Annual modulation from secular variations: not relaxing DAMA?

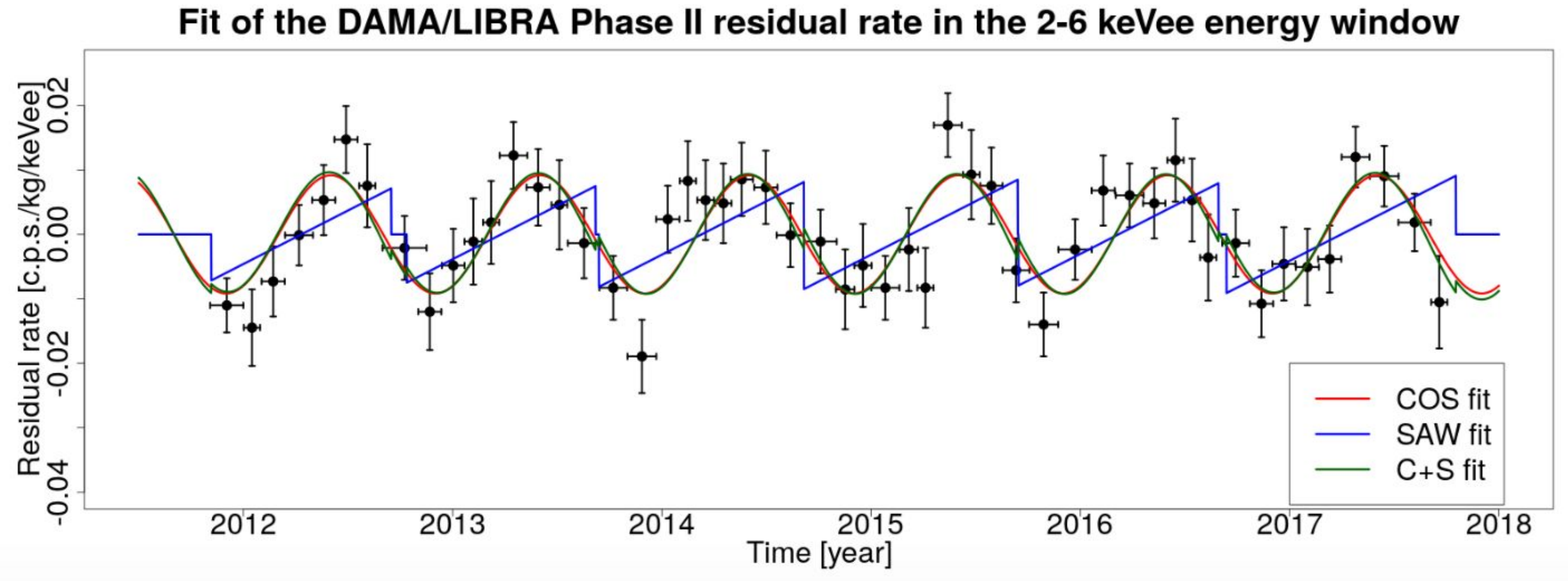
March 27th, 2020

A. Messina, M. Nardecchia, S. Piacentini

[ArXiv:2003.03340]

On-line “Newton 1665” seminars
Phenomenology / theory / astro / cosmo

Why?



- Because by eye the sawtooth [Buttazzo et al. ArXiv:2002.00459] cannot reproduce much more than the period of the data!
- Can we be quantitative?

DAMA/NaI and DAMA/LIBRA data

- **DAMA/NaI:** 100kg NaI(TI), 7 yr, exposure = 0.29 ton yr

R. Bernabei et al., Phys. Lett. B480 23-31 (2000).

- **DAMA/LIBRA I:** 250 kg NaI(TI), 7 yr, exposure = 1.04 ton yr

R. Bernabei et al., Phys. J. C56, 333 (2008), arXiv:0804.2741 [astro-ph].

- **DAMA/LIBRA II:** exposure 1.13 ton yr

R. Bernabei et al., Nucl. Phys. At. Energy 19 (2018) 307, arXiv:1805.10486 [hep-ex].

Single-hit residual rate definition

$$flat_{jk} = < r_{ijk} >_i$$

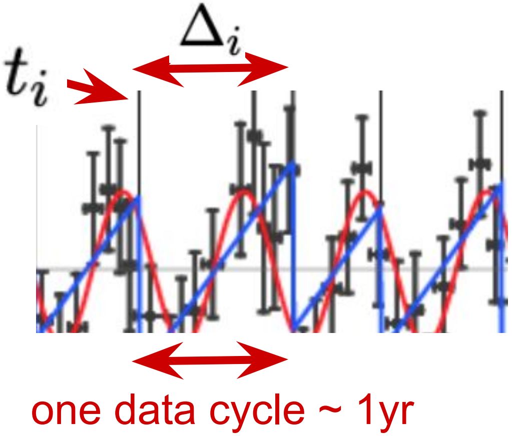
$$r_i = < r_{ijk} - flat_{jk} >_{jk}$$

R. Bernabei et al., Riv. Nuovo Cim. 26N.1, 1-73 (2003) [arXiv:astro-ph/0307403].

k : energy index
j : detector index
i : time index

'flat_jk' is the background specific to each crystal, it subtracted before combining them. However, there is no reason to compute flat_jk on the whole cycle. If 'flat_ijk' is linear for instance, then one generates a sawtooth-like signal with period equal to the cycle.

Model comparison: models



Naive likelihood:

$$\mathcal{L}(\{\mu_i\},\{\sigma_i\};\{D_i\}) = \prod_{i=i}^{n} \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left[-\frac{(y_i-\mu_i)^2}{2\sigma_i^2}\right],$$

2 simple, well defined models:

- **COS:**

$$S_{COS}(t) = A\cos\left(\frac{2\pi}{T}(t-t_0)\right),$$

1 free intensity parameter

Fixed to DM

- **SAW:**

$$S_{SAW}(t) = B\,(t-t_i) \quad \text{with} \quad t_i - \frac{\Delta_i}{2} < t < t_i + \frac{\Delta_i}{2},$$

Model comparison: (Bayesian) strategy

- Assign to each model a prior probability: $\pi(COS),\ \pi(SAW)$
- Use the data to update the odds ratio:

$$\frac{p(COS|D)}{p(SAW|D)} = \frac{\mathcal{L}(COS;D)}{\mathcal{L}(SAW;D)} \times \frac{\pi(COS)}{\pi(SAW)}$$

Posterior odds | Bayes Factor | Prior odds

- For parametric models, $\mathcal{L}$ is the likelihood averaged over the parameters (not the best fit $\hat{\mathcal{L}}$ (profiled)), for symmetric cases: $\mathcal{L} \simeq \hat{\mathcal{L}} \cdot OF$, OF: ockham's factor (parameters dependent)

