Dark matter waves

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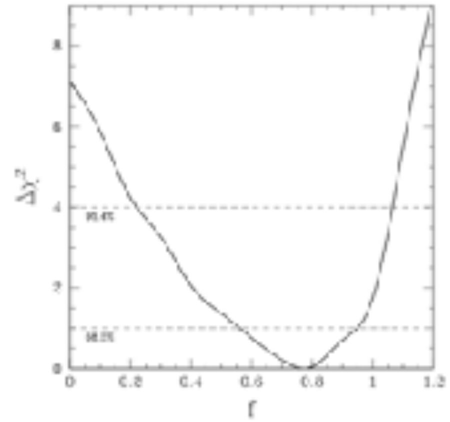
Work done with
Jerry Ostriker, Scott Tremaine, Edward Witten 1610.08297
Xinyu Li, Greg Bryan 1810.01915
Dan Kabat, Xinyu Li, Luca Santoni, Sam Wong 1904.12803
Austin Joyce, Michael Landry, Xinyu Li 2004.01188

Rich evidence for dark matter - from its gravitational effects

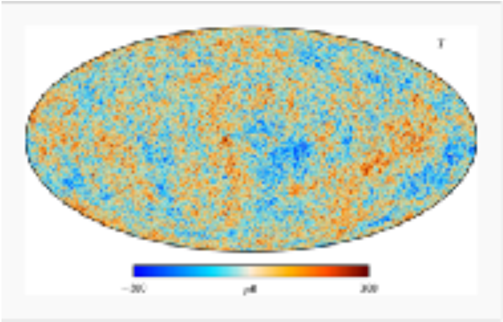
- Dynamical measurements.

- Gravitational lensing measurements.

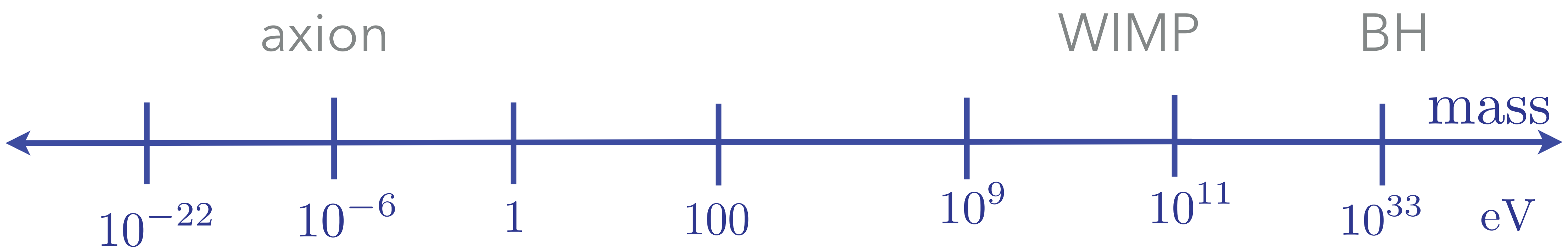


Hoekstra, Yee, Gladders

- Growth of perturbations.



An enormous range in possible DM mass:



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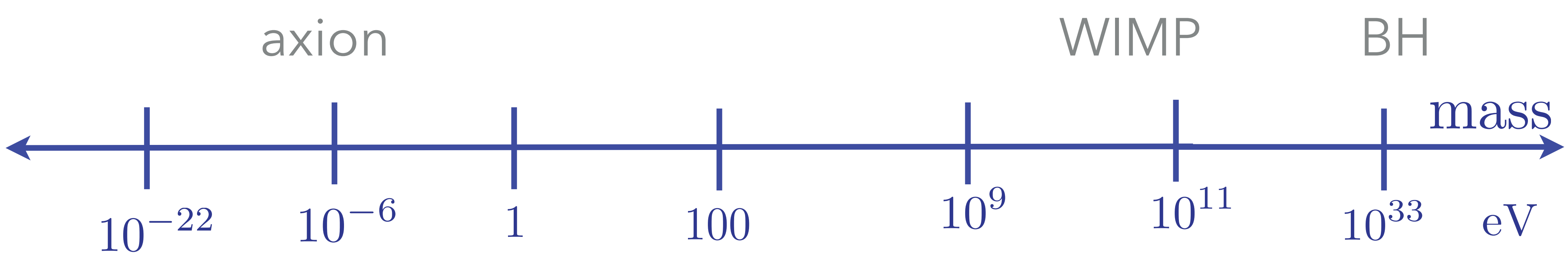

What we know: mass density in solar neighborhood is $0.3\ \mathrm{GeV/cm^3}$

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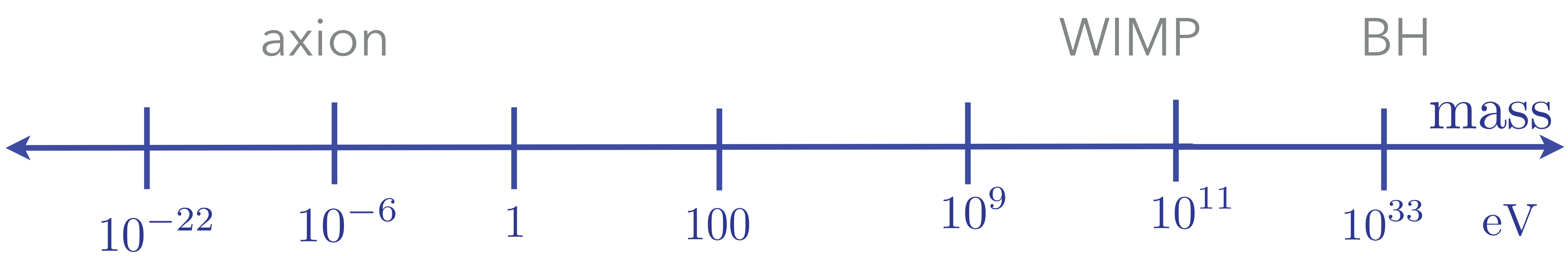

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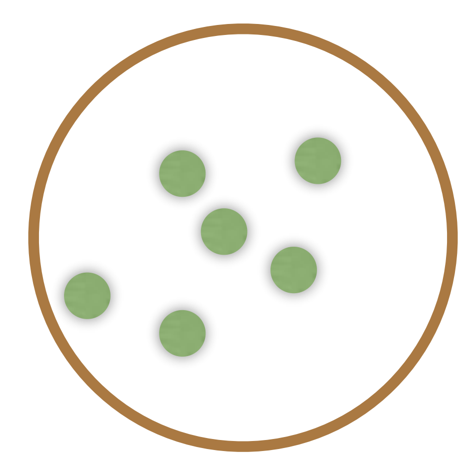

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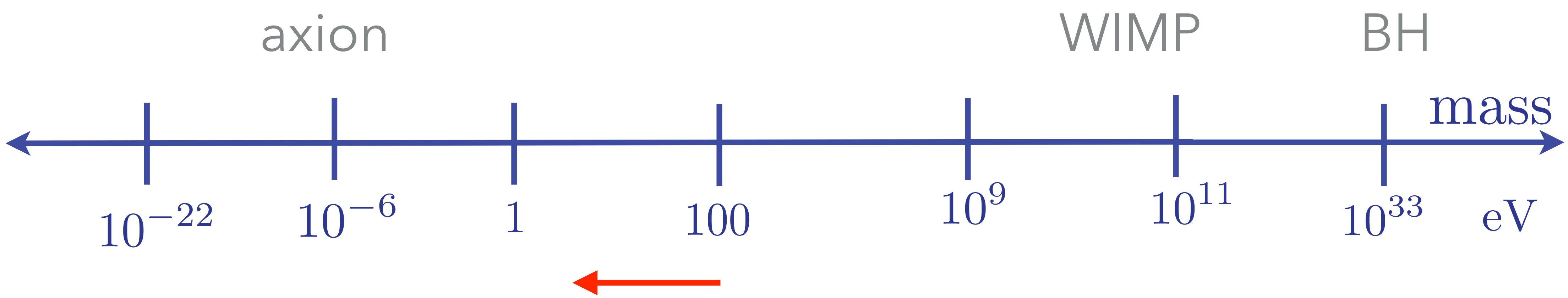

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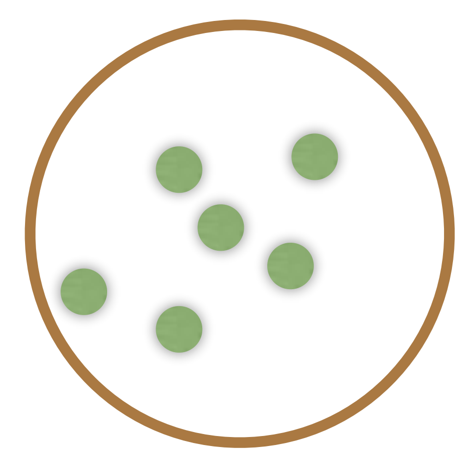
If mean interparticle separation < de Broglie wavelength (1/mv):

wave regime $\quad m < 100\,\mathrm{eV}$



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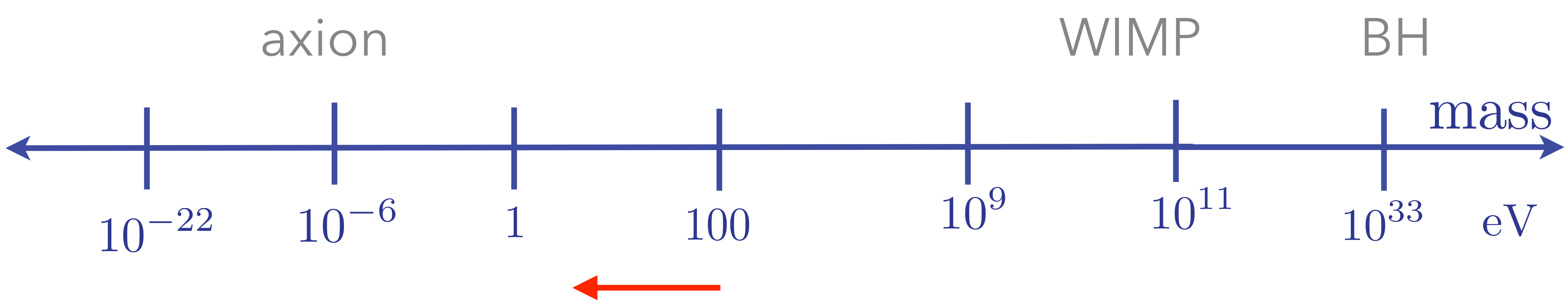

What we know: mass density in solar neighborhood is $0.3\ \mathrm{GeV/cm^3}$

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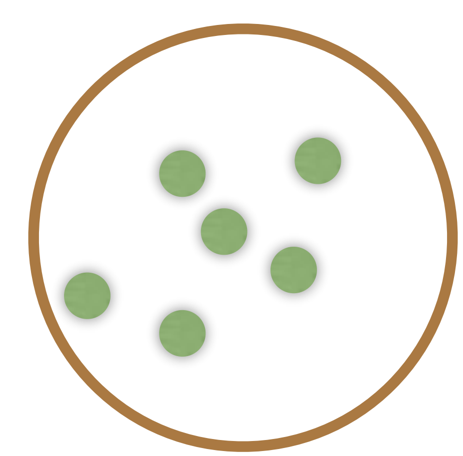
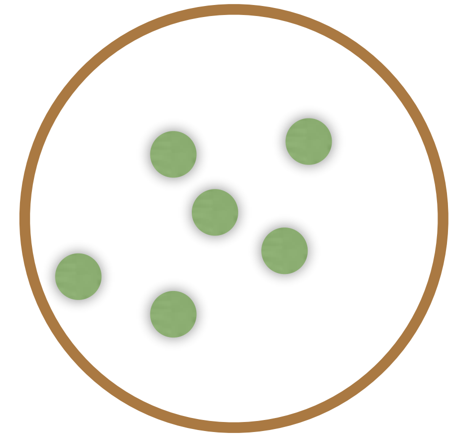
$$1/mv \sim 10^{-3}\,\mathrm{cm} \quad \text{for} \quad m = 10\,\mathrm{eV}$$
$$10^{4}\,\mathrm{cm} \quad \text{for} \quad m = 10^{-6}\,\mathrm{eV}$$
$$100\,\mathrm{pc} \quad \text{for} \quad m = 10^{-22}\,\mathrm{eV}$$





Outline

Particle physics motivations

Wave dynamics and phenomenology

Astrophysical implications (ultra-light DM)

Experimental implications (light DM)

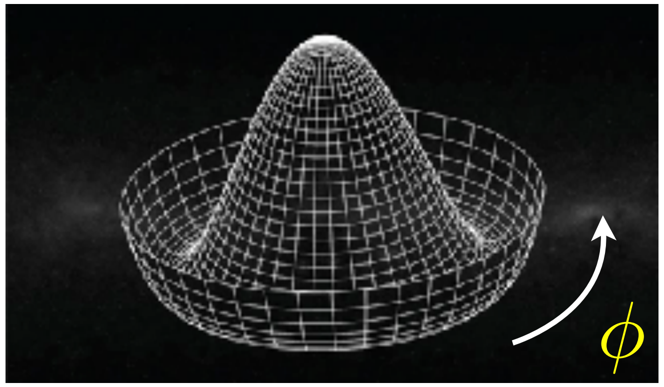
$$\begin{aligned} 1/mv \sim 10^{-3}\,\mathrm{cm} \quad &\mathrm{for} \quad m = 10\,\mathrm{eV} \\ 10^{4}\,\mathrm{cm} \quad &\mathrm{for} \quad m = 10^{-6}\,\mathrm{eV} \\ 100\,\mathrm{pc} \quad &\mathrm{for} \quad m = 10^{-22}\,\mathrm{eV} \end{aligned}$$

QCD axion (for $m = 10^{-6}\,\mathrm{eV}$)

Fuzzy DM (Hu, Barkana, Gruzinov) (for $m = 10^{-22}\,\mathrm{eV}$)

Particle physics motivations

- A natural candidate for a light (scalar) particle is a pseudo-Nambu-Goldstone boson.



