New 't Hooft anomalies and the phases of gauge theories

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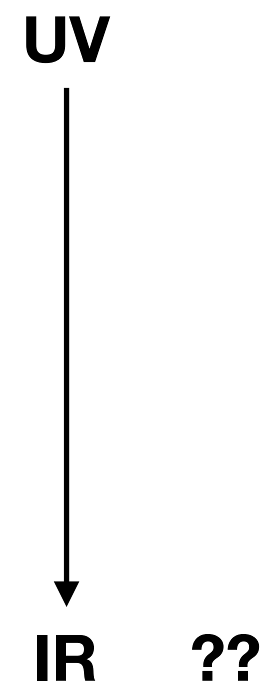
goal: informal intro & ***few*** ***(or one)*** examples from my work with

Mohamed Anber
(1805.12290, 1807.00093, 1811.10642, 1909.09027, 2001.03631, 2002.02037)
and
Thomas Ryttov
(1904.11640)



anomaly matching

limits fantasies about IR!

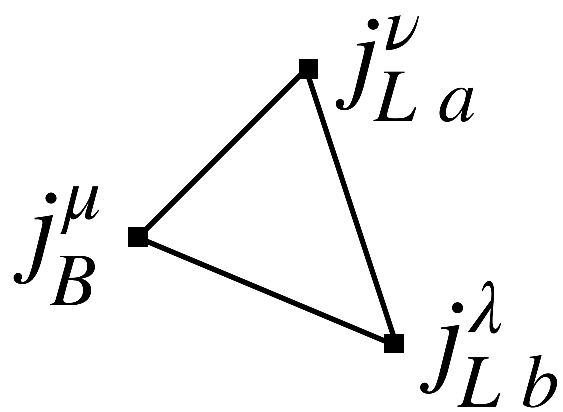
reminder on ’t Hooft anomalies:

SU(3) QCD with 2 massless flavors of fundamental quarks

exact global symmetry $SU(2)_L \times SU(2)_R \times U(1)_B$

Ex.: $U(1)_B \, SU(2)_L^2$

UV:



quarks, $Q_B = \frac{1}{3}$

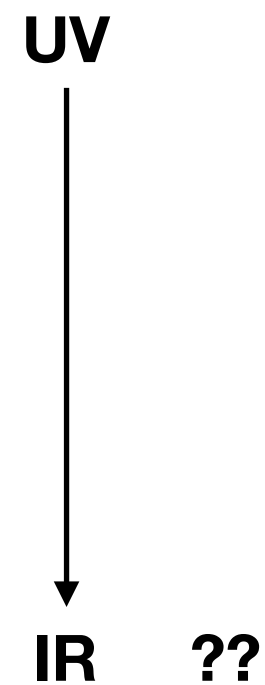
anomaly RG invariant

IR:

→ single massless (p, n) $SU(2)_L$ doublet, $Q_B = 1$

→ massless Goldstones (π^+, π^-, π^0)

IR physics “nontrivial”



thought anomaly matching was set in stone since ca. 1980
"0-form", or "traditional", anomalies played major role in, say, "preon" models (1980's), Seiberg dualities (1990's)

new "generalized 't Hooft anomaly matching"

Gaiotto, Kapustin, Komargodski, Seiberg, Willett … 2014-

“generalized ’t Hooft anomaly matching”

“anomalies of exact global (discrete) symmetries revealed upon turning on background gauge fields for global symmetries, **compatible with their faithful action** (interpret some as “gauging higher-form symmetry”)”

currently active area of research, across fields

“generalized ’t Hooft anomaly matching”

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currently active area of research, across fields

condensed matter, mathematical physics, **high-energy theory**

- classification
- general theorems
- **examples and dynamical implications in QFT**

impossible to review all!

"learn by example": here, vectorlike theories,
see Konishi's talk for chiral

SU(N) gauge theory:
N_f Dirac flavors $\Psi = (\psi, \tilde{\psi}) \sim (R, R^*)$ of N-ality n_c

($n_c = 1$ fundamental; $n_c = 2$ two-index S/AS; …; $n_c = N$ adjoint)

global symmetry:

$$\underbrace{SU(N_f)_L \times SU(N_f)_R \times U(1)_B}_{\text{usual stuff}} \times \underbrace{\mathbb{Z}^{(0)}_{2N_f T_R}}_{\text{anomaly free part of axial U(1)}}$$

$U(1)_A : \psi \to e^{i\alpha}\psi, \tilde{\psi} \to e^{i\alpha}\tilde{\psi}$

so $m \ \mathrm{tr}\tilde{\psi} \cdot \psi$ violates $U(1)_A$

$$\mathscr{D}\Psi \to \mathscr{D}\Psi \ e^{i\alpha 2N_f T_R Q_{top}}$$

$$\alpha = \frac{2\pi}{2N_f T_R}, \text{ so } U(1) \to \mathbb{Z}^{(0)}_{2N_f T_R}$$

not in QCD: $T_F = 1$, $\mathbb{Z}^{(0)}_{2N_f}$ part of $U(1)_B$ and centers of $SU(N_f)_{L,R}$

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usual stuff

anomaly free part of axial U(1)

$$U(1)_A : \psi \to e^{i\alpha}\psi, \tilde{\psi} \to e^{i\alpha}\tilde{\psi} \qquad \mathscr{D}\Psi \to \mathscr{D}\Psi \; e^{i\alpha 2N_fT_RQ_{top}}$$

so, under anomaly free $\mathbb{Z}^{(0)}_{2N_fT_R}$: $\mathscr{D}\Psi \to \mathscr{D}\Psi \; e^{i2\pi Q_{top}}$

phase = 1, $Q_{top} \in \mathbb{Z}$

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global symmetry:

"1-form" symmetry

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times \mathbb{Z}^{(0)}_{2N_f T_R} \times \mathbb{Z}^{(1)}_{p=\gcd(N,n_c)}$$

↓

Ex.: $p = \gcd(N, n_c)$ **adj:** $p = N, \ \mathbb{Z}^{(1)}_N$

fundamental (F) quark probes can not be screened in **adjoint** theory; $\mathbb{Z}^{(1)}_N$ means that N F-quarks can be screened in adjoint theory:

- fundamental strings unbreakable
- their number conserved mod(N)

SU(N) gauge theory:

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Ex.: $p = \gcd(N, n_c)$

AS/S N**-even:** $p = 2, \ \mathbb{Z}_2^{(1)}$

fundamental (F) quark probes can not be screened in **AS/S** theory; $\mathbb{Z}_2^{(1)}$ means that 2 F-quarks can be screened in AS/S 2-index, even-N theory:

- fundamental strings unbreakable
- their number conserved mod(2)

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AS/S N**-even:** $p = 2, \ \mathbb{Z}^{(1)}_2$

"1-form" global symmetry acts on topologically nontrivial line operators (Wilson loops winding around, say, the torus) - classic probe of deconfinement, for example…

global symmetry:

$$\frac{SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times \mathbb{Z}^{(0)}_{2N_f T_R}}{\mathbb{Z}_{\frac{N}{p}} \times \mathbb{Z}_{N_f}} \times \mathbb{Z}^{(1)}_{p=\gcd(N,n_c)}$$

↑

discrete identifications (eliminate redundancies) important…

"anomalies of exact global (discrete) symmetries revealed upon turning on background gauge fields for global symmetries, **compatible with their faithful action** (interpret some as "gauging higher-form symmetry")"

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"New 't Hooft anomalies" (example of):

idea goes like…

- put the theory on some (large $\gg \Lambda^{-1}$) manifold, say $\mathbb{T}^4$ (or $\mathbb{CP}^2$)
- turn on **general** global symmetry backgrounds on $\mathbb{T}^4$ (or $\mathbb{CP}^2$)
- these lead to an anomaly in discrete symmetry if $Q = Q_{bckgd} \neq \mathbb{Z}$

$$\mathbb{Z}^{(0)}_{2N_f T_R}: \quad \mathscr{D}\Psi \to \mathscr{D}\Psi\, e^{i2\pi Q}$$

- this phase $e^{i2\pi Q}$ **is the anomaly** - RG invt, to be reproduced at any scale (and at any volume, incl. $V \to \infty$): IR can not be "trivially gapped", i.e. have unique vacuum with a mass gap

two points remain to illustrate during rest of talk:

1. what are these backgrounds with $Q \neq 0$

- ’*t Hooft fluxes* and their generalizations, on $\mathbb{T}^4, \mathbb{CP}^2 \ldots$

2. what constraints do new anomalies place?

- usually require $\mathbb{Z}^{(0)}_{2N_f T_R}$ be (partially) broken (or CFT)
- constrain the physics of domain walls
- limit/forbid scenarios where massless composites saturate all the usual ‘traditional’ ’t Hooft anomalies
- constrain finite-T phases (eg ordering of phase transitions, interfaces…)

DISCLAIMER:

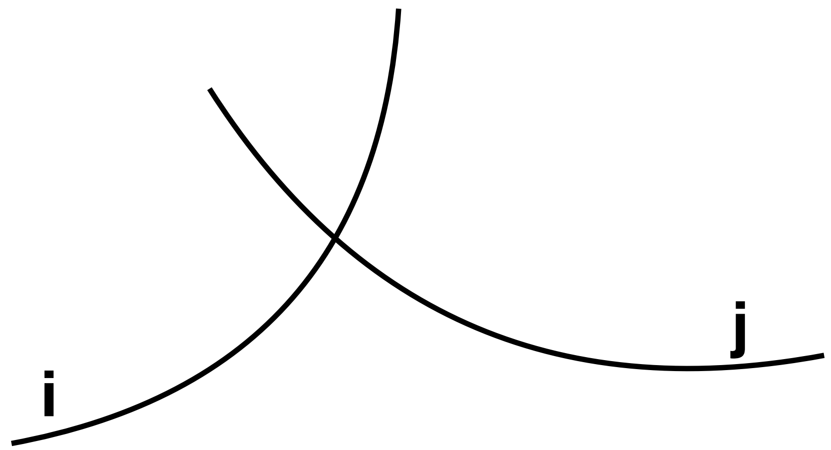
- “generalized” anomalies do not tell us which consistent IR scenario is realized