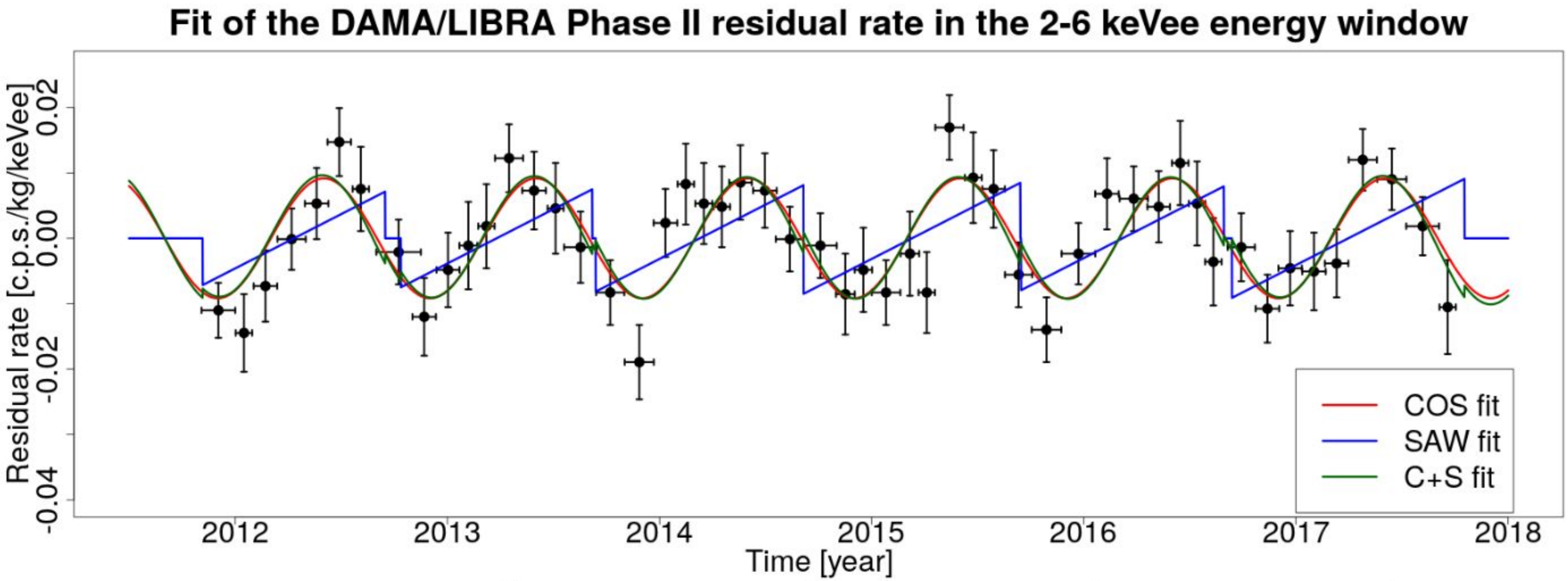
Model comparison: Bayes Factor

Maximum likelihood ratio

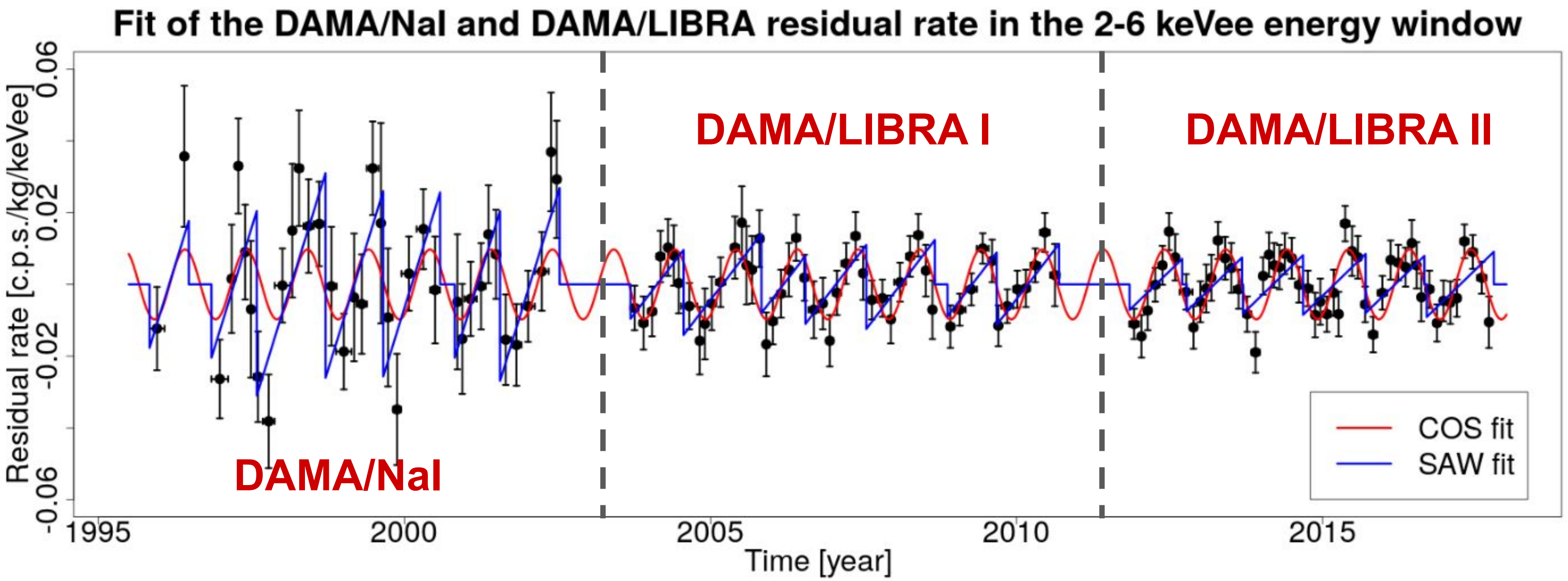
$$BF_{A,B} = \frac{\mathcal{L}(A;D)}{\mathcal{L}(B;D)} \simeq LR_{A,B} \cdot OF_{A,B}$$

- For the frequentistic: $-2log(LR_{AB}) = \Delta\chi^2_{AB}$
- OF: is the Ockham's factor that penalises models with unnecessary complexity (parameters)

$BF_{A,B}$ =100 means that after having seen the fit you 'should' prefer 100 times more model A than what you did before.



Model comparison	BF [dB]	LR [dB]	OF [dB]	ΔBIC
COS vs SAW	86.5	84.7	1.8	−39.0
COS vs C+S	11.3	−1.4	18.0	−3.31
SAW vs C+S	−69.9	−86.2	16.3	35.7



$$A = (0.00973 \pm 0.00078)\ \mathrm{cpd/kg/keVee} \qquad \chi^2/dof = 116.0/138$$

$$\begin{cases} B_{NaI} & = (0.0553 \pm 0.0085)\ \mathrm{cpd/kg/keVee/yr} \\ B_{LIBRAI} & = (0.0222 \pm 0.0032)\ \mathrm{cpd/kg/keVee/yr} \\ B_{LIBRAII} & = (0.0166 \pm 0.0028)\ \mathrm{cpd/kg/keVee/yr} \end{cases} \qquad \chi^2/dof = 145.8/136$$

$$BF = 88.8\ \mathrm{dB},$$
$$LR = 64.7\ \mathrm{dB},$$
$$OF = 24.1\ \mathrm{dB},$$

Fit *t0, T, A* of the COS+SAW model

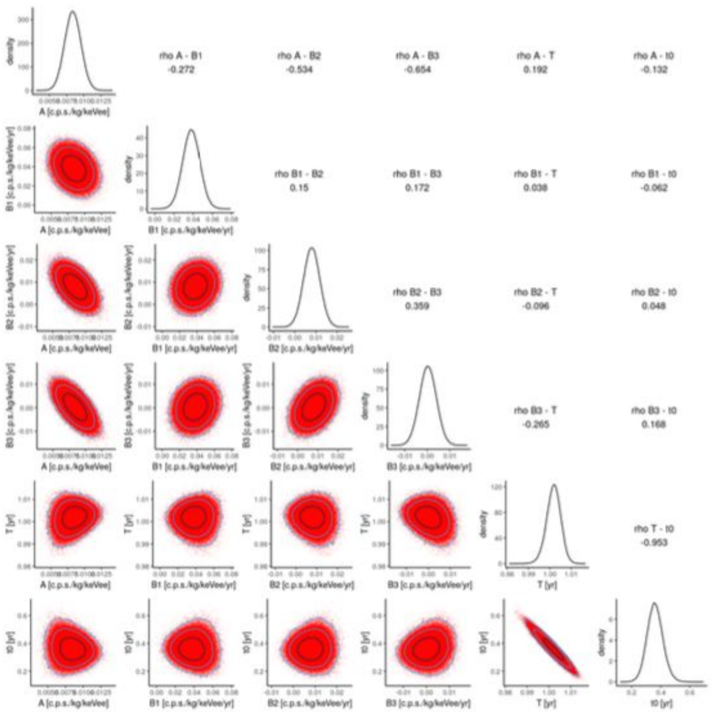
$$
\begin{cases}
A & = (0.00981 \pm 0.00079)\ \mathrm{cpd/kg/keVee} \\
t_0 & = (0.382 \pm 0.037)\ \mathrm{yr} \\
T & = (1.0008 \pm 0.0023)\ \mathrm{yr}
\end{cases}
$$

$$
BF = 62.4\ \mathrm{dB}, \quad LR = 71.7\ \mathrm{dB}, \quad OF = -9.3\ \mathrm{dB},
$$

Compatible with:

T= 1 yr,

t0 = 2nd July (0.418 yr)



A *B1* *B2* *B3* *T* *t0*

Possible bias in the signal subtraction

To remove bkgd, the rate is averaged over time and subtracted:

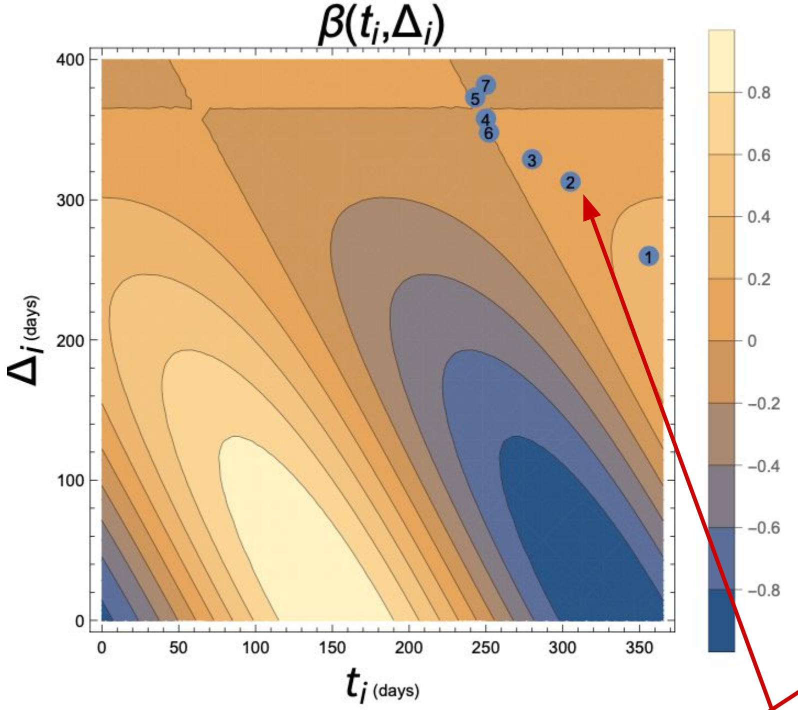
$$S(t) \equiv r(t) - \langle r(t) \rangle_\Delta = A \cos\left(\frac{2\pi t}{T} - \phi\right)$$

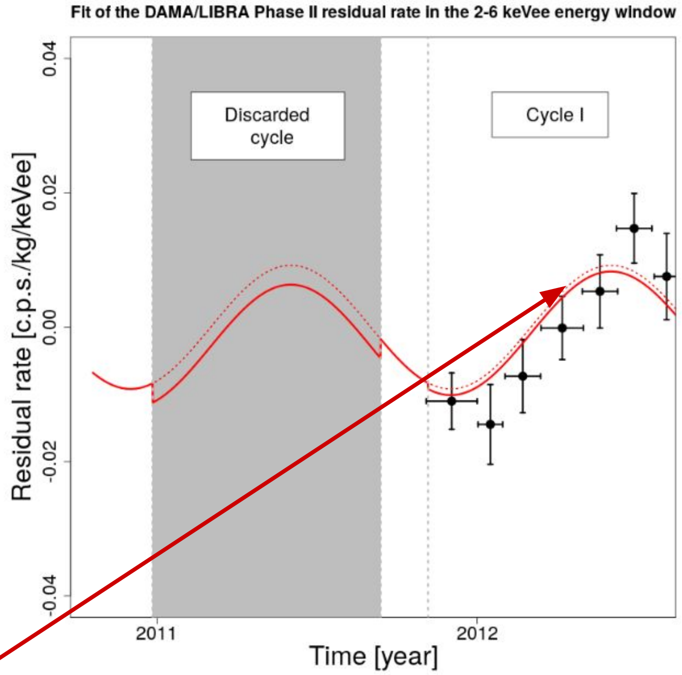
If the time interval (ti, Delta) is different than 1 yr or not 'symmetric', you subtract some signal as well:

$$\beta(t_i, \Delta_i) = \frac{1}{\Delta_i} \int_{t_i}^{t_i+\Delta_i} \cos\left(\frac{2\pi}{T}(t' - t_0)\right) dt'.$$

DAMA does removes asymmetric datasets to avoid this problem

You know what you are subtracting and thus can correct for it!





