Utrecht University

Spinoza Institute

The Cellular Automaton Interpretation of Quantum Mechanics

arxiv:2005.06374

On-line “Newton 1665” seminars
phenomenology/theory/astro/cosmo

CERN, Geneva, webinar

Gerard ’t Hooft

May 19, 2020

Utrecht University

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▶ *Progress on*

The Cellular Automaton Interpretation of Quantum Mechanics

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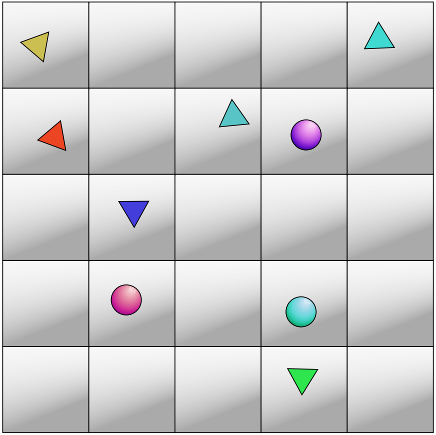
The Cellular Atomaton: Only *classical* evolution equations.



Quantum field lattice: same with *quantum* evolution equations.

Conjecture:

- Every cellular automaton is mathematically equivalent to a genuine quantum field theory on a lattice.
- Every lattice quantum field theory can be accurately approximated with a *classical* cellular automaton.

But we have the objection raised by Bell's theorem, and the puzzles posed by "paradoxes" such as
Greenberger - Horne - Zeilinger (GHZ), etc.

To be discussed after the lecture

These objections are philosophical. The arguments are known not to be infinitely accurate. And that's why they fail. The arguments in favor of this 'hidden variable' conjecture are much stronger.

I'll show you how they go.

Operators are arranged in the following classes:

- Beables,

  refer to things that are 'truly there'.

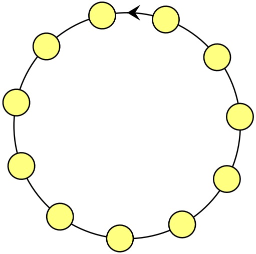
  All beable operators commute with one another, at all times.

- Changeables,

  transform beables into other beables,

- Superimposables,

  all other operators.



1. The periodic chain.
Ontological states:

$$|0\rangle,\ |1\rangle,\ \dots\ |N-1\rangle$$

Evolution law:
$|k\rangle_{t+\delta t} = U(\delta t)\,|k\rangle_t$

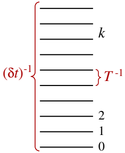
$$U(\delta t)|k\rangle = |k+1\rangle$$

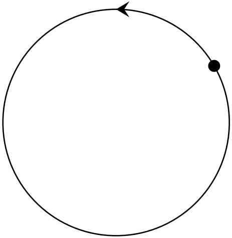
$$U(\delta t) = e^{-iH\,\delta t}\ , \qquad \frac{\mathrm{d}|\psi\rangle}{\mathrm{d}t} = -i\,H|\psi\rangle$$

$$|n\rangle^E \stackrel{\text{def}}{=} \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} e^{2\pi ikn/N}|k\rangle^{\text{ont}}\ , \qquad k = 0,\cdots,\ N-1\ ;\quad n = 0,\cdots,\ N-1\ .$$

$$|k\rangle^{\text{ont}} = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} e^{-2\pi ikn/N}|n\rangle^E\ .$$

$$H = \frac{2\pi}{N\,\delta t}\,n = \omega n$$





2. The continuum limit.

Ontological states: $|\phi\rangle$

Evolution law:

$$\frac{\mathrm{d}}{\mathrm{d}t}|\phi\rangle_t = \omega$$

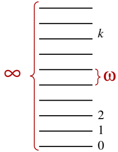
$$U(\delta t)|\phi\rangle = |\phi + \omega t\rangle$$