- I think, we do not yet know what is the complete set of consistency requirements

1. what are these backgrounds that have $Q \neq 0$

- '*t Hooft fluxes* and their generalizations, on $\mathbb{T}^4, \mathbb{CP}^2 \ldots$

$$A_i^c = (\Omega_{ij}^c)^{-1}(A_j^c + i\mathrm{d})\Omega_{ij}^c$$



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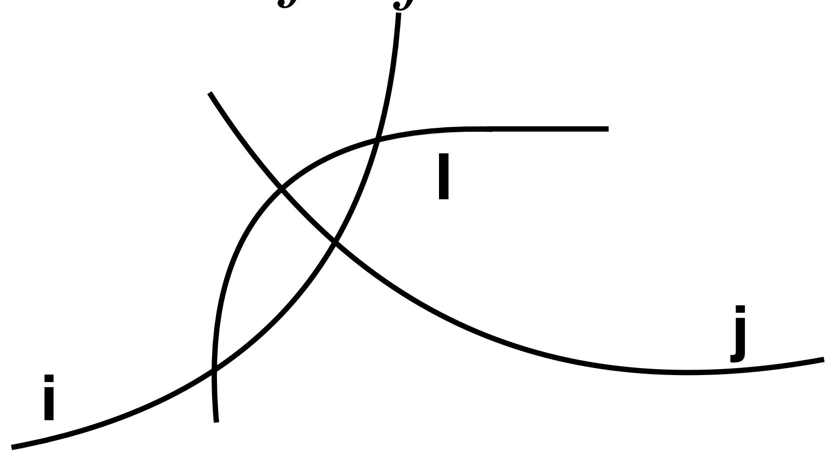

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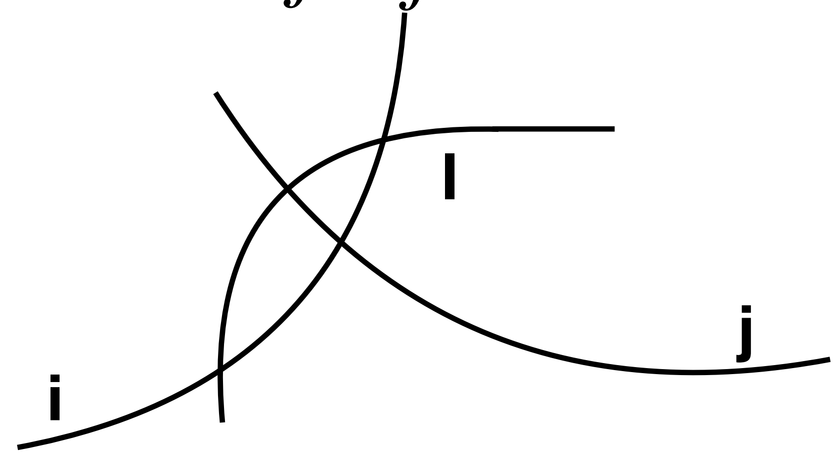

global 1-form center

$$\mathbb{Z}_N^{(1)} : \Omega_{ij}^c \to e^{i\frac{2\pi}{N}m_{ij}}\, \Omega_{ij}^c$$

$$m_{ij} + m_{jl} + m_{li} = 0 \mod N$$

action nontrivial on winding
(around the "world")
Wilson loops only

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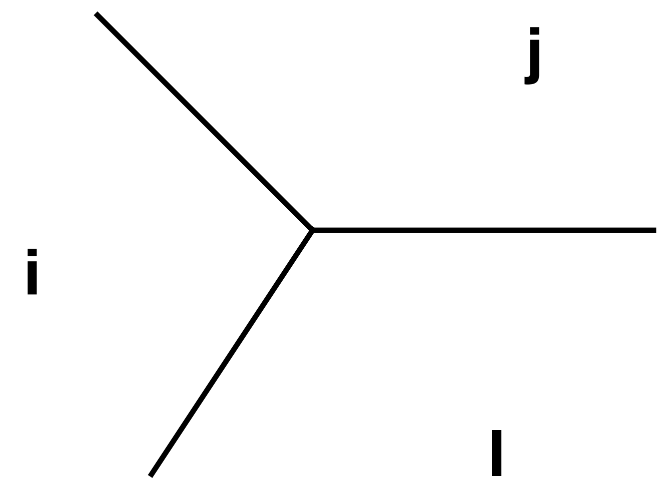

