Pertubative analysis of the walking phase transition

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Introduction

- One of the possible solutions to the Higgs hierarchy problem occurs in the models with new strong dynamics (Higgs boson is a bound state of new interactions)
- The constraints from flavour physics require the scale of generation of the fermion mass operators to be high, the only way we can make the top mass heavy enough is by means of the walking dynamics.
- At high temperature (early universe) the system will be presumably in the deconfined phase

There will be confined /deconfined phase transition in the early universe. Very intriguing phenomena which can lead to interesting phenomenological features

Studies of the walking phase transitions

- Ideally since we are dealing with the strong dynamics we should rely on the lattice simulations (1807.08411)
- Another way to analyze this phase transition can be done in holographic models, results are obtained in the light dilaton limit hep-th/0107141(we have only one light field, and its vev acts as order parameter)

- **recently it was proposed that the walking dynamics occurs when the beta function of a system has has a complex fixed point, with the real part $\gg$ imaginary part** 0905.4752 Kaplan et al; 1807.11512 Gorbenko et al.
- Such scenario can be realized in perturbative toy gauge model with scalar and fermion fields 1908.04325 Benini et al.

Gauge theories near conformal window

- Consider a gauge theory with fermions, then at fixed number of colors there will be a conformal window

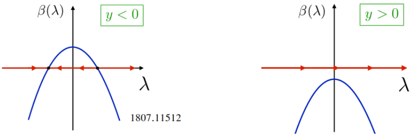
$$N_f^- < N_f < N_f^+$$

- near N_f^+ theory is weakly coupled and there is a perturbative fixed point, once N_f^+ is crossed we lose UV freedom.
- Near N_f^- theory is strongly coupled and below it theory loses conformality. **How we can describe the disappearance of the fixed point?** One possibility was conjectured in 0507171/hep-th Gies and Jaeckel;0905.4752 Kaplan et al;1807.11512 Gorbenko et al. that there is two fixed near fixed points merge and become complex .

Fixed points in the complex plane and walking

$$\frac{d\lambda}{d\ln\mu} = -y - \lambda^2$$

If y is negative we have two fixed points $\lambda = \pm\sqrt{-y}$, however once y becomes positive these fixed points become complex.



The scale separation for the $y > 0$ case becomes

$$\sim \log\left[\frac{\Lambda_{UV}}{\Lambda_{IR}}\right] \simeq \frac{\pi}{\sqrt{y}} \sim \frac{1}{\sqrt{\beta_{walking}}} \Rightarrow$$

exponential scale separation

Perturbative model of walking dynamics 1908.04325 Benini et al

The model is based on $SU(N_c) \times U(N_s) \times SU(N_f)^2 \times U(1)$ global symmetry, where both ϕ and ψ are fundamentals of $SU(N_c)$

$$\mathcal{L} = -\frac{1}{4} F^A_{\mu\nu} F^{\mu\nu}_A + i\mathrm{Tr}\bar{\psi}\not{D}\psi + \mathrm{Tr} D_\mu \phi^\dagger D^\mu \phi - \tilde{h}\mathrm{Tr}\phi^\dagger\phi\phi^\dagger\phi - \tilde{f}(\mathrm{Tr}\phi^\dagger\phi)^2$$

RGE evolution for the 't Hooft's couplings becomes

$$\lambda \equiv \frac{N_c g^2}{16\pi^2}, \qquad h \equiv \frac{N_c \tilde{h}}{16\pi^2}, \qquad f \equiv \frac{N_c N_s \tilde{f}}{16\pi^2}, \quad x_{s,f} = \frac{N_{s,f}}{N_c}$$

$$\beta_\lambda = -\frac{22-4x_s-x_f}{3}\lambda^2 + \lambda^3\left(\frac{2}{3}(4x_s + 13x_f - 34) - 2\frac{x_s+x_f}{N_c^2}\right),$$

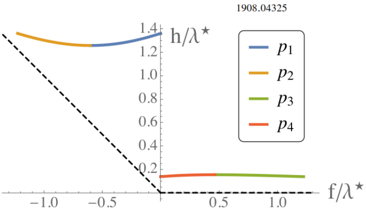
$$\beta_h = 4(1+x_s)h^2 + \frac{24}{N_c N_s} fh - \left(6 - \frac{6}{N_c^2}\right)\lambda h + \left(\frac{3}{4} - \frac{3}{N_c^2}\right)\lambda^2,$$

$$\beta_f = 4\left(1 + \frac{4}{N_s N_c}\right)f^2 + 8(1+x_s)fh - \left(6 - \frac{6}{N_c^2}\right)\lambda f + 12x_s h^2 + \frac{3x_s}{4}\left(1 + \frac{2}{N_c^2}\right)\lambda^2$$

we have four fixed points .