Up to 10% effects in the data cycles used by DAMA

Suggested fitting procedure

$$\mu_i = A\cos\left(\frac{2\pi}{T}(t_i - t_0)\right) - \frac{A}{\Delta}\int_{t^*}^{t^*+\Delta}\cos\left(\frac{2\pi}{T}(t' - t_0)\right)dt' + B\left(t_i - \frac{\Delta}{2}\right)$$

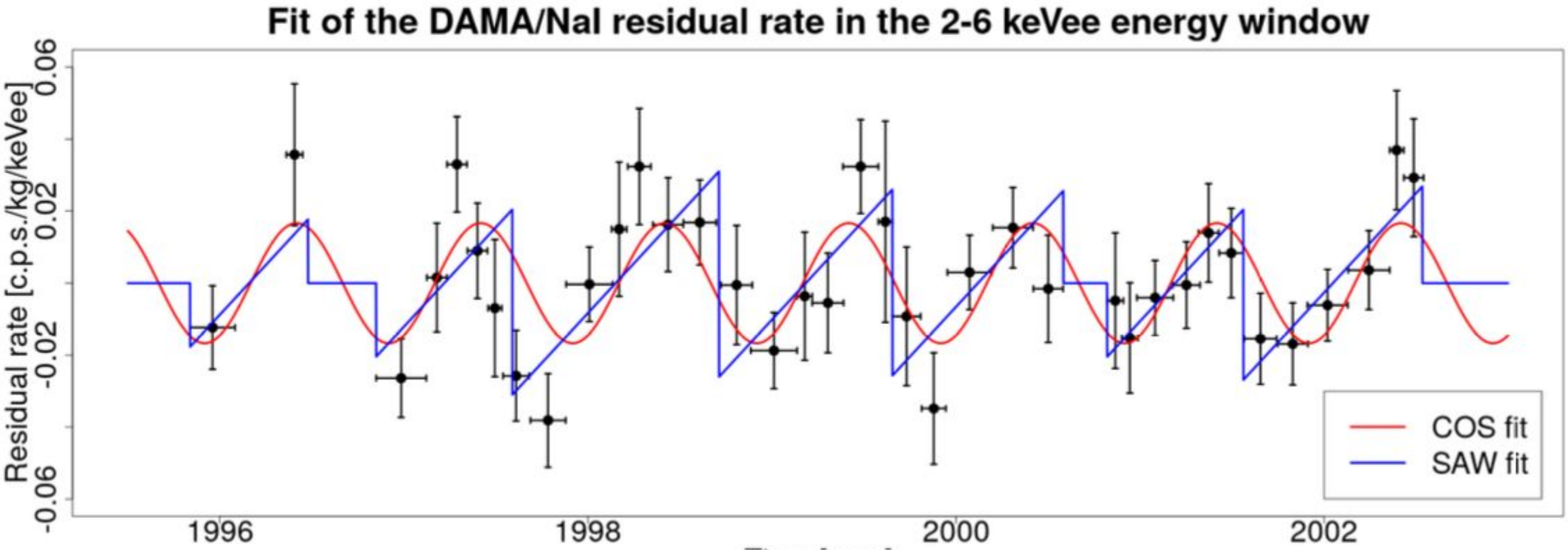
	T, t0 fixed to DM	*Free T, t0*
	F1	F2
A [cpd/kg/keVee]	0.0084 ± 0.0011	0.0084 ± 0.0012
B1 [cpd/kg/keVee/yr]	0.0371 ± 0.0089	0.0381 ± 0.0090
B2 [cpd/kg/keVee/yr]	0.0078 ± 0.0038	0.0080 ± 0.0038
B3 [cpd/kg/keVee/yr]	-0.0006 ± 0.0035	-0.0003 ± 0.0038

Conclusions

- The DAMA residual modulation cannot be possibly explained by a slowly time-varying background (BF~1E8)
- There is no need to have data taking cycles of one year duration as DAMA has done in the past
- We suggest to include the 2 above effects in the fit!

