3} \partial^2 \pi (\partial \pi)^2 + \frac{1}{\Lambda_2^2} \dot{\gamma}_{ij} \partial_i \pi \partial_j \pi + \frac{1}{M_{\text{Pl}}} \pi \dot{\gamma}_{ij}^2 \right]$$

$$\alpha_B = \frac{\dot{\phi} X G_{3,X}}{H G_4}$$

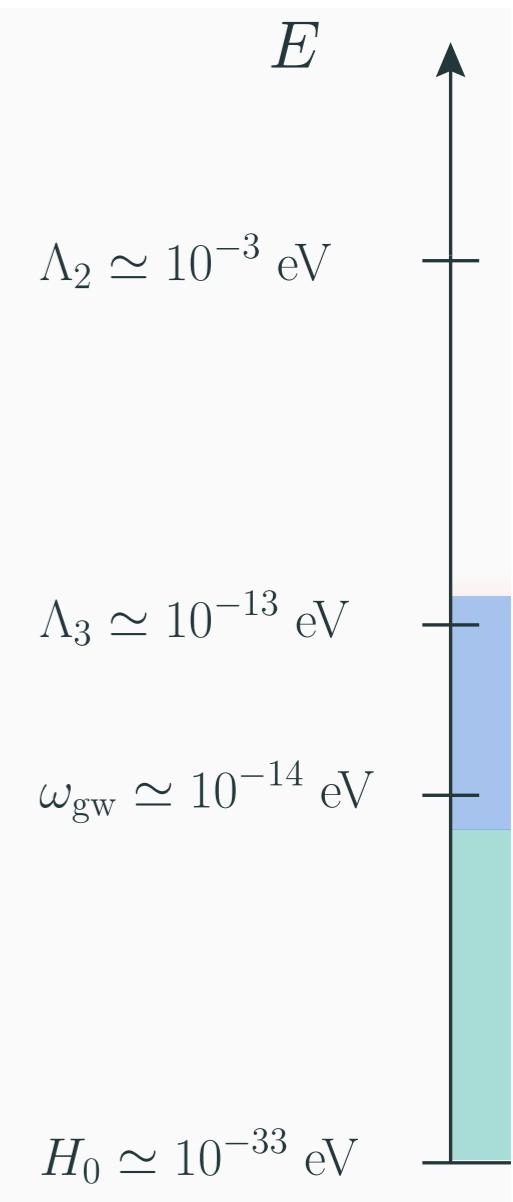
$$\Lambda_2 \equiv (H_0 M_{\text{Pl}})^{1/2}$$

Same calculation but with $\beta = \frac{|\alpha_B|}{\alpha c_s^2} \frac{\omega}{H} h_0^+$

Exponential growth **quenched** by large self-couplings of π .

Kills the effect? Simulations \sim preheating

No clear constraints on α_B ...



Expanded action for α_B

$$\mathcal{L} = \frac{1}{2} (\dot{\pi}^2 - c_s^2 (\partial_k \pi)^2) + \frac{1}{4} ((\dot{\gamma}_{ij})^2 - (\partial_k \gamma_{ij})^2)$$

$$\alpha_B = \frac{\dot{\phi} X G_{3,X}}{H G_4}$$

$$+ \alpha_B \left[\frac{1}{\Lambda_3^3} \partial^2 \pi (\partial \pi)^2 + \frac{1}{\Lambda_2^2} \dot{\gamma}_{ij} \partial_i \pi \partial_j \pi + \frac{1}{M_{\text{Pl}}} \pi \dot{\gamma}_{ij}^2 \right]$$

The regime $\beta > 1$ seems problematic:

$$\ddot{\pi} + c_s^2 [k^2 + \beta \cos(\omega u) \epsilon_{ij}^+ k^i k^j] \pi = 0$$