- The most well motivated example is the QCD axion, which solves the strong CP problem.
  (Peccei, Quinn; Weinberg; Wilczek; Kim; Shifman, Vainshtein, Zakharov, Zhitnitsky; Dine, Fischler, Srednicki; Preskill, Wise, Wilczek; Abbott, Sikivie)

- There are also plenty of axion-like-particles (ALP) in string theory, from compactification.
  (Svrcek, Witten; Arvanitaki, Dubovsky et al.; Bachlechner, Eckerle, Janssen, Kleban)

Ultra -light version

mass m $\leftarrow$ $10^{-22}\,\mathrm{eV}$ $\rightarrow$

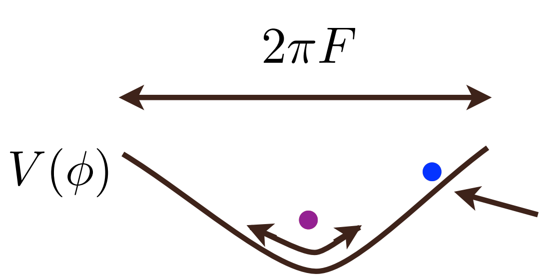
Fuzzy dark matter (FDM)
Hu, Barkana, Gruzinov
Amendola, Barbieri

- Consider an angular field (a pseudo Nambu-Goldstone) of periodicity $2\pi F$ i.e. an axion-like field with a potential from non-perturbative effects (not QCD axion).

$$\mathcal{L} \sim -\frac{1}{2}(\partial\phi)^2 - \Lambda^4(1 - \cos[\phi/F]) \qquad m \sim \Lambda^2/F$$

(candidates: Arvanitaki et al.
Svrcek, Witten)

- Relic abundance matches dark matter abundance.



$\phi \sim F$ at early times until $H \sim m$

$$\Omega_{\mathrm{matter}} \sim 0.1 \left(\frac{F}{10^{17}\,\mathrm{GeV}}\right)^2 \left(\frac{m}{10^{-22}\,\mathrm{eV}}\right)^{1/2}$$

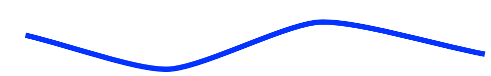
(Preskill, Wise, Wilczek; Abbot, Sikivie; Dine, Fischler, with constant m)

Dynamics of wave dark matter:

- Ignoring self-interactions $\longrightarrow$ $-\Box\phi + m^2\phi = 0$

$$\phi = \frac{1}{\sqrt{2m}}\left[\psi e^{-imt} + \psi^* e^{imt}\right]$$

Non-relativistic limit $\longrightarrow$ $i\dot{\psi} = \left[-\frac{\nabla^2}{2m} + m\Phi_{\rm grav.}\right]\psi$



- An alternative viewpoint: ψ as a (classical) fluid. $\psi = \sqrt{\rho/m}\, e^{i\theta}$ i.e. $\rho = m\,|\psi|^2$

mass conservation $\dot{\rho} + \nabla\cdot\rho v = 0$ where $v = \frac{1}{m}\nabla\theta$

superfluid
(see also Berezhiani, Khoury)

Euler equation $\dot{v} + v\cdot\nabla v = -\nabla\Phi_{\rm grav.} + \frac{1}{2m^2}\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$

21–4The meaning of the wave function

When Schrödinger first discovered his equation he discovered the conservation law of Eq. (21.8) as a consequence of his equation. But he imagined incorrectly that P was the electric charge density of the electron and that $\boldsymbol{J}$ was the electric current density, so he thought that the electrons interacted with the electromagnetic field through these charges and currents. When he solved his equations for the hydrogen atom and calculated ψ, he wasn't calculating the probability of anything—there were no amplitudes at that time—the interpretation was completely different. The atomic nucleus was stationary but there were currents moving around; the charges P and currents $\boldsymbol{J}$ would generate electromagnetic fields and the thing would radiate light. He soon found on doing a number of problems that it didn't work out quite right. It was at this point that Born made an essential contribution to our ideas regarding quantum mechanics. It was Born who correctly (as far as we know) interpreted the ψ of the Schrödinger equation in terms of a probability amplitude—that very difficult idea that the square of the amplitude is not the charge density but is only the probability per unit volume of finding an electron there, and that when you do find the electron some place the entire charge is there. That whole idea is due to Born.

The wave function $\psi(\boldsymbol{r})$ for an electron in an atom does not, then, describe a smeared-out electron with a smooth charge density. The electron is either here, or there, or somewhere else, but wherever it is, it is a point charge. On the other hand, think of a situation in which there are an enormous number of particles in exactly the same state, a very large number of them with exactly the same wave function. Then what? One of them is here and one of them is there, and the probability of finding any one of them at a given place is proportional to $\psi\psi^*$. But since there are so many particles, if I look in any volume $dx\,dy\,dz$ I will generally find a number close to $\psi\psi^*\,dx\,dy\,dz$. So in a situation in which ψ is the wave function for each of an enormous number of particles which are all in the same state, $\psi\psi^*$ *can* be interpreted as the density of particles. If, under these circumstances, each particle carries the same charge q, we can, in fact, go further and interpret $\psi^*\psi$ as the density of *electricity*. Normally, $\psi\psi^*$ is given the dimensions of a probability density, then ψ should be multiplied by q to give the dimensions of a charge density. For our present purposes we can put this constant factor into ψ, and take $\psi\psi^*$ itself as the electric charge density. With this understanding, $\boldsymbol{J}$ (the current of probability I have calculated) becomes directly the electric current density.

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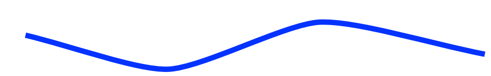

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(see also Berezhiani, Khoury)

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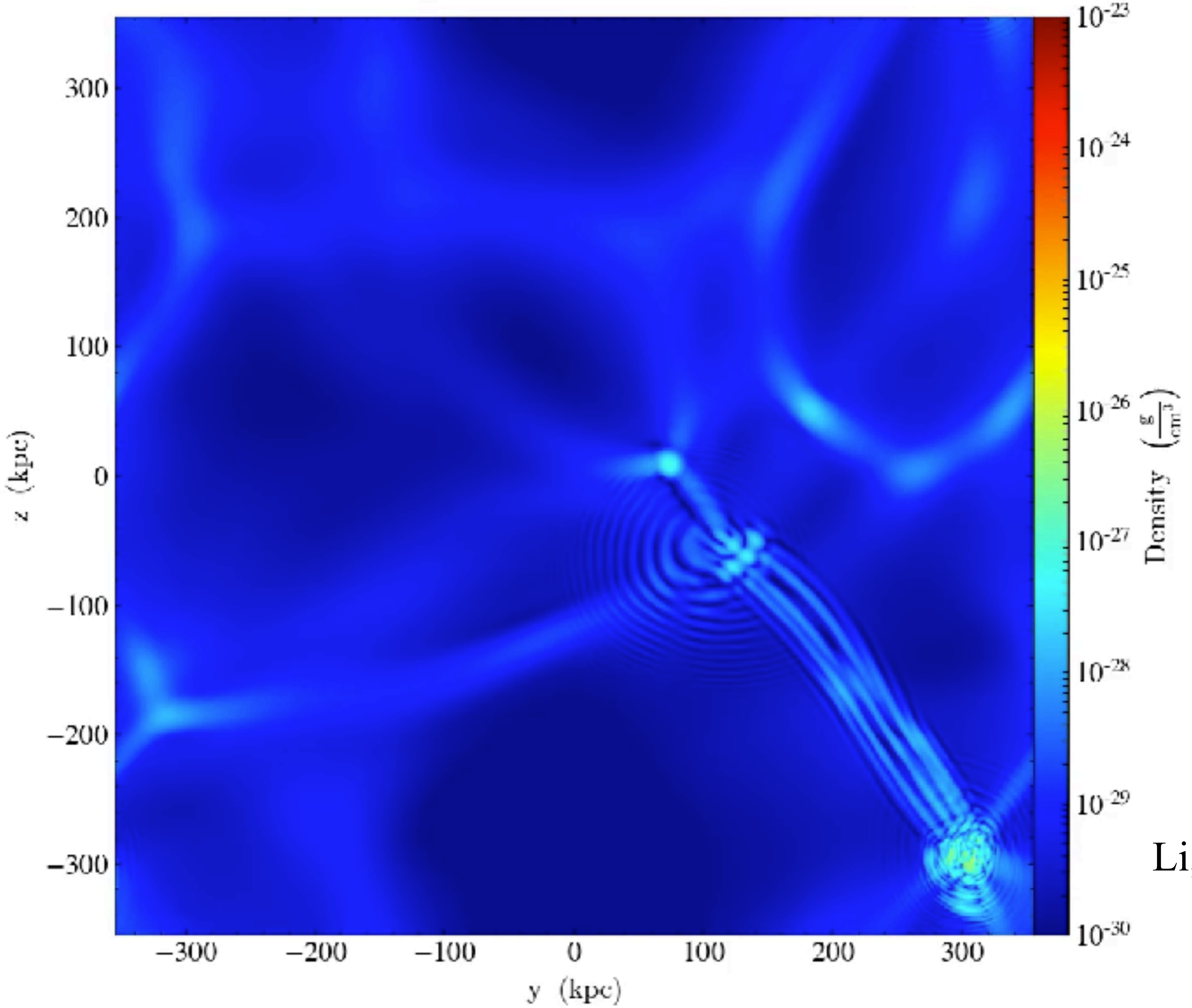
Wave effects in a cosmological simulation

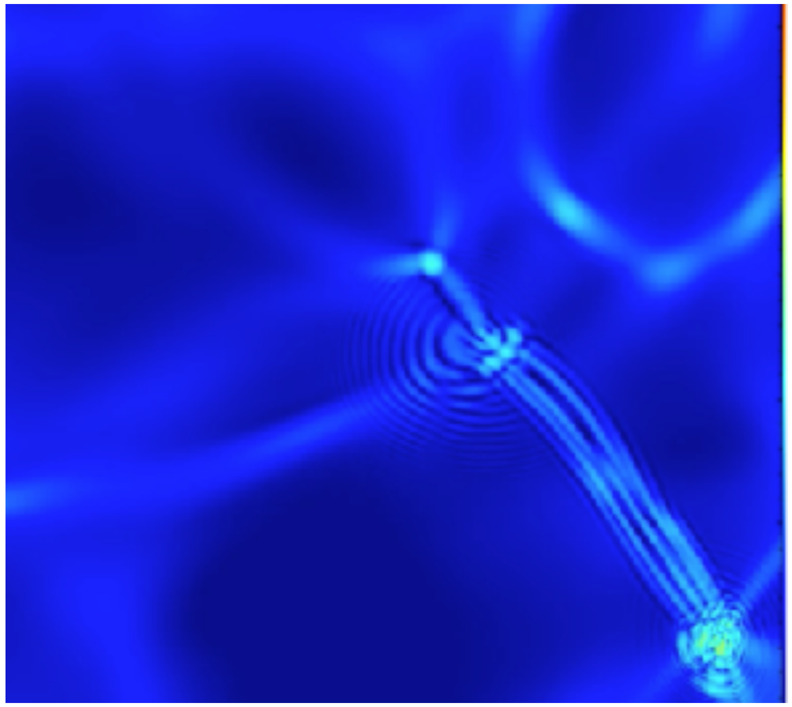


Li, LH, Bryan

See Schive, Chiueh, Broadhurst; Veltmaat, Niemeyer; Schwabe, Niemeyer, Engels; Mocz et al.; Nori, Baldi

Wave effects from light/ultra-light DM:

- dynamical friction
- evaporation of sub-halos by tunneling
- interference
- tidal streams and gravitational lensing
- Lyman-alpha forest
- direct detection
- detection by pulsar timing array



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- dynamical friction

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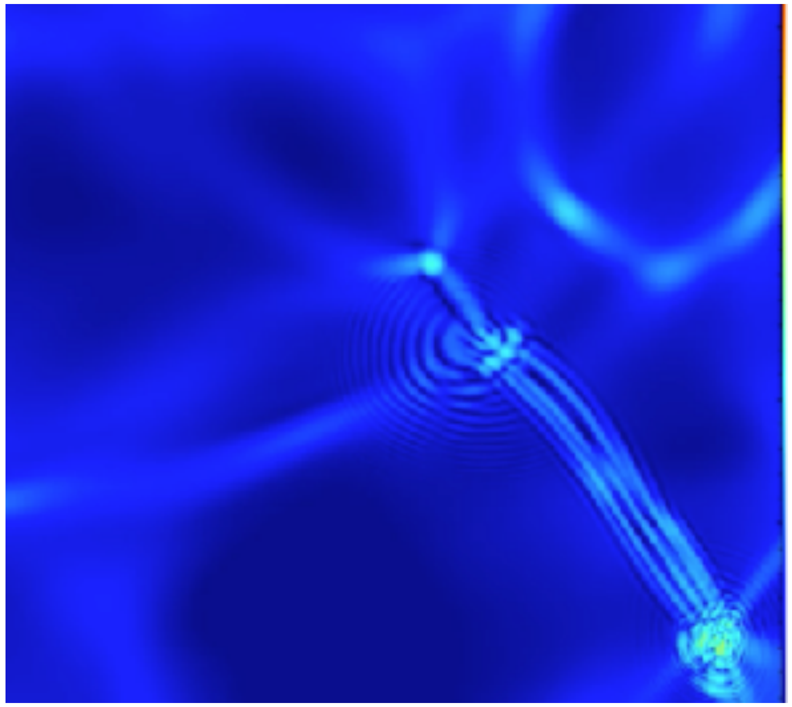
- vortices (and walls)

- black hole hair



Vortices

- Consider again fluid formulation: $\psi = \sqrt{\rho/m}\, e^{i\theta}$

$$\dot\rho + \nabla \cdot \rho v = 0 \quad \text{where} \quad v = \frac{1}{m}\nabla\theta$$

$$\dot v + v \cdot \nabla v = -\nabla\Phi_{\text{grav.}} + \frac{1}{2m^2}\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$$

- Naively, vorticity cannot exist, because the velocity field is a gradient flow. In addition, one might think Kelvin's theorem should hold i.e. no vorticity is generated if there's no vorticity to begin with.

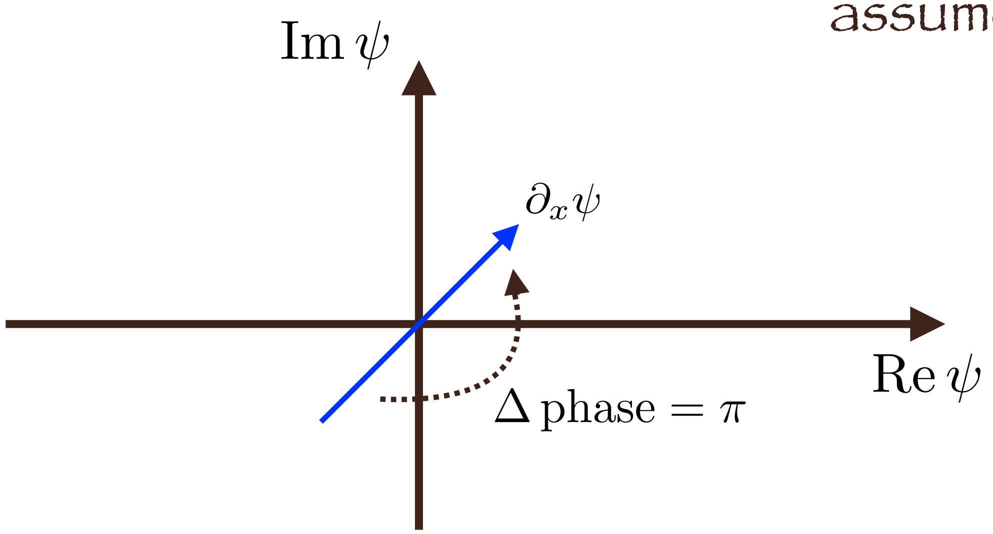
- The loophole: where $\rho = 0$.

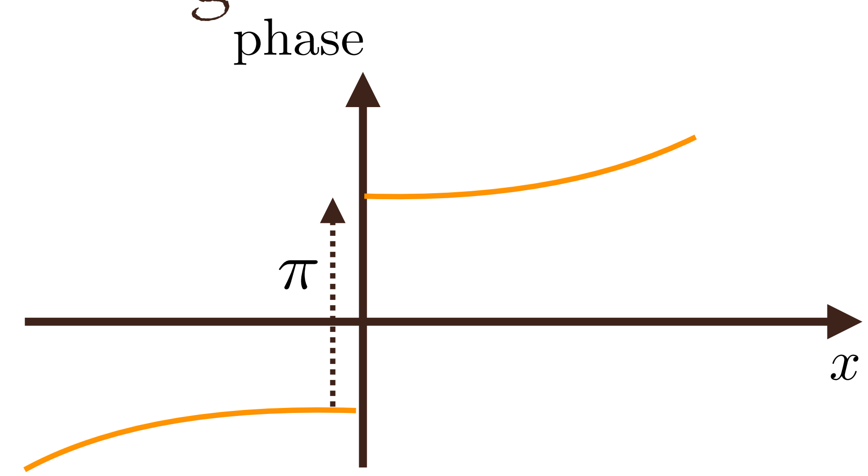
- A simpler example first: a wall defect in 1D

Consider ψ in one spatial dimension. Suppose it vanishes at some point, say x=0.

$$\psi(x) \sim \cancel{\psi(0)} + x\,\partial_x \psi\Big|_0 + \ldots$$

assumed non-vanishing





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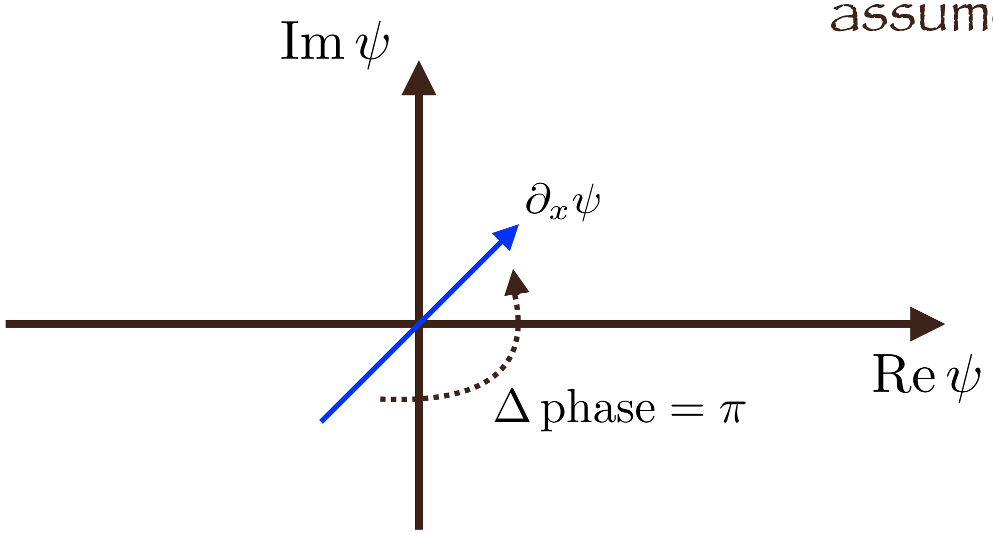
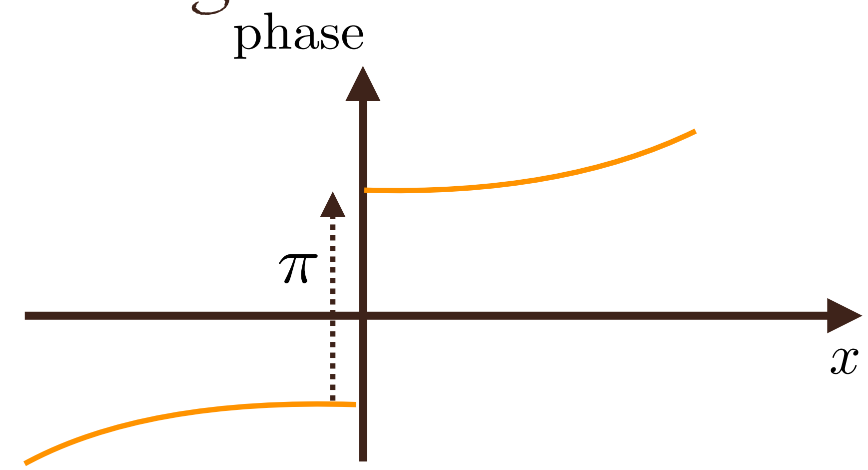
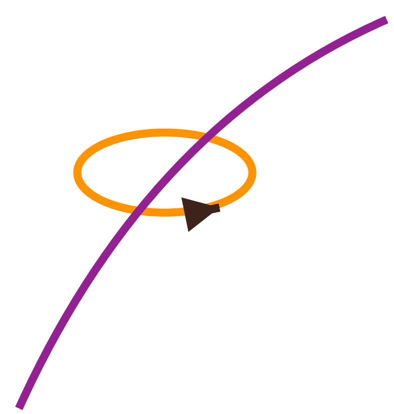




- ## Vortex

Argument generalizes to higher dimensions. In 3D, vanishing of both real & imaginary parts implies intersection of 2 surfaces i.e. a line/string defect -> vortex.

$$\psi(\vec{x}) \sim \cancel{\psi(0)} + \vec{x}\cdot\vec{\partial}\psi\Big|_0 + \ldots$$

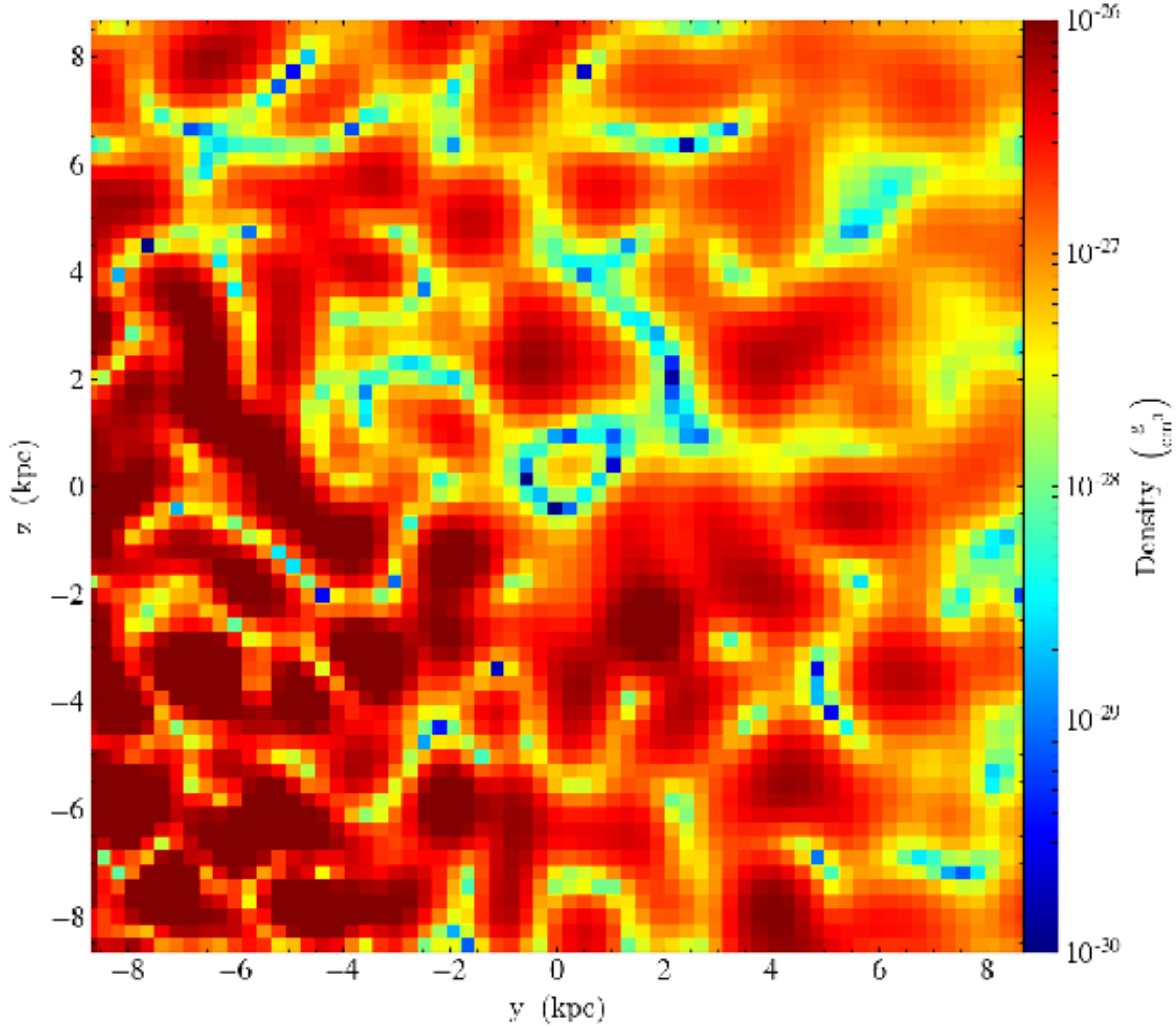


phase wraps by 2π (or $2\pi n$)

