My home in Pisa
14/05/2020

New ’t Hooft Anomalies and Phases of Chiral Gauge Theories

K. Konishi (Univ. Pisa/INFN, Pisa)

Plan of the talk

I. Chiral Gauge Theories

II. Generalized 't Hooft (Mixed) anomalies and

Phases of Chiral Gauge Theories

Bolognesi, KK, Shifman PRD '18
Bolognesi, KK PRD '19
Bolognesi, KK, Luzio JHEP '19
Bolognesi, KK, Luzio ArXiv '20

I. Chiral gauge theories

Why ?

(i) Macro and Molecular level Nature: $O(10^{-6}) \sim O(10^{0})\ cm$

nontrivial chiral properties e.g., DNA

(ii) Microscopic world very precisely described by $O(10^{-16})\ cm$

$$SU(3)_{QCD} \times (SU(2)_L \times U_Y(1))_{GWS}$$

Parity violation

* Lorentz group

$(\frac{1}{2}, 0), (0, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}), \ldots$

(iii) Grand Unified theories SU(5), SO(10) … ? $O(10^{-29}\ cm)$

- (ii), (iii) are chiral gauge theories * : they are weakly coupled, consistent in perturbation theory

Gauge-anomaly cancellation

- Perfectly reasonable (good) theories, as low-energy effective actions

As such, they leave mysteries; masses, families, neutrinos, axions, dark matter, the origin of the electroweak symmetry breaking, naturalness, etc. etc.

Significant results on strongly-coupled asymptotically-free gauge theories

- QCD SU(3) YM w. quarks

  confinement, XSB,

  40 years (!) of lattice simulations (w/ advanced computers),

- $\mathcal{N}$=2 susy gauge theories (w/ advanced mathematics),

  monopoles and duality
  SCFT, XSB and confinement

  25 years since Seiberg-Witten

- All refer to vectorlike gauge theories

  cfr. N=1 supersymmetric chiral theories; susy breaking

- Surprisingly little is known today about strongly-coupled (AF) chiral gauge theories

A challenge for theorists: understand them better!
Nature may be making use of them in a so-far unknown (to us) way

Earlier studies on chiral gauge theories

- Tumbling (MAC)

$$G \overset{\langle\psi_1\psi_2\rangle=\Lambda_1^3}{\longrightarrow} G' \overset{\langle\psi_1'\psi_2{}'\rangle=\Lambda_2^3}{\longrightarrow} \ldots$$

- Complementarity

  $\langle\phi\rangle \neq 0$, $\phi \sim \text{fund.rep.}$ Confinement ~ Higgs

- ♦ 't Hooft's anomaly matching constraints

- Appelquist-Cohen-Schmaltz criterion (free energy), $f_{IR} < f_{UV}$

- "a" theorem (RG flow)

- Large N

No truly significant progress so far ….

☞ Renewed efforts

SU(N) models

Bolognesi, KK, Shifman '18
Bolognesi, KK '19

- AF, no masses, potential, no θ angle, unique vacuum
- Confinement ? / Dynamical Higgs ?
- Flavor symmetry breaking ? In which pattern ?
- Infrared CFT ?
- Hierarchical mass scales generated (tumbling) ?

Model	$3b_0$
$(1,1)$	$9N-8$
$(1,0)$	$9N-6$
$(2,0)$	$7N-12$
$(3,0)$	$5N-18$
$(0,1)$	$9N+6$
$(0,2)$	$7N+12$
$(0,3)$	$5N+18$
$(2,1)$	$7N-14$
$(1,-1)$	$7N$

(N_ψ, N_χ) models

(i) $\psi^{\{ij\}}\,, \quad \eta_i^B\,, \qquad (i,j=1,2,\ldots,N\,, \quad B=1,2,\ldots,N+4)\,,$

$\square\!\square \oplus (N+4)\,\bar{\square}$

(1,0) model

(0,1) model

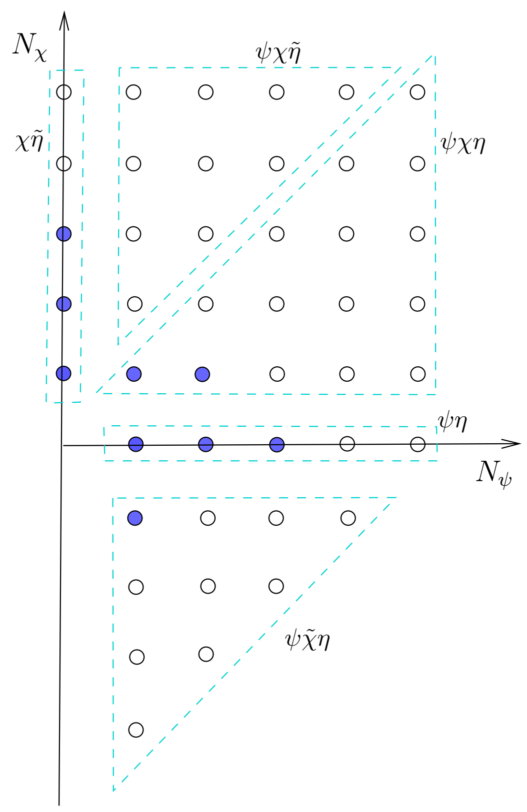
(ii) $\chi_{[ij]}\,, \qquad \tilde{\eta}^{B\,j}\,, \qquad B=1,2,\ldots,(N-4)\,,$ $\quad \bar{\begin{smallmatrix}\square\\\square\end{smallmatrix}} + (N-4)\,\square$

(iii) $\psi^{\{ij\}}\,, \qquad \chi_{[ij]}\,, \qquad \eta_i^A\,, \qquad A=1,2,\ldots 8\,,$

(1,1) model

How to realize SU(8) symmetry ?!?

(iv) … …



Lessons

Bolognesi, KK, Shifman '18
Bolognesi, KK '19

- 't Hooft anomaly matching requirement severely restricts possible phases / flavor symmetry realization

- (Partial) color-flavor locking, dynamical Higgs vacua +

  (partial) dynamical Abelianization :

  useful tools for finding the (in general, non unique) solutions

- Tumbling or MAC ? Not assumed, but looks natural in some cases

- A new mass hierarchy ? $F_\pi \ll \Lambda ?$

- Still, need of more powerful theoretical arguments badly felt ….