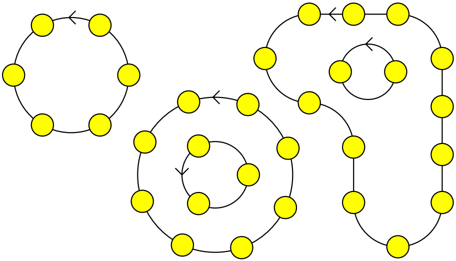
$$U(\delta t) = e^{-iH\,\delta t}\ , \qquad \frac{\mathrm{d}|\psi\rangle}{\mathrm{d}t} = -i\,H|\psi\rangle$$

$$|n\rangle^E \ \stackrel{\text{def}}{=}\ \frac{1}{\sqrt{2\pi}}\oint e^{i\phi n/N}|\phi\rangle^{\text{ont}}\ , \qquad 0 \le \phi < 2\pi\ ;\quad n = 0, \cdots,\ \infty\ .$$

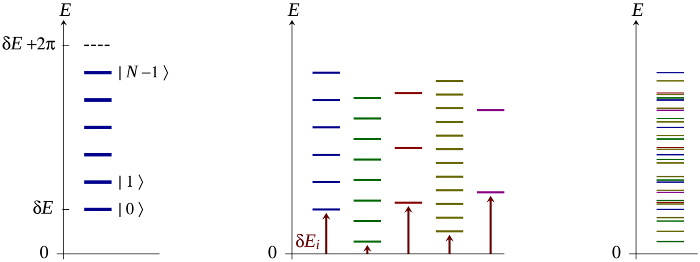
$$|\phi\rangle^{\text{ont}} \ =\ \frac{1}{\sqrt{2\pi}}\sum_{n=0}^{\infty} e^{-i\phi n/N}|n\rangle^E\ .$$

$$H = \omega\, n$$

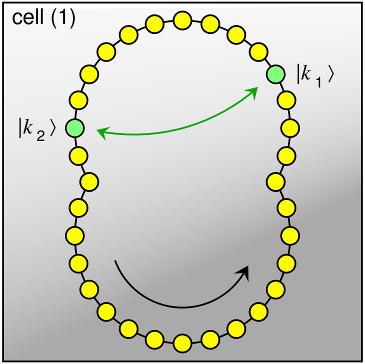




Finite,
deterministic,
time reversible
models



The ‘exchange interaction’



H contains $|k_1\rangle\langle k_2|$

Calculation:

$$H = p + \pi|\psi\rangle\langle\psi| \ ,$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|k_1\rangle^{(1)} - |k_2\rangle^{(1)}\right)$$

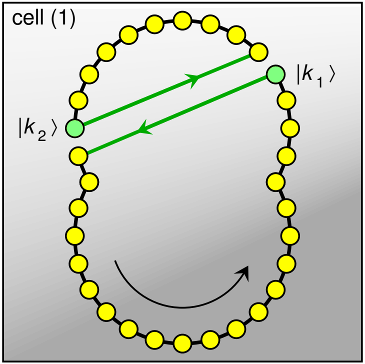
footnotes:

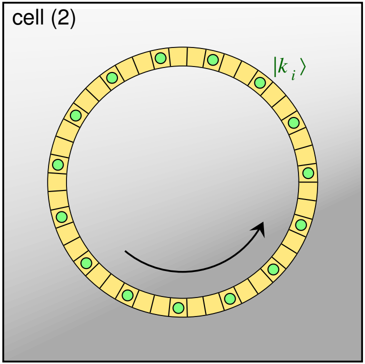
1. easier in continuum limit.
2. Uses $e^{\pi i} = -1$.

and 3. Separates states odd and even in $k_1 \leftrightarrow k_2$.

But this does not (yet) give quantum mechanics ...

Now let cell (2) act as a *sieve*





$$H = p^{(1)} + p^{(2)} + \pi|\psi\rangle^{(1)}\,\langle\psi|^{(1)}\,\left(\sum_i |k_i\rangle^{(2)}\,\langle k_i|^{(2)}\right)$$

We have $\quad H = p^{(1)} + p^{(2)} + \pi|\psi\rangle^{(1)}\,\langle\psi|^{(1)}\left(\sum_i |k_i\rangle^{(2)}\,\langle k_i|^{(2)}\right)$

$\sum_i |k_i\rangle^{(2)}\,\langle k_i|^{(2)}$ is a projection operator. Its vacuum expectation value is small; this operator only becomes sizeable if we include all very high energy states:

$$\left\langle\ \sum_i |k_i\rangle^{(2)}\,\langle k_i|^{(2)}\ \right\rangle_{\rm vac} = \alpha\ , \qquad |\alpha| \ll 1$$

So, at low energies, we get as effective interaction Hamiltonian:

$$H^{\rm int} = \varepsilon|\psi\rangle\,\langle\psi| \quad \text{with} \quad \varepsilon = \pi\alpha, \quad |\varepsilon| \ll 1\ .$$

And this is real quantum mechanics!