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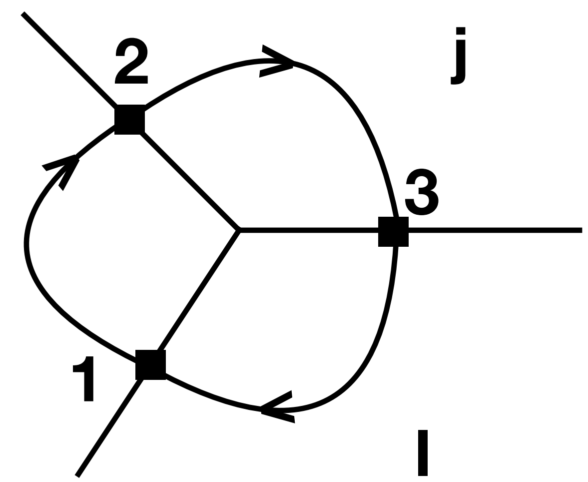
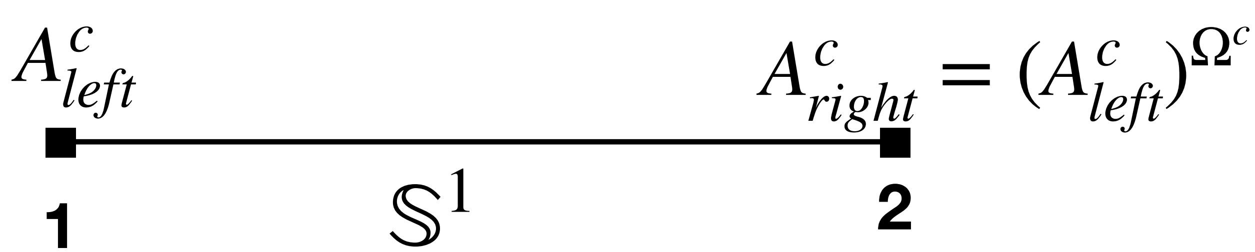
contractible loop
$\mathbb{Z}_N^{(1)}$ **invariant**



$$\mathrm{Tr}[\ e^{i\int_1^2 A_i}\, \Omega_{ij}\, e^{i\int_2^3 A_j}\, \Omega_{jl}\, e^{i\int_3^1 A_l}\, \Omega_{li}\ ]$$

noncontractible loop
$\mathbb{Z}_N^{(1)}$: $\times\, e^{i\frac{2\pi}{N}}$



$$\mathrm{Tr}[\ e^{i\int_1^2 A_i}\, \Omega]$$

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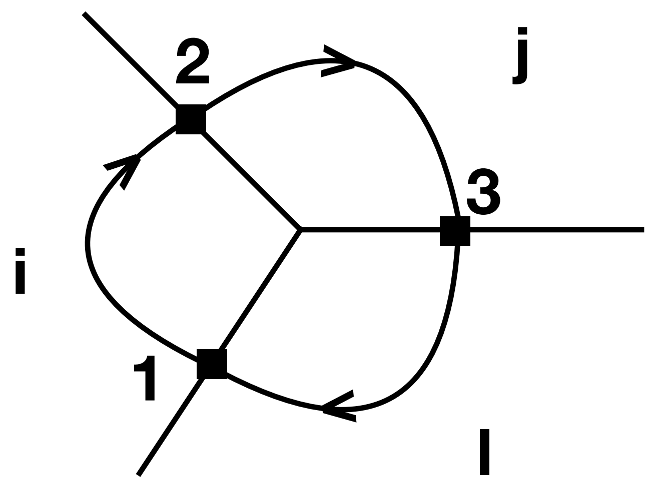

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introducing a $\mathbb{Z}_N^{(1)}$ background: relax cocycle condition

$$\mathrm{Tr}[\; e^{i\int_1^2 A_i} \, \Omega_{ij} \, e^{i\int_2^3 A_j} \, \Omega_{jl} \, e^{i\int_3^1 A_l} \, \Omega_{li} \;]$$

(**formalism of 2-form $\mathbb{Z}_N$ gauge field** **continuum,** lattice, triangulation $\quad NB^{(2)} = dB^{(1)}; \oint B^{(1)} = 2\pi\mathbb{Z}; \oint B^{(2)} = \frac{2\pi\mathbb{Z}}{N} \ldots$)

less abstract: '*t Hooft fluxes* as examples of $\mathbb{Z}_N^{(1)}$ backgrounds

1. what are these backgrounds that have $Q \neq 0$

- '*t Hooft fluxes* and their generalizations, on $\mathbb{T}^4, \mathbb{CP}^2 \ldots$

