Entropy Bound and Unitarity

or

(From Black Holes to Baryons and Back)

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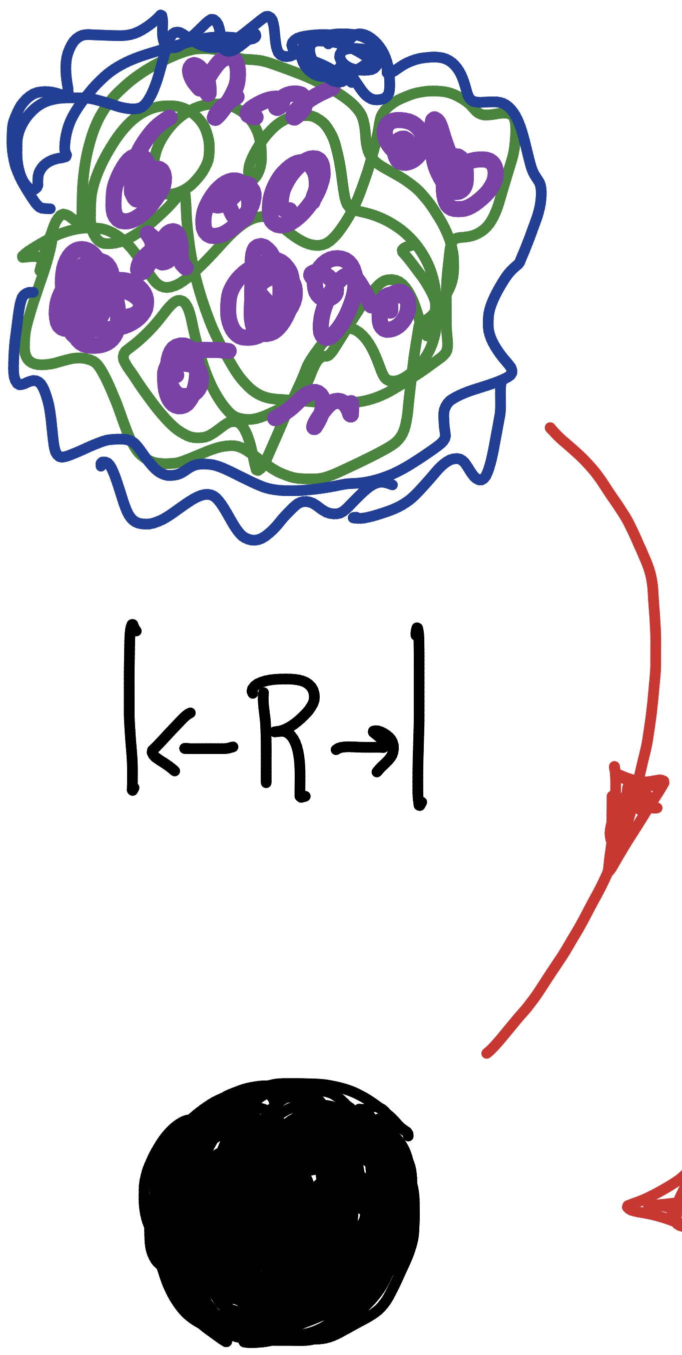
Based on: hep-th: 2003.05546
1907.07332
1906.03530

and earlier

Any object of mass M has associated <u>Schwarzschild</u> radius:

$$R \equiv 2 G_N M$$

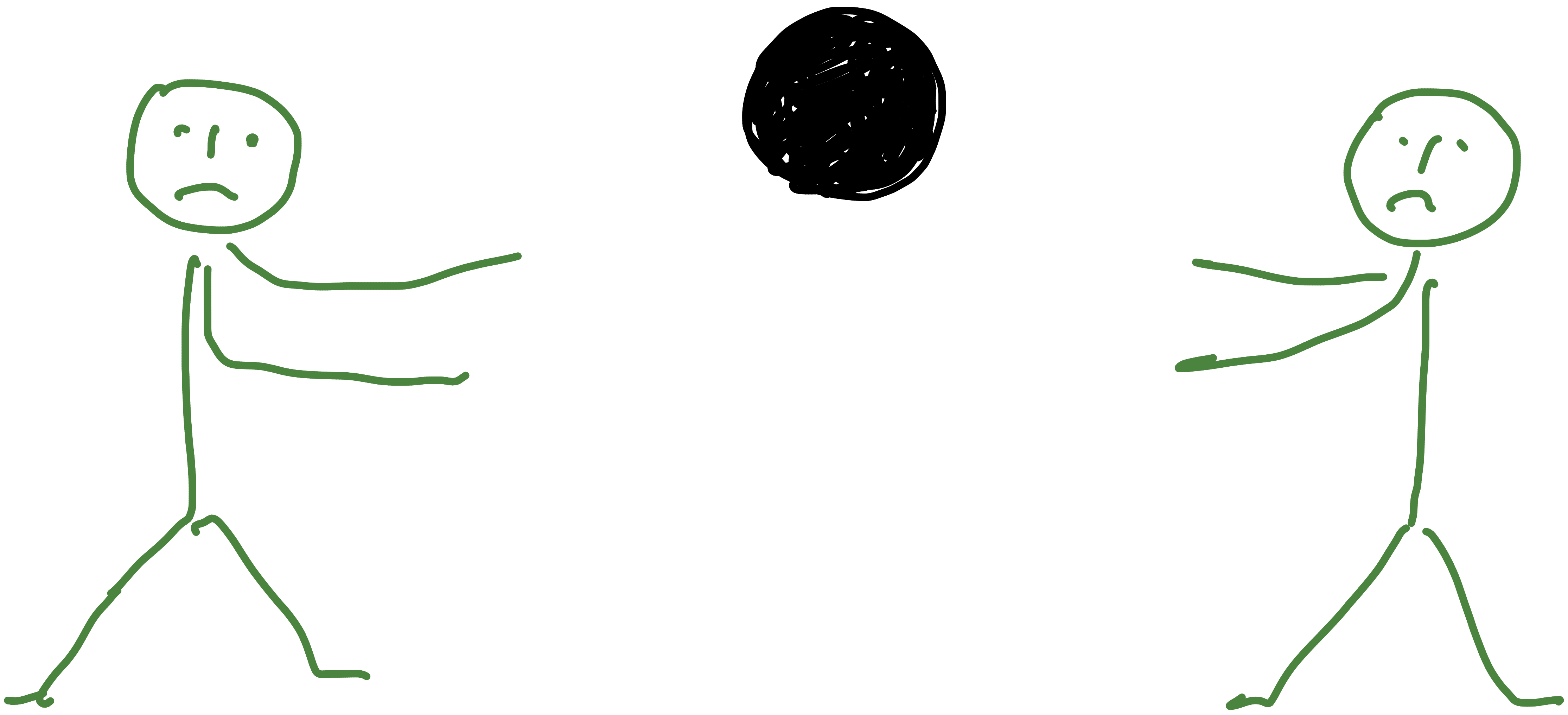


Black hole

Classically <u>featureless</u>.

So carries no information?