We can generate every term $H^{\rm int}_{ij}$ with $i \neq j$

And we can modify the diagonal terms on $H^{\rm int}$ as well.

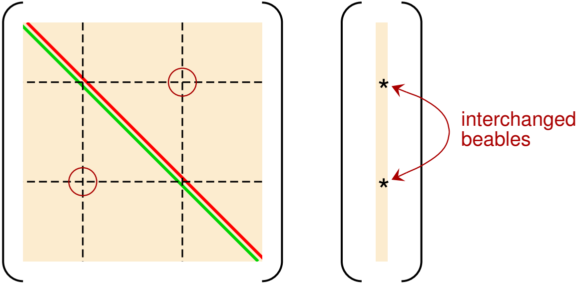
Just modify the velocities of the ontological variables $|k\,\rangle$ for each cell to depend on $\vec{k}$.

Only at the highest energies ($E \to \mathcal{O}(\delta t^{-1})$ we loose this freedom. The projection operators do not stay small.

A new kind of constraints for our 'unified theories'?
Quantum gravity?

Diagrammatic description of relation between off-diagonal components of Hamiltonian, and the exchange operators.
The Hamiltonian is built from ‘changeables’:

$H =$



The sieves must be sufficiently tight but not too tight.

Locality: $H = \sum_{\vec{x}} \mathcal{H}(\vec{x}\,)$

At fixed time t, when $\vec{x} \neq \vec{x}\,'$ (outside the light cone):

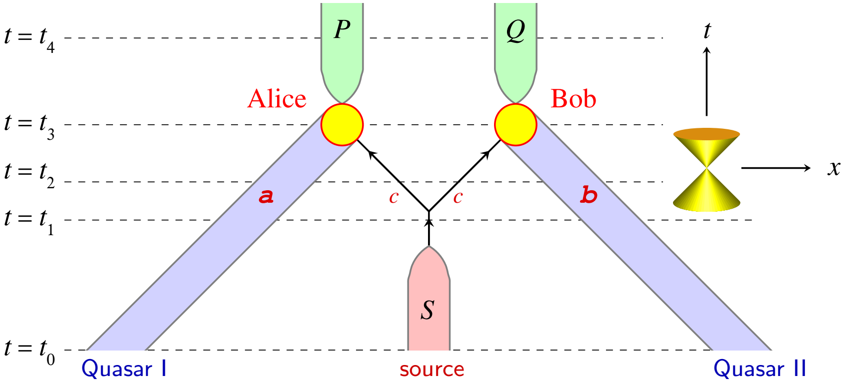
$$[\mathcal{H}(\vec{x}\,),\ \mathcal{H}(\vec{x}\,'\,)] = 0\ .$$

This means that the interchanges of beables must commute outside the light cone, which is easy to guarantee in classical models (classical locality)

Questions yet to be answered:

- Continuous symmetries. In particular
  Lorentz invariance (special relativity)
- Free particles (non-interacting quantum fields)
- More general, more efficient procedures
- General relativity

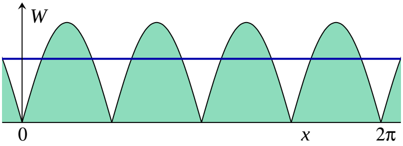
$$=\approx= \infty =\approx=$$



In the Bell experiment, at $t = t_0$, one must demand that those degrees of freedom that later force Alice and Bob to make their decisions, and the source that emits two entangled particles, have *3 - body correlations of the form*

$$W(a,\ b,\ c) \propto |\sin(2(a+b)-4c)| \qquad \text{(or worse)}$$

The 3 - body correlation:



Is this "conspiracy"?

The ontological nature of a physical state is *conserved in time*. If a photon is observed, at late times, to be in a given polarization state, it has been in *exactly the same state* the moment it was emitted by the source.

Non-observable hidden variables?

If a is fixed, then b and c are *not* statistically independent.

Strange, but true.

(example of *spacelike correlations*, common in QFT)

Physical laws should in principle not depend on statistical regularities

If you believe in determinism, you have to believe it all the way: