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Standard Model Meets Gravity: Electroweak Symmetry Breaking and Inflation

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Outline

- Introduction and motivations, naturalness
- Semiclassical relation between the Fermi and Planck scales
- Inflation
- Conclusions

based on works

arXiv:1803.08907 + arXiv:1804.06376 M.S. and Andrey Shkerin

arXiv:2001.09088 + arXiv:2002.07105 M.S. , Andrey Shkerin, and Sebastian Zell

Introduction and motivations, naturalness

Triumph of the SM in particle physics

- The Standard Model in now complete: the last particle - Higgs boson, predicted by the SM, has been found
- No significant deviations from the SM have been observed
- With experimental values of the masses of the top quark and of the Higgs boson the SM is a self-consistent effective field theory all the way up to the quantum gravity Planck scale M_P.

Naturalness – rather technical criterion:

Physics at the electroweak scale or right above it should be organised in such a way that quadratic divergencies in the Higgs boson mass are eliminated, to remove sensitivity of m_H to physics at very high energy scale Λ (e.g. GUT).

If this does not happen, the theory is called unnatural and fine-tuned

right above EW scale

The original source of the naturalness requirement: hierarchy problem in Grand Unified theories

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Gauge-symmetry hierarchies*

Eldad Gildener
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 15 June 1976)

It is shown that one cannot artifically establish a gauge hierarchy of any desired magnitude by arbitrarily adjusting the scalar-field parameters in the Lagrangian and using the tree approximation to the potential; radiative corrections will set an upper bound on such a hierarchy. If the gauge coupling constant is approximately equal to the electromagnetic coupling constant, the upper bound on the ratio of vector-meson masses is of the order of $\alpha^{-1/2}$, independent of the sclar-field masses and their self-couplings. In particular, the usual assumption that large scalar-field mass ratios in the Lagrangian can induce large vector-meson mass ratios is false. A thus far unsuccessful search for natural gauge hierarchies is briefly discussed. It is shown that if such a hierarchy occurred, it would have an upper bound of the order of $\alpha^{-1/2}$.

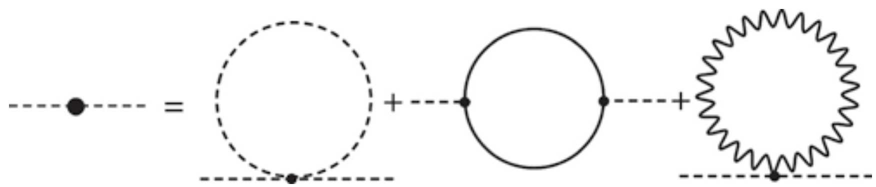
Extra GUT particles beyond the SM – leptoquarks (vector and scalar) must be very heavy, $M_X > 10^{15}$ GeV

- this is required by the gauge coupling unification
- this is needed for stability of matter, proton lifetime $\tau_p > 10^{34}$ years

Hierarchy: $(\frac{M_X}{M_W})^2 \simeq 10^{28}$

Two faces of hierarchy

- Ad hoc tuning between the parameters (masses and couplings of different multiplets) at the tree level with an accuracy of 26 orders of magnitude
- Stability of the Higgs mass against radiative corrections Gildener, '76



$$\delta m_H^2 \simeq \alpha_{GUT}^n M_X^2$$

Tuning is needed up to 14th order of perturbation theory!

Proposed solutions

Stability of EW scale – requirement of “naturalness”: absence of quadratic divergencies in the Higgs mass

- Low energy SUSY: compensation of bosonic loops by fermionic loops
- Composite Higgs boson - new strong interactions
- Large extra dimensions

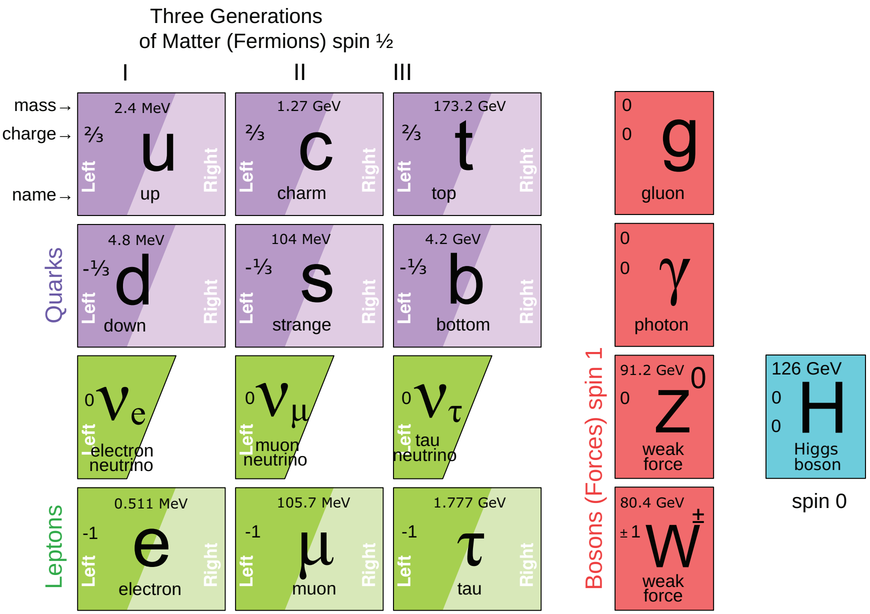
All require new physics right above the Fermi scale, which was expected to show up at the LHC, but it did not!

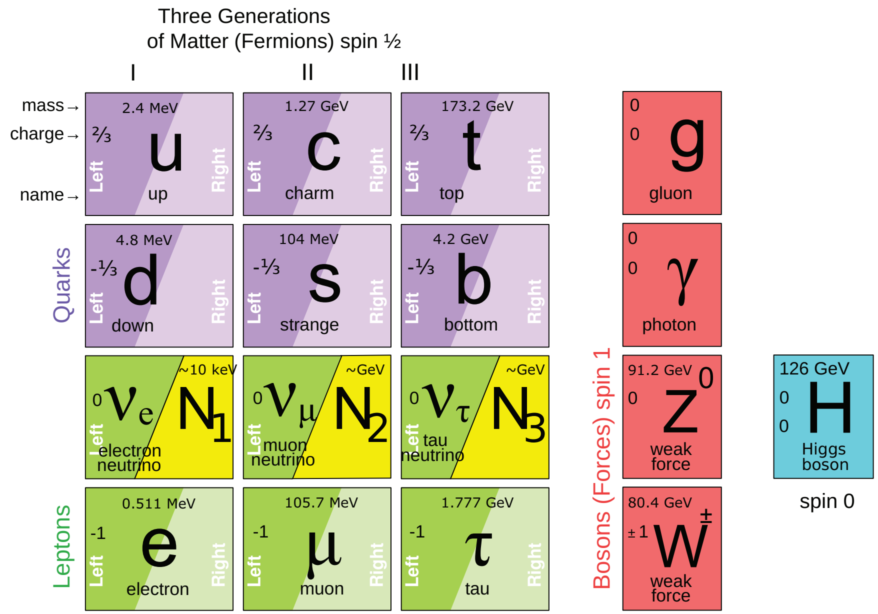
Change of paradigm ?

The source of the hierarchy problem - heavy particles (with substantial coupling to the Higgs boson) Vissani; MS; Farina, Pappadopulo, Strumia. No heavy particles - no large perturbative contributions - perhaps, no fine tuning? All new physics: below the Planck scale.

UV physics (gravity?) should be organised in such a way that the Fermi scale is much smaller than the Planck scale. (M_P is not a mass of any particle, it gives the strength of interaction!)

Example of complete theory: the νMSM





νMSM $\equiv$ Neutrino minimal Standard Model, Asaka, MS

$\equiv$ Minimal low scale see-saw model with 3 singlet fermions

Role of $N_2,\ N_3$ with mass in 100 MeV – GeV region: "give" masses to neutrinos and produce baryon asymmetry of the Universe.

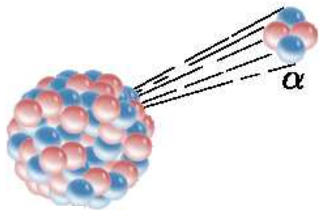
Role of N_1 with mass in keV region: dark matter.

Role of the Higgs boson: break the symmetry and inflate the Universe

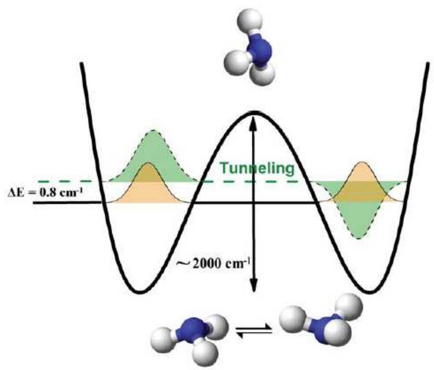
Why the week scale is so much smaller than the Planck scale?

Very small numbers in quantum physics: quantum tunnelling

- 1928, Gamow theory of alpha decay,
  $^{238}U \rightarrow ^{234} Th + \alpha, \Gamma = E_{\text{bound}} e^{-S} \ll E_{\text{bound}}$



- 1951, Townes, Ammonia Maser,
  $\omega = \omega_0 e^{-S} \ll \omega_0$



- Perhaps, $m_H^2 = M_P^2 e^{-S}$ with $S = 72$?

Semiclassical relation between the Fermi and Planck scales?

Higgs-Planck hierarchy: the ratio of the two scales is exponentially small,

$$\frac{m_H}{M_P} \simeq 10^{-16} \simeq e^{-S}, \quad S \simeq 36$$

Proposal: there is only one fundamental scale in Nature, M_P and the electroweak scale is generated from it non-perturbatively

non-perturbatively $\equiv$ semi-classically (?) $\Longrightarrow$ Large S

Requirements to the theory

- Perturbatively $m_H = 0$, symmetry protection (?)
- Existence of (semi) classical configurations leading to $\langle H \rangle \neq 0, \ \langle H \rangle \ll M_P$

If the mass of the Higgs boson is put to zero in the SM, the classical Lagrangian has a wider symmetry: it is scale and conformally invariant: Dilatations - global scale transformations ($\sigma = const$)

$$\Psi(x) \rightarrow \sigma^n \Psi(\sigma x) ,$$

$n = 1$ for scalars and vectors and $n = 3/2$ for fermions.

It is tempting to use this symmetry for solution of the hierarchy problem Bardeen '95: why the Higgs boson mass is so small in comparison with the Planck scale?

Attractive features of classically scale invariant/conformal theories: no explicit mass parameters, only dimensionless couplings

Higgs mass can be predicted, more precisely, can be expressed via vector boson mass!

Radiative Corrections as the Origin of Spontaneous Symmetry Breaking*

Sidney Coleman

and

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(Received 8 November 1972)

We investigate the possibility that radiative corrections may produce spontaneous symmetry breakdown in theories for which the semiclassical (tree) approximation does not indicate such breakdown. The simplest model in which this phenomenon occurs is the electrodynamics of massless scalar mesons. We find (for small coupling constants) that this theory more closely resembles the theory with an imaginary mass (the Abelian Higgs model) than one with a positive mass; spontaneous symmetry breaking occurs, and the theory becomes a theory of a massive vector meson and a massive scalar meson. The scalar-to-vector mass ratio is computable as a power series in e, the electromagnetic coupling constant. We find, to lowest order, $m^2(S)/m^2(V) = (3/2\pi)(e^2/4\pi)$. We extend our analysis to non-Abelian gauge theories, and find qualitatively similar results. Our methods are also applicable to theories in which the tree approximation indicates the occurrence of spontaneous symmetry breakdown, but does not give complete information about its character. (This typically occurs when the scalar-meson part of the Lagrangian admits a greater symmetry group than the total Lagrangian.) We indicate how to use our methods in these cases.

The Coleman-Weinberg primer

- Take the Abelian Higgs model with scalar self coupling $\lambda \sim g^4$ and the Higgs mass m_H equal to zero.
- The scale invariance is anomalous due to “dimensional transmutation”: the renormalisation-group running of the parameters leads to a non-vanishing trace of the energy-momentum tensor, which enters the divergence of the scale current. The physical quantities depend on the renormalisation scale only logarithmically. Take a scale-independent renormalisation, e.g. DimReg. No counter-term is needed to renormalise the scalar mass! RG equation has a fixed point at $m_H = 0$, $\mu \frac{\partial}{\partial \mu} m_H^2 \propto m_H^2$.

The Coleman-Weinberg primer

- Important: any power-like divergent contributions to the Higgs boson mass are purely technical and are introduced by explicitly breaking the conformal invariance by regulators. (Original CW paper: normalisation conditions are: $\frac{d^2V}{d\phi^2}|_0 = 0, \quad \frac{d^4V}{d\phi^4}|_M = \lambda$.)
- Compute the CW effective potential and discover that the U(1) theory is in the Higgs phase.
- Read off the ratio between the Higgs boson mass and the vector boson mass, $\frac{m_H^2}{m_W^2} = \frac{3e^2}{8\pi^2}$.

Radiative symmetry breaking in the SM

Lagrangian is invariant at the classical level, and scale symmetry is broken by quantum corrections (conformal anomaly) a'la Coleman-Weinberg: Linde '76; Weinberg '76; Buchmuller, Dragon '88; Hempfling '96; Meissner, Nicolai '06; Foot et al '07, '11; Iso, et al '09; Boyle et al '11; Wetterich '11, Salvio, Strumia '14; Lindner et al, '14, '15, '17

Does not work for the SM:

- If the top quark mass $m_t \lesssim m_t^{crit}$, then the minimum of the effective potential is generated at $\langle H \rangle \simeq 100$ MeV due to chiral symmetry breaking in QCD
- If the top quark mass $m_t \gtrsim m_t^{crit}$, then an extra minimum of the effective potential is generated at $\langle H \rangle \gtrsim M_P$ due to top quark loops

$m_t^{crit} = 170 - 174$ GeV accounting for uncertainties in the relation between the MC and pole masses of the top quark.

The idea fails, and enlarging of the SM is necessary. Extra gauge bosons? Extra fermions? Extra scalars?

But we do have the breaking of scale invariance! Gravity comes with a dimensionful parameter $M_P \gg m_H$, and this must be taken into account!

Perturbatively, with mass-independent regularisation (such as DimReg) : no contribution to the Higgs mass, and all gravity corrections are suppressed by the Planck mass. The RG equation $\mu\frac{\partial}{\partial\mu}m_H^2 \propto m_H^2$ remains in force!

Gravity + conformally invariant SM is an ideal playground for looking at non-perturbative generation of the weak scale

SM and gravity, scalar sector

Scalar theory plus gravity in Palatini formulation with non-minimal coupling, $\xi > 0$:

$$\frac{\mathcal{L}_{\varphi,g}}{\sqrt{g}} = -\frac{1}{2}(M_P^2 + \xi\varphi^2)R + \frac{1}{2}(\partial\varphi)^2 + V(\varphi)$$

Scale invariant "matter" with

$$V(\varphi) = \frac{\lambda}{4}\varphi^4$$

Palatini formulation of gravity (Palatini'1919, Einstein'1925): the metric and the Christoffel connection are treated as independent variables. If $\xi = 0$, Palatini gravity is exactly equivalent to the standard metric Einstein gravity. This is not the case for non-minimal coupling!

Fermi scale generation

We want to compute the Higgs vev:

$$\langle \varphi \rangle \sim \int \mathcal{D}\varphi \mathcal{D}g_{\mu\nu} \varphi e^{-S_E} \ .$$

S_E is the euclidean action of the model.

Remarks:

- Euclidean path integral for gravity may not be well defined due to the problem with the conformal factor of the metric
- We will ignore this problem and follow the crowd: Hawking; Coleman, de Luccia; Veneziano; ..., Isidori, Rychkov, Strumia, Tetradis; ... Branchina, Messina, Sher;...

Small $\varphi \ll M_P$ - gravity is irrelevant – no contribution to the vev of the Higgs from scalar loops.

Challenge: account for contributions with $\varphi \gg M_P$.