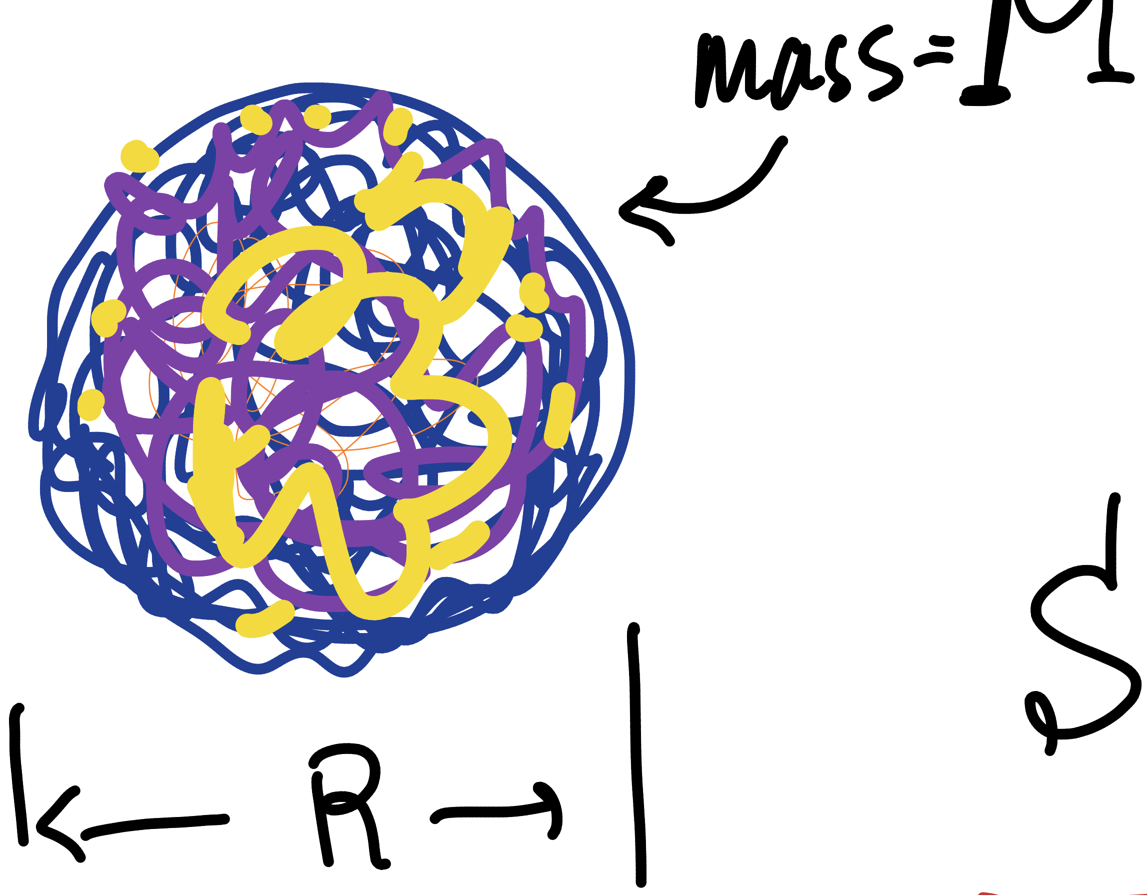

Hi, what's up



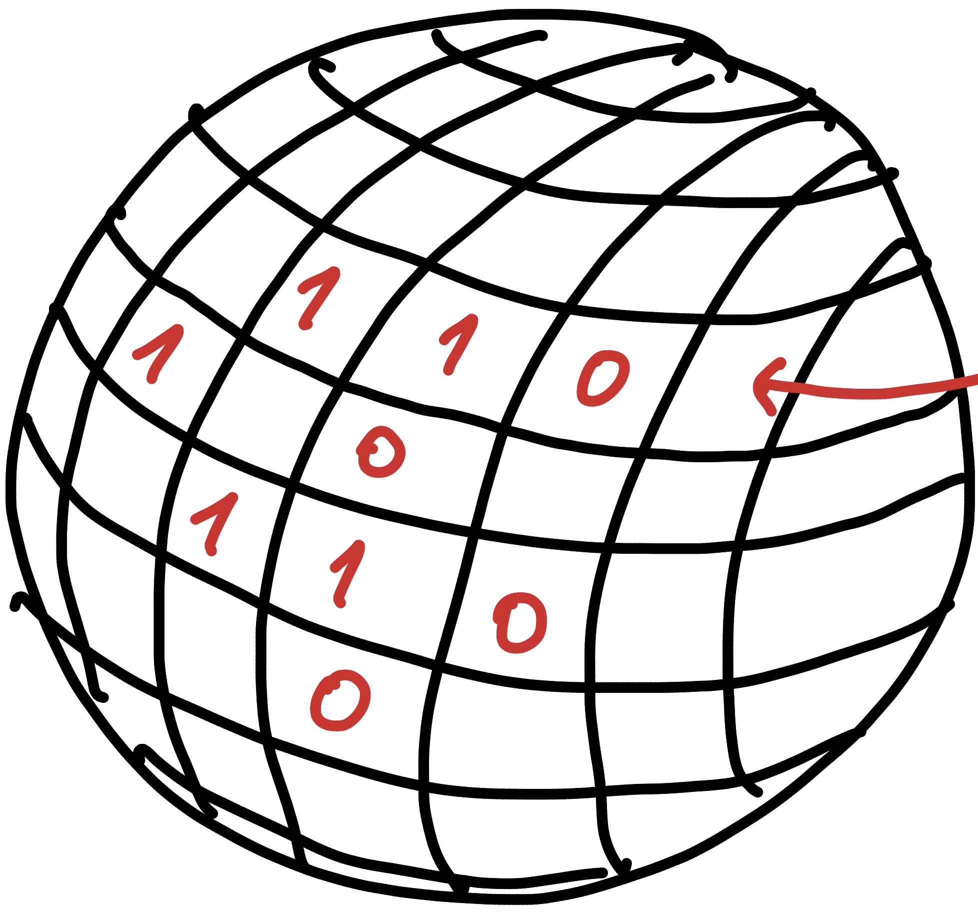
Black hole "mysteries":

In quantum theory black hole carries a maximal information:

Black hole saturates Bekenstein's entropy bound.



$$S_{max} = MR$$



Planck-area pixels L_p^2

$$S_{BH} = \frac{R^2}{L_p^2}$$

Alice

Saturates Bekenstein's entropy bound

$$S \leq S_{Bek} = 2\pi E R$$

$$E = M_{BH} = \frac{R}{2G}$$

$$\downarrow$$

$$S_{BH} = S_{Bek} = \frac{4\pi R^2}{4G} = \frac{Area}{4G}$$

Second mystery:

Why information is not coming out?

(Is it not?)

Hawkings thermal radiation:

$$T = \frac{\hbar}{R}$$

However, we sometimes forget that Hawkings computation is exact only in the limit:

$$M \to \infty, \quad G_N \to 0$$

$$R = 2MG_N = \text{finite}$$

But, in this limit Black hole is <u>eternal</u>.

What happens for finite M ?

Microscopic theory needed.

But, let us first ask:

Are black holes unique ?