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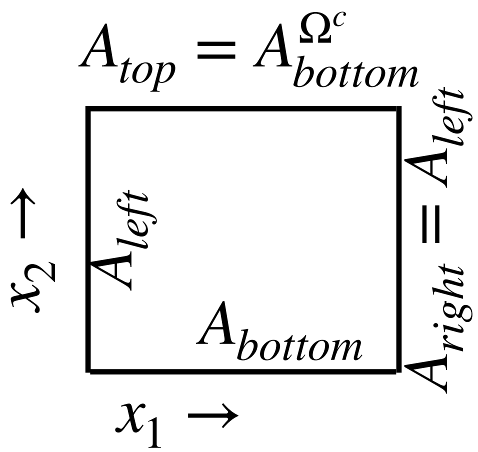

$$A_1(x_2) = \frac{2\pi x_2}{L^2}\mathrm{diag}(\frac{1}{N}, \ldots, \frac{1}{N}, -1 + \frac{1}{N})$$

unit 't Hooft flux in $x_1 x_2$: gauging $\mathbb{Z}_N^{(1)}$; $F_{12} = const$.

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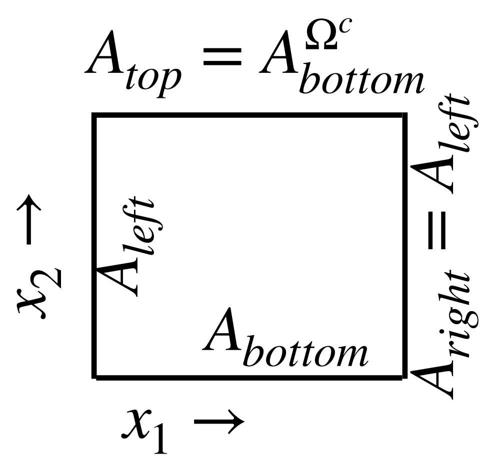

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$$\Omega^c(L) = e^{i\frac{2\pi}{N}}\Omega^c(0)$$

periodicity (=cocycle) only up to center, not allowed in SU(N) theory

unit 't Hooft flux in $x_1 x_2$: gauging $\mathbb{Z}_N^{(1)}$; $F_{12} = const$.

$\updownarrow$

$$\left(\oint B^{(2)} = \frac{2\pi\mathbb{Z}}{N} \right)$$

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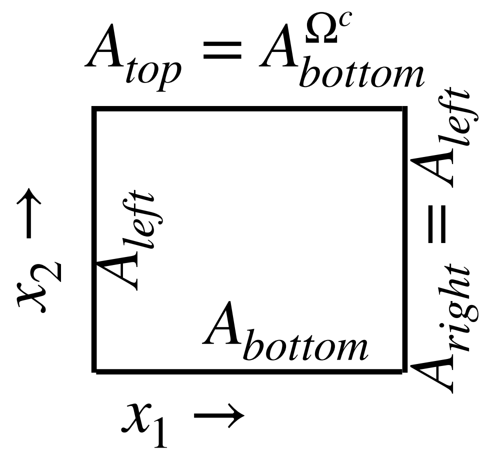

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unit 't Hooft flux in $x_1 x_2$:
gauging $\mathbb{Z}_N^{(1)}$; $F_{12} = const$.
add same in $x_3 x_4$, compute

$Q_{top}^c = 1 - \frac{1}{N}$ **- anomaly!!**

mixed $\mathbb{Z}_{2N_f N}^{(0)}$-$\mathbb{Z}_N^{(1)}$chiral/center anomaly in QCD(adjoint)

Both UV and candidate IR theories can be put on $\mathbb{T}^4$ in same global symmetry background - and anomaly of $\mathbb{Z}_{2N_f T_R}^{(0)}$ should be the same!

I. what are these backgrounds that have $Q \neq 0$

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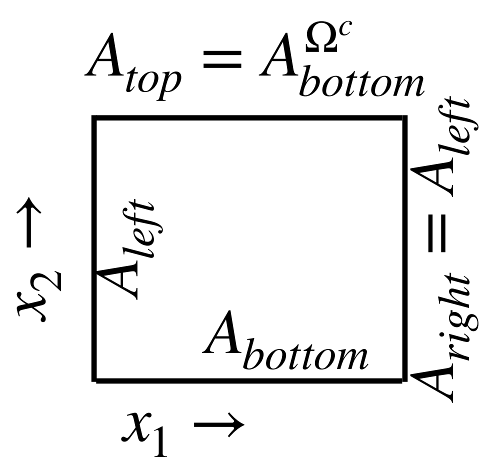

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$$\Omega^c(L) = e^{i\frac{2\pi}{N}}\Omega^c(0)$$

not good for $\psi, \tilde{\psi}$
of $n_c < N$, not single valued
but add similar fluxes
for F, B to compensate!