Theory for large φ:

$$\mathcal{L} = -\frac{1}{2}\xi\varphi^2 R + \frac{1}{2}(\partial\varphi)^2 + \frac{\lambda}{4}\varphi^4$$

- Scale-invariant
- Planck scale is dynamical, $\propto \sqrt{\xi}\varphi$

Conjecture: contribution of large Higgs fields $\varphi > M_P/\sqrt{\xi}$ to path integral is better to be found in the Einstein frame. Conformal transformation (note: in Palatini gravity $R_{\mu\nu} \to R_{\mu\nu}$):

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \;, \quad \Omega^2 = 1 + \frac{\xi\varphi^2}{M_P^2}$$

Redefinition of the Higgs field to make canonical kinetic term

$$\frac{d\chi}{d\varphi} = \frac{1}{\Omega} \quad \Longrightarrow \; \varphi = \frac{M_P}{\sqrt{\xi}} \sinh\left(\frac{\sqrt{\xi}\chi}{M_P}\right)$$

Resulting action

$$S = \int d^4x\sqrt{-\hat{g}}\left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu\chi\partial^\mu\chi}{2} + \frac{\lambda M_P^4}{4\xi^2}\sinh^4\left(\frac{\sqrt{\xi}\chi}{M_P}\right) \right\}$$

Most important:

$$\langle\varphi(x)\rangle \sim \int \mathcal{D}\mathcal{A}\mathcal{D}\varphi(x)\mathcal{D}g_{\mu\nu}\varphi(x)e^{-S_E} \Longrightarrow \int \mathcal{D}\mathcal{A}\mathcal{D}\chi\mathcal{D}\hat{g}_{\mu\nu}e^{\frac{\sqrt{\xi}\chi(x)}{M_P}-S_E} .$$

Modification of the action and equations of motion!

Equations of motion for χ contain a source term $\delta(x)$ $\Longrightarrow$ new classical solutions. Similar to

- computation of $\int dx x^N e^{-x^2}$ for large N,
- to computation of multi-particle production Khlebnikov, Rubakov, Tinyakov '91,
- to proof of confinement in 3D Georgi-Glashow model, $\langle\exp(\int A_\mu dv^\mu)\rangle$ in Polyakov '76.

Schematically, modification of the right-hand side for scalar field equation:

$$\Box\chi + ... = \sqrt{\xi}\delta(x)/M_P$$

Path integral:

$$\int_{\varphi \gtrsim M_P/\sqrt{\xi}} \mathcal{D}\varphi \varphi e^{-S_E} \to M_P \int_{\chi \gtrsim M_P \log(1/\sqrt{\xi})} \mathcal{D}\chi J e^{-W} \ ,$$

where $W = \sqrt{\xi}\chi(0)/M_P + S_E$ and J is the corresponding Jacobian.

The Higgs vev

$$\langle \varphi \rangle \approx M_P e^{-\bar{W}} \ ,$$

may be much smaller than M_P because the action $\bar{W}$ on the saddle point

$$\bar{W} \gg 1 \ .$$

Condensed matter analogue: exponentially small mass gap in superconductors.

Computation

Field equations in the Einstein frame for maximally $O(4)$ symmetric metric $d\tilde{s}^2 = f^2(r)dr^2 + r^2 d\Omega_3^2$

$$\partial_r \left(\frac{r^3 \chi'}{f} \right) - r^3 f U'(\chi) = -\frac{\sqrt{\xi}}{M_P} \delta(r)$$

$$6 - 6f^2 + \frac{2r^2 f^2 U(\chi)}{M_P^2} - \frac{r^2 \chi'^2}{M_P^2} = 0$$

Boundary conditions at infinity $r \to \infty$: $f^2(r) \to 1 \, , \chi(r) \to 0 \, .$

Solution for small r: $\chi = -\sqrt{6} M_P \log{(r M_P)} + c$,
$f = \sqrt{6}(r M_P)^2 / \sqrt{\xi} \; ,$ where c is a constant to match the asymptotic at large r. Coincides with Hawking-Turok instanton, but the singularity near $r = 0$ is fixed by the source term.