II. Generalized ’t Hooft (Mixed) anomalies and the Phases of chiral gauge theories

II. Generalized 't Hooft (Mixed) anomalies and the Phases of chiral gauge theories

Part A

Bolognesi, KK, Luzio, JHEP '19

We study (Part A) :

- Consequences of gauging exact 1-form $\mathbb{Z}_k^C$ center symmetries in several simple SU(N) gauge theories w/ chiral (or vectorlike) fermions (k = a divisor of N)

- Models: (here fermions are not acted upon by $\mathbb{Z}_k^C$)

(i) Fermions in self-adjoint, antisymmetric single-column irreps, $\ydiagram{1,1,1}$

SU(6) w/ N_F =1,2,…; SU(N)

(ii) Adjoint QCD SU(N) with N_F λ in adjoint repr $\ydiagram{2,1,1,1,1,1,1}$

(iii) "QCD" with N_F pairs of "quarks" in

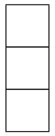
$$\psi, \tilde{\psi} = \ydiagram{2} \oplus \overline{\ydiagram{2}}$$

$$\psi, \tilde{\psi} = \ydiagram{1,1} \oplus \overline{\ydiagram{1,1}}$$

(iv) Fermions in $\frac{N-4}{k}\,\ydiagram{2} \oplus \frac{N+4}{k}\,\overline{\ydiagram{1,1}}\,,$

e.g., SU(6) with $\ydiagram{2} \oplus 5\,\overline{\ydiagram{1,1}}$ SU(8) w $\ydiagram{2} \oplus 3\,\overline{\ydiagram{1,1}}$

Results

(i) SU(6) theory with a single Weyl fermion in <u>20</u> 

(0-form) $\mathbb{Z}_6^{\psi}$ & (1-form) $\mathbb{Z}_3^{C}$ symmetries!

Mixed anomaly implies XSB

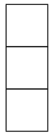
$$\mathbb{Z}_6^{\psi} \longrightarrow \mathbb{Z}_2^{\psi}$$

IR: $\langle\psi\psi\rangle \sim \Lambda^3 \neq 0$ but $\langle\psi\psi\rangle_{\underline{1}} = 0$ so, <u>prob.ly</u> $\langle\psi\psi\rangle_{\underline{35}} \neq 0$

Dynamical Abelianization

or $\langle\psi\psi\psi\psi\rangle \neq 0\,,$ or $\langle\bar{\psi}\bar{\psi}\psi\psi\rangle \neq 0$

Yamaguchi

(ii) SU(N) theory with a single Weyl fermion in  $\}\ \frac{N}{2}$

(a) $N = 4$: no mixed anomalies

(b) $N = 4\ell,\ \ell \geq 2$ $\quad \frac{4}{N} = \frac{1}{\ell}$ mixed anomaly $\quad \mathbb{Z}^{\psi}_{2T_R} \longrightarrow \mathbb{Z}^{\psi}_{\frac{2T_R}{\ell}}$

(c) $N = 4\ell + 2$ $\quad 2T_R \cdot \frac{4}{N} = 2T_R \cdot \frac{2}{2\ell+1}$ mixed anomaly $\quad \mathbb{Z}^{\psi}_{2T_R} \longrightarrow \mathbb{Z}^{\psi}_{\frac{2T_R}{2\ell+1}}$

Generally

$$U(1)_{\psi} \longrightarrow \mathbb{Z}^{\psi}_{2T_R N_f} \longrightarrow \mathbb{Z}^{\psi}_{2T_R N_f \frac{\gcd(4,N)}{N}}$$

instantons mixed anomaly

$2T_R = \binom{N-2}{N/2-1}$

(ii) Adjoint QCD SU(N) with N_F λ in adjoint repr

Poppitz's talk

(iii) "QCD" with N_F pairs of "quarks" in $\psi, \tilde{\psi} = \square\!\square \oplus \overline{\square\!\square}$

$\psi, \tilde{\psi} = \begin{smallmatrix}\square\\\square\end{smallmatrix} \oplus \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$

(iv) Fermions in $\frac{N-4}{k}\, \square\!\square \oplus \frac{N+4}{k}\, \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$,

SU(6) with $\square\!\square \oplus 5\, \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$

- G_F (classical) = $SU(5) \times U(1)_\psi \times U(1)_\chi$
- Instantons : $U(1)_\psi \times U(1)_\chi \to \frac{U(1)_{\psi\chi} \times \mathbb{Z}_8^\psi \times \mathbb{Z}_{20}^\chi}{\mathbb{Z}_{40}} \sim U(1)_{\psi\chi} \times \mathbb{Z}_4$
- Conventional 't Hooft constr. : $SU(5) \times U(1)_{\psi\chi}$ broken
- Gauging the 1-form $\mathbb{Z}_2^C$: $\mathbb{Z}_4 \to \mathbb{Z}_2$
- At least two of the condensates $\psi\chi$, $\psi\psi$, $\chi\chi$

The systematics ?

- Weyl fermions ψ_i in representations R_i and multiplicities $N_{f,i}$ with $i = 1, \ldots, n_R$
- $U(1)_{\psi_i} \longrightarrow \mathbb{Z}^{\psi}_{2T_{R_i} N_{f,i}}$ due to instantons
- $n(R_i)$ be the N-ality
- Define $k = \gcd\left(N, n(R_1), n(R_2), \ldots, n(R_{n_R})\right)$
- If k > 1 then we have a nontrivial 1-form center symmetry $\mathbb{Z}_k^C$

Not sufficient to have some interesting effects

- We need $\tilde{k} > 1$ to get an interesting effects, where

$$\boxed{\tilde{k} = \frac{N}{\gcd\left(\frac{N^2}{k^2}, N\right)} = \frac{kn}{\gcd\left(n^2, kn\right)} = \frac{k}{\gcd\left(n, k\right)} \in \mathbb{Z}}$$

$N = k\,n\,, \qquad n \in \mathbb{Z}$

- <u>The Toron charge</u>

$n_{12} = n_{34} = \frac{N}{k}$ units of 't Hooft fluxes

$$\frac{n_{12}\, n_{34}}{N} = \frac{1}{N}\frac{N^2}{k^2} = \frac{N}{k^2} = \frac{n}{k} = \frac{n/\gcd(n,k)}{k/\gcd(n,k)} = \frac{m}{k/\gcd(n,k)} = \frac{m}{\tilde{k}}\,, \qquad m \equiv \frac{n}{\gcd(n,k)} \in \mathbb{Z}$$

combined with instantons gives the minimum topological charge

$$\exists p, \exists q \in \mathbb{Z}\,, \qquad \frac{N}{k^2}\cdot p + q = \frac{m}{\tilde{k}}\cdot p + q = \frac{mp + \tilde{k}q}{\tilde{k}} = \frac{1}{\tilde{k}}$$

Bézout's lemma

II. Generalized ’t Hooft (Mixed) anomalies and the Phases of chiral gauge theories

Part B

Bolognesi, KK, Luzio, ArXiv ’20

We study (Part B *) :

Bolognesi, KK, Luzio ,'20

♦ Consequences of gauging the 1-form $\mathbb{Z}_N^C$ center symmetries in some simple SU(N) gauge theories w/ chiral fermions including those in the fundamental representaiton *

- Models:

(i) $\psi^{\{ij\}}\,, \quad \eta_i^B\,, \qquad (i,j=1,2,\ldots,N\,, \quad B=1,2,\ldots,N+4)$

"$\psi\eta$" model
$(1,0)$ model
Bars – Yankielovicz models

in repr $\quad \square\!\square \oplus (N+4)\,\bar{\square}$

(ii) $\chi_{[ij]}\,, \qquad \eta^{B\,j}\,, \qquad B=1,2,\ldots,(N-4)$

in repr $\quad \bar{\begin{smallmatrix}\square\\\square\end{smallmatrix}} + (N-4)\,\square$

* ♦ Ordinarily fermions in the fund. irrep of SU(N) would simply breaks the center symmetry. Use color-flavor locked $\mathbb{Z}_N^C$ symmetry!