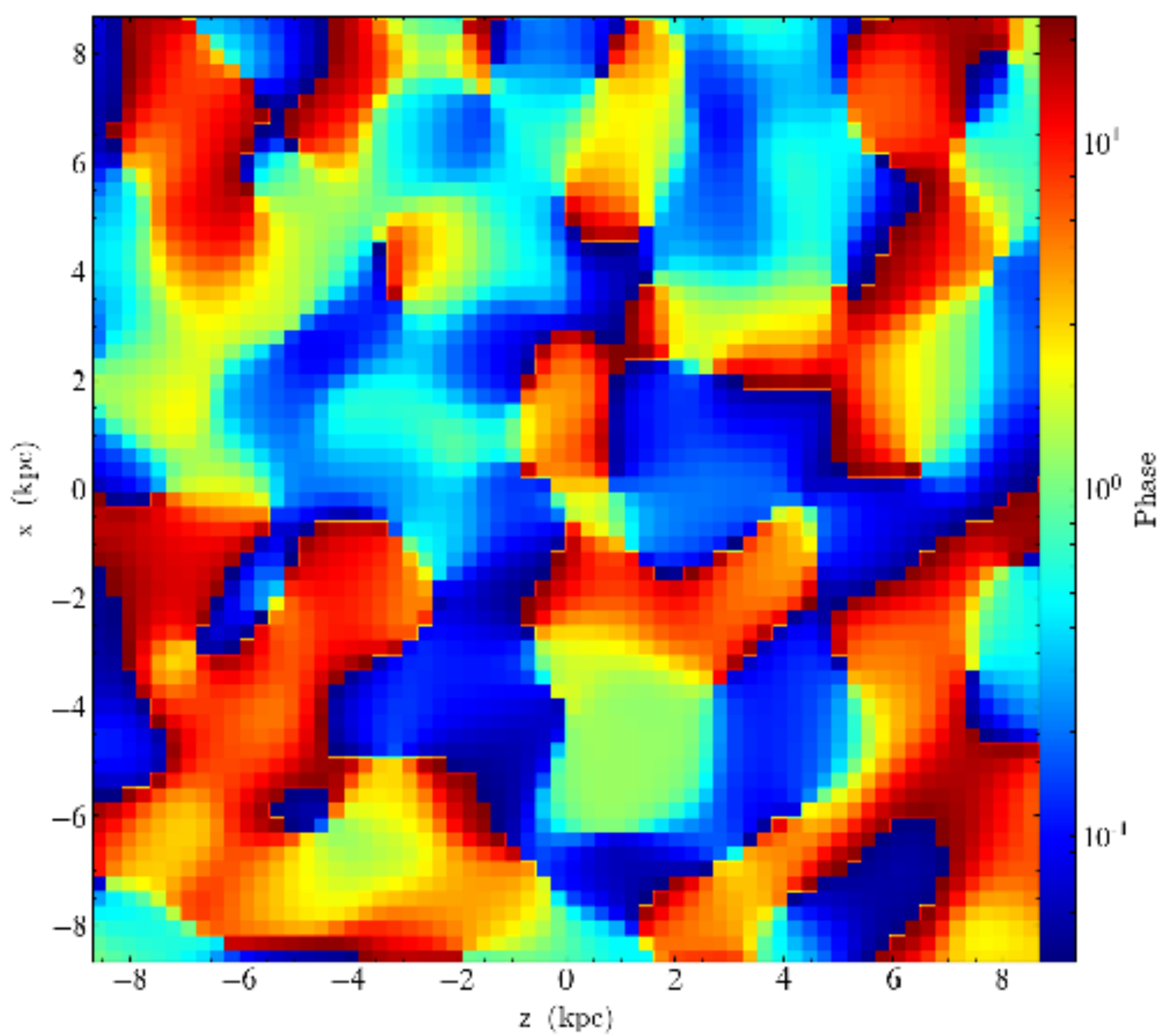
$$\oint \vec{v}\cdot d\vec{\ell} = 2\pi n/m$$

$$\rho \propto r^{2n} \quad \& \quad v \propto 1/r$$

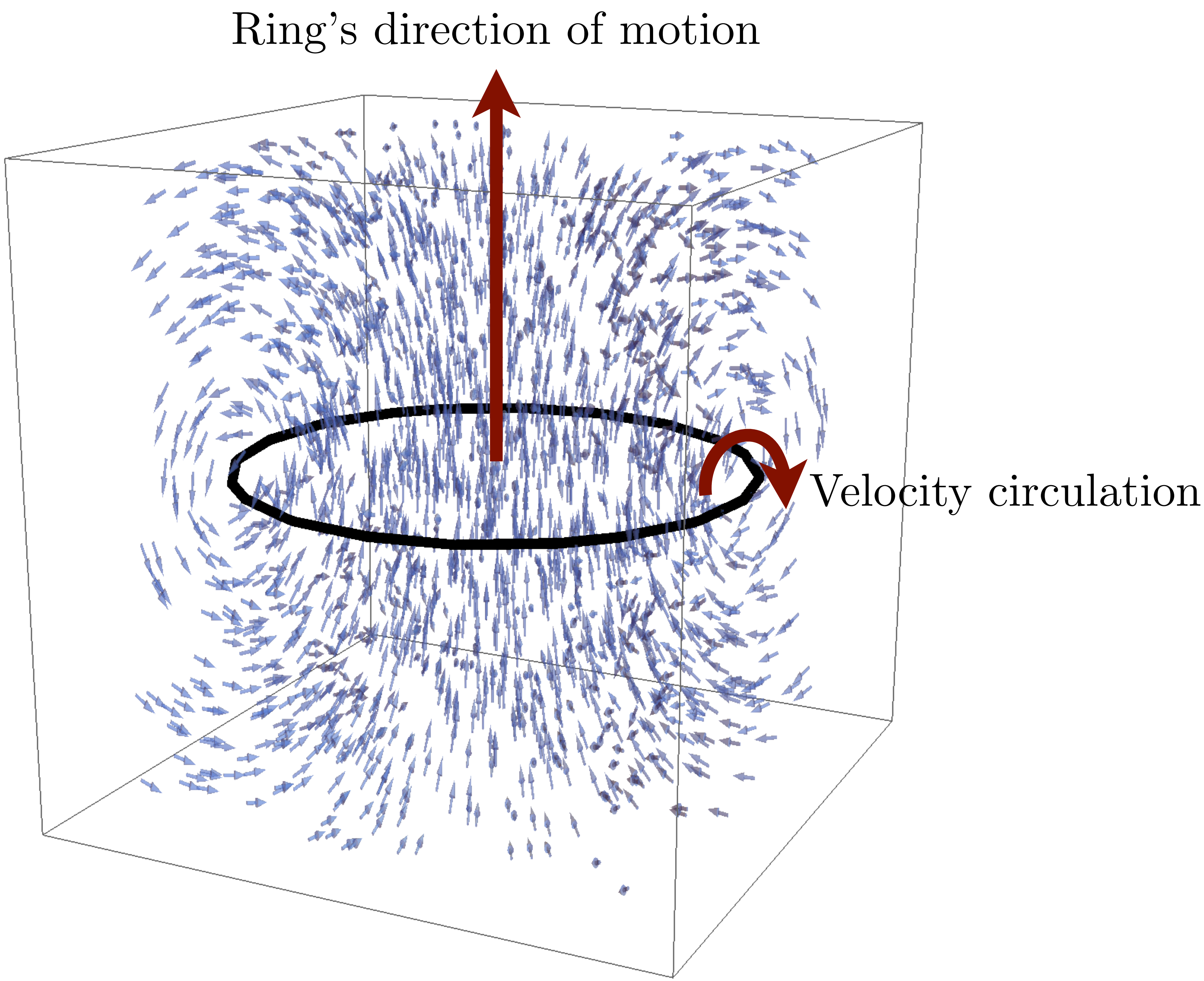
Note: this is not the usual axion string.