$$\beta = \frac{|\alpha_B|}{\alpha c_s^2} \frac{\omega}{H} h_0^+$$

gradient instability < 0

Expanded action for α_B

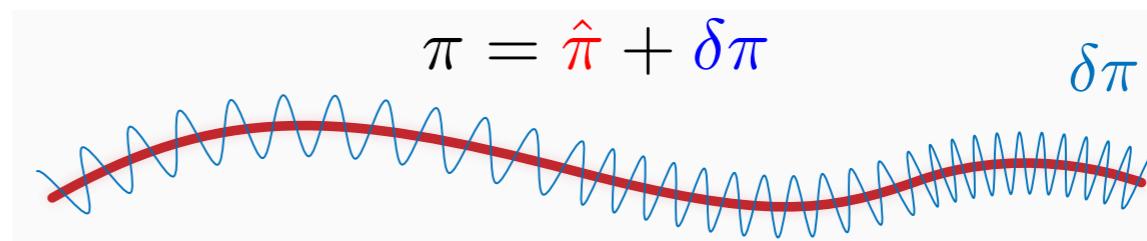
$$\mathcal{L} = \frac{1}{2} (\dot{\pi}^2 - c_s^2 (\partial_k \pi)^2) + \frac{1}{4} ((\dot{\gamma}_{ij})^2 - (\partial_k \gamma_{ij})^2) + \alpha_B \left[\frac{1}{\Lambda_3^3} \partial^2 \pi (\partial \pi)^2 + \frac{1}{\Lambda_2^2} \dot{\gamma}_{ij} \partial_i \pi \partial_j \pi + \frac{1}{M_{\text{Pl}}} \pi \dot{\gamma}_{ij}^2 \right]$$

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We must check whether this is true even when we include nonlinearities