Let's study $\psi\eta$ models - SU(N) theory with

$$\psi^{\{ij\}}\ , \quad \eta_i^B\ , \qquad (i,j=1,2,\dots,N\ , \quad B=1,2,\dots,N+4)$$

Appelquist, Cohen,Schmaltz,Shrock '99, Appelquist,Duan, Sannino,'00 Bolognesi, KK, Shifman '18

in repr $\square\!\square \oplus (N+4)\,\bar{\square}$

- G_F = $SU(N+4)\times U(1)_{\psi\eta}$ (*)

(actually a covering group of the true symmetry group)

♦ Interesting as the conventional 't Hooft anomaly matching constraints allow both

(i) Chirally symmetric confining vacuum:

no condensates, and with massless composite fermions ("baryons") $\frac{(N+4)(N+3)}{2}$

$$\mathcal{B}^{[AB]}=\psi^{ij}\eta_i^A\eta_j^B\ , \qquad A,B=1,2,\dots,N+4\ ,$$

and

(ii) Color-flavor locked dynamical Higgs phase, with condensates

$$\langle\psi^{\{ij\}}\eta_i^B\rangle = c\,\Lambda^3\delta^{jB}\neq 0\ , \qquad j,B=1,2,\dots N\ , \qquad c\sim O(1)$$

with $\frac{N^2+7N}{2}$ massless baryons and 8N+1 NG bosons

- G_F = $SU(N)_{\rm cf}\times SU(4)_{\rm f}\times U(1)'$

(i) Chirally symmetric confining vacuum

fields	$SU(N)_{\mathrm{c}}$	$SU(N+4)$	$U(1)_{\psi\eta}$	$(\mathbb{Z}_2)_F$
ψ	$\yng(2)$	$(\cdot)$	$\frac{N+4}{2}$	$+1$
η	$\bar{\yng(1)}$	$\yng(1)$	$-\frac{N+2}{2}$	-1
B^{AB}	$(\cdot)$	$\yng(1,1)$	$-\frac{N}{2}$	-1

(ii) Color-flavor locked dynamical Higgs phase

	fields	$SU(N)_{\mathrm{cf}}$	$SU(4)_{\mathrm{f}}$	$U(1)'$	$(\mathbb{Z}_2)_F$
UV	ψ	$\yng(2)$	$\frac{N(N+1)}{2} \cdot (\cdot)$	$N+4$	1
	η^{A_1}	$\bar{\yng(2)} \oplus \bar{\yng(1,1)}$	$N^2 \cdot (\cdot)$	$-(N+4)$	-1
	η^{A_2}	$4 \cdot \bar{\yng(1)}$	$N \cdot \yng(1)$	$-\frac{N+4}{2}$	-1
IR	$\mathcal{B}^{[A_1B_1]}$	$\bar{\yng(1,1)}$	$\frac{N(N-1)}{2} \cdot (\cdot)$	$-(N+4)$	-1
	$\mathcal{B}^{[A_1B_2]}$	$4 \cdot \bar{\yng(1)}$	$N \cdot \yng(1)$	$-\frac{N+4}{2}$	-1

Wish to find out if the generalized ’t Hooft mixed-anomaly-matching gives a stronger constraint

- Be more careful about the symmetry:

  the true symmetry group of the system turns out to be: (N=even)

$$G = SU(N) \times \frac{U(1)_{\psi\eta} \times SU(N+4) \times (\mathbb{Z}_2)_F}{\mathbb{Z}_N \times \mathbb{Z}_{N+4}}$$ cfr. (*)

- Gauge the 1-form $\mathbb{Z}_N \subset SU(N) \cap (U_{\psi\eta}(1) \times (\mathbb{Z}_2)_F)$ symmetry

  - Introduce

    - A: $U(1)_{\psi\eta}$ 1-form gauge field,
    - $A_2^{(1)}$: $(\mathbb{Z}_2)_F$ 1-form gauge field,
    - $A_{\rm f}$: $SU(N+4)$ 1-form gauge field,
    - $B_{\rm c}^{(2)}$: $\mathbb{Z}_N$ 2-form gauge field,
    - $B_{\rm f}^{(2)}$: $\mathbb{Z}_{N+4}$ 2-form gauge field.

a : SU(N) gauge field

$$\boxed{N B_{\rm c}^{(2)} = dB_{\rm c}^{(1)} \; ; \qquad (N+4) B_{\rm f}^{(2)} = dB_{\rm f}^{(1)} \; .}$$

- $$\frac{1}{2\pi}\int_{\Sigma_2} B_{\rm c}^{(2)} = \frac{n_1}{N} \,, \qquad n_1 \in \mathbb{Z}_N \,, \qquad \frac{1}{2\pi}\int_{\Sigma_2} B_{\rm f}^{(2)} = \frac{m_1}{N+4} \,, \qquad m_1 \in \mathbb{Z}_{N+4}$$

- Impose invariance under:

$$B_{\rm c}^{(2)} \to B_{\rm c}^{(2)} + {\rm d}\lambda_{\rm c}\ , \qquad B_{\rm c}^{(1)} \to B_{\rm c}^{(1)} + N\lambda_{\rm c}\ ,$$

$$B_{\rm f}^{(2)} \to B_{\rm f}^{(2)} + {\rm d}\lambda_{\rm f}\ , \qquad B_{\rm f}^{(1)} \to B_{\rm f}^{(1)} + (N+4)\lambda_{\rm f}\ , \qquad \text{(\&)}$$

$$\widetilde{a} \to \widetilde{a} + \lambda_{\rm c}\ ,$$
$$A \to A - \lambda_{\rm c} - \lambda_{\rm f}\ ,$$
$$A_2^{(1)} \to A_2^{(1)} + \frac{N}{2}\lambda_{\rm c} + \frac{N+4}{2}\lambda_{\rm f}\ ,$$
$$\widetilde{A}_{\rm f} \to \widetilde{A}_{\rm f} + \lambda_{\rm f}\ .$$

where
$$\widetilde{a} = a + \frac{1}{N}B_{\rm c}^{(1)},$$
$$\widetilde{A}_{\rm f} = A_{\rm f} + \frac{1}{N+4}B_{\rm f}^{(1)}$$

- Fermion kinetic terms

$$\overline{\psi}\gamma^\mu\left(\partial + \mathcal{R}_{\rm S}(\widetilde{a}) + \frac{N+4}{2}A + A_2\right)_\mu P_{\rm L}\psi$$
$$+\overline{\eta}\gamma^\mu\left(\partial + \mathcal{R}_{\rm F^*}(\widetilde{a}) + \widetilde{A}_{\rm f} - \frac{N+2}{2}A - A_2\right)_\mu P_{\rm L}\eta\ .$$

with <u>replacement</u>

$$\mathcal{R}_S(\tilde{a}) \to \mathcal{R}_S(\tilde{a}) - \frac{2}{N}B_c^{(1)} \qquad \mathcal{R}_{F*}(\tilde{a}) \to \mathcal{R}_S(\tilde{a}) + \frac{1}{N}B_c^{(1)}$$

$$A \to A + \frac{1}{N}B_c^{(1)} + \frac{1}{N+4}B_f^{(1)} \qquad A_2 \to A_2 - \frac{1}{2}B_c^{(1)} - \frac{1}{2}B_f^{(1)}$$

each term is now invariant under (&)

