Axing minimal pre-inflationary axion models

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Work in progress

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The aim:

We revisit the **cosmological history** of **pre-inflationary axion models** with a **standard inflationary sector**, whose compatibility with **axion isocurvature bounds** requires **axion-inflaton interactions**.

The novelty:

We **discard invalid assumptions** on the dynamics of isocurvature perturbations

Detailed simulations of the evolution of **super-horizon axion isocurvature** modes, including nonperturbative effects with **lattice** computations

The plan:

Motivation of pre-inflationary axion models

The trouble with isocurvature fluctuations

Improved calculations

Motivating pre-inflationary axion models

Why the axion?

QCD admits a CP violating interaction

$$\int d^4x \, \mathcal{L}_{\theta} = \int d^4x \, \bar{\mathrm{Tr}} \, \frac{g^2 \theta}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} = \theta n_{\mathrm{top}},$$

 θ is an angle, and unphysical, yet there is a **physical CP** - **violating phase**

 $\theta_{\rm phys} = \theta + \operatorname{Arg} \, \det M_q$

Measurements of the neutron's dipole moment imply $|\theta_{phys}| < 10^{-11}$ [Kim 09]

Can be explained if θ_{phys} becomes a **dynamical** field -the **axion** [Peccei & Quinn, Weinberg, Wilczek]

Need a spontaneously broken U(1) with QCD anomaly, the axion A is pseudo-Goldstone:

$$\theta_{\rm phys}(x) = \theta + \frac{A(x)}{f_A} + \operatorname{Arg} \, \det M'_q$$

Nonperturbative QCD effects generate an even potential for $\theta_{phys}(x)$, stabilized at 0!

The nonperturbative potential only arises at low temperatures. Axion field can end up oscillating, and the **time-averaged oscillations** interact with gravity like **presureless dark matter**!



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Pre-inflationary scenarios



For illustration purposes only. Resemblance to the actual product might be limited

Pre-inflationary axion dark matter

For a given axion mass/ $f_{\scriptscriptstyle A}$ the DM abundance depends on the misalignment angle θ_I

$$\theta_I^2 = 3.4 \times 10^{-7} \left(\frac{\Omega_{DM} h^2}{0.12}\right) \left(\frac{f_A}{3 \times 10^{17} \text{GeV}}\right)^{-1.165}$$

[Borsanyi et al]

Post-inflationary scenarios



For illustration purposes only. Resemblance to the actual product might be limited

Post-inflationary axion dark matter

For a given axion mass/ f_A the DM abundance is predicted!

 $\Omega_A h^2 \approx 0.12 \Rightarrow 3 \times 10^{10} \,\text{GeV} \lesssim f_A \lesssim 1.2 \times 10^{11} \,\text{GeV} \quad ; \quad 50 \,\mu\text{eV} \lesssim m_A \lesssim 200 \,\mu\text{eV}$

[Borsanyi et al][Redondo et al][Villadoro et al][Hindmarsh et al][Safdi et al][Kawasaki et al]...

(String decay contribution still under debate)

Mass windows and experiments



How can one theoretically motivate the large values of f_A in pre-inflationary models?

Pre-inflationary models from axion GUTs

Axion GUTs can offer an intriguing connection between the axion scale f_A and the grand unification scale.

This can result in **complementarities** between **axion searches** and **proton decay** experiments.

Motivates targets for low-mass axion experiments like ABRACADABRA and CASPER.



The trouble with isocurvature





1.19

There are **curvature fluctuations** which correspond to **variations in the total density**



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Both types of perturbations generate different features in CMB temperature maps

Measurements can constraint the isocurvature fraction



Axion isocurvature perturbations

With the universe comprising axions and radiation, one can define the **isocurvature perturbation**

$$S_{A\gamma} = \left. \frac{\delta n_A}{n_A} - \frac{\delta n_{\rm rad}}{n_{\rm rad}} \right|_{\delta\rho=0} = \left. \frac{\delta n_A}{n_A} - 3\frac{\delta T}{T} \right|_{\delta\rho=0}, \quad n_A \propto \theta^2$$

This is to be contrasted with the **density or curvature perturbation** *R*.