$$\psi_i = (\Omega_{ij}^c)^{n_c}\, \Omega_{ij}^F\, \Omega_{ij}^B\, \psi_j$$

$$\prod (\Omega^c)^{n_c}\, \Omega^F\, \Omega^B = 1$$

tripple overlaps

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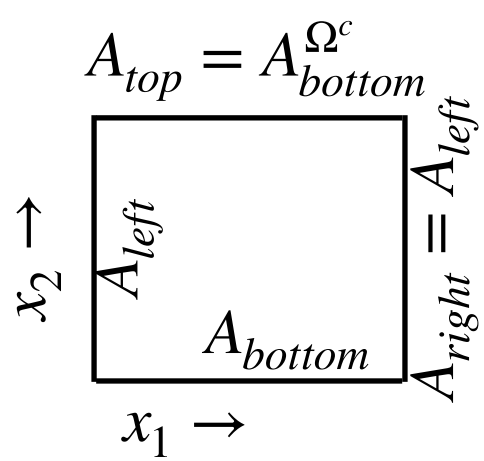

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$$\prod_{\text{tripple overlaps}} (\Omega^c)^{n_c}\, \Omega^F\, \Omega^B = 1$$

$$B_1(x_2) = \frac{2\pi x_2}{L^2}\left(-\frac{n_c}{N}\right)$$

$$B_1(L) = B_1(0) + i\Omega^{B\dagger}\partial_1\Omega^B$$

$$\Omega^B(x_1) = e^{i\frac{2\pi}{L}x^1(-\frac{n_c}{N})}$$

$$\Omega^B(L) = e^{-i\frac{2\pi n_c}{N}}\Omega^B(0)$$

$(\Omega^c)^{n_c}\Omega^B$ - periodic, single valued Ψ

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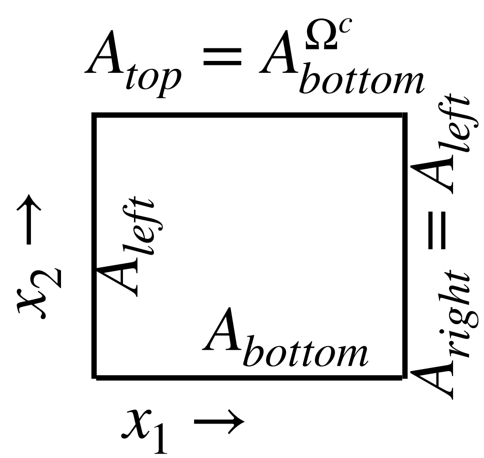

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but add similar fluxes
for F, B to compensate!

$$\psi_i = (\Omega_{ij}^c)^{n_c}\, \Omega_{ij}^F\, \Omega_{ij}^B\, \psi_j$$

$$\prod_{\text{tripple overlaps}} (\Omega^c)^{n_c}\, \Omega^F\, \Omega^B = 1$$

get non-integer topological charges for F,B,C - and anomalies, of course, e.g.:

$$Q_{top}^c = 1 - \frac{1}{N} \qquad Q_{top}^F = 1 - \frac{1}{N_F} \qquad Q_{top}^B = (\frac{n_c}{N} + \frac{1}{N_f})^2$$

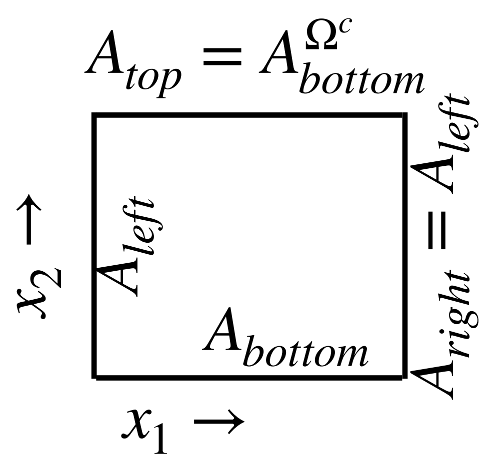
others, different context
Anber, EP 1909.09027

Again, idea is that both UV and IR theories can be put on $\mathbb{T}^4$ in same global symmetry background - and anomaly of $\mathbb{Z}_{2N_f T_R}^{(0)}$ should be the same!