Computation

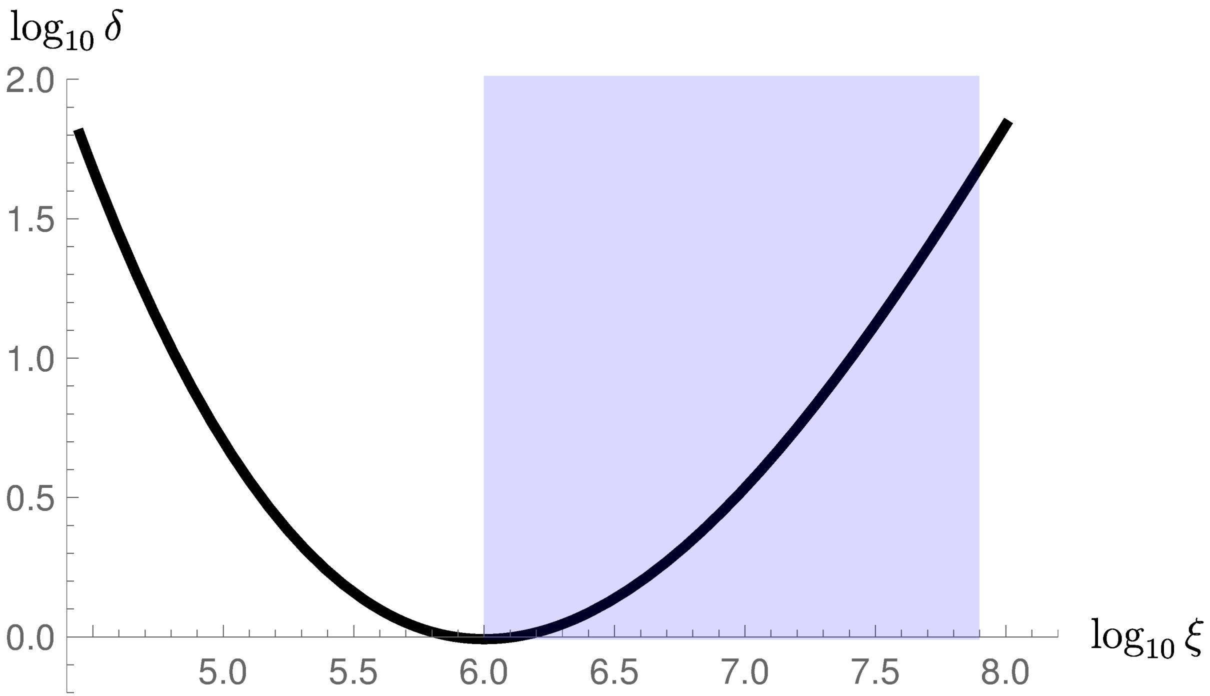
The action is singular, since $\chi \to \infty$ when $r \to 0$. Add to the action higher dimensional operators which can be generated by radiative corrections. We also require that these operators:

- do not introduce new degrees of freedom
- do not spoil asymptotic scale invariance, when $h \to \infty$ (automatic)
- do not violate positivity conditions Adams et al '2006

Example

$$\delta_\delta = \frac{\delta}{M_P^8 \Omega^8} (\partial_\mu \varphi)^6 \,, \Omega^2 = 1 + \frac{\xi \varphi^2}{M_P^2} \,.$$

This is just one example, the results are largely insensitive to the choice!



Values of the non-minimal coupling $\boldsymbol{\xi}$ and the sextic derivative coupling $\boldsymbol{\delta}$, for which $\boldsymbol{B = \ln(M_P/(\sqrt{\xi}M_F))}$. Admissible values of $\boldsymbol{\xi}$ are within the blue area, the left bound coming from inflation and the right bound coming from top quark measurements.

The hierarchy between the Planck and the Fermi scale is a natural phenomenon when the SM is classically conformal and $\boldsymbol{\xi}$ is large!

Inflation

The very same theory with large ξ leads to inflation!

Inflationary potential:

$$V = \frac{\lambda M_P^4}{4\xi^2} \sinh^4 \left(\frac{\sqrt{\xi}\chi}{M_P} \right) \Bigg\}$$

Predictions:

Metric Higgs inflation Bezrukov, MS '07 , $\xi = 49000\sqrt{\lambda}$

$$n_s = 1 - \frac{2}{N} \simeq 0.97\,, \;\; r_M = \frac{12}{N^2} \simeq 0.0033\;,$$

Palatini Higgs Inflation Bauer, Demir '08, $\xi = 1.1 \times 10^{10}\lambda$

$$n_s = 1 - \frac{2}{N} \simeq 0.97\,, \quad r_P = \frac{2}{\xi N^2} \ll r_M\;,$$

Reheating: almost instantaneous

Metric: due to creation of longitudinal vector bosons, parametric resonance DeCross et al' 18

Palatini: tachyonic production of Higgs excitations Rubio, Tomberg' 19

UV behaviour

Metric: UV cutoff for zero background: $\frac{M_P}{\xi} \ll E_{inf} \simeq \frac{\lambda^{\frac{1}{4}} M_P}{\sqrt{\xi}}$