Invariant tensors felt by the fermions

Stora-Zumino

ψ :

$$\mathcal{R}_{\rm S}\big(F(\widetilde{a})\big) + \frac{N+4}{2}{\rm d}A + {\rm d}A_2 = \mathcal{R}_{\rm S}\big(F(\tilde{a}) - B_{\rm c}^{(2)}\big) + \frac{N+4}{2}\left[{\rm d}A + B_{\rm c}^{(2)} + B_{\rm f}^{(2)}\right] + \left[{\rm d}A_2^{(1)} - \frac{N}{2}B_{\rm c}^{(2)} - \frac{N+4}{2}B_{\rm f}^{(2)}\right] ,$$

η :

$$\mathcal{R}_{\rm F^*}\big(F(\widetilde{a})\big) + F(\widetilde{A}_{\rm f}) - \frac{N+2}{2}{\rm d}A - {\rm d}A_2 = -[F(\widetilde{a}) - B_{\rm c}^{(2)}] + [F(\widetilde{A}_{\rm f}) - B_{\rm f}^{(2)}] - \frac{N+2}{2}\left[{\rm d}A + B_{\rm c}^{(2)} + B_{\rm f}^{(2)}\right] - \left[{\rm d}A_2^{(1)} - \frac{N}{2}B_{\rm c}^{(2)} - \frac{N+4}{2}B_{\rm f}^{(2)}\right] .$$

....

$\mathcal{B}$:

$$\mathcal{R}_{\rm A}\big(F(\widetilde{A}_{\rm f})\big) - \frac{N}{2}{\rm d}A - {\rm d}A_2 = \mathcal{R}_{\rm A}[F(\widetilde{A}_{\rm f}) - B_{\rm f}^{(2)}] - \frac{N}{2}\left[{\rm d}A + B_{\rm c}^{(2)} + B_{\rm f}^{(2)}\right] - \left[{\rm d}A_2^{(1)} - \frac{N}{2}B_{\rm c}^{(2)} - \frac{N+4}{2}B_{\rm f}^{(2)}\right] .$$

Calculation of mixed anomalies ; generalized 't Hooft anomaly matching criteria

Results:

- **Mixed $(\mathbb{Z}_2)_F - [\mathbb{Z}_N]^2$ anomaly**

$$
\begin{aligned}
&-\frac{\mathrm{N}+2}{\mathrm{N}} + \frac{(\mathrm{N}+1)(\mathrm{N}+4)^2}{8\mathrm{N}} - \frac{(\mathrm{N}+4)(\mathrm{N}+1)}{4} + \frac{\mathrm{N}(\mathrm{N}+1)}{8} \\
&+\frac{\mathrm{N}+4}{\mathrm{N}} - \frac{(\mathrm{N}+4)(\mathrm{N}+2)^2}{4\mathrm{N}} + \frac{(\mathrm{N}+2)(\mathrm{N}+4)}{2} - \frac{\mathrm{N}(\mathrm{N}+4)}{4} = 1\,.
\end{aligned}
$$

UV:

$$
S_{\mathrm{UV}}^{(5)} = 1 \cdot \frac{1}{8\pi^2} \int_{\Sigma_5} N^2 (B_{\mathrm{c}}^{(2)})^2 \cdot A_2^{(1)}\,.
$$

$$
\Delta S_{\mathrm{UV}}^{(4)} = \pm \mathrm{i}\pi\mathbb{Z}\,, \qquad \delta A_2^{(1)} = d\,\frac{1}{2}\delta A_2^{(0)}\,, \qquad \delta A_2^{(0)} = \pm 2\pi\,.
$$

♦ The partition fn changes sign under $\psi \to -\psi, \quad \eta \to -\eta$ Z$_2$ anomaly !

♦ N.B. without gauging the 1-form symmetry $B_{\mathrm{c}}^{(2)} = B_{\mathrm{c}}^{(1)} = B_{\mathrm{f}}^{(2)} = B_{\mathrm{f}}^{(1)} = 0\,.$

would have found: $\delta S_{\mathrm{UV}}^{4} = -\dfrac{2}{8\pi^2} \displaystyle\int_{\Sigma_4} \mathrm{tr}[F(a)]^2\, \frac{\delta A_2^{(0)}}{2}$ $\therefore$ No Z$_2$ anomaly!

IR: $0 \cdot N^2 (B_{\mathrm{c}}^{(2)})^2 A_2^{(1)} = 0$ No Z$_2$ anomaly

♦ Assuming the chirally symmetric vacuum

♦ Mixed $(\mathbb{Z}_2)_F - [\mathbb{Z}_N]^2$ 't Hooft anomaly does not UV-IR match

$\therefore$ The chirally symmetric vacuum is inconsistent

- **Mixed** $(\mathbb{Z}_2)_F - [\mathbb{Z}_{N+4}]^2$ **anomaly**

UV: No Z_2 anomaly

IR: $(N+4)^2 \int (B_{\rm f}^{(2)})^2 A_2^{(1)}$ term present with coefficient

the situation opposite to the previous; but the conclusion is the same

$$-\frac{(N+3)(N+4)}{8} + \frac{N+2}{N+4} - \frac{N^2(N+3)}{8(N+4)} + \frac{N(N+3)}{4} = -1 \ .$$

♦ Mixed $(\mathbb{Z}_2)_F - [\mathbb{Z}_{N+4}]^2$ 't Hooft anomaly does not UV-IR match

.^.. The chirally symmetric vacuum is inconsistent

- **Mixed** $(\mathbb{Z}_2)_F - \mathbb{Z}_N - \mathbb{Z}_{N+4}$ **anomaly** matches in UV and IR
- **Mixed** $(\mathbb{Z}_2)_F - \mathbb{Z}_N - U(1)_{\psi\eta}$ **anomaly** matches in UV and IR
- **Mixed** $(\mathbb{Z}_2)_F - \mathbb{Z}_{N+4} - U(1)_{\psi\eta}$ **anomaly** matches in UV and IR

Conclusions / discussions / reflections

- $\mathbf{Mixed}\ (\mathbb{Z}_2)_F - [\mathbb{Z}_N]^2\ \mathbf{anomaly}$ found above in the UV theory (absent in the IR): the mixed 't Hooft anomaly does not match in the UV and IR, in the chirally symmetric confining vacuum: it cannot be the correct vacuum of the theory.

- No difficulties in the dynamical Higgs phase

$$\langle \psi^{\{ij\}} \eta_i^B \rangle = c\,\Lambda^3 \delta^{jB} \neq 0\,, \qquad j, B = 1, 2, \dots N\,, \qquad c \sim O(1)$$

more likely to be the correct vacuum.

- Several conceptually new extensions in this analysis

Tanizaki, et. al.' 18

$$\mathbb{Z}_N = [\mathbb{Z}_N \subset SU(N)] \cap [U_V(1)]$$

in the standard QCD with N_F quarks

  - Gauge the color-flavor locked $\mathbb{Z}_N$

$$\mathbb{Z}_N = [\mathbb{Z}_N \subset SU(N)] \cap [U_{\psi\eta}(1) \times \mathbb{Z}_2]$$

  - Mixed anomaly involving $(\mathbb{Z}_2)_F$

$$\psi \to -\psi, \qquad \eta \to -\eta$$

Space rotation of 360 degrees!