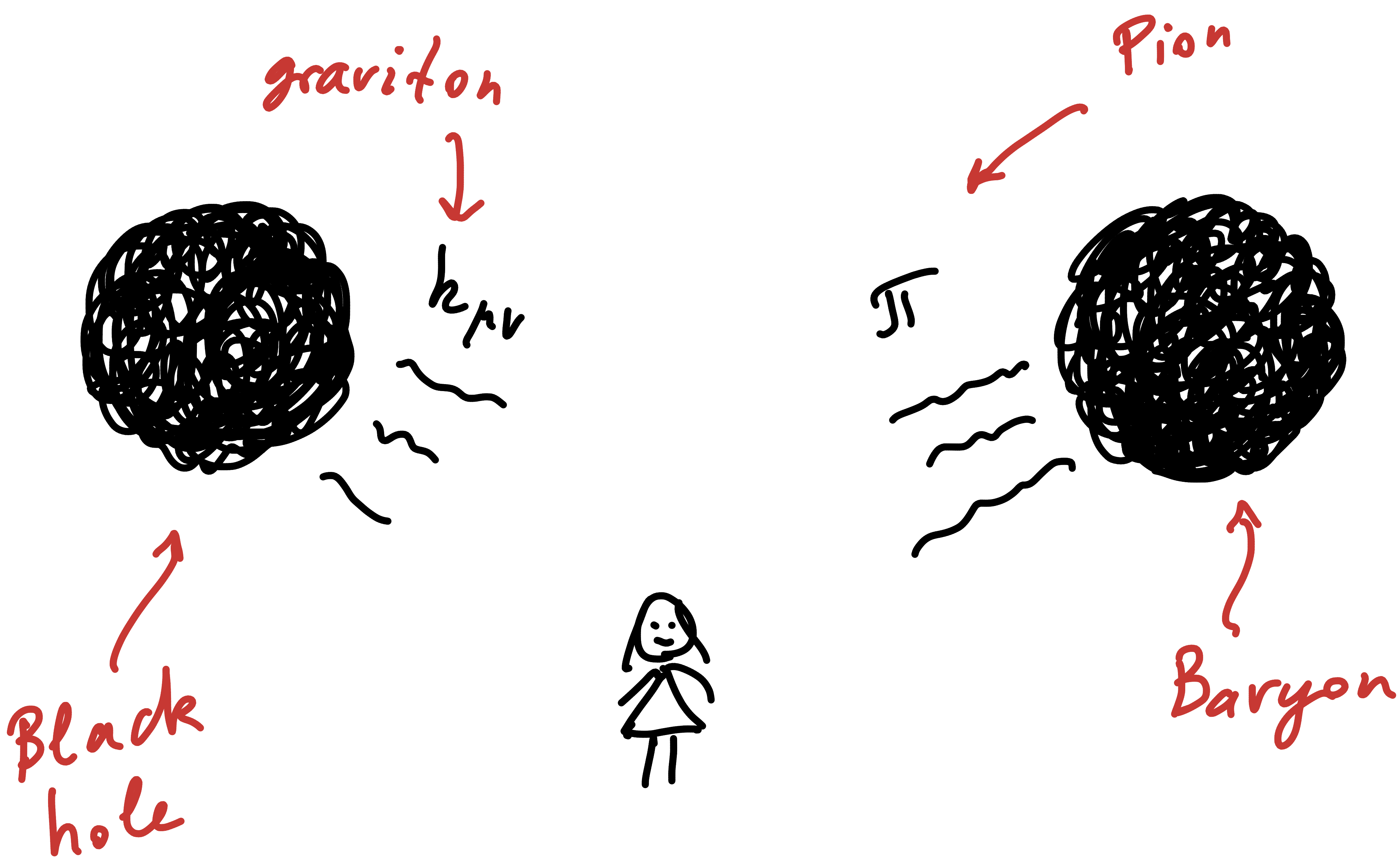
It turns out that the answer is no.

All objects behave in similar way at the point of saturation of entropy bound!

We just never looked.

Here is what we have discovered:

All QFT objects (such as solitons, baryons, instantons or simply lumps of classical fields) at the saturation point behave like black holes.



⊛ The entropy is equal to area:

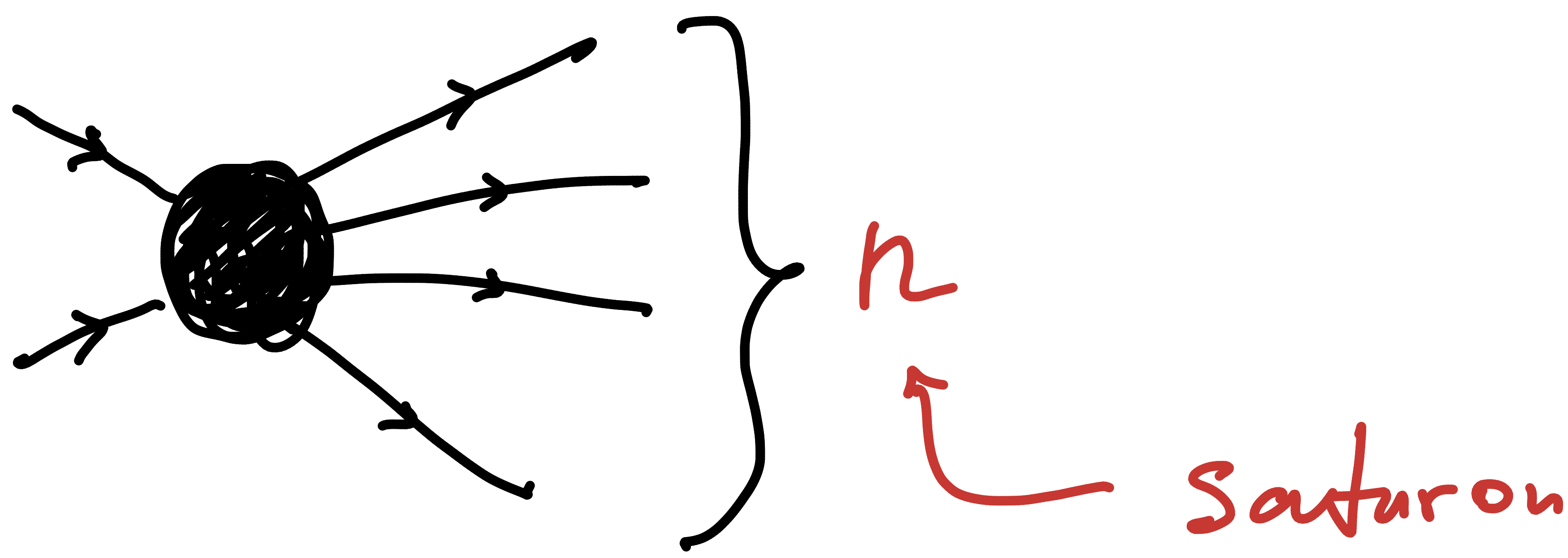
$$S = \text{Area}$$

⊛ In semi-classical limit, the object possesses information horizon.