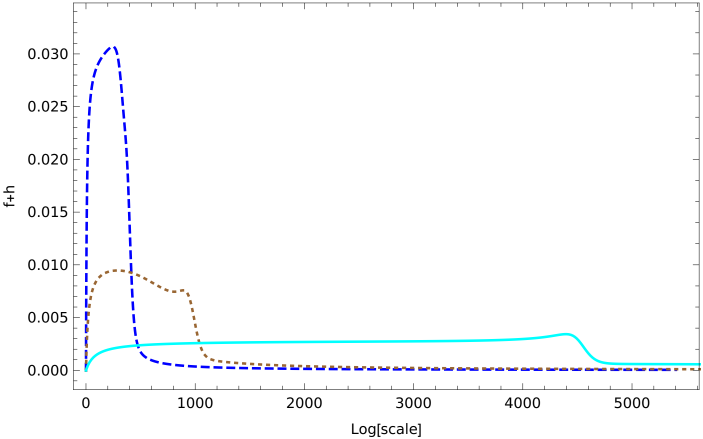
Fixed point merger

Choosing N_f, N_s, N_c we make the fixed point pertubative (Banks-Zaks)



The two real fixed point merger occurs when N_s/N_c becomes smaller than ~ 0.073

The model has exactly the fixed point merger phenomena and near it will have a walking behavior



$N_c = 25, N_s = 2, N_f = 120, 130, 135$

We study phase transition in this scenario which is under perturbative control ☺

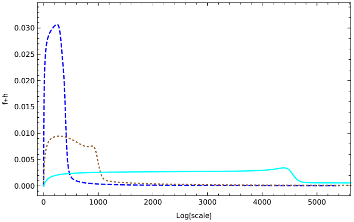
Of course the it is not clear how the results can be applied to more motivated models without scalars ☹

Symmetry breaking in perturbative walking dynamics

- The potential of the model is

$$V = \tilde{h}\mathrm{Tr}\phi^\dagger\phi\phi^\dagger\phi + \tilde{f}(\mathrm{Tr}\phi^\dagger\phi)^2$$

  once the coupling combination $\tilde{f}/N_s + \tilde{h}$ becomes negative the symmetry of the model will be broken $SU(N_c) \to SU(N_c - N_s)$ (analog of chiral symmetry breaking)
- At the instance when $\tilde{f}/N_s + \tilde{h} = 0$ there will be one mode "scalon" with zero mass at tree level, however it will get a mass at one loop level



$$V_{CW}(v) = g_i \frac{m_i^4(v)}{64\pi^2}\left[\log\left(\frac{m_i^2(v)}{\mu_R^2}\right) - c_i\right],$$

$$m^2_{\mathrm{scalon}} = \frac{16\pi^2}{N_c N_s}\beta^{\mathrm{sb}}_{f+h} v^2 = \beta^{\mathrm{sb}}_{\tilde{f}+\tilde{h}/N_s} v^2.$$

potential is controlled by β^{sb} function at the scale of symmetry breaking

Comparison with holographics results

The potential controlling the phase transition is just a CW potential and is very similar to the results found in holographics approach with dilaton hep-th/0107141,1104.479,1812.06996,1910.06238(scalon in our case)

$$V^{GW}_{\text{dilaton}} = \frac{N^2}{16\pi^2}\phi^4 \left[\lambda_0 + \lambda_0' g_{UV} \left(\frac{\phi}{\Lambda_{UV}}\right)^{\epsilon}\right] \qquad \epsilon \sim \beta_{walking},$$

	PWD	RS with light dilaton
scale separation $\Lambda_{UV}/\Lambda_{IR}$	$\sim O(1)^{\frac{1}{\sqrt{\beta_{walking}}}}$	$\sim O(1)^{\frac{1}{\beta_{walking}}}$
β function at symm. breaking	$\gg \beta_{walking}$	$\sim \beta_{walking}$

The potential at phase transition is not controlled by the $\beta_{\rm walking}$ but by $\beta_{\rm symmetry\ breaking}$ phase transition properties can be very different!