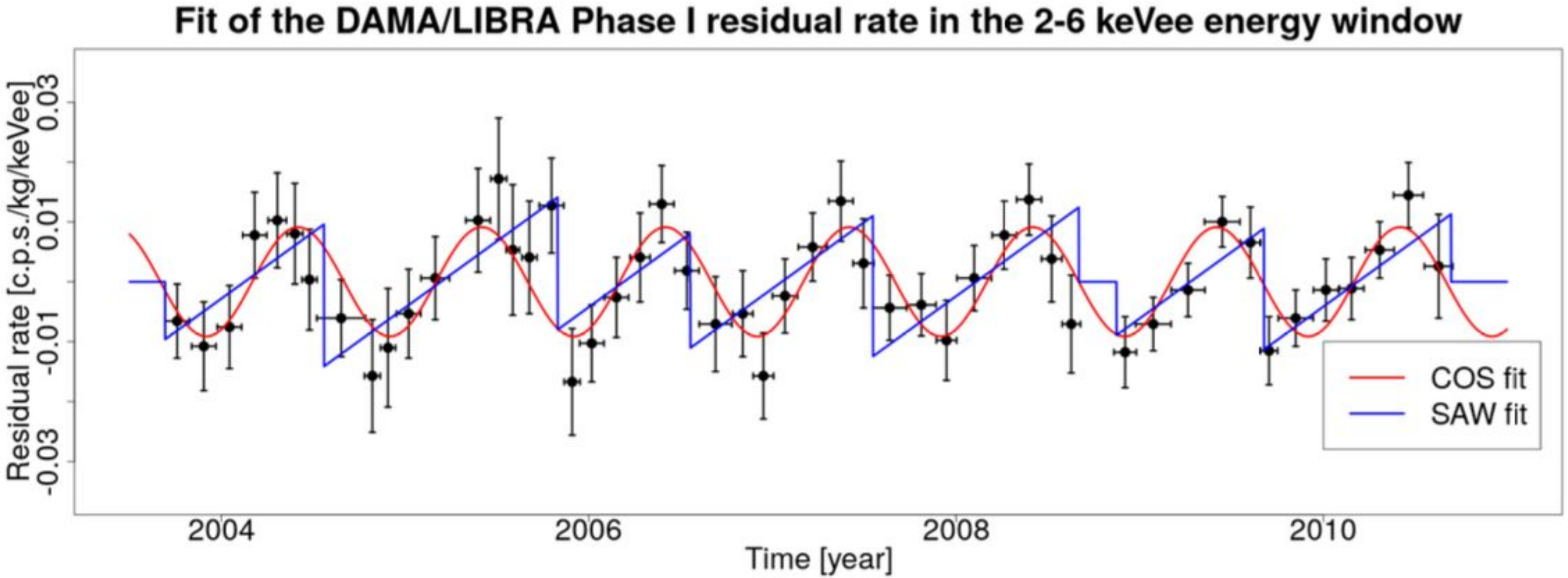
Additional material

DAMA/NaI fit



$BF_{C,S}$ [dB]	$LR_{C,S}$ [dB]	$OF_{C,S}$ [dB]	ΔBIC
-16.7	-18.1	1.4	8.34

DAMA/LIBRA I fit



$BF_{C,S}$ [dB]	$LR_{C,S}$ [dB]	$OF_{C,S}$ [dB]	ΔBIC
14.0	12.0	2.0	-5.53

Comparative results

DAMA phase	Fit to COS model		Fit to SAW model	
	A [cpd/kg/keVee]	$\chi^2_{\text{cos}}/\text{dof}$	B [cpd/kg/keVee/yr]	$\chi^2_{\text{saw}}/\text{dof}$
DAMA/NaI	0.0168 ± 0.0029	36.7/36	0.0552 ± 0.0085	28.3/36
LIBRA I	0.0092 ± 0.0013	29.6/49	0.0222 ± 0.0032	35.1/49
LIBRA II	0.0092 ± 0.0011	43.3/51	0.0166 ± 0.0029	82.3/51

DAMA phase	$BF_{C,S}$ [dB]	$LR_{C,S}$ [dB]	$OF_{C,S}$ [dB]	ΔBIC
DAMA/NaI	−16.7	−18.1	1.4	8.34
LIBRA Phase I	14.0	12.0	2.0	−5.53
LIBRA Phase II	86.5	84.7	1.8	−39.0
All	88.8	64.7	24.1	−39.7

