The Invariant Set Model of Quantum Physics

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Discretization of the Bloch sphere, fractal invariant sets and Bell's theorem

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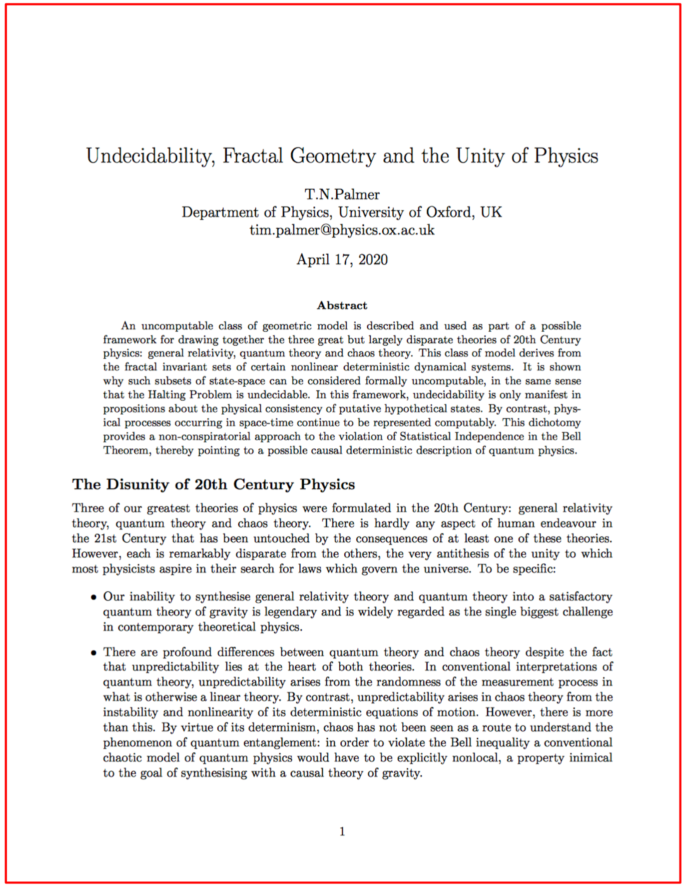
An arbitrarily dense discretization of the Bloch sphere of complex Hilbert states is constructed, where points correspond to bit strings of fixed finite length. Number-theoretic properties of trigonometric functions (not part of the quantum-theoretic canon) are used to show that this constructive discretized representation incorporates many of the defining characteristics of quantum systems: completementarity, uncertainty relationships and (with a simple Cartesian product of discretized spheres) entanglement. Unlike Meyer's earlier discretization of the Bloch Sphere, there are no orthonormal triples, hence the Kocken–Specker theorem is not nullified. A physical interpretation of points on the discretized Bloch sphere is given in terms of ensembles of trajectories on a dynamically invariant fractal set in state space, where states of physical reality correspond to points on the invariant set. This deterministic construction provides a new way to understand the violation of the Bell inequality without violating statistical independence or factorization, where these conditions are defined solely from states on the invariant set. In this finite representation, there is an upper limit to the number of qubits that can be entangled, a property with potential experimental consequences.

1. Introduction

The fields $\mathbb{R}$ and $\mathbb{C}$ are deeply embedded in the formalism of both classical and quantum theories of physics. However, the status of these continuum fields is fundamentally different in the two classes of theory.



Undecidability, Fractal Geometry and the Unity of Physics

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April 17, 2020

Abstract

An uncomputable class of geometric model is described and used as part of a possible framework for drawing together the three great but largely disparate theories of 20th Century physics: general relativity, quantum theory and chaos theory. This class of model derives from the fractal invariant sets of certain nonlinear deterministic dynamical systems. It is shown why such subsets of state-space can be considered formally uncomputable, in the same sense that the Halting Problem is undecidable. In this framework, undecidability is only manifest in propositions about the physical consistency of putative hypothetical states. By contrast, physical processes occurring in space-time continue to be represented computably. This dichotomy provides a non-conspiratorial approach to the violation of Statistical Independence in the Bell Theorem, thereby pointing to a possible causal deterministic description of quantum physics.