$$Z_{\mu\nu}[\hat{\pi}(x)] \partial^\mu \partial^\nu \delta\pi = 0$$



- **Gradient instabilities:** imaginary solution of $Z_{\mu\nu} k^\mu k^\nu = 0$ for k^μ
- **Ghost instabilities:** $Z_{00} < 0$

Expanded action for α_B

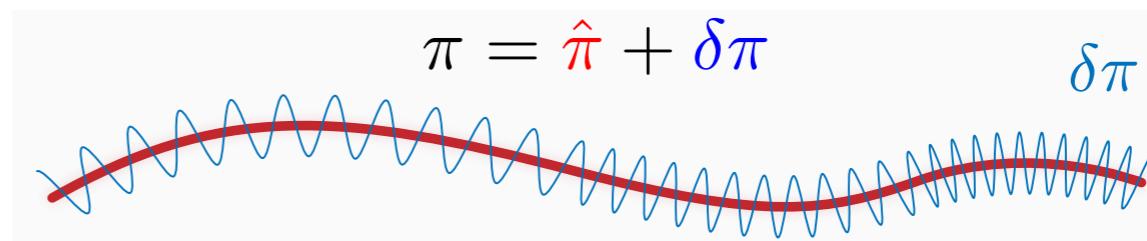
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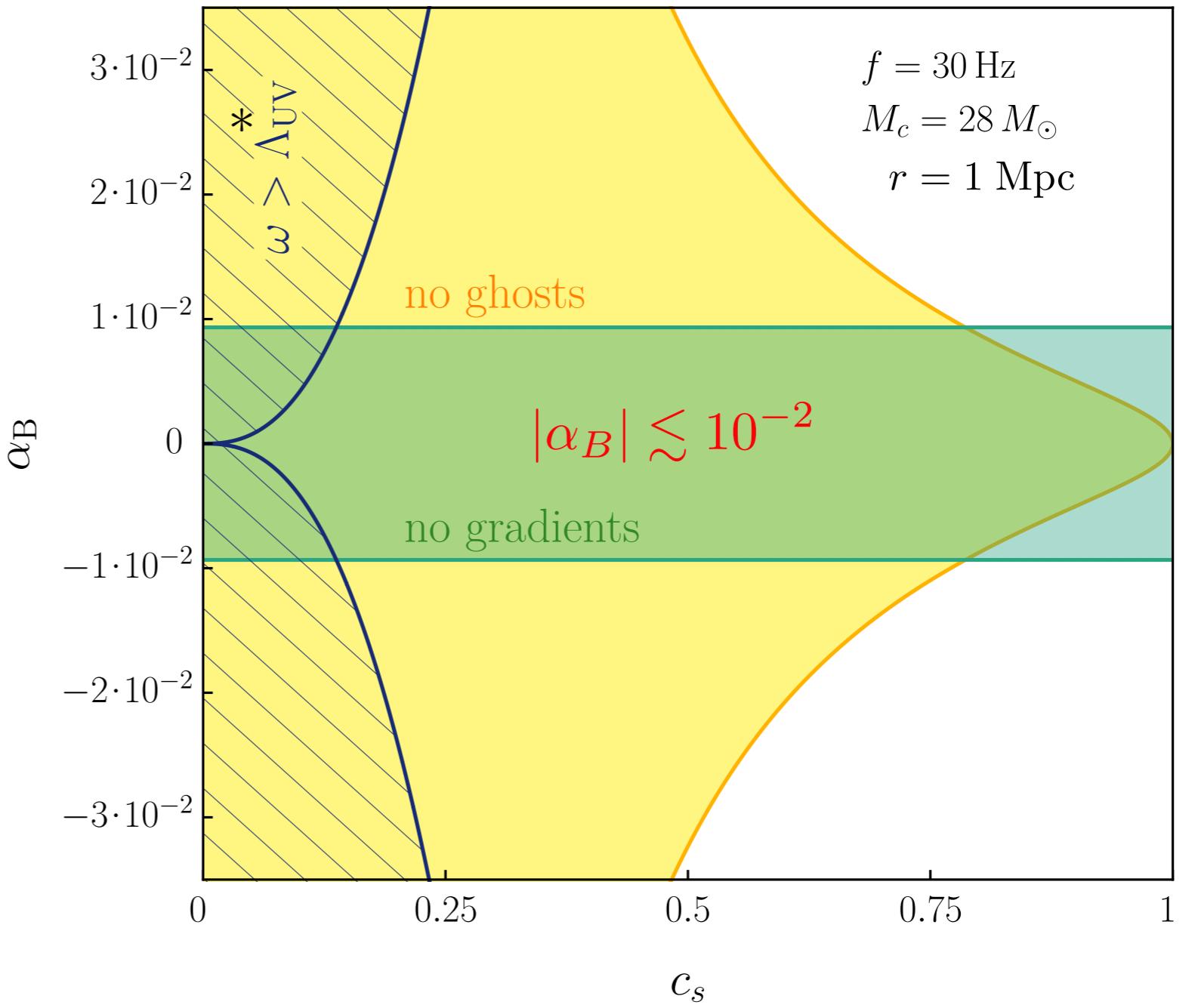
- **Gradient instabilities:** imaginary solution of $Z_{\mu\nu} k^\mu k^\nu = 0$ for $k^\mu \quad \beta > 1$
- **Ghost instabilities:** $Z_{00} < 0 \quad \beta^2 > (1 - c_s^2) c_s^{-4}$