What will be phenomenological implications of the such phase transition?

The transition will be of the first order! It can lead to significant stochastic signal in gravitational waves

Perturbative model is far from being realistic, we have a huge number of fermions fields remaining massless during the phase transition

Pheno properties CW-like phase transitions

Phase transitions in the early universe lead to the stochastic GW signals, which can be detected by the current or future GW experiments

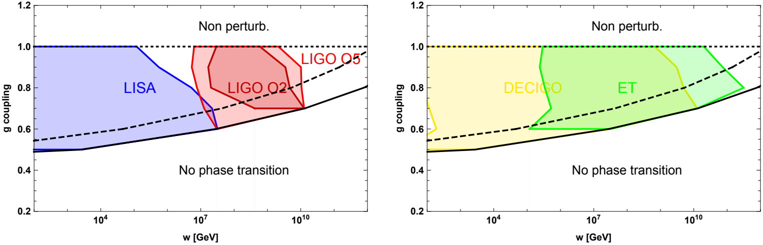


Figure: prospects of GW experiments in detecting simple U(1) model with Coleman-Weinberg radiative potential 1910.01124

If phase transition is of the first order we can detect new physics up to $\gtrsim 10^{11}$GeV scale, energies way out of reach for any collider experiment !

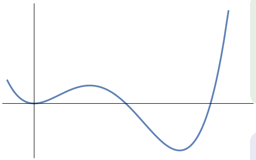
Phase transition in pertubative walking dynamics

In order to calculate the phase transition properties in the early universe we need to take into account thermal effects, we resum thermal masses.

$$V(v, T) = \sum_i V_{CW}(m_i^2 + \Pi_i) + V_T(m_i^2 + \Pi_i)$$

$\Pi \propto N_c g^2 T^2$ are hard thermal masses

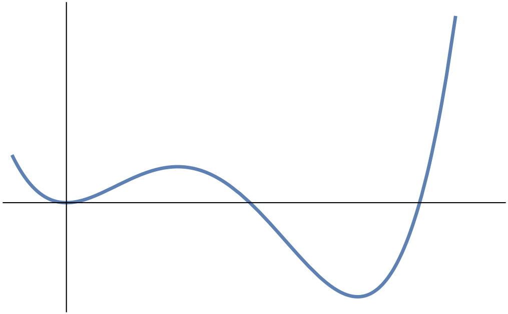
We need to know the properties of potential far away from the true minimum



The thermal loop expansion parameters becomes $\sim g\sqrt{N_c}$, **we are pushed towards smaller couplings**

Theory has a Landau pole in the deep IR, we need the couplings at the scale $\Lambda \sim g\sqrt{N}T_{\text{min pert}}$ to remain perturbative

Calculating the tunneling probability

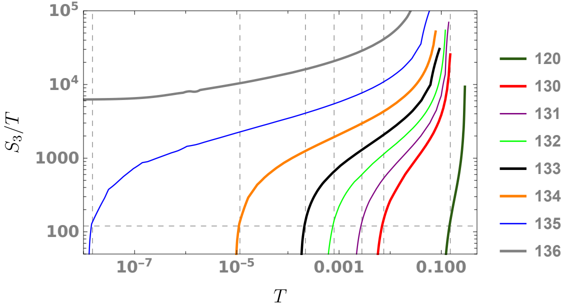


$$\Gamma(T) \sim \max\left[T^4\left(\frac{S_3}{2\pi T}\right)^{3/2}\mathrm{Exp}(-S_3/T),\, R_0^{-4}\left(\frac{S_4}{2\pi}\right)^2\mathrm{Exp}(-S_4)\right]$$

The model contains more than one scalar field, however all of them except the scalon direction will have a tree level potential so that the tunneling will be one dimensional