The Disunity of 20th Century Physics

Three of our greatest theories of physics were formulated in the 20th Century: general relativity theory, quantum theory and chaos theory. There is hardly any aspect of human endeavour in the 21st Century that has been untouched by the consequences of at least one of these theories. However, each is remarkably disparate from the others, the very antithesis of the unity to which most physicists aspire in their search for laws which govern the universe. To be specific:

- Our inability to synthesise general relativity theory and quantum theory into a satisfactory quantum theory of gravity is legendary and is widely regarded as the single biggest challenge in contemporary theoretical physics.
- There are profound differences between quantum theory and chaos theory despite the fact that unpredictability lies at the heart of both theories. In conventional interpretations of quantum theory, unpredictability arises from the randomness of the measurement process in what is otherwise a linear theory. By contrast, unpredictability arises in chaos theory from the instability and nonlinearity of its deterministic equations of motion. However, there is more than this. By virtue of its determinism, chaos has not been seen as a route to understand the phenomenon of quantum entanglement: in order to violate the Bell inequality a conventional chaotic model of quantum physics would have to be explicitly nonlocal, a property inimical to the goal of synthesising with a causal theory of gravity.

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https://fqxi.org/data/essay-contest-files/Palmer_FXQi_Palmer_1.pdf

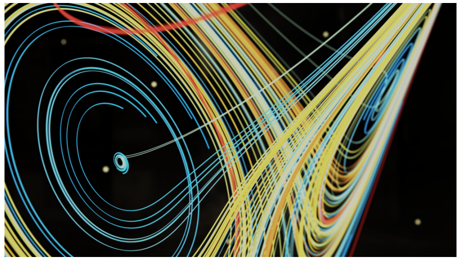
There is no compelling reason to believe that quantum mechanics is a fundamental theory of physics

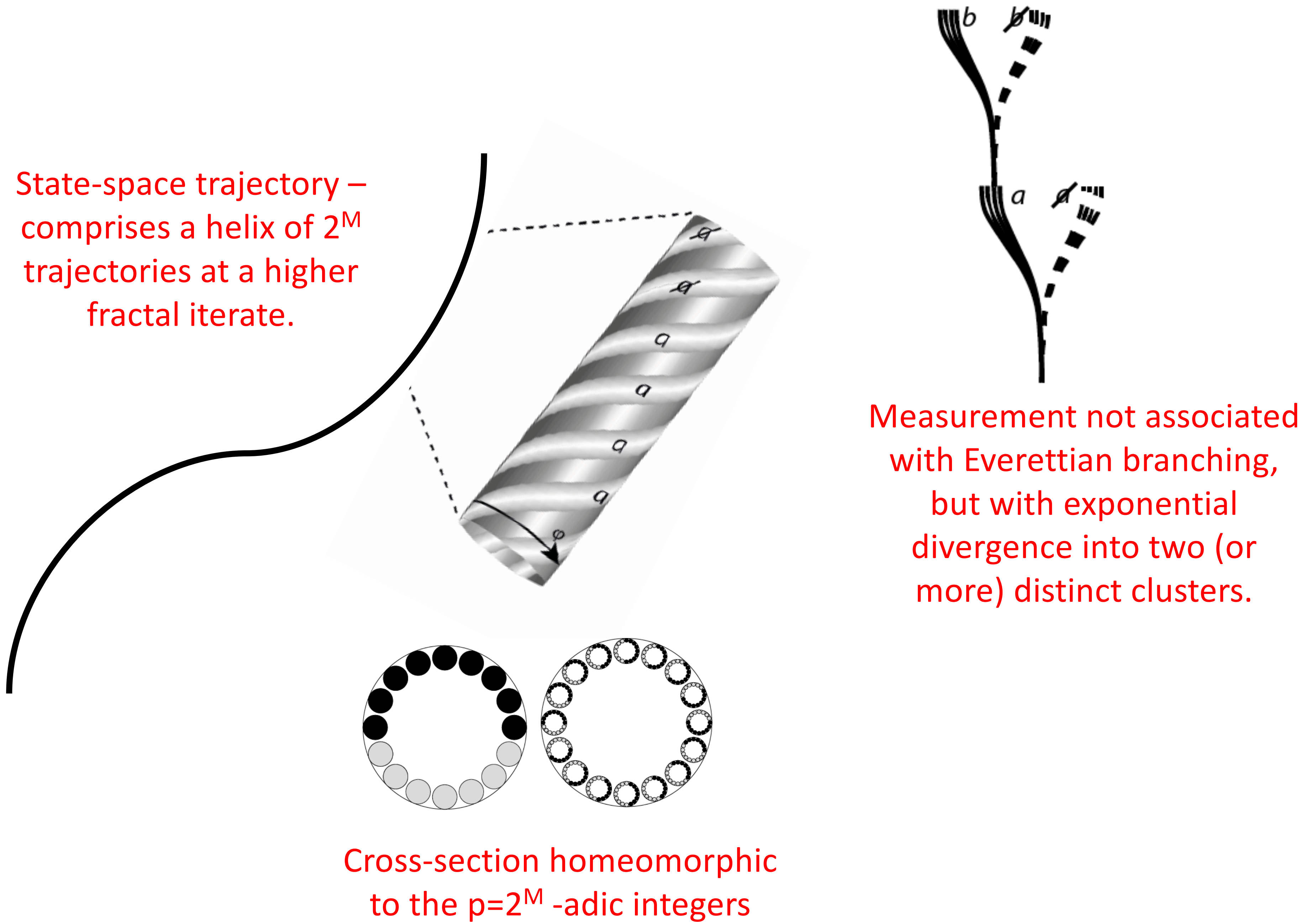
- The close formal similarity of the Schrödinger and Liouville equations suggests that linearity and indeterminism are not fundamental features of quantum physics.
- The continuum plays a more vital role in quantum mechanics (c.f. Hardy's Continuity Axiom) than it does in classical theory. If we seek a finite theory of quantum physics, it will not approximate quantum theory.
- The property of nonlocality makes unification with GR deeply problematic.
- Here I want to show that a finite theory of quantum physics can evade the conclusion that physics is not locally causal.