One can define **power spectra** and an **isocurvature fraction**

$$\langle S_{A\gamma}(\mathbf{k}) \mathbf{S}_{\mathbf{A}\gamma}(\mathbf{k}') \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \mathbf{P}_{\mathbf{A}\gamma}(|\mathbf{k}|) \langle R(\mathbf{k}) \mathbf{R}(\mathbf{k}') \rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \mathbf{P}_{\mathbf{R}}(|\mathbf{k}|)$$

$$P_X(|\mathbf{k}|) \equiv \frac{2\pi^2}{|\mathbf{k}|^3} \Delta_{\mathbf{X}}(|\mathbf{k}|)$$

$$\beta_{\rm iso}(|\mathbf{k}|) = \frac{\mathbf{P}_{\mathbf{A}\gamma}(|\mathbf{k}|)}{\mathbf{P}_{\mathbf{A}\gamma}(|\mathbf{k}|) + \mathbf{P}_{\mathbf{R}}(|\mathbf{k}|)} = \frac{\mathbf{\Delta}_{\mathbf{A}\gamma}(|\mathbf{k}|)}{\mathbf{\Delta}_{\mathbf{A}\gamma}(|\mathbf{k}|) + \mathbf{\Delta}_{\mathbf{R}}(|\mathbf{k}|)}$$

CMB measurements imply

 $\beta_{\rm iso}(0.002 {\rm Mpc}^{-1}) < 0.035$

[Planck 2018]

Dark matter has dark consequences

Axion field *A* a combination of scalar phases. With a single complex scalar:

$$\phi = \frac{f}{\sqrt{2}}e^{i\theta} = \frac{f}{\sqrt{2}}e^{iA/f}. \quad n_A \propto A^2 = f^2\theta^2$$

Deep in the radiation era, the **isocurvature** is **dominated by the axion**

$$\rho_{\rm rad} \gg \rho_A \Rightarrow S_{A\gamma} \approx \frac{\delta n_A}{n_A} \qquad \theta = \langle \theta \rangle + \delta \theta \Rightarrow \langle S_{A\gamma} S_{A\gamma} \rangle \approx \frac{4}{\langle \theta \rangle^2} \langle \delta \theta \delta \theta \rangle = \frac{4}{\theta_I^2} \langle \delta \theta \delta \theta \rangle = \frac{4}{\theta_I^2} \Delta_{\delta \theta} \delta_{\delta \theta}$$

If the axion is a massless field during inflation, power spectrum freezes for k < aH

Power spectrum fixed by the only relevant dimensionful scale, H

$$\Delta_{\delta\theta} = \frac{1}{f^2} \Delta_{\delta A} = \frac{1}{f^2} \left(\frac{H}{2\pi}\right)^2 \qquad \qquad \Delta_{A\gamma} \approx \frac{4}{\theta_I^2} \Delta_{\delta\theta} = \frac{4}{f^2 \theta_I^2} \left(\frac{H}{2\pi}\right)^2$$

Substituting relic abundance constraint and imposing CMB bound,

$$H \lesssim \frac{5 \times 10^{13} \,\text{GeV}}{(f(\text{GeV}))^{0.165}}$$
 e.g. $H < 7 \times 10^{11} \,\text{GeV}$ for $f > 10^{11} \,\text{GeV}$

Dark matter has dark consequences

But the simplest inflationary models favour larger values of H during inflation!



Non-standard inflation?

f or mass dynamical (and large) during inflation?

Need inflaton-axion interactions

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Non-standard inflation?

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Model A: Axion embedded into inflaton

1 complex scalar with PQ charge, containing axion and inflaton

[Fairbairn & Marsh]

Axion still treated as massless, but f becomes Planckian inflaton value

$$\Delta_{A\gamma} \approx \frac{4}{f^2 \theta_I^2} \left(\frac{H}{2\pi}\right)^2 \to \frac{4}{\rho_{\inf}^2 \theta_I^2} \left(\frac{H}{2\pi}\right)^2$$

 $f_A < 1.4 \times 10^{14} \mathrm{GeV}$

[Fairbairn & Marsh] [Ballesteros, Redondo, Ringwald, CT]

Incompatible with GUTs?