⊛ In quantum theory the information is retrieved very slowly:

$$t \gtrsim S R$$

(*) The object saturates unitarity in scattering



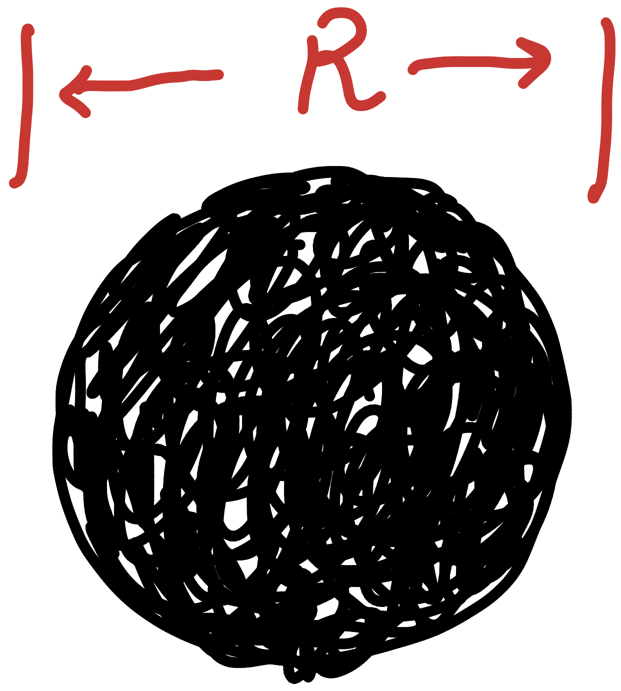
We call such objects "saturons".

The key point is an universal connection between entropy and unitarity.

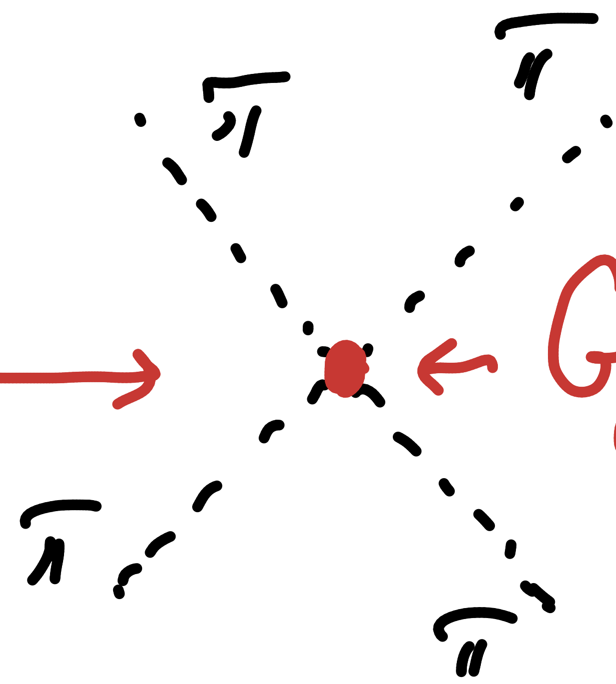
Unitarity imposes the following upper bounds on entropy of any QFT object of radius R:

(I) Area-law bound:

$$S_{max} = \frac{Area}{G_{gold}} = \frac{R^2}{f^2}$$



Goldstone coupling



$G_{gold} = \frac{1}{f^2}$

Goldstone boson of spontaneously broken symmetry: (1) Space-translation;
(2) Internal.

II Inverse-coupling bound:

$$S_{max} = \frac{1}{\alpha(q)}$$

running coupling at $q = \frac{1}{R}$ for interaction of range R

Notice, for Goldstone:

$$\alpha_{Gold}(q) = \frac{q^2}{f^2} \longrightarrow \frac{1}{\alpha_{Gold}\left(\frac{1}{R}\right)} = R^2 f^2$$