Constraints for stellar-mass BHs

$$\beta = \frac{|\alpha_B|}{\alpha c_s^2} \frac{\omega}{H} h_0^+$$

$$h_0^+ \sim \frac{1}{\sqrt{2}} \cdot \frac{4}{r} (GM_c)^{5/3} (\pi f)^{2/3}$$

- $\beta > 1$: **gradient inst.**
- $\beta^2 > (1 - c_s^2) c_s^{-4}$: **ghost inst.**

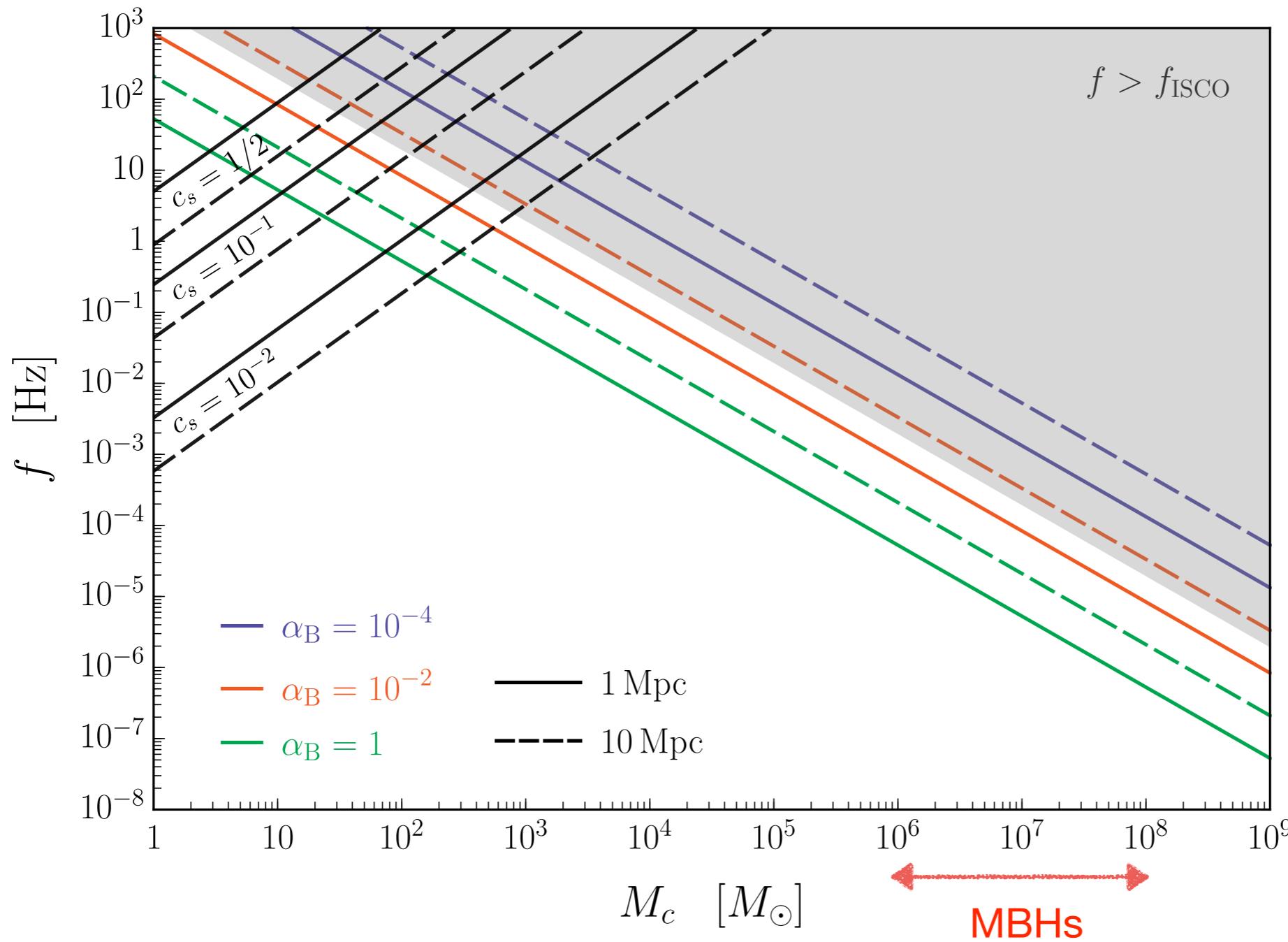


$$* \quad \Lambda_{UV} \sim \frac{\alpha^{1/2} c_s^{11/6}}{\alpha_B^{1/3}} \Lambda_3$$