Model B: Axion very massive during inflaton

2 complex scalars with PQ charges, inflaton mostly in ϕ_1 , axion contained in ϕ_2

[Nakayama, Takimoto]

During inflation, lightest direction mostly aligned with ϕ_1 : axion isocurvature fluctuations suppressed



After inflation: Avoid ϕ_1 VEV so that QCD axion contained in ϕ_2 , which had suppressed fluctuations.

Compatible with GUTs?

Invalid assumptions?

For model A it was assumed:

Axion field is massless during inflation

If axion interacts with inflaton, the field is not at its minimum during inflation: Goldstone theorem does not apply and the axion is massive

For models A, B it was assumed

Primordial angular perturbations fixed after horizon crossing during inflation

Not true for massive fields or with inflaton interactions! Non-perturbative phenomena during reheating are known to affect angular perturbations



In previous work we showed that the PQ symmetry is restored by nonperturbative effects for type A models with low f.

We hypothesized that PQ could remain broken for $f > 10^{16}$ GeV, but did not confirm this with dedicated simulations

[Ballesteros, Redondo, Ringwald, CT]



Improved calculations

To-do list

For models A and B

Solve numerically the **evolution of inflationary perturbations**, accounting for nonzero masses

Use the above as initial conditions for **multi-field lattice simulations** for complex scalars + Higgs) including effects of **Higgs decays** and **SM radiation**

Derive power spectra for isocurvature perturbation and extrapolate to CMB scales

Model A setup

$$\sqrt{-g}\mathcal{L} \supset \sqrt{-g}\left\{ \left(\frac{M_P^2}{2} + \xi\left(\phi^{\dagger}\phi - \frac{v_{\phi}^2}{2}\right)\right) R - \lambda_{\phi}\left(\phi^{\dagger}\phi - \frac{v_{\phi}^2}{2}\right) - \lambda_h\left(h^2 - \frac{v_h^2}{2}\right) - 2\lambda_{h\phi}\left(\phi^{\dagger}\phi - \frac{v_{\phi}^2}{2}\right)\left(h^2 - \frac{v_h^2}{2}\right)\right\}$$

 $\phi = \phi_1 + i\phi_2$ Inflaton field, containing axion

h : Higgs field

Evolution of perturbations during inflation

With $\delta \phi = \delta \phi_1 + i \delta \phi_2$ we separate the fluctuations components into

 $\delta \phi_{\parallel}$ aligned with inflationary background $\delta \phi_{\perp}$ orthogonal to background

Potential admits straight inflationary trajectories, with misalignment angle θ_I

$$\delta\phi_{\parallel} = \cos\theta_I \delta\phi_1 + \sin\theta_I \delta\phi_2, \quad \delta\phi_{\perp} = \cos\theta_I \delta\phi_2 - \sin\theta_I \delta\phi_1$$

For straight trajectories, perturbations in flat spatial curvature gauge satisfy

$$\delta\ddot{\phi}_{\parallel} + 3H\delta\dot{\phi}_{\parallel} + \left(\frac{\mathbf{k}^{2}}{a^{2}} + m_{\parallel}^{2} - \frac{1}{M_{P}^{2}a^{3}}\frac{d}{dt}\left(\frac{a^{3}\dot{\phi}_{\parallel}^{2}}{H}\right)\right)\delta\phi_{\parallel} = 0$$

$$\delta\ddot{\phi}_{\perp} + 3H\delta\dot{\phi}_{\perp} + \left(\frac{\mathbf{k}^{2}}{a^{2}} + m_{\perp}^{2}\right)\delta\phi_{\perp} = 0$$
[Gordon et al]