Burgess, Lee, Trott ; Barbon, Espinosa '09

Palatini: UV cutoff for zero background: $\frac{M_P}{\sqrt{\xi}} \gtrsim E_{inf}$ Bauer Demir '11

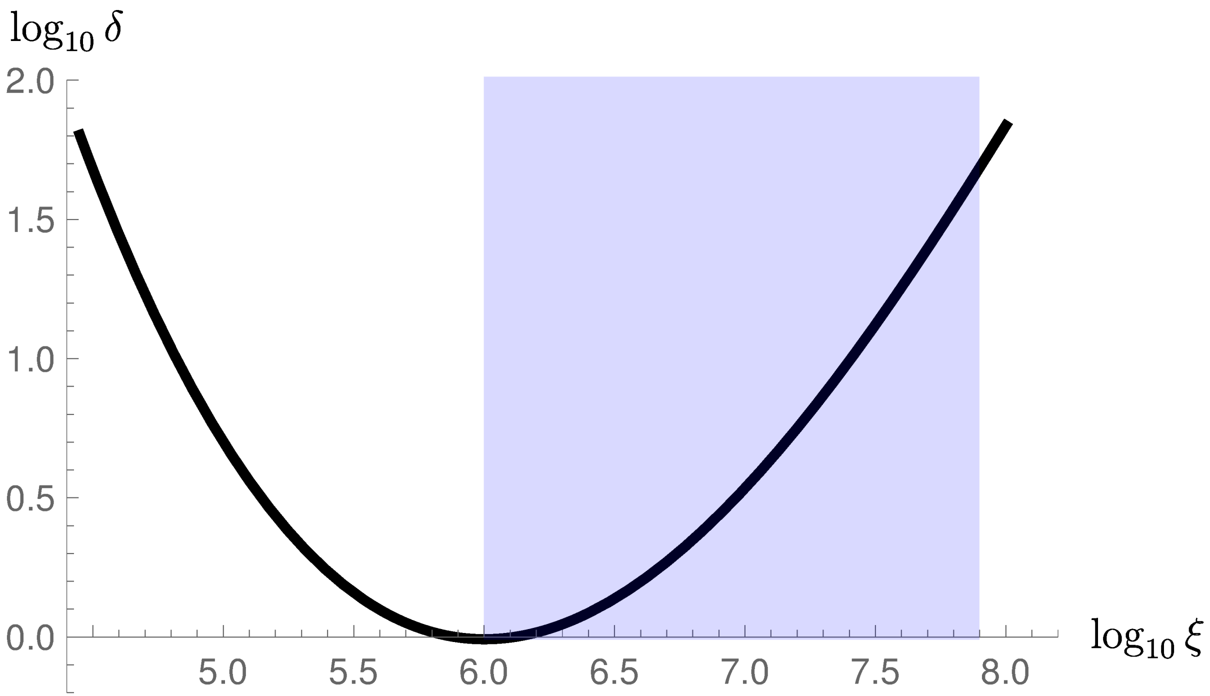
In both cases in the inflationary background $\chi \neq 0$ the cutoff scale is larger than the inflation scale Bezrukov et al '10

The relation between low energy and inflationary parameters is more robust in Palatini Higgs inflation:

From experimental conservative lower bound $m_t \gtrsim 170$ GeV:

$\xi < 7.9 \cdot 10^7$

From inflationary requirement $V' > 0$: $\xi > 1.0 \cdot 10^6$.



This leads to prediction $r_P = 7 \times 10^{-11\pm1}$

Conclusions

- Very small m_H/M_P ratio is (perhaps) telling us that
  - There are no new particles with masses between the Fermi and Planck scale
  - The smallness of the Fermi scale is a semiclassical non-perturbative UV effect associated with gravity and new type of instantons
  - The gravitational degrees of freedom are best described in Palatini formulation
  - Palatini Higgs inflation is different from metric Higgs inflation, most notably in prediction of $r_P \ll r_M$ and in removing the "low cutoff" problem.

- Open problems
  - Unfortunately, we can make no prediction of the ratio m_H/M_P, as this depends on details of UV theory.
  - We cannot estimate the contribution of the effects other than perturbative and semiclassical.