  - Roles of $U(1)_{\psi\eta}$ and $(\mathbb{Z}_2)_F$ symmetries and their external gauge fields are formally similar:

    A, A_2 transform nontrivially under the 1-form gauge transformation (&)

    Both affected by the 2-form gauge fields

- Important differences with anomalies involving only continuous symmetries:

  - All triangles with $U_{\psi\eta}(1), SU(N+4)$ are matched by

    $C_{\rm UV} = C_{\rm IR}$ before the gauging of the 1-form center symmetry:

    they continue to be matched after the replacement , automatically

    No new info by the gauging of the 1-fom Z_N

    N=odd

  - A different story for discrete $(Z_2)_F$ anomaly: before the gauging of the 1-form center symmetry (absence of) the anomaly matched by

    $C_{\rm UV} = C_{\rm IR}$ modulo 2, and by the integer instanton numbers

$$\frac{1}{8\pi^2}\int_{\Sigma_4} F^2 \in \mathbb{Z} \ .$$

    So the discrete $(Z_2)_F$ anomaly may arise after gauging of the 1-form $\mathbb{Z}_N$ and the UV-IR matching may be ruined. Our calculations show it indeed does.

    fractional topological charge

  - BTW, the result "C=1" we found is the only meaningful and nontrivial answer.

    C= a fractional number, 1/2, 1/N, .. : nonsense.
    C= 2, 4, .. : well it's fine but implies nothing.

    (Even if it does not guarantee in itself that everything is OK)

- Our result is formally similar to the mixed anomaly

$$CP - (\mathbb{Z}_N)^2$$

found in the pure SU(N) YM at $\theta = \pi$

Gaiotto, Kapustin, Komargodski, Seiberg,.'17

But the way the failure of the t Hooft matching manifests itself in the IR is different!

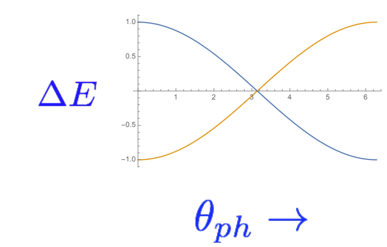
- In the pure SU(N) YM at $\theta = \pi$ CP(T) is spontaneously broken by doubly degenerate vacua

T.D. Lee

- In our case, not $(\mathbb{Z}_2)_F$ (space 2π rotation) but $U_{\psi\eta}(1)$ therefore

$\mathbb{Z}_N = [\mathbb{Z}_N \subset SU(N)] \cap [U_{\psi\eta}(1) \times \mathbb{Z}_2]$ is spontaneously broken by

$$\langle \psi^{\{ij\}} \eta_i^B \rangle = c\,\Lambda^3 \delta^{jB} \neq 0$$



- Conclusion:

  - The chirally symmetric confining vacuum does not survive our more refined consistency check: it cannot be realized dynamically in the IR

at least for N even

  - The dynamical Higgs vacuum looks OK

THE END

Thank you for your attention !

Stuart Raby, Savas Dimopoulos, and Leonard Susskind, *Tumbling Gauge Theories*, Nucl. Phys. **B169**, 373 (1980).

G. 't Hooft, *Naturalness, Chiral Symmetry, and Spontaneous Chiral Symmetry Breaking,* in *Recent Developments In Gauge Theories,* Eds. G. 't Hooft, C. Itzykson, A. Jaffe, H. Lehmann, P. K. Mitter, I. M. Singer and R. Stora, (Plenum Press, New York, 1980) [Reprinted in *Dynamical Symmetry Breaking*, Ed. E. Farhi et al. (World Scientific, Singapore, 1982) p. 345.

I. Bars and S. Yankielowicz, *Composite quarks and leptons as solutions of anomaly constraints,* Phys. Lett. 101B(1981) 159.

C. Q. Geng and R. E. Marshak, "Two Realistic Preon Models With SU(N) Metacolor Satisfying Complementarity," Phys. Rev. D **35**, 2278 (1987). doi:10.1103/PhysRevD.35.2278.

T. Appelquist, A. G. Cohen, M. Schmaltz and R. Shrock, *New constraints on chiral gauge theories,* Phys. Lett. B **459**, 235 (1999) [hep-th/9904172].

T. Appelquist, Z. y. Duan and F. Sannino, *Phases of chiral gauge theories,* Phys. Rev. D **61**, 125009 (2000) [hep-ph/0001043].

Y. L. Shi and R. Shrock, $A_k\bar{F}$ *chiral gauge theories,* Phys. Rev. D **92**,105032 (2015) [arXiv:1510.07663 [hep-th]]; Y. L. Shi and R. Shrock, *Renormalization-Group Evolution and Nonperturbative Behavior of Chiral Gauge Theories with Fermions in Higher-Dimensional Representations,* Phys. Rev. D **92**, 125009 (2015) [arXiv:1509.08501 [hep-th]].

E. Poppitz and Y. Shang, *Chiral Lattice Gauge Theories Via Mirror-Fermion Decoupling: A Mission (im)Possible?*, Int. J. Mod. Phys. A **25**, 2761 (2010) [arXiv:1003.5896 [hep-lat]].

A. Armoni and M. Shifman, *A Chiral SU(N) Gauge Theory Planar Equivalent to Super-Yang-Mills,* Phys. Rev. D **85**, 105003 (2012) [arXiv:1202.1657 [hep-th]].

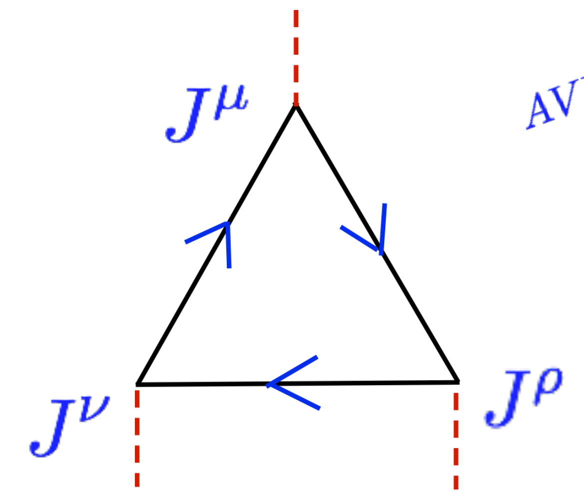
E. Eichten, R. D. Peccei, J. Preskill and D. Zeppenfeld, *Chiral Gauge Theories in the 1/n Expansion,* Nucl. Phys. B **268**, 161 (1986).