Power spectra at the end of inflation

Boundary conditions: match result in the **local Minkowski frame well below the horizon:**

$$\phi_{i,\mathbf{k}}(t) \rightarrow \frac{e^{-i\omega\tau(t)}}{(2\omega)^{1/2}}, \ t \rightarrow -\infty, \quad \omega = \sqrt{\mathbf{k}^2 + a(t)^2 m_i^2}$$

These solutions with this **normalization** are **associated with quantized fields**

$$\hat{\phi}_i(\mathbf{k}, \mathbf{t}) = \phi_{\mathbf{i}, \mathbf{k}}(\mathbf{t}) \mathbf{a}_{\mathbf{k}} + \phi_{\mathbf{i}, \mathbf{k}}(\mathbf{t})^* \mathbf{a}_{\mathbf{k}}^{\dagger}, \quad [\mathbf{a}_{\mathbf{i}\mathbf{k}}, \mathbf{a}_{\mathbf{i}\mathbf{k}'}^{\dagger}] = (\mathbf{2}\pi)^{\mathbf{3}} \delta(\mathbf{k} - \mathbf{k}')$$

such that the corresponding power spectra are determined by the $\phi_{i,\mathbf{k}}(t)$

$$\left\langle \hat{\phi}_i(\mathbf{k}, \mathbf{t}) \hat{\phi}_i(\mathbf{k}'.\mathbf{t}) \right\rangle = (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}') \mathbf{P}_{\phi_i}(|\mathbf{k}|.\mathbf{t}), \quad \mathbf{P}_{\phi_i}(|\mathbf{k}|.\mathbf{t}) = |\phi_{i,\mathbf{k}}(\mathbf{t})|^2$$

We can infer power spectra from properly normalized solutions of the equations of motion

Power spectra at the end of inflation



Assumed: $\xi = 0.1, \quad \lambda_{h\phi} = 10^{-6}$

Isocurvature power spectrum at the end of inflation

From the previous result we can estimate the **power spectrum of the isocurvature** perturbation at the end of inflation



Assumed: $f_A = 5 \times 10^{17} \text{GeV}, \quad \xi = 0.1$

2 different lattices with 512³ points

Isocurvature power spectrum at the end of inflation

The previous results show that the **(small) axion mass suppresses the power spectrum at super-horizon scales,** but the effect is very mild

Do the isocurvature modes decay further after inflation?

What happens when the background crosses the origin and the ϕ_2 mass becomes negative? Is there any instability/growth?



Lattice simulations of model A

We simulate **3 real scalars** $\phi_1(t, \mathbf{x}), \phi_2(t, \mathbf{x}), h(t, \mathbf{x})$ with *h* decaying into a **relativistic bath of SM particles** with density $\rho_{SM}(t)$, in an **expanding universe** with scale factor a(t)

$$\begin{split} \ddot{\phi}_{k} + 3\frac{\dot{a}}{a}\dot{\phi}_{k} - \frac{1}{a^{2}}\vec{\nabla}^{2}\phi_{k} + \frac{\partial V(\phi_{l})}{\partial\phi_{k}} &= 0, \ k = 1, 2, \\ \ddot{h} + 3\frac{\dot{a}}{a}\dot{h} - \frac{1}{a^{2}}\vec{\nabla}^{2}h + \frac{\partial V(\phi_{l})}{\partial h} + \Gamma_{h}\dot{h} &= 0, \\ \dot{\rho}_{SM} + 4\frac{a}{a}\rho_{SM} &= \Gamma_{h}\dot{h}^{2}, \\ \left(\frac{\dot{a}}{a}\right)^{2} &= \frac{1}{3M_{P}^{2}}\left(\rho_{SM} + V + \frac{1}{2}(\dot{\phi}_{1}^{2} + \dot{\phi}_{2}^{2} + \dot{h}^{2}) + \frac{1}{2a^{2}}\left((\nabla\phi_{1})^{2} + (\nabla\phi_{2})^{2} + (\nabla h)^{2}\right)\right) \end{split}$$

We use a modified version of "latticeeasy" [Felder, Tkachev]. Changes account for:

Higgs decays SM radiation and impact on scale factor evolution Modified initial conditions for super-horizon modes

We fix $\xi = 0.1$, $\lambda_{\phi} = 1.27 \times 10^{-11}$ and vary f_A , $\lambda_{h\phi}$

Relevant results for $f=10^{11}$ GeV



Relevant results for $f=10^{13}$ GeV



Relevant results for $f=10^{15}$ GeV



similar results for f_A all the way up to...