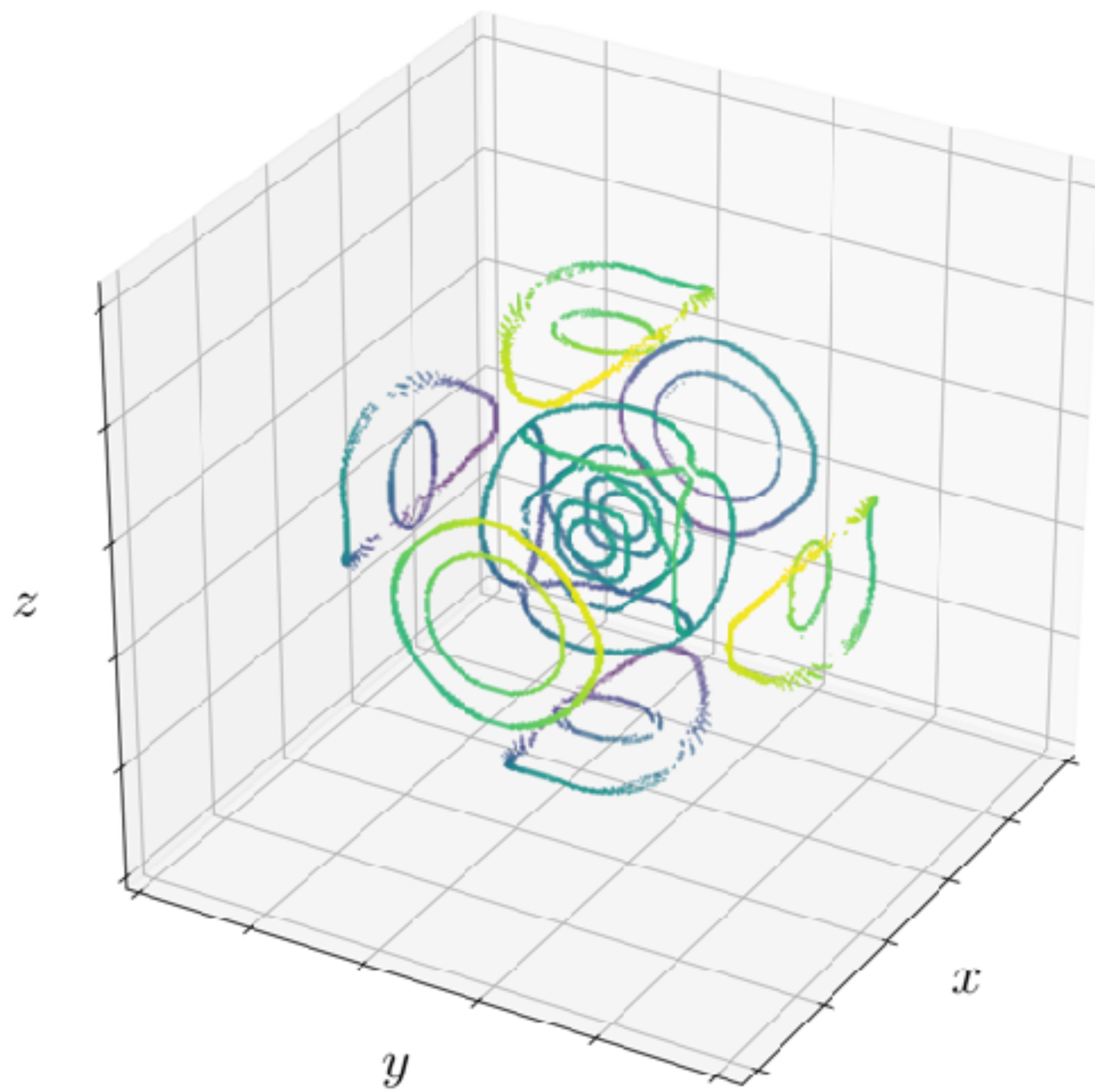


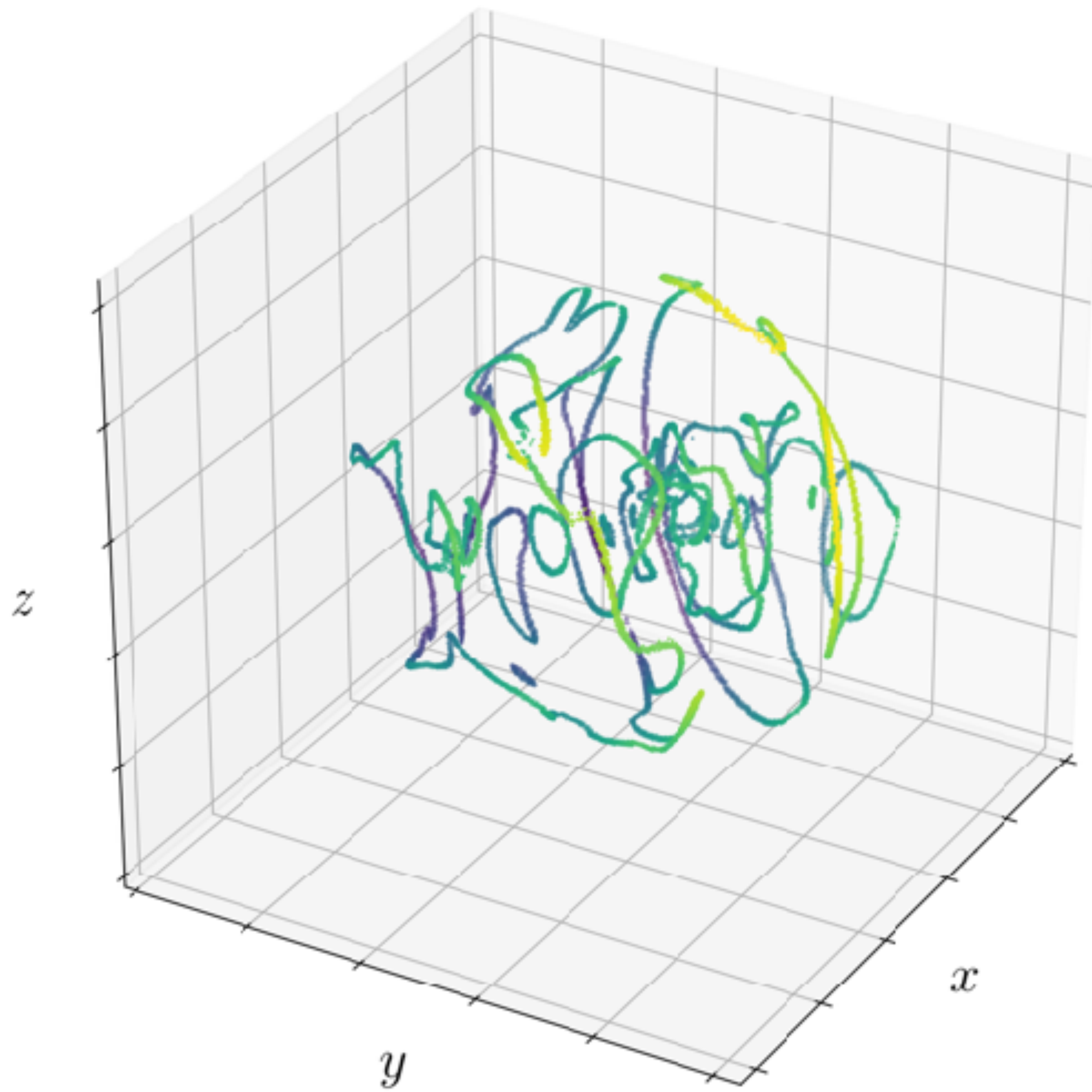
z (kpc)
y (kpc)
Density $\left(\frac{g}{cm^3}\right)$
10^{-26}
10^{-27}
10^{-28}
10^{-29}
10^{-30}



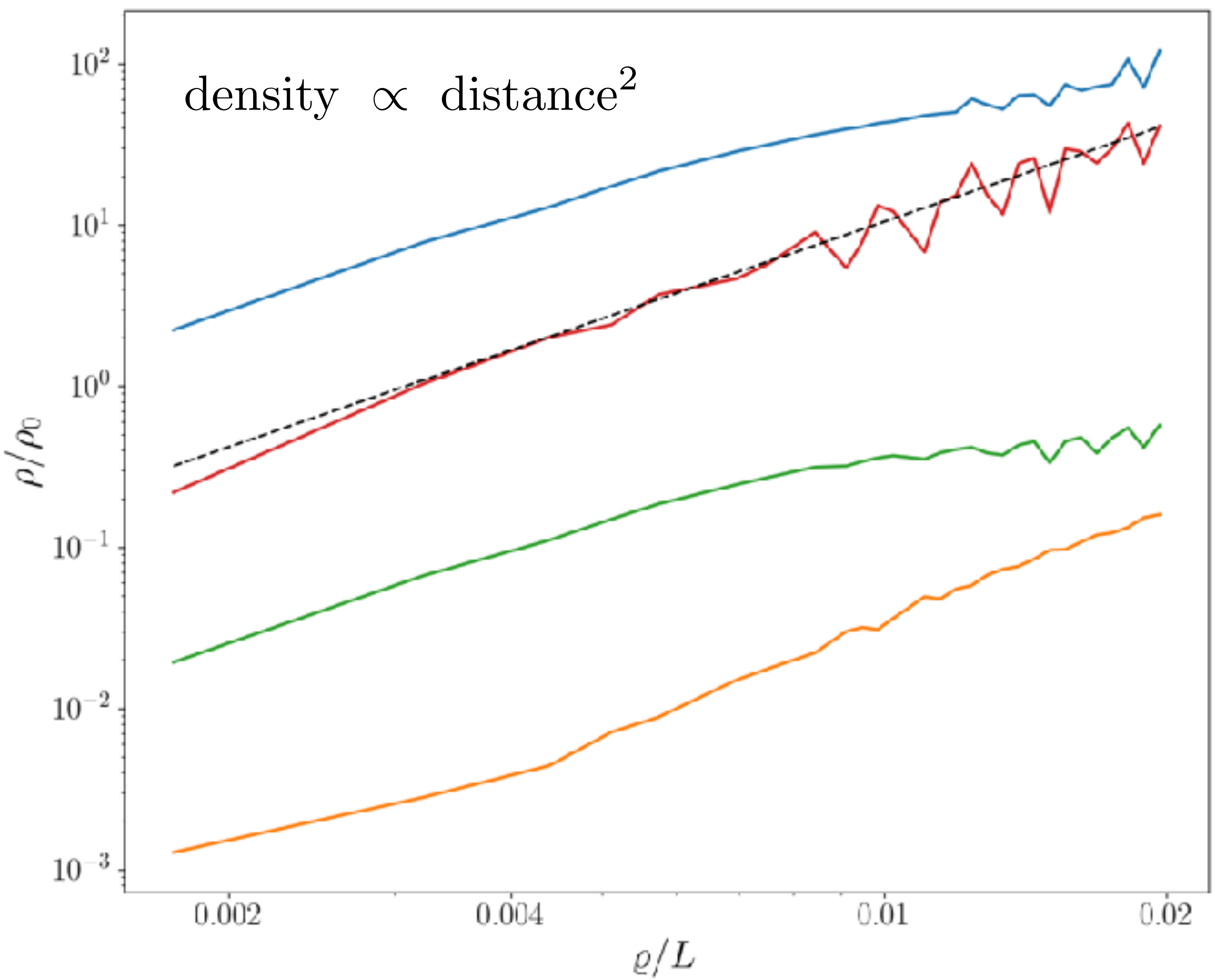
x (kpc)
z (kpc)
Phase
10^1
10^0
10^{-1}
8
6
4
2
0
−2
−4
−6
−8