Gradient instability, $\beta > 1$, for α_B

$$\beta = \frac{|\alpha_B|}{\alpha c_s^2} \frac{\omega}{H} h_0^+$$

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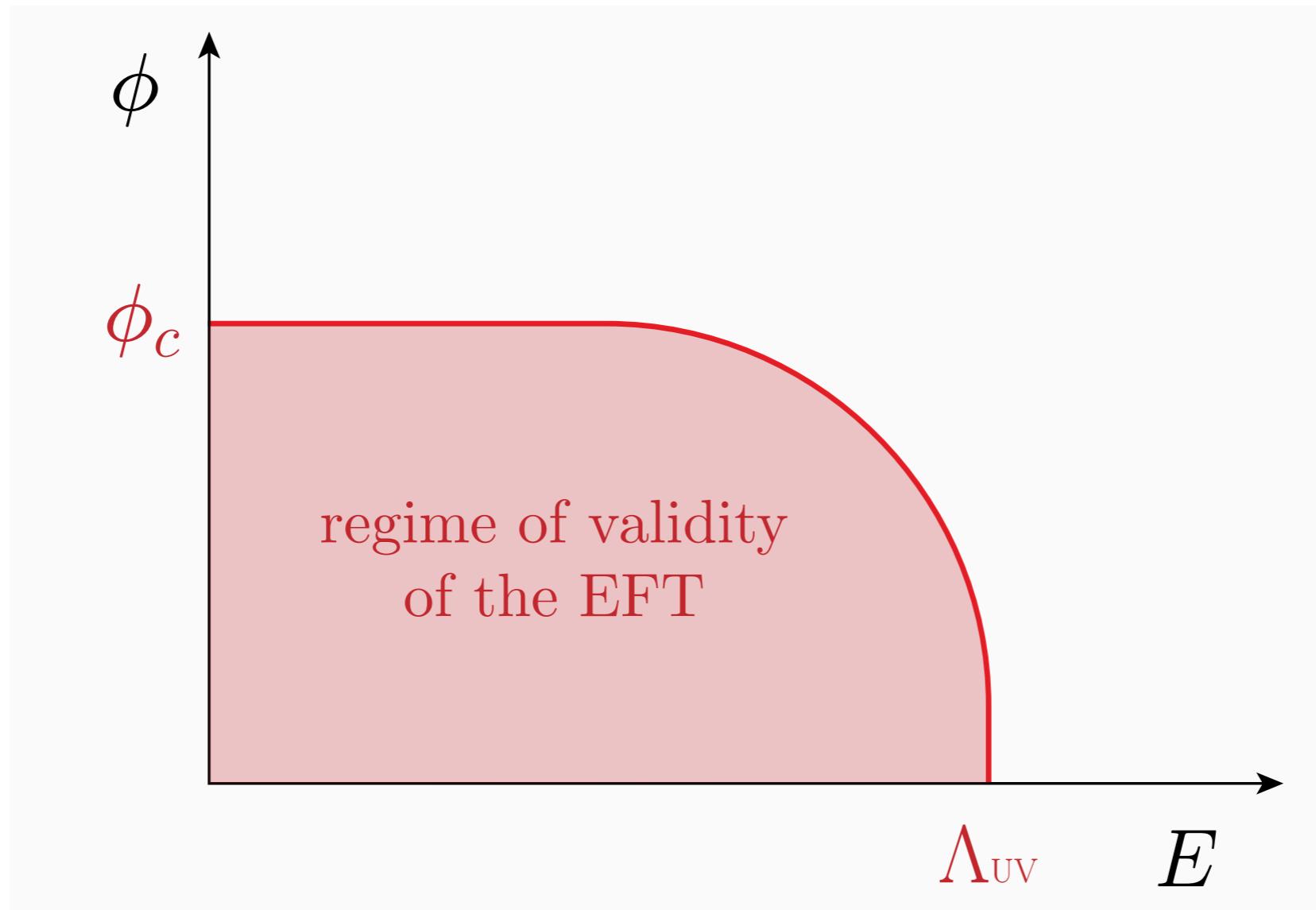
$$|\alpha_B| \lesssim 10^{-2}$$

Population of MBHs is
enough to globally
trigger the instability

Down to 10^{10} Km

Fate of instability

Is the instability real or artefact of EFT? Gradient and ghost instabilities can appear in the low energy EFT of stable UV complete theories



Fate of instability depends on the (unknown) UV completion of these theories

To trust the EFT: $|\alpha_B| \lesssim 10^{-2}$. Interestingly close to constraints from the large-scale structure

Summary and conclusion

Gravitational waves probe modified gravity as light probes material

In many cases very effectively, more than what large-scale structure can do

$$\begin{aligned}\mathcal{L} = & G_4(\phi, X)R + G_2(\phi, X) + G_3(\phi, X)\square\phi \\ & - 2G_{4,X}(\phi, X)\left[(\square\phi)^2 - (\phi_{;\mu\nu})^2\right] \\ & + G_5(\phi, X)G^{\mu\nu}\phi_{;\mu\nu} + \frac{1}{3}G_{5,X}(\phi, X)\left[(\square\phi)^3 - 3\square\phi(\phi_{;\mu\nu})^2 + 2(\phi_{;\mu\nu})^3\right] \\ & - F_4(\phi, X)\epsilon^{\mu\nu\rho}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu}\phi_{;\nu'}\phi_{;\rho}\phi_{;\rho'} \\ & - F_5(\phi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\phi_{;\mu}\phi_{;\mu'}\phi_{;\nu}\phi_{;\nu'}\phi_{;\rho}\phi_{;\rho'}\phi_{;\sigma}\phi_{;\sigma'}\end{aligned}$$

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