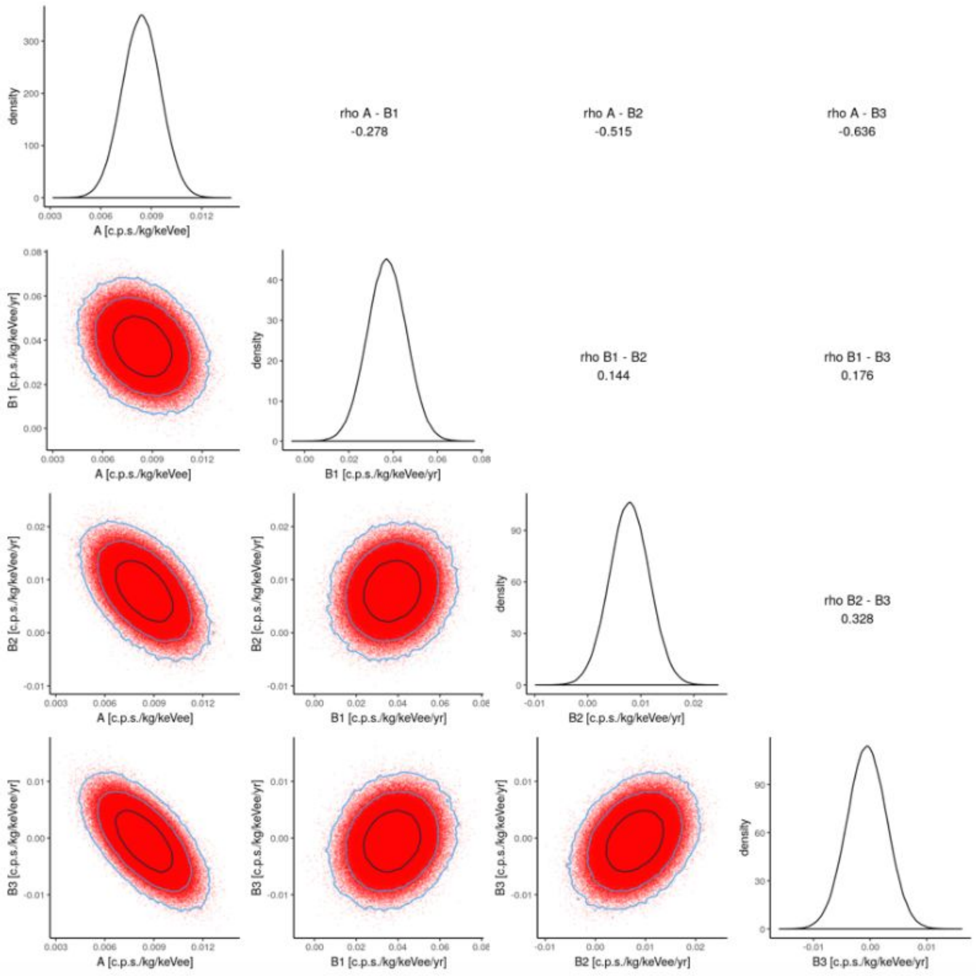
Comparative results

Fit results	A [cpd/kg/keVee]	B [cpd/kg/keVee/yr]	χ^2/dof
C+S fit	0.0102 ± 0.0016	-0.0035 ± 0.0042	42.7/50

Model comparison	BF [dB]	LR [dB]	OF [dB]	ΔBIC
COS vs SAW	86.5	84.7	1.8	−39.0
COS vs C+S	11.3	−1.4	18.0	−3.31
SAW vs C+S	−69.9	−86.2	16.3	35.7

Table 3. **Top**: *Results of the fit of the cosine amplitude A and the sawtooth coefficient B of the C+S model obtained on the DAMA residuals in the (2-6) keVee energy window during the DAMA/LIBRA Phase II, together with the corresponding χ^2/d.o.f.* **Bottom**: *Comparison between the various models in the DAMA/LIBRA Phase II dataset in terms of Bayes factor (BF), likelihood ratio (LR), Ockham's factor (OF) and difference of Bayesian Information Criterion (ΔBIC). For all these three metrics (BF, LR, ΔBIC) the SAW model is largely disfavoured.*

Marginal posterior for COS+SAW on DAMA/LIBRA II
(*T*,*t0* fixed)



Priors, posteriors and likelihood for COS and SAW on DAMA/LIBRA II