The Invariant Set Postulate

The universe is a deterministic dynamical system evolving precisely on a fractal invariant set in state space.

The laws of physics at their most primitive derive from the geometry of the invariant set.




State-space trajectory – comprises a helix of 2^M trajectories at a higher fractal iterate.
Measurement not associated with Everettian branching, but with exponential divergence into two (or more) distinct clusters.
Cross-section homeomorphic to the $p=2^M$ -adic integers
b
a

Complex Hilbert vectors.

The symbolically labelled helix of trajectories can be described probabilistically by Hilbert Vectors of the form

$$|\psi\rangle = \cos\frac{\theta}{2}|a\rangle + e^{i\phi}\sin\frac{\theta}{2}|\not a\rangle$$

where $\cos^2\frac{\theta}{2} = \frac{n_1}{2^M} \in \mathbb{Q} \Rightarrow \cos\theta \in \mathbb{Q}$

and $\frac{\phi}{2\pi} = \frac{n_2}{2^M} \in \mathbb{Q}$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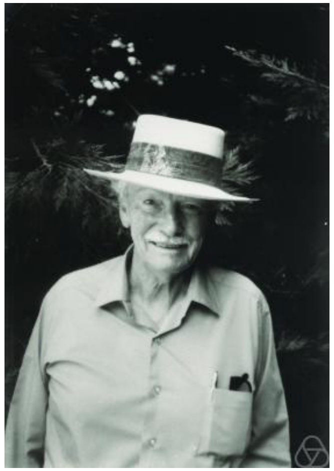
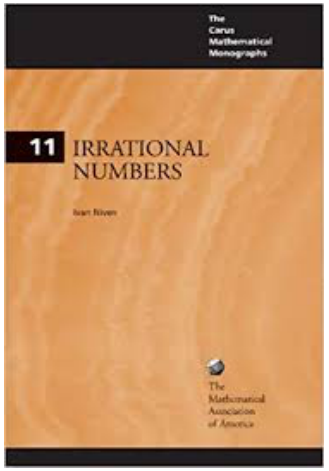
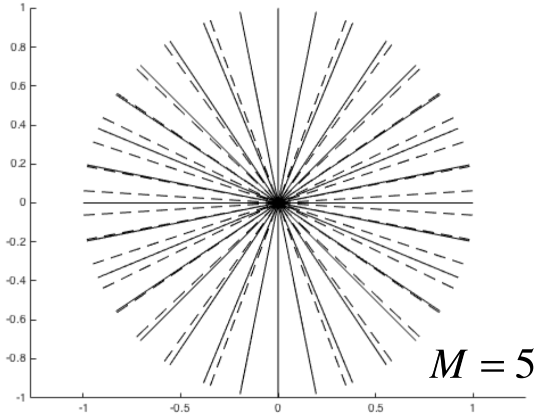
Niven's Theorem