J. Goity, R. D. Peccei and D. Zeppenfeld, *Tumbling and Complementarity in a Chiral Gauge Theory,* Nucl. Phys. B **262**, 95 (1985).

back to p.4

Fukuda-Miyamoto '49 Steinberger '49, Schwinger '51,
Adler '69, Bell, Jackiw '69, Bardeen '69, Wess-Zumino, '71 …

Axial / chiral anomalies



AVV, AAA, LLL, etc

- No conservation in all channels
- Observable (calculable) effects

  $\pi^0 \to 2\gamma \qquad m_\eta \gg m_\pi$

- Gauge vertices:
  - Gauge symmetry destroyed (inconsistency)
  - Anomaly cancellation needed $\quad \sum_{q's,\ell's} \mathrm{Tr}(\{t^a t^b\} t^c) = 0$

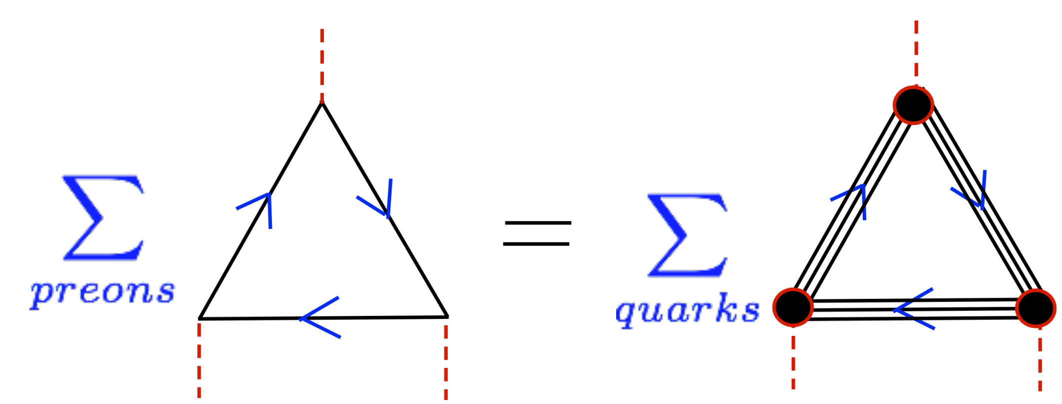
OK in $SU_{QCD}(3) \times (SU_L(2) \times U_Y(1))_{WS}$

't Hooft anomaly :
obstruction to gauging

- External "gauge" fields:

  Abelian / nonAbelian anomalies (NOT an inconsistency)
  - observable effects $K\bar{K} \to \pi\pi\pi$ etc. Wess-Zumino-Witten (5D) action

- 't Hooft anomaly matching conditions



- Mixed 't Hooft's anomalies

- Other new anomalies

Dai-Freed '15
Garcia-Etxebarria-Montero '18, Wang-Wen-Witten '18, Wan-Wang '19;

Back to p.9

Generalized symmetries

Seiberg, Kapustin, Aharony, Gaiotto, Tachikawa, Willet, Komargodski, .. '05-'18

Poppitz, Kikuchi, Tanizaki, Sakai, Shimizu, Yonekura,

- From 0-form symmetries (acting on local operators) to k-form symmetries (acting on line, surface, etc operators)
  - e.g. the center symmetry in SU(N) YM (k=1)

$$e^{i\oint_\gamma A} \to \Omega_N \, e^{i\oint_\gamma A} \,, \qquad \Omega_N = e^{2\pi i/N} \mathbb{1} \in \mathbb{Z}_N$$

Wilson loop
Polyakov loop
Criteria for different phases

Phases of the system not described by the VEV of local operators

$$\langle \phi(x) \rangle \,, \qquad \langle \bar{\psi}(x) \psi(x) \rangle$$

Nambu-Jona Lasinio
Landau-Ginzburg

: a change of PARADIGM !

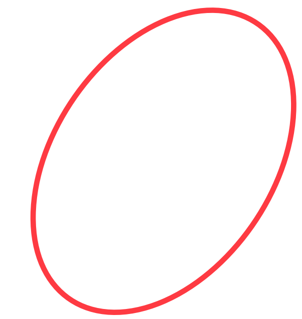
#SPT

Symmetry-protected -topological order

Haldane '83
Wen, '89
Wang, … …

- Higher form symmetries are all Abelian

“Gauging” the 1-form discrete Z_N symmetry in SU(N) YM :



(*) $$g(x+L) = e^{2\pi i/N} g(x) \qquad \exp i \oint_L a \to e^{2\pi i/N} \exp i \oint_L a$$

- Gauging a discrete symmetry = identifying the configurations connected by it and eliminating the double counting

cfr. “ordinary” gauging of a continuous symmetry

- Introduce the “U(N)” gauge field

$$a \to \tilde{a} \equiv a + \frac{1}{N} B_c^{(1)} \mathbb{1}_N \; ; \qquad F(a) \to \tilde{F}(\tilde{a}) = F(a) + B_c^{(2)} \mathbb{1}_N$$

imposing the 1-form gauge invariance

$$\tilde{a} \to \tilde{a} + \lambda \mathbb{1}_N \qquad B^{(1)} \to B^{(1)} + N\lambda \, , \qquad B^{(2)} \to B^{(2)} + d\lambda$$

$$\boxed{N B^{(2)} = d B^{(1)}} \longrightarrow \text{SU(N)} / Z_N \text{ gauge theory}$$

$$\frac{1}{8\pi^2} \int_{\Sigma_4} \mathrm{tr}\, F^2 \longrightarrow \frac{1}{8\pi^2} \int_{\Sigma_4} \mathrm{tr}\, (\tilde{F}(\tilde{a}) - B_c^{(2)})^2$$

$$= \frac{1}{8\pi^2} \int \mathrm{tr}\, (\tilde{F}(\tilde{a}))^2 - \frac{N}{8\pi^2} \int (B_c^{(2)})^2 = \mathbb{Z} - \frac{\mathbb{Z}}{N}$$

’t Hooft’s fractional magnetic flux

→ fractional topological charge

$$a \equiv t_R^b A_\mu^b dx^\mu \qquad F^2 \equiv F \wedge F = \tfrac{1}{2} F^{\mu\nu} F^{\rho\sigma} dx_\mu dx_\nu dx_\rho dx_\sigma = \tfrac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} d^4x$$

Differential form notation

Gauging exact 1-form $\mathbb{Z}_k^C$ symmetries in an SU(N) model: k= divisor of N

- Introduce the pair $(B_c^{(2)}, B_c^{(1)})$

$$kB_{\rm c}^{(2)} = dB_{\rm c}^{(1)}$$

$$B_{\rm c}^{(2)} \mapsto B_{\rm c}^{(2)} + d\lambda_{\rm c}, \qquad B_{\rm c}^{(1)} \mapsto B_{\rm c}^{(1)} + k\lambda_{\rm c}$$

- Imbed SU(N) in U(N) $\quad \widetilde{a} = a + \frac{1}{k} B_{\rm c}^{(1)}$ $\quad$ with inv under $\quad \tilde{a} \to \tilde{a} + \lambda_c$

$$F(a) \to \tilde{F}(\tilde{a}) - B_{\rm c}^{(2)}$$

- Fermion kinetic term

$U_\psi(1)$ gauge field A_ψ

$\psi \to e^{i\alpha}\psi$

$$\bar{\psi}\,\gamma^\mu(\partial + R(\tilde{a}) - \frac{n(R)}{k} B_c^{(1)} + A_\psi)_\mu P_L\,\psi$$