1. what are these backgrounds that have $Q \neq 0$

- '*t Hooft fluxes* and their generalizations, on $\mathbb{T}^4, \mathbb{CP}^2 \ldots$

$$A_i^c = (\Omega_{ij}^c)^{-1}(A_j^c + i\mathrm{d})\Omega_{ij}^c$$



$$A_1(x_2) = \frac{2\pi x_2}{L^2}\mathrm{diag}(\frac{1}{N}, \ldots, \frac{1}{N}, -1 + \frac{1}{N})$$

$$A_1(L) = A_1(0) - i\Omega^{c\dagger}(x_1)\partial_1\Omega^c(x_1)$$

$$\Omega^c(x_1) = e^{i\frac{2\pi}{L}x^1\mathrm{diag}(\frac{1}{N},\frac{1}{N},\ldots,-1+\frac{1}{N})}$$

$$\Omega^c(L) = e^{i\frac{2\pi}{N}}\Omega^c(0)$$

unit 't Hooft flux in x_1x_2:
gauging $\mathbb{Z}_N^{(1)}$; $F_{12} = const$.
add same in x_3x_4, compute

$Q^c_{top} = 1 - \dfrac{1}{N}$ **- anomaly!!**

$$\Omega^c(L) = e^{i\frac{2\pi}{N}}\Omega^c(0)$$

not good for $\psi, \tilde{\psi}$
of $n_c < N$, not single valued
but add similar fluxes
for F, B, L to compensate!

$$\psi_i = (\Omega_{ij}^c)^{n_c}\ \Omega_{ij}^F\ \Omega_{ij}^B\ \Omega_{ij}^{Lorentz}\ \psi_j$$

$$\prod_{\text{tripple overlaps}} (\Omega^c)^{n_c}\ \Omega^F\ \Omega^B\ \Omega^{Lorentz} = 1$$

if, $\mathbb{T}^4 \rightarrow \mathbb{CP}^2$: ***("non-spin manifold") more constraining phases to match!***

$$Q^c_{top} = \frac{1}{2}(1 - \frac{1}{N}) \quad Q^F_{top} = \frac{1}{2}(1 - \frac{1}{N_F}) \quad Q^B_{top} = \frac{1}{2}(\frac{1}{2} + \frac{1}{N_F} + \frac{n_c}{N})^2 \quad Q^{grav.}_{top} = -\frac{1}{8}$$

Cordova, Dumitrescu 1806.09592

Anber, EP 2002.02037

global symmetry:

$$\frac{SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times \mathbb{Z}^{(0)}_{2N_f T_R}}{\mathbb{Z}_{\frac{N}{p}} \times \mathbb{Z}_{N_f}} \times \mathbb{Z}^{(1)}_{p=\mathrm{gcd}(N,n_c)}$$

"New 't Hooft anomalies" (example of):

- turn on **general** global symmetry backgrounds on $\mathbb{T}^4$ or $\mathbb{CP}^2$
- these lead to an anomaly in discrete symmetry $Q = Q_{bckgrd} \neq \mathbb{Z}$

$\mathbb{Z}^{(0)}_{2N_f T_R}$: $\mathcal{D}\Psi \to \mathcal{D}\Psi \, e^{i2\pi Q}$

$$e^{i2\pi Q}\Big|_{UV} = e^{i2\pi \frac{1}{N_f T_R}[N_f \, T_R \, Q^c_{top} + \mathrm{dim}_R \, T_F \, Q^F_{top} + \mathrm{dim}_R \, N_f \, (Q^B_{top} + Q^{grav.}_{top})]}$$

- above $e^{i2\pi Q}$ is the **"BCF anomaly"** - to be reproduced by theory at any scale (and at any volume, incl. $V \to \infty$, incl. T>0)

2. what constraints do they place?

global symmetry:

$$\frac{SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times \mathbb{Z}^{(0)}_{2N_f T_R}}{\mathbb{Z}_{\frac{N}{p}} \times \mathbb{Z}_{N_f}} \times \mathbb{Z}^{(1)}_{p=\gcd(N,n_c)}$$

$$\mathbb{Z}^{(0)}_{2N_f T_R}: \ \mathcal{D}\Psi \to \mathcal{D}\Psi \, e^{i2\pi Q}$$

$$e^{i2\pi Q}\Big|_{UV} = e^{i2\pi \frac{1}{N_f T_R}[N_f \, T_R \, Q^c_{top} + \dim_R T_F \, Q^F_{top} + \dim_R N_f \, (Q^B_{top} + Q^{grav.}_{top})]}$$

an exercise *(in "number theory", use either* $\mathbb{T}^4$ or $\mathbb{CP}^2$*)*

"theorem": *suppose a set of massless composites matches all 'traditional' anomalies;*

*then, **they can not, by themselves** match the new anomaly if either*

$$\gcd(N, N_f) > 1 \qquad \textit{or} \qquad \gcd(N, n_c) > 1$$

(otherwise, they do match anomaly)

rules out the massless composites as the "sole player" in the IR

need other IR "d.o.f.": typically symmetry breaking and associated domain walls/TQFT/: **Example**

2. what constraints do they place?

Ex1: SU(2) QCD(adj) with one Dirac flavor = two Weyl

Anber, EP, 1805.12290+…

A. "vanilla phase" with broken $SU(2)_F$

anomaly implications in IR:
Cordova, Dumitrescu 1806.09592

B. massless composite Dirac fermion, doublet of $SU(2)_F$

motivation:

saturates all "traditional" 0-form anomalies; spectrum = $\mathbb{R}^3 \times \mathbb{S}^1$ solution

Unsal 2007

$$\gcd(N, n_c = N) = N > 1$$

B. in IR: $\text{Tr}\, \lambda^3 \sim SU(2)_F$ doublet

2. what constraints do they place?