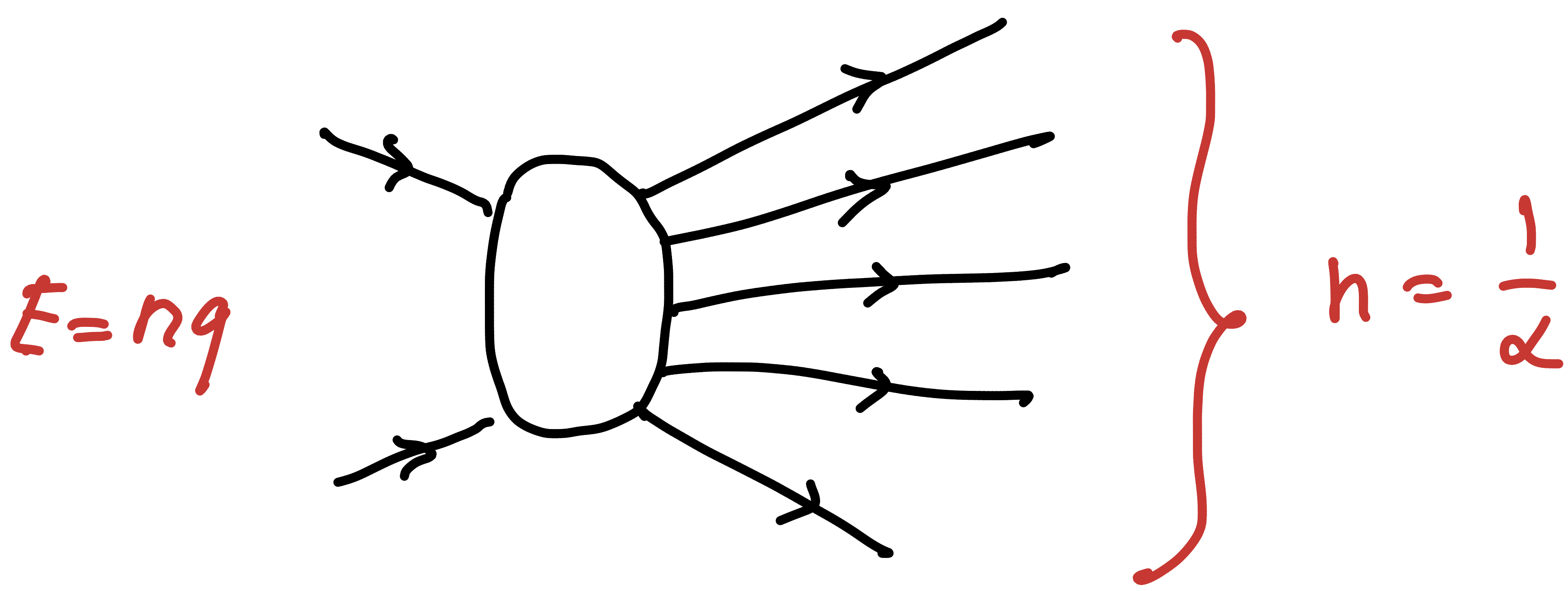
Area

Saturation of these entropy bounds is in one-to-one correspondence with saturation of unitarity by

$2 \rightarrow n$ scattering amplitudes

for $\quad n = \frac{1}{\alpha}$

The point of optimal truncation.