Ring's direction of motion
Velocity circulation

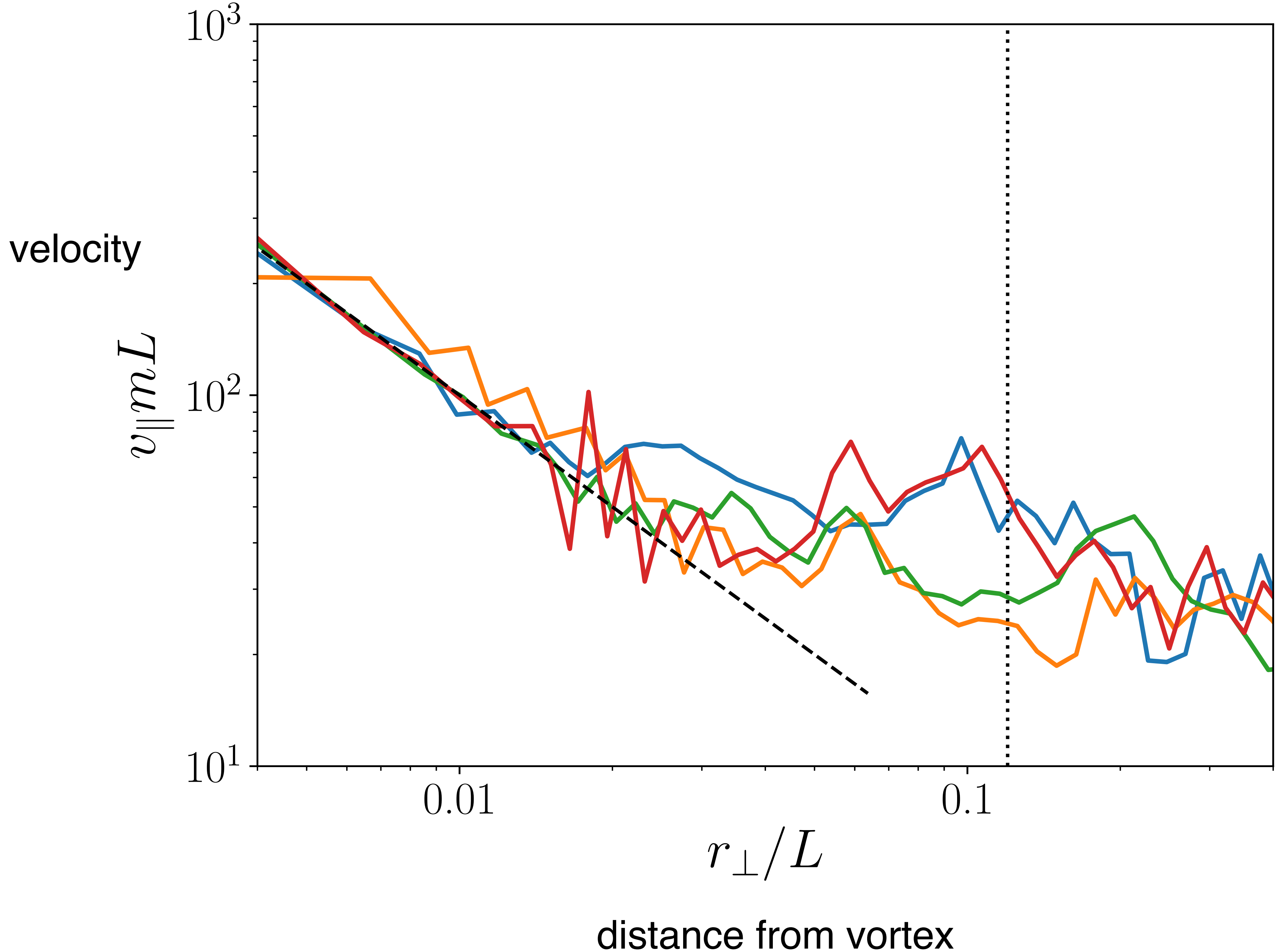

z
y
x

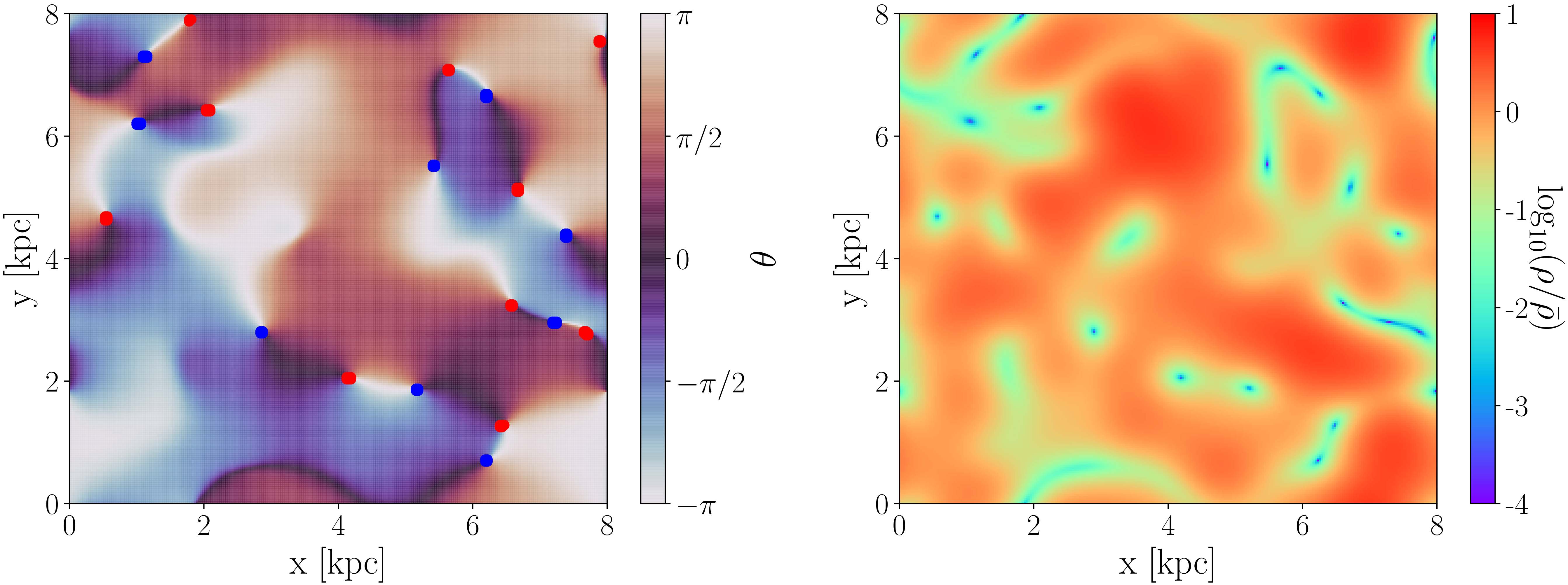

z
y
x

density

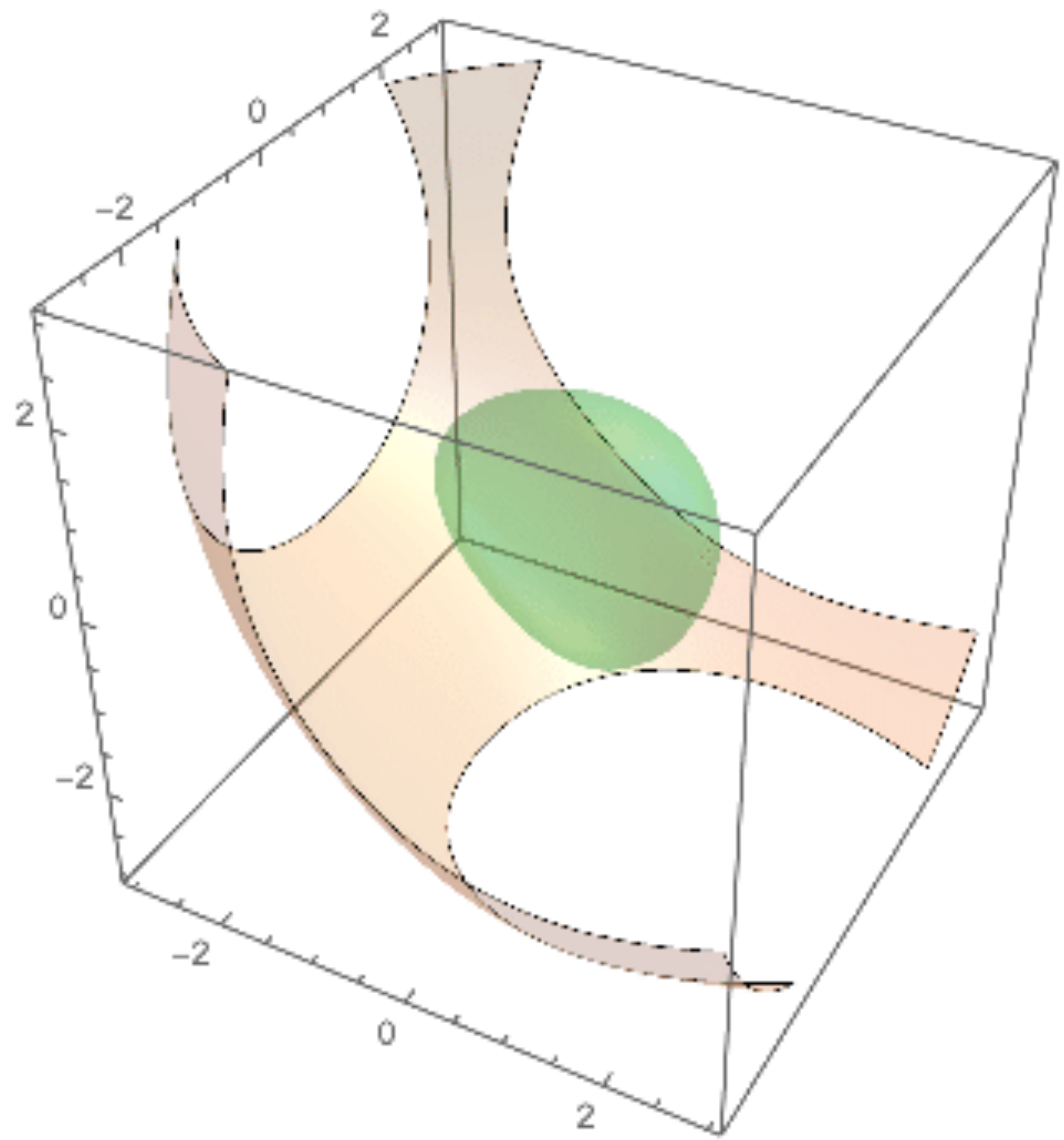


distance from vortex


velocity
$v_{\parallel} m L$
10^3
10^2
10^1
0.01
0.1
$r_{\perp}/L$
distance from vortex



A 2D example built from a superposition of waves with random phases

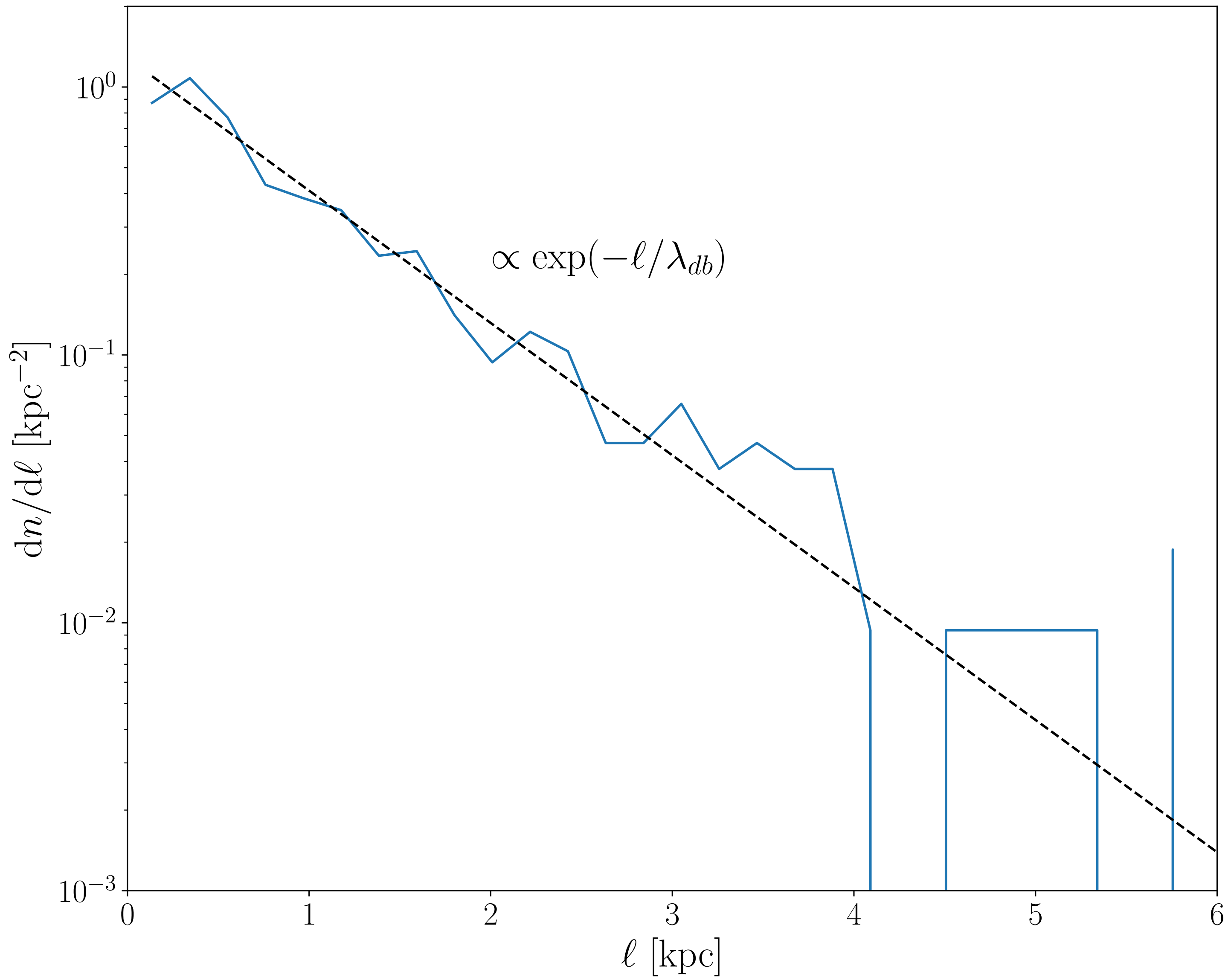

2
0
−2
2
0
−2
−2
0
2

Additional comments:

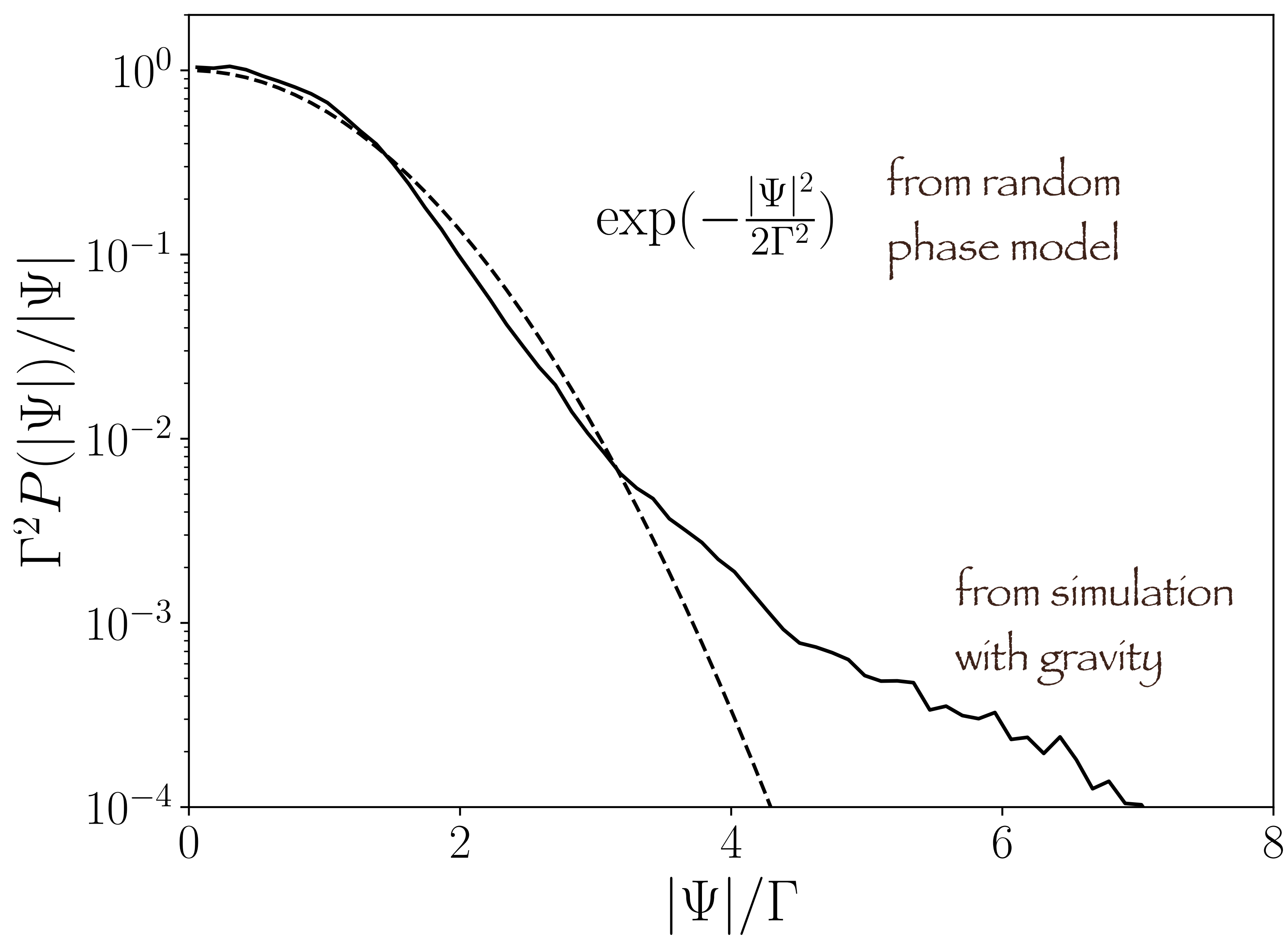
- Should defects be rare? No - roughly one vortex ring per de Broglie volume. Can compute this analytically for a model halo composing of a superposition of waves with random phases: essentially looking for zero-crossing.