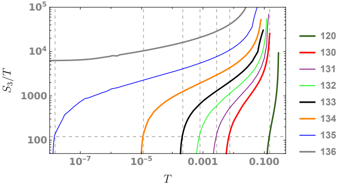
Numerical results

- We have chosen minimal number of fields $N_c = 25, N_s = 2$ for which the complex perturbative fixed point appears.
- Varying the number of fermions N_f we can get very different couping values at the confinement scale



we can get very different behaviours of the tunnelling action

Phase transition properties



phase transitionTroughly occurs when $\Gamma \gtrsim H^4 \Rightarrow S_3/T \sim 120$

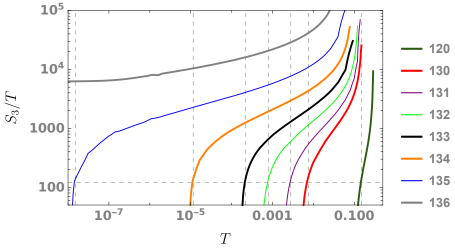
The GW signal properties mainly depend on the two parameters:
$\beta^{-1} = \frac{d}{dT}\left(\frac{S3}{T}\right) \sim R_{bubble}$ is timescale of the phase transition and
$\alpha = \frac{\Delta V}{\rho_{rad}}$ parametrizes the latent heat released during the phase transition

$$\boxed{\Omega_{GW}^{scalar\ \ field} \propto \left(\frac{H}{\beta}\right)^2 \left(\frac{\alpha}{1+\alpha}\right)^2}$$

The faster (larger couplings) is the phase transition, the weaker will be the signal.

Light dilaton limit

- Let us consider $N_f = 136$ case, we can see that the tunneling action is too large for transition to occur
- We are in the limit when the walking is occurring for small values of coupling (theory is close to loosing UV freedom)
- Small couplings at walking $\Rightarrow$ we have small couplings at symmetry breaking scale and scalon/dilaton is light. Situation is exactly the same as in holographic models hep-th/0107141, where additional mechanism was suggested to trigger the phase transition 1711.11554,1812.06996.



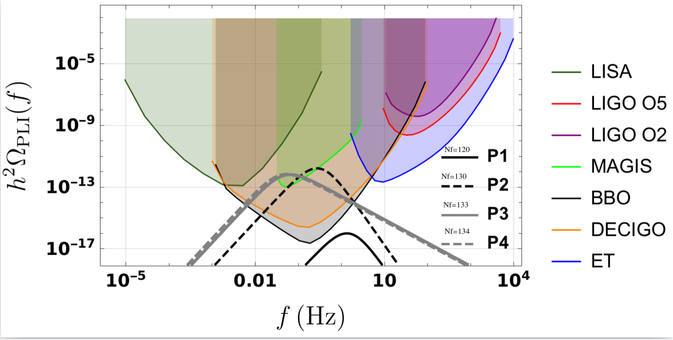
Calculating the GW signal

We have considered $N_f = 120, 130, 133, 134,\ N_s = 2, N_c = 25$

- **bubble wall (scalar field), sound waves,turbulence**(recommended to be ignored 1910.131125) three sources of GW signal
- The bubble expansion is always relativistic $\Delta V \gtrsim P_{LO} \propto T^2 \Delta m^2$
- The energy distribution **(scalar field), sound waves** contributions will depend strongly on whether the bubble has reached the terminal velocity. For the constant velocity expansion

$$\frac{\text{Energy}^{\text{bubble shell}}}{\text{Energy}^{\text{total}}} \propto \frac{R^2}{R^3} \to 0$$

- NLO pressure 1703.08215 can become important in preventing the accelerated expansion .



For the points **P1,P2** the signal is dominated by the sound wave component (the bubbles have reached constant velocity expansion) and for **P4,P5** the signal is bubble wall dominated (bubbles are accelerating at the instance of collision)

Summary

- We have studied the phase transitions in the perturbative models of the walking dynamics
- Generically we can get very different scenarios for the phenomenology depending on the exit from walking dynamic conditions
- The model remains perturbative only in the region where there is a signal
- However there is a clear tendency of reduction/vanishing of the signal once larger couplings are considered

Of course this is just a toy model, however it seems that some feature will remain in realistic models as well.