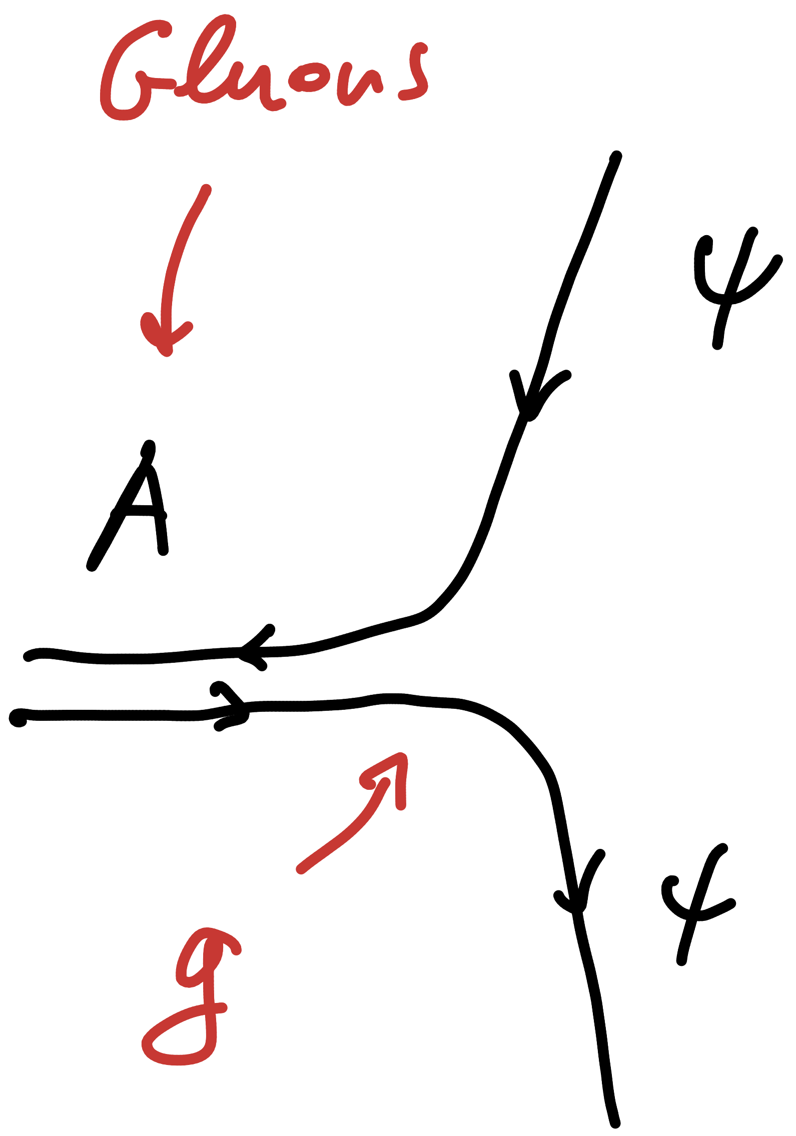
momentum-transfer $q = \frac{1}{R}$

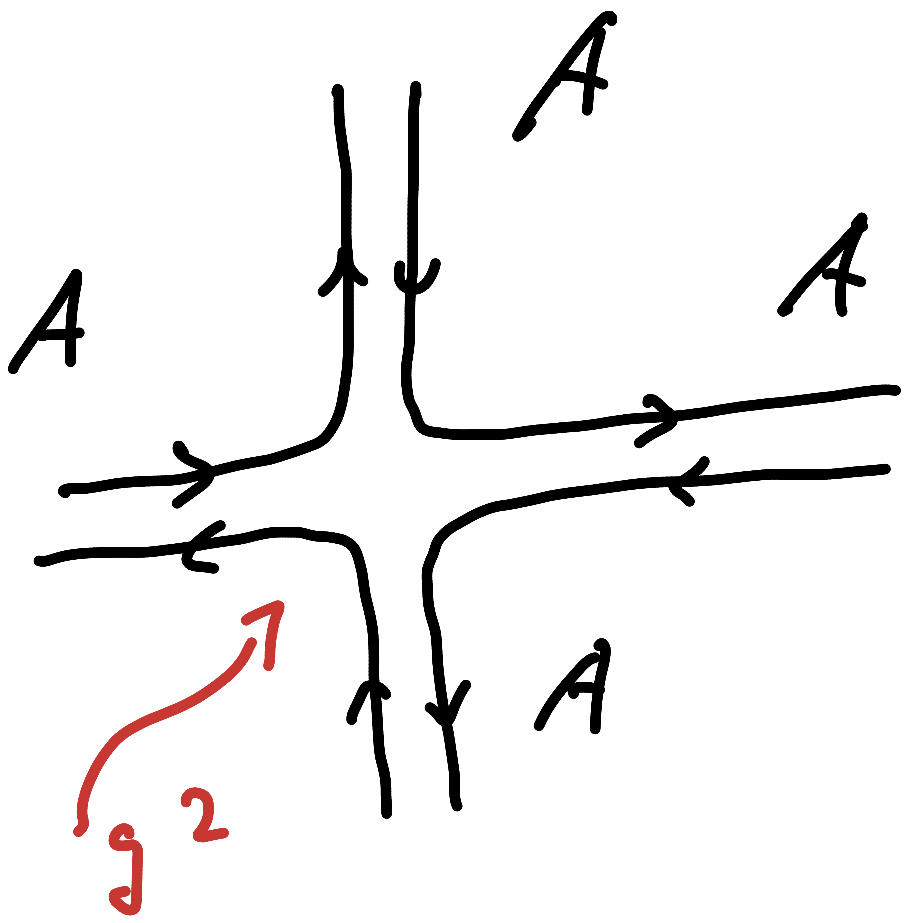
SU(N) QCD with N_F flavors of quarks

ψ_L , ψ_R

$U(N_F)_L \otimes U(N_F)_R$

Flavor group





QCD coupling $\alpha \equiv \frac{g^2}{4\pi}$

't Hooft coupling (for color):

$$\lambda_t \equiv \alpha N$$

't Hooft coupling for flavor:

$$\lambda_F \equiv \alpha N_F$$

't Hooft limit (and Veneziano for N_F)

$$\alpha \to 0, \; N \to \infty, \; N_F \to \infty$$

$$\lambda_t = \text{finite} \qquad \lambda_F = \text{finite}$$

Also:

$$\Lambda_{QCD} = \text{finite}$$

In this theory we have confinement and chiral symmetry breaking

$$\langle \bar{\psi} \psi \rangle \neq 0$$

$$\downarrow$$

$$U(N_F)_L \otimes U(N_F)_R \longrightarrow U(N_F)$$

There emerge $N_F^2 - 1$ massless Nambu-Goldstone bosons, pions

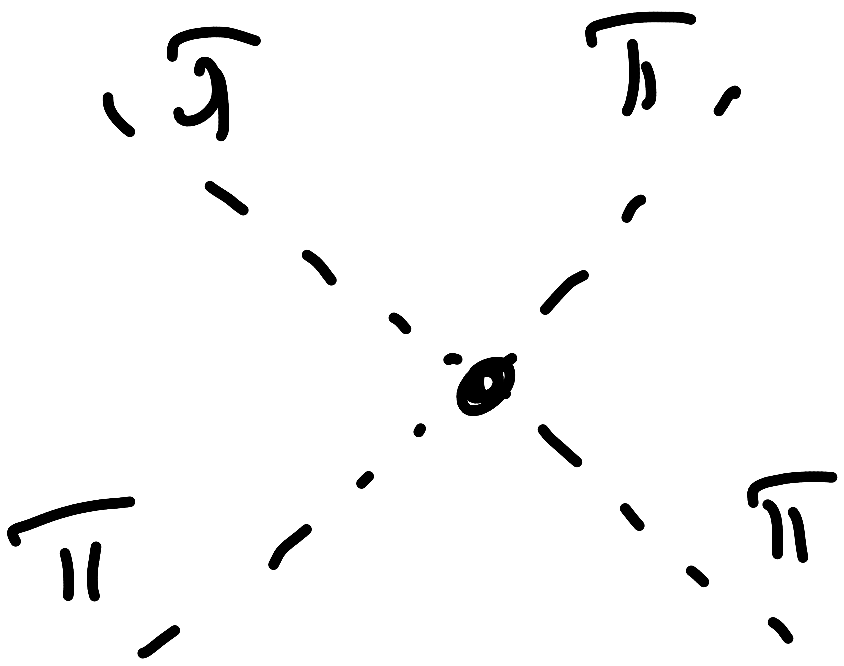
$$\pi_\alpha^{\;\beta} \qquad \alpha, \beta = 1, \dots N_F$$

flavor index of $U(N_F)$

(There is also η' with mass$^2 \sim \frac{1}{N}$

Witten; Veneziano)