Relevant results for $f = 5 \times 10^{17} \text{ GeV}$



First conclusions

The avoidance of the restoration of the PQ symmetry from short-range fluctuations favours GUT-scale values for f_A

As hypothesized by [Ballesteros, Redondo, Ringwald, CT], the **trapping** of inflaton **in potential well** is key, but one needs larger f_A than initially thought

Still to do

Estimate isocurvature fraction at CMB reference scale. Need:

Power spectra of isocurvature

Extrapolation to low scales

Results for isocurvature spectra

Super-horizon isocurvature modes have an initial exponential growth but then decay as $1/a(t)^2!$



Lower momenta in red, higher momenta in blue In gray: $10^6 \phi_1(\tau_{\rm pr})/M_P$

Note correlation of isocurvature peaks with crossings of the origin by the inflaton

Why the $1/a(t)^2$ decay?

Follows when the fields become **small fluctuations around the VEV** of the PQ field.

Massless perturbations in Im ϕ see a negligible potential and redshift as 1/a(t)

The initial exponential growth is larger for even lower modes

Power spectrum at CMB scales and times will be a result of the competition between initial amplification and later decay

As $\Delta_{A\gamma}(|\mathbf{k}|, \tau)a(\tau)^2/\Delta_{A\gamma}(|\mathbf{k}|, 0)$ seems to always peak and then oscillate around a constant value, we can estimate a bound

$$\Delta_{A\gamma}(|\mathbf{k}|,\tau) \lesssim \frac{\Delta_{A\gamma}(|\mathbf{k}|,0)}{a(\tau)^2} \operatorname{Max}\left(\frac{\Delta_{A\gamma}(|\mathbf{k}|,\tau)a(\tau)^2}{\Delta_{A\gamma}(|\mathbf{k}|,0)}\right)$$

From initial conditions

Amplification factor estimated from lattice simulations



The plot shows 13 lattice simulations with different grid (N=64, 128) and box sizes.

Now we only have to extrapolate along \sim 51 orders of magnitude to reach CMB scales :)!



Cleanup: retain only larger grids, remove high and low momenta of each simulation, average over simulations with overlapping momenta.

For chosen parameters this gives upper bound on isocurvature power spectrum at CMB scale/time of $O(1) \times (value at end of inflation)$. Ruled out by Planck!

Can go **beyond upper bounds** by simulating for longer times, and computing averages of enhancement factors.



Have found no viable model, though it is hard to scan parameter space...

Effect of Higgs portal coupling

Amplification factor of lowest mode, from small to large values of Higgs portal.



Portal coupling can source axion fluctuations at intermediate times

Have found no viable model, though it is hard to scan parameter space...

Model B

In model B, requiring axion to be aligned with field 2 gives potential energy valley with positive effective mass \rightarrow **no effective potential well**!



With **no well**, **enhancement of fluctuations** can be more efficient than for model A... Adding a VEV for ϕ_1 would give back an axion-inflaton model of type A.

Model B



Much higher amplification than in model A

Conclusions

The life of an axion isocurvature mode is richer than it was thought.

Traditional arguments for computing axion isocurvature perturbations are based on **invalid assumptions** of **masslessness** and **freezout** at horizon crossing

The axion **isocurvature** perturbation can evolve rather dramatically after inflation, with **early exponential growth and late-time decay**

GUT-scale f_A helps in avoiding restoration of PQ symmetry

The **simplest models don't seem to work** as the extrapolation of the isocurvature power spectrum to CMB scales gives values incompatible with Planck bounds

We are axing simple pre-inflationary axion models