Let $0 < \phi < \dfrac{\pi}{2}$ and $\dfrac{\phi}{2\pi} = \dfrac{n_1}{2^M}$, then $\cos\phi \notin \mathbb{Q}$



Example

$$|\psi_1\rangle = \cos\frac{\phi}{2}|a\rangle + \sin\frac{\phi}{2}|\not{a}\rangle$$

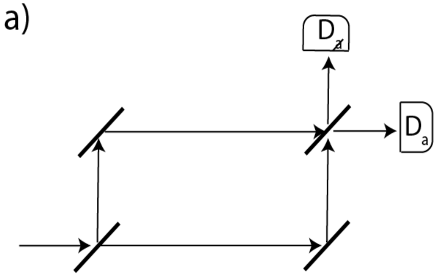
$$\updownarrow U_H$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|a\rangle + e^{i\phi}|\not{a}\rangle)$$

In quantum theory, $|\psi_1\rangle$ and $|\psi_2\rangle$ are both well defined states on the Bloch sphere.
In the proposed discretisation, by Niven's Theorem if $|\psi_1\rangle$ is defined, then $|\psi_2\rangle$ is not and vice versa.
Doesn't matter how big M is, i.e. no convergence to the quantum theoretic continuum limit as $M \to \infty$. I.e. $M = \infty$ is a singular limit.

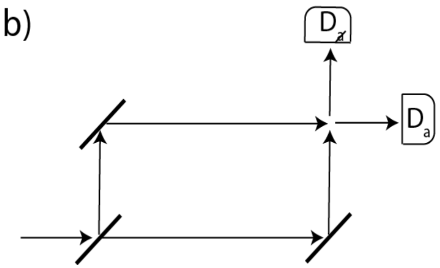
What does this mean in practice?

a)



$$|\psi_1\rangle = \cos\frac{\phi}{2}|a\rangle + \sin\frac{\phi}{2}|\not a\rangle$$

b)



$$|\psi_2\rangle = \frac{1}{\sqrt{2}}(|a\rangle + e^{i\phi}|\not a\rangle)$$

Perform experiment a) on an ensemble of particles on Monday $\Rightarrow \cos\phi \in \mathbb{Q}$

Perform experiment b) on an ensemble of particles on Tuesday $\Rightarrow \phi / 2\pi \in \mathbb{Q}$

Ask what I would have observed had I performed Tuesday's experiment on Monday's ensemble.

The counterfactual outcome is undefined!

I.e. cannot simultaneously perform an interferometric and a which-way measurement.

Or, cannot simultaneously perform a position and momentum measurement.

A number-theoretic description of Bohr's principle of complementarity?

Applying Similar Number-Theoretic Arguments to Bell's Theorem

There is no algorithm for deciding which states lie on the invariant set.

Hence no algorithm for predicting which experimental set up Alice and Bob will choose.

$$\rho(\lambda|00)=\rho_0\neq 0$$

$$\rho(\lambda|11)=\rho_0\neq 0$$

$$\rho(\lambda|01)=0$$

$$\rho(\lambda|10)=0$$

Counterfactual Experiments.

The bottom two do not lie on the invariant set.

They correspond to states which are p-adically distant from the invariant set.

Being based on (rational) Hilbert vectors (and tensor products), Invariant Set Model violates Bell inequality exactly as does quantum theory.

The (superdeterministic) violation of statistical independence implies a novel locally causal interpretation of Bell's Theorem.

“My own view is that, to understand quantum non-locality, we shall require a **radically new theory**. This new theory will not just be a slight modification of quantum mechanics but something as different from standard quantum mechanics as General Relativity is from Newtonian Gravity. It would have to be something which has a completely different conceptual framework.”

Roger Penrose. The Large, the Small and the Human Mind, 1997

“One can always hope that there will be future developments which will lead to **a drastically different theory** from the present quantum mechanical theory and for which there may be a partial return of determinism.”

P.A.M. Dirac, The Development of Quantum Mechanics, Conferenza Tenuta il, 14 Aprile 1972, Roma [Conference held on 14 April 1972, Rome], Accademia Nazionale dei Lincei, 1974.