Dynkin

$\mathrm{tr}\big(t_R^a t_R^b\big) = T_R\,\delta^{ab}$

- Standard $U_\psi(1)$ anomaly

$$\delta S_{\delta A_\psi^{(0)}} = \frac{2T(R)}{8\pi^2}\int \mathrm{tr}F^2\,\delta\alpha = 2\,T(R)\,\mathbb{Z}\,\delta\alpha$$

$\mathbb{Z}_{2T(R)} \subset U_\psi(1)$

non Anomalous

- Mixed anomaly

$$\frac{1}{8\pi^2}\int_{\Sigma_4} \mathrm{tr}\big(\tilde{F} - B_c^{(2)}\big)^2 = \frac{1}{8\pi^2}\int_{\Sigma_4}\{\underbrace{\mathrm{tr}\tilde{F}^2}_{\in\mathbb{Z}} - N(B_c^{(2)})^2\} \qquad -\frac{N}{8\pi^2}\int_{\Sigma_4}\big(B_c^{(2)}\big)^2 = \frac{N}{k^2}\,\mathbb{Z}$$

In general fractional! "toron"

$$a \equiv t_R^b A_\mu^b dx^\mu \qquad F^2 \equiv F\wedge F = \tfrac{1}{2}F^{\mu\nu}F^{\rho\sigma}dx_\mu dx_\nu dx_\rho dx_\sigma = \tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}d^4x$$

back to p.10

“Large” gauge transformations

$$\mathbb{Z}_N \qquad \psi \to \mathrm{e}^{\frac{4\pi \mathrm{i}}{N}} \psi \,, \qquad \eta \to \mathrm{e}^{-\frac{2\pi \mathrm{i}}{N}} \eta \,,$$

$$(\mathbb{Z}_2)_F \times U(1)_{\psi\eta}$$

$$\psi \to (-1)\, \mathrm{e}^{\mathrm{i} \frac{N+4}{2} \frac{2\pi}{N}} \psi = \mathrm{e}^{-\mathrm{i} \frac{N}{2} \frac{2\pi}{N}}\, \mathrm{e}^{\mathrm{i} \frac{N+4}{2} \frac{2\pi}{N}} \psi \,, \qquad \eta \to (-1)\, \mathrm{e}^{-\mathrm{i} \frac{N+2}{2} \frac{2\pi}{N}} \eta = \mathrm{e}^{\mathrm{i} \frac{N}{2} \frac{2\pi}{N}}\, \mathrm{e}^{-\mathrm{i} \frac{N+2}{2} \frac{2\pi}{N}} \eta \,.$$

Generalized cocycle condition in the triple overlapping region:

$SU(N)/Z_N \sim PSU(N)$ bundle

back to

Calculation of anomalies by Stora-Zumino descent procedure

6D anomaly functionals

ψ :

$$
\begin{aligned}
&\frac{1}{24\pi^2}\,\mathrm{tr}\Bigg(\mathcal{R}_{\mathrm{S}}\big(F(\tilde{a})-B_{\mathrm{c}}^{(2)}\big)+\frac{N+4}{2}\left[\mathrm{d}A+B_{\mathrm{c}}^{(2)}+B_{\mathrm{f}}^{(2)}\right] \\
&\qquad +\left[\mathrm{d}A_2^{(1)}-\frac{N}{2}B_{\mathrm{c}}^{(2)}-\frac{N+4}{2}B_{\mathrm{f}}^{(2)}\right]\Bigg)^3 \\
= \;& \frac{(N+4)}{24\pi^2}\,\mathrm{tr}\left[\big(F(\tilde{a})-B_{\mathrm{c}}^{(2)}\big)^3\right] \\
&+\frac{(N+2)(N+4)}{16\pi^2}\,\mathrm{tr}\left[\big(F(\tilde{a})-B_{\mathrm{c}}^{(2)}\big)^2\right]\wedge\left[\mathrm{d}A+B_{\mathrm{c}}^{(2)}+B_{\mathrm{f}}^{(2)}\right] \\
&+\frac{N(N+1)}{2\cdot 24\pi^2}\left(\frac{N+4}{2}\right)^3\left[\mathrm{d}A+B_{\mathrm{c}}^{(2)}+B_{\mathrm{f}}^{(2)}\right]^3 \\
&+\frac{N+2}{8\pi^2}\,\mathrm{tr}\big(F(\tilde{a})-B_{\mathrm{c}}^{(2)}\big)^2\left[\mathrm{d}A_2^{(1)}-\frac{N}{2}B_{\mathrm{c}}^{(2)}-\frac{N+4}{2}B_{\mathrm{f}}^{(2)}\right] \\
&+\frac{1}{8\pi^2}\left(\frac{N+4}{2}\right)^2\frac{N(N+1)}{2}\left[\mathrm{d}A+B_{\mathrm{c}}^{(2)}+B_{\mathrm{f}}^{(2)}\right]^2\left[\mathrm{d}A_2^{(1)}-\frac{N}{2}B_{\mathrm{c}}^{(2)}-\frac{N+4}{2}B_{\mathrm{f}}^{(2)}\right] \\
&+\frac{1}{8\pi^2}\left(\frac{N+4}{2}\right)\frac{N(N+1)}{2}\left[\mathrm{d}A+B_{\mathrm{c}}^{(2)}+B_{\mathrm{f}}^{(2)}\right]\left[\mathrm{d}A_2^{(1)}-\frac{N}{2}B_{\mathrm{c}}^{(2)}-\frac{N+4}{2}B_{\mathrm{f}}^{(2)}\right]^2 \\
&+\frac{1}{24\pi^2}\frac{N(N+1)}{2}\left[\mathrm{d}A_2^{(1)}-\frac{N}{2}B_{\mathrm{c}}^{(2)}-\frac{N+4}{2}B_{\mathrm{f}}^{(2)}\right]^3\,. \qquad (5.18)
\end{aligned}
$$

Calculation of anomalies: Stora-Zumino descent procedure

η :