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Cordova, Dumitrescu 1806.09592

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saturates all "traditional" 0-form anomalies; **spectrum =** $\mathbb{R}^3 \times \mathbb{S}^1$ **solution**

Unsal 2007

but not the "new" $\mathbb{Z}_8^{(0)}$chiral- $\mathbb{Z}_2^{(1)}$center anomaly

on $\mathbb{T}^4$ OK, with four-fermi condensate $\mathbb{Z}_8^{(0)} \to \mathbb{Z}_4^{(0)}$

no bilinear condensate, as on $\mathbb{R}^3 \times \mathbb{S}^1$ **!**

B. in IR: $\mathrm{Tr}\, \lambda^3 \sim SU(2)_F$ doublet

$\langle \det \lambda^2 \rangle \neq 0; SU(2)_F$ singlet, $\mathbb{Z}_8^{(0)} \to \mathbb{Z}_4^{(0)}$

2. what constraints do they place?

Ex1: SU(2) QCD(adj) with one Dirac flavor = two Weyl

Anber, EP, 1805.12290+…

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Unsal 2007

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no bilinear condensate, as on $\mathbb{R}^3 \times \mathbb{S}^1$!

on $\mathbb{CP}^2$ unbroken part of $\mathbb{Z}_4^{(0)}$ not matched,
$\mathbb{Z}_2$-valued, need an extra IR $\mathbb{Z}_2$ TQFT

Cordova, Dumitrescu 1806.09592;
Bi, Senthil 1808.07465;
Wan, Wang 1812.11955;
Cordova, Ohmori 1912.13069;
Anber, EP 2002.02037

Will not speculate on **A.** vs **B.** Lattice studies Jena, MIT on…latest 1912.11723
Illustrates utility of new anomaly matching.

2. what constraints do they place?

Ryttov, EP, 1904.11640;
Cordova, Ohmori 1912.13069;
Anber,EP 2002.02037

Ex1.1: SU(N) QCD(adj) with N_F Weyl flavors

A. "vanilla phase" with broken $SU(N_f)$ or CFT…

B. massless composites = gauge invariant copy of UV fermions

saturate all "traditional" 0-form anomalies

but not the "new" $\mathbb{Z}^{(0)}_{2NN_f}$ chiral- $\mathbb{Z}^{(1)}_{N}$ center anomaly

on $\mathbb{T}^4$ OK, with multi-fermi condensate $\mathbb{Z}^{(0)}_{2NN_f} \to \mathbb{Z}^{(0)}_{2N_F}$

on $\mathbb{CP}^2$ unbroken part of $\mathbb{Z}^{(0)}_{2N_f}$ not matched (even Nf),

need an extra IR $\mathbb{Z}_2$ TQFT - shown to exist…

Will not speculate on **A.** vs **B.**

Illustrates utility of new anomaly matching.

2. what constraints do they place?

Ex 2: SU(6) [or SU(4k+2)] QCD(AS) with single Dirac

Anber,EP 1909.09027, 2002.02037

A. "vanilla phase" with broken $\mathbb{Z}_8^{(0)} \to \mathbb{Z}_2^{(0)}$

domain wall physics nontrivial, e.g. w/ light axion

Anber,EP 2001.03631

B. massless composite Dirac

saturates all "traditional" 0-form anomalies

but not the "new" $\mathbb{Z}_8^{(0)}$chiral-B and C 't Hooft fluxes

on $\mathbb{T}^4$ OK, with 4-fermi condensate $\mathbb{Z}_8^{(0)} \to \mathbb{Z}_4^{(0)}$

on $\mathbb{CP}^2$ unbroken part of $\mathbb{Z}_4^{(0)}$ not matched (seen w/ only B flux, not $\mathbb{Z}_2^{(1)}$)

need an extra IR TQFT - argued to exist…

Cordova, Ohmori 1912.13069; Thorngren 2001.11938

Will not speculate on **A.** vs **B.**

Illustrates utility of new anomaly matching.

Conclusion:

New 't Hooft anomalies are an exciting development.

what constraints do they place?

- gave examples

- usually require $\mathbb{Z}^{(0)}_{2N_f T_R}$ be (partially) broken (or CFT)
- constrain the physics of domain walls
- limit/forbid scenarios where massless composites saturate all the usual 'traditional' 't Hooft anomalies
- constrain finite-T phases (eg ordering of phase transitions, interfaces…)

- "generalized" anomalies do not tell us which consistent IR scenario is realized

- I think, we do not yet know what is the complete set of consistency requirements