- Smaller rings move faster: $v \sim \dfrac{1}{mR}$

- Minimal connection with angular momentum - vortices exist without net rotation of the halo; having angular momentum also does not by itself imply existence of vortices (i.e. can always superimpose s-wave with others).

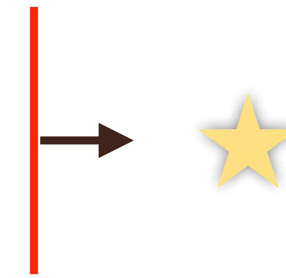


Number density distribution of vortex rings as a function of ring size destructive interference



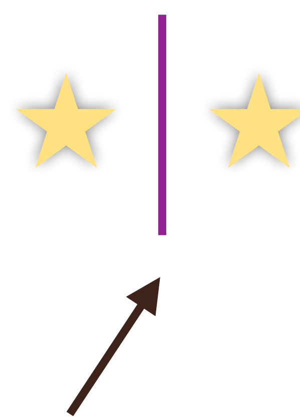
Probability distribution of $|\Psi|$ from simulation and from analytic random phase model (see also Centers et al.)

Observational signatures (for ultralight DM):

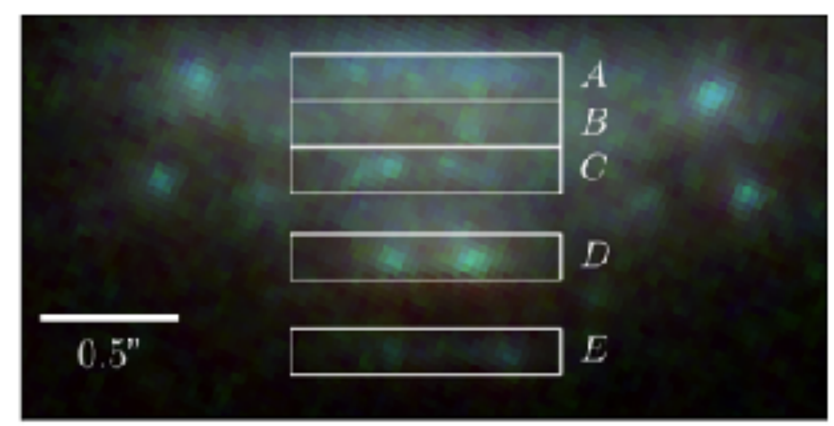


- Gravitational lensing by a vortex can lead to 10^{-4} arcsec displacement of distant sources in 10^5 years. (Mishra-Sharma, Van Tilburg, Weiner)

- In lensing events with extreme magnification (> 100), interference substructure can lead to fluctuations at the 10 percent level.



critical line



Dai et al.: strongly lensed arc

(See also: Dalal, Kochanek; Alexander et al.; Chan et al.; Broadhurst et al.)

- Heating of tidal streams.

A second topic: BH + scalar DM

Image of central region in M87 from the Event Horizon Telescope

Black hole hair from oscillating scalar

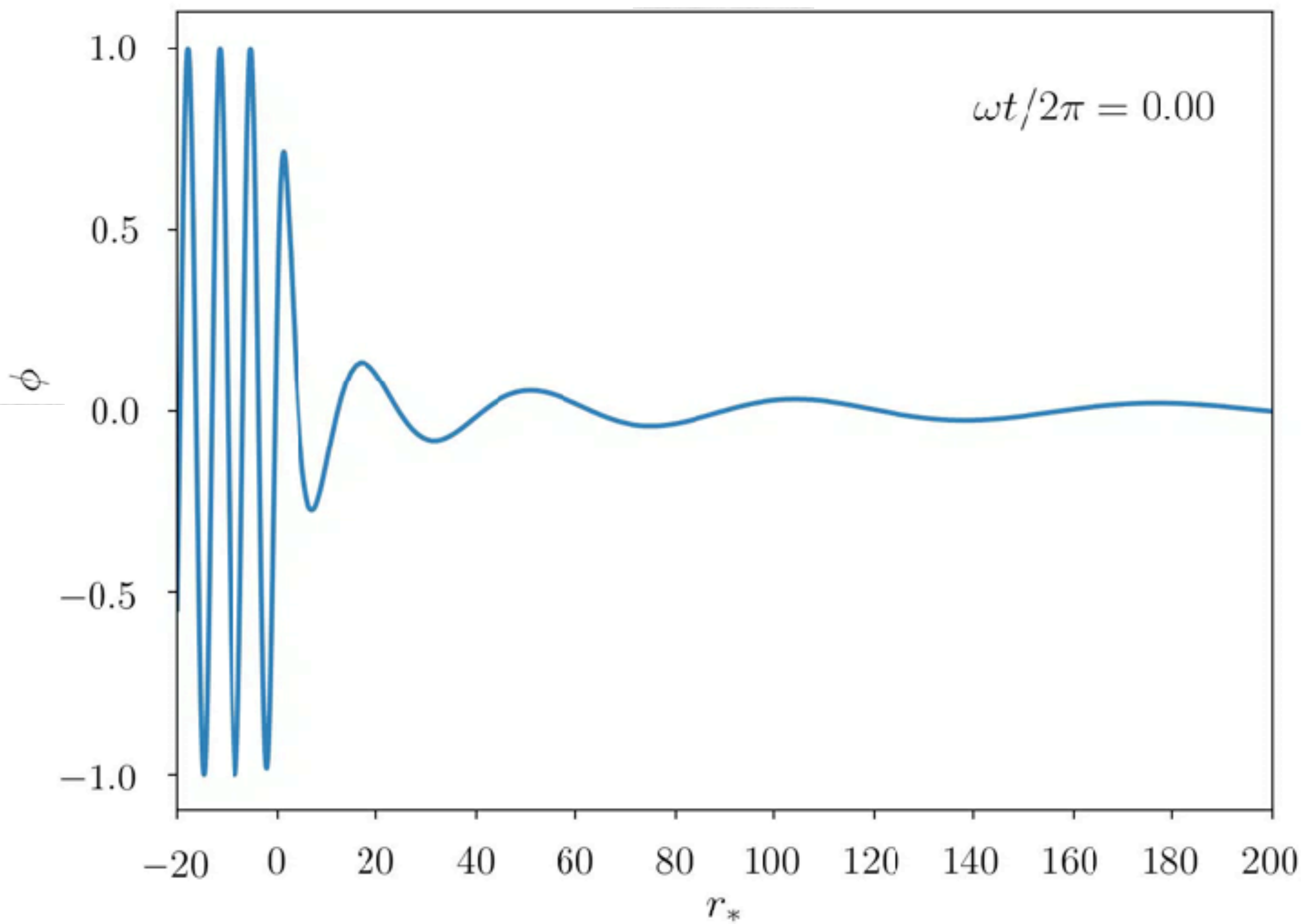
- Bekenstein's no (scalar) hair theorem can be violated in several ways:

An example: violate the vanishing boundary condition at infinity.
Jacobson (1999) showed that assuming $\phi \propto t$ far away for a massless scalar is sufficient to endow black hole with hair. The scalar charge is proportional to the time derivative, which is small for a cosmologically evolving scalar.
(See also Horbatsch & Burgess.)

- This can be generalized to an oscillatory time dependence, such as in the context of a black hole surrounded by dark matter consisting of an oscillating scalar (with non-zero mass).
Or a more mundane description: a stationary accretion flow of dark matter.

- Note: this is distinct from super-radiance.

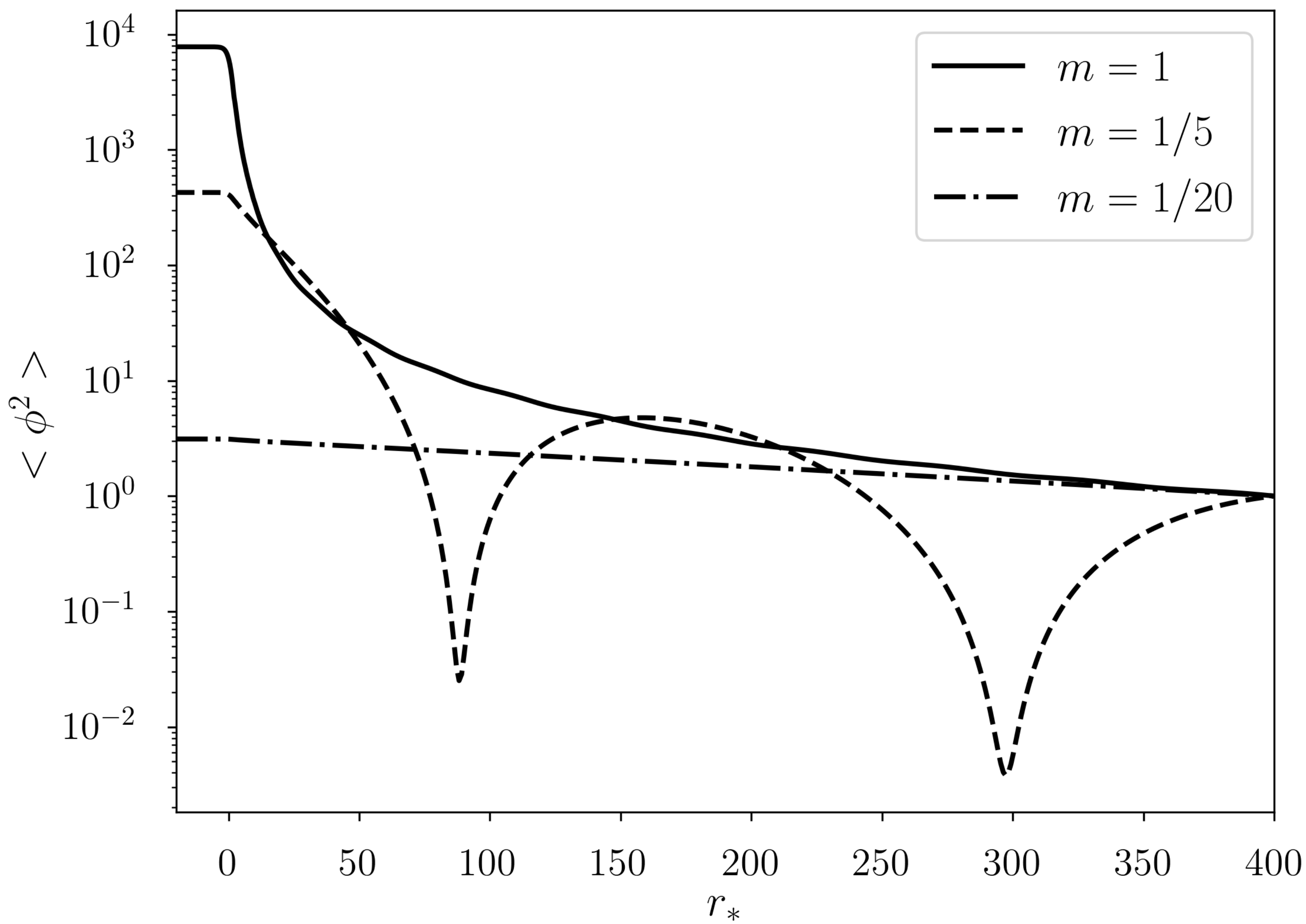
$(-\Box + m^2)\phi = 0 \quad$ in Schwarz. bgd.



$m = 1/r_{\mathrm{Schwarz.}}$

$r_* = r + \ln(r-1) \qquad (r_{\mathrm{Schwarz.}} = 1)$

similar pile − up occurs for $m > 10^{-3}\, r_{\mathrm{Schwarz.}}^{-1}$



$m = 1$
$m = 1/5$
$m = 1/20$
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
$< \phi^2 >$
0
50
100
150
200
250
300
350
400
r_*

Additional comments:

- Different scalar profile depending on scalar mass in relation to horizon size.
- Gravitational backreaction is negligible.
- Self-interaction (for an axion) might be interesting? - $(\phi/F)^2 \sim 10^{-7} - 10^{-3}$
- Kerr? Orbital angular momentum?