$$
\begin{aligned}
&\frac{1}{24\pi^2}\,\mathrm{tr}\,\{-[F(\widetilde{a})-B_{\mathrm{c}}^{(2)}]+[F(\widetilde{A}_{\mathrm{f}})-B_{\mathrm{f}}^{(2)}]-\frac{N+2}{2}\left[\mathrm{d}A+B_{\mathrm{c}}^{(2)}+B_{\mathrm{f}}^{(2)}\right]\\
&-\left[\mathrm{d}A_2^{(1)}-\frac{N}{2}B_{\mathrm{c}}^{(2)}-\frac{N+4}{2}B_{\mathrm{f}}^{(2)}\right]\}^3\\
&=-\frac{(N+4)}{24\pi^2}\,\mathrm{tr}\left[\left(F(\tilde{a})-B_{\mathrm{c}}^{(2)}\right)^3\right]+\frac{N}{24\pi^2}\mathrm{tr}\left[\left(F(\tilde{A}_{\mathrm{f}})-B_{\mathrm{f}}^{(2)}\right)^3\right]\\
&\quad-\frac{(N+2)(N+4)}{16\pi^2}\,\mathrm{tr}\left[\left(F(\tilde{a})-B_{\mathrm{c}}^{(2)}\right)^2\right]\wedge\left[\mathrm{d}A+B_{\mathrm{c}}^{(2)}+B_{\mathrm{f}}^{(2)}\right]\\
&\quad-\frac{N}{8\pi^2}\frac{N+2}{2}\,\mathrm{tr}[F(\widetilde{A}_{\mathrm{f}})-B_{\mathrm{f}}^{(2)}]^2\left[\mathrm{d}A+B_{\mathrm{c}}^{(2)}+B_{\mathrm{f}}^{(2)}\right]\\
&\quad-\frac{N(N+4)(N+2)^3}{8\cdot 24\pi^2}\left[\mathrm{d}A+B_{\mathrm{c}}^{(2)}+B_{\mathrm{f}}^{(2)}\right]^3\\
&\quad-\frac{N+4}{8\pi^2}\,\mathrm{tr}\big(F(\tilde{a})-B_{\mathrm{c}}^{(2)}\big)^2\left[\mathrm{d}A_2^{(1)}-\frac{N}{2}B_{\mathrm{c}}^{(2)}-\frac{N+4}{2}B_{\mathrm{f}}^{(2)}\right]\\
&\quad-\frac{N}{8\pi^2}\,\mathrm{tr}[F(\widetilde{A}_{\mathrm{f}})-B_{\mathrm{f}}^{(2)}]^2\left[\mathrm{d}A_2^{(1)}-\frac{N}{2}B_{\mathrm{c}}^{(2)}-\frac{N+4}{2}B_{\mathrm{f}}^{(2)}\right]\\
&\quad-\frac{1}{8\pi^2}\left(\frac{N+2}{2}\right)^2N(N+4)\left[\mathrm{d}A+B_{\mathrm{c}}^{(2)}+B_{\mathrm{f}}^{(2)}\right]^2\left[\mathrm{d}A_2^{(1)}-\frac{N}{2}B_{\mathrm{c}}^{(2)}-\frac{N+4}{2}B_{\mathrm{f}}^{(2)}\right]\\
&\quad-\frac{1}{8\pi^2}\left(\frac{N+2}{2}\right)N(N+4)\left[\mathrm{d}A+B_{\mathrm{c}}^{(2)}+B_{\mathrm{f}}^{(2)}\right]\left[\mathrm{d}A_2^{(1)}-\frac{N}{2}B_{\mathrm{c}}^{(2)}-\frac{N+4}{2}B_{\mathrm{f}}^{(2)}\right]^2\\
&\quad-\frac{1}{24\pi^2}N(N+4)\left[\mathrm{d}A_2^{(1)}-\frac{N}{2}B_{\mathrm{c}}^{(2)}-\frac{N+4}{2}B_{\mathrm{f}}^{(2)}\right]^3\ .
\end{aligned}
\tag{5.19}
$$

Calculation of anomalies: <u>Stora-Zumino</u> descent procedure

Baryons

$$
\begin{aligned}
&\frac{1}{24\pi^2}\,\mathrm{tr}\left(\mathcal{R}_{\mathrm{A}}(F(\widetilde{A}_{\mathrm{f}})-B_{\mathrm{f}}^{(2)})-\frac{N}{2}\left[\mathrm{d}A+B_{\mathrm{c}}^{(2)}+B_{\mathrm{f}}^{(2)}\right]\right.\\
&\qquad\qquad\left.-\left[\mathrm{d}A_2^{(1)}-\frac{N}{2}B_{\mathrm{c}}^{(2)}-\frac{N+4}{2}B_{\mathrm{f}}^{(2)}\right]\right)^3\\
=\;&\frac{N+4-4}{24\pi^2}\mathrm{tr}\left[\left(F(\tilde{A}_{\mathrm{f}})-B_{\mathrm{f}}^{(2)}\right)^3\right]\\
&-\frac{N^3(N+4)(N+3)}{16\cdot 24\pi^2}\left[\mathrm{d}A+B_{\mathrm{c}}^{(2)}+B_{\mathrm{f}}^{(2)}\right]^3\\
&-\frac{N+2}{8\pi^2}\frac{N}{2}\mathrm{tr}[F(\widetilde{A}_{\mathrm{f}})-B_{\mathrm{f}}^{(2)}]^2\left[\mathrm{d}A+B_{\mathrm{c}}^{(2)}+B_{\mathrm{f}}^{(2)}\right]\\
&-\frac{N+2}{8\pi^2}\mathrm{tr}[F(\widetilde{A}_{\mathrm{f}})-B_{\mathrm{f}}^{(2)}]^2\left[\mathrm{d}A_2^{(1)}-\frac{N}{2}B_{\mathrm{c}}^{(2)}-\frac{N+4}{2}B_{\mathrm{f}}^{(2)}\right]\\
&-\frac{1}{8\pi^2}\left(\frac{N}{2}\right)^2\frac{(N+4)(N+3)}{2}\left[\mathrm{d}A+B_{\mathrm{c}}^{(2)}+B_{\mathrm{f}}^{(2)}\right]^2\left[\mathrm{d}A_2^{(1)}-\frac{N}{2}B_{\mathrm{c}}^{(2)}-\frac{N+4}{2}B_{\mathrm{f}}^{(2)}\right]\\
&-\frac{1}{8\pi^2}\frac{N}{2}\frac{(N+4)(N+3)}{2}\left[\mathrm{d}A+B_{\mathrm{c}}^{(2)}+B_{\mathrm{f}}^{(2)}\right]\left[\mathrm{d}A_2^{(1)}-\frac{N}{2}B_{\mathrm{c}}^{(2)}-\frac{N+4}{2}B_{\mathrm{f}}^{(2)}\right]^2\\
&-\frac{1}{24\pi^2}\frac{(N+4)(N+3)}{2}\left[\mathrm{d}A_2^{(1)}-\frac{N}{2}B_{\mathrm{c}}^{(2)}-\frac{N+4}{2}B_{\mathrm{f}}^{(2)}\right]^3\;. \qquad (5.21)
\end{aligned}
$$

Determination of the mixed anomalies involving $(\mathbb{Z}_2)_F$

$$S^{6D} = \int_6 [...] \left[\mathrm{d}A_2^{(1)} - \frac{N}{2} B_{\mathrm{c}}^{(2)} - \frac{N+4}{2} B_{\mathrm{f}}^{(2)} \right] \; .$$

- Integrate to give 5D boundary WZW action:

$$S^{5D} = \int_5 [...] \left[A_2^{(1)} - \frac{1}{2} B_{\mathrm{c}}^{(1)} - \frac{1}{2} B_{\mathrm{f}}^{(1)} \right]$$

- Consider variations

$$\delta[A_2^{(1)} - \frac{1}{2} B_{\mathrm{c}}^{(1)} - \frac{1}{2} B_{\mathrm{f}}^{(1)}] = \frac{1}{2}\, \mathrm{d}\, \delta A_2^{(0)} \; ,$$

$$\delta S^{4D} = \frac{1}{2} \int_4 [...]\, \delta A_2^{(0)} \; , \qquad \delta A_2^{(0)} = \pm 2\pi \, .$$

one gets various 4D anomalies (“anomaly inflow”)

☞ Results