Experimental implications (light DM e.g. QCD axion):

$$\mathcal{L} \sim \frac{\phi}{f} F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\partial_\mu \phi}{f} \bar{\Psi}\gamma^5\gamma^\mu\Psi$$

Reviews: Sikivie 2003
Graham et al. 2015, Marsh 2016

- Coupling to EM

ADMX (cavity) - photon from axion in magnetic field ϕ^2

ABRACADABRA - magnetic flux from axion in magnetic field $\dot{\phi}$

ADBC - rotation of polarization of photon propagating in axion $\Delta\phi$

- Coupling to spin $\hat{H} \sim \vec{\nabla}\phi \cdot \hat{\sigma}$

CASPEr - spin precession like in NMR $\vec{\nabla}\phi$

Eot-Wash - torsional spin pendulum $\vec{\nabla}\phi$

Experimental implications (light DM e.g. QCD axion):

$$\mathcal{L} \sim \frac{\phi}{f} F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\partial_\mu \phi}{f}\bar{\Psi}\gamma^5\gamma^\mu\Psi$$

Reviews: Sikivie 2003
Graham et al. 2015, Marsh 2016

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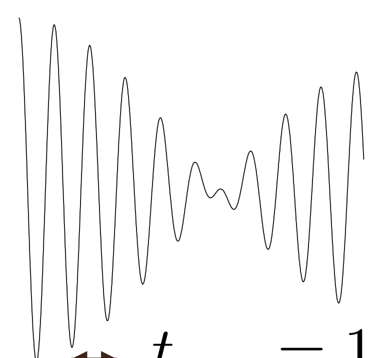
- $$\phi \sim \psi e^{-imt} + \psi^* e^{imt}$$

slow fast



$t_{\text{osc.}} = 1/m \sim 10^{-9}\,\text{s}$

$t_{\text{coherent}} = 1/(m\,v^2) \sim 10^{-3}\,\text{s}$

Derevianko; Foster, Rodd, Safdi; Centers et al.

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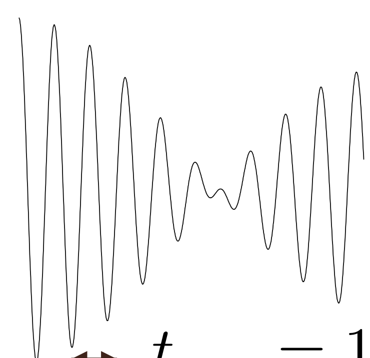

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- Measure correlation functions e.g.

$\langle \phi(t)^2 \phi(t')^2 \rangle - \langle \phi^2 \rangle^2 \sim [|t-t'|/t_{\mathrm{coherent}}]^{-3} + \mathrm{osc.}$ (or even space-time correlations).

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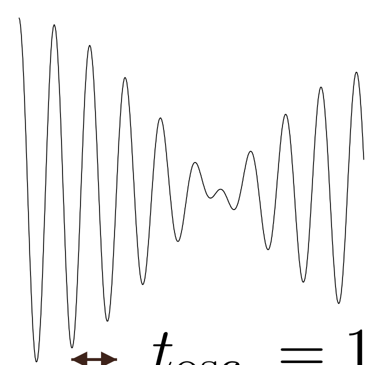
- $$\phi \sim \underbrace{\psi}_{\text{slow}} \underbrace{e^{-imt}}_{\text{fast}} + \psi^* e^{imt}$$



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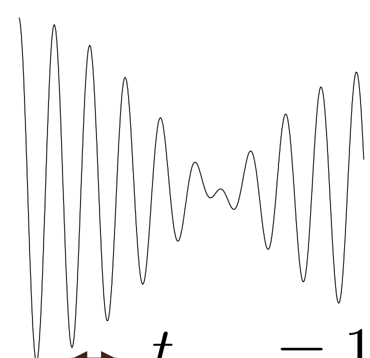

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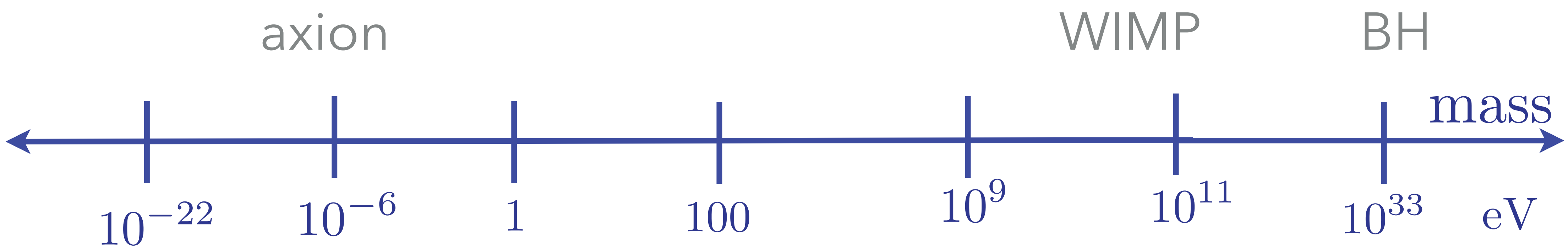
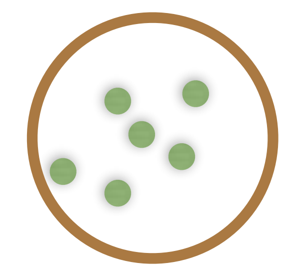
- At vortices $\phi = 0$ but $\vec{\nabla}\phi \neq 0$.
- Phase of oscillation might be interesting: $\phi \sim |\psi| \cos(mt - \theta)$.

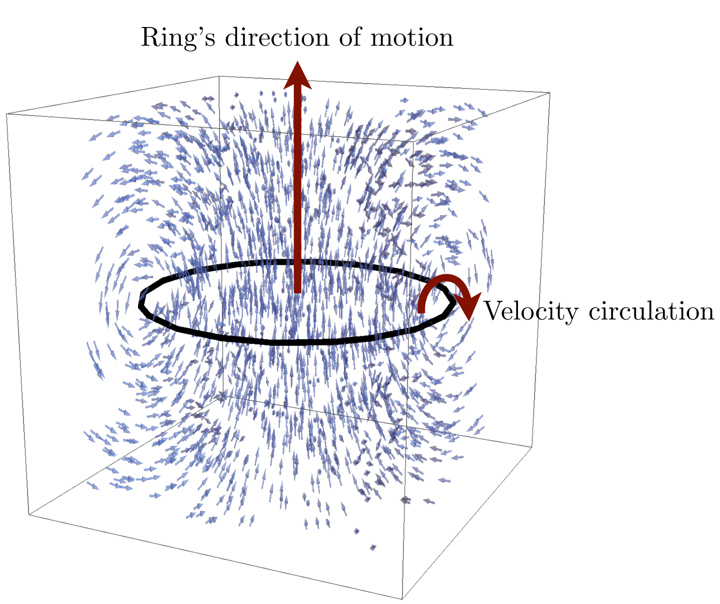
An enormous range in possible DM mass:

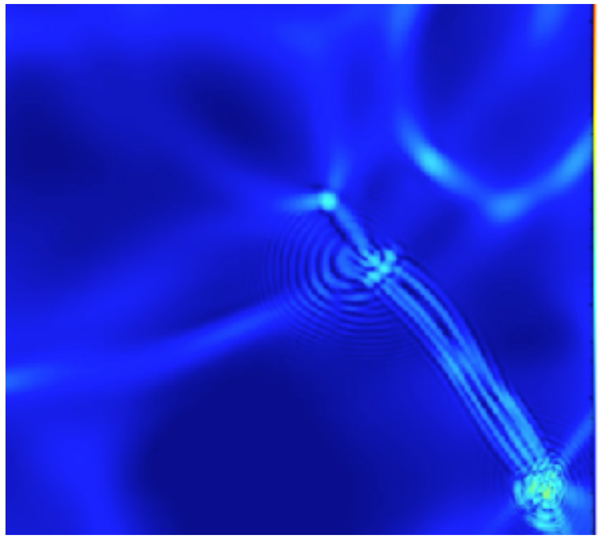


wave regime $\quad m < 100\,\mathrm{eV}$

$$
\begin{aligned}
1/mv \sim 10^{-3}\,\mathrm{cm} &\quad \text{for} \quad m = 10\,\mathrm{eV} \\
10^{4}\,\mathrm{cm} &\quad \text{for} \quad m = 10^{-6}\,\mathrm{eV} \\
100\,\mathrm{pc} &\quad \text{for} \quad m = 10^{-22}\,\mathrm{eV}
\end{aligned}
$$







additional slide

Recent developments in light scalar dark matter

Vortices in superfluid DM
- with Austin Joyce, Xinyu Li, Michael Landry

Black hole scalar hair
- with Dan Kabat, Xinyu Li, Luca Santoni, Sam Wong

Irsic, Viel, Haehnelt, Bolton, Becker 2017

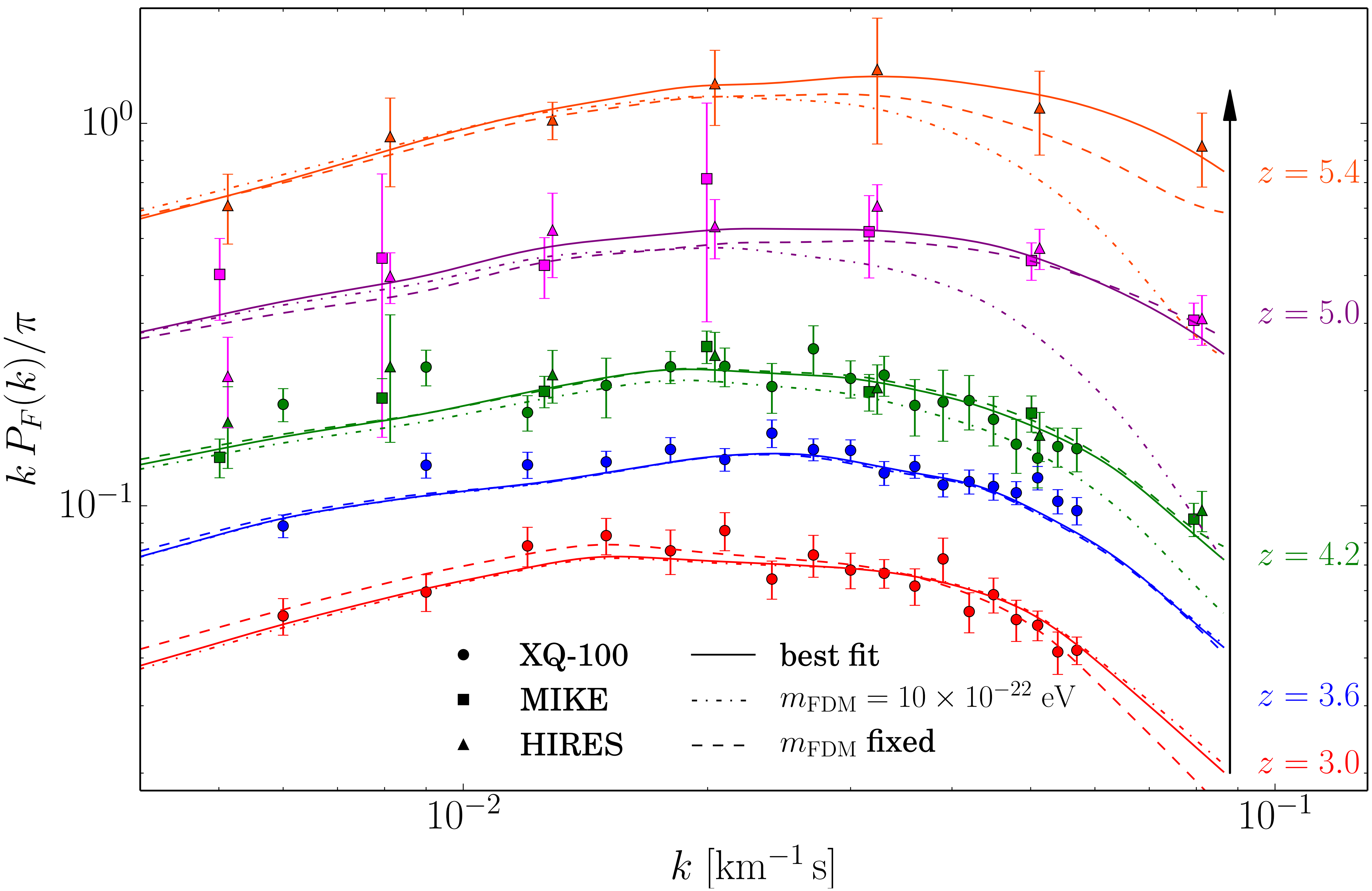


Figure thanks to Vid Irsic and Matteo Viel

Importance of ionizing background and reionization history fluctuations?