Pion coupling



$$\alpha_\pi = \frac{g^2}{f_\pi^2} = \frac{g^2}{N\Lambda_{QCD}^2}$$

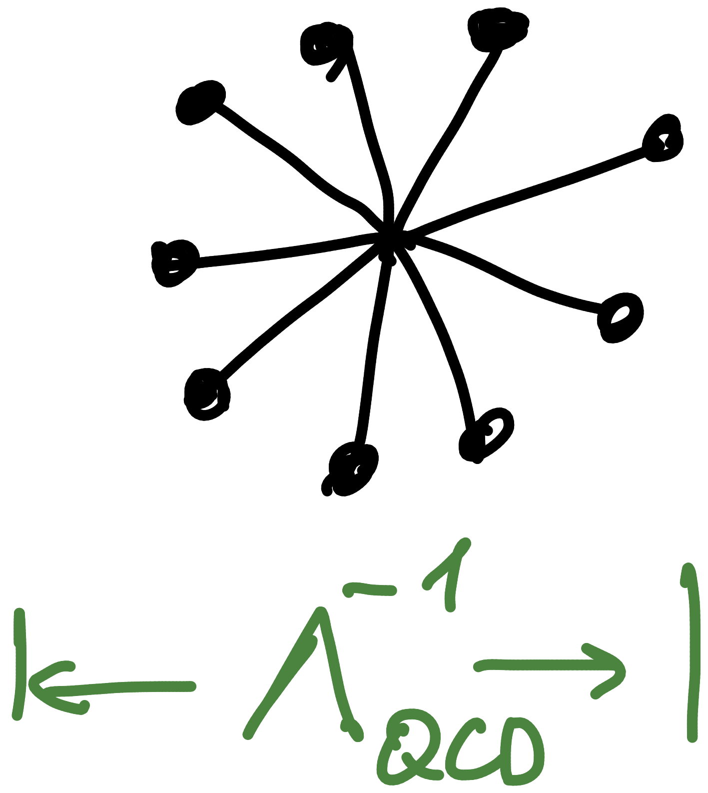
Pion decay constant

$$f_\pi = \sqrt{N}\,\Lambda_{QCD}$$

In particular, for $g \sim \Lambda_{QCD}$

$$\alpha_\pi = \frac{1}{N}$$

Baryons in $SU(N)$ Witten

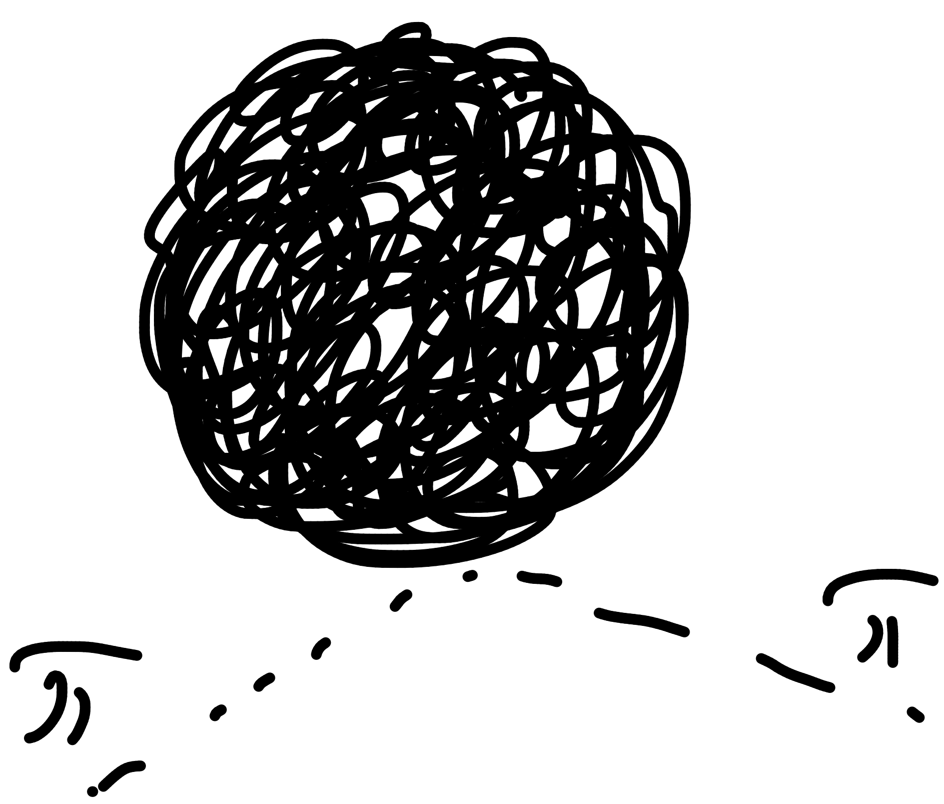


$$M_B = N \Lambda_{QCD}$$

$$R_B = \Lambda_{QCD}^{-1}$$

$$\alpha N = 1 \rightarrow \lambda_t = 1$$

Baryons can be viewed as solitonic bound-states of N pions, Skyrmions.



$$\alpha_\pi \cdot N = 1$$

Baryon entropy (e.g. maximal spin).

Baryon forms a multiplet under the flavor symmetry $SU(N_F)$ of dimensionality

$$n_{st} = (N+1)\binom{N+N_F-1}{N}$$

The micro-state entropy:

$$S_B = \ln(n_{st}) \simeq N \ln\left\{\left(1+\frac{1}{\lambda_F}\right)^{\lambda_F}\left(1+\lambda_F\right)\right\}$$

$$N = \frac{1}{\alpha} = \frac{1}{\alpha_\pi(R_B)}$$

Saturates the bound $S_{max} = \frac{1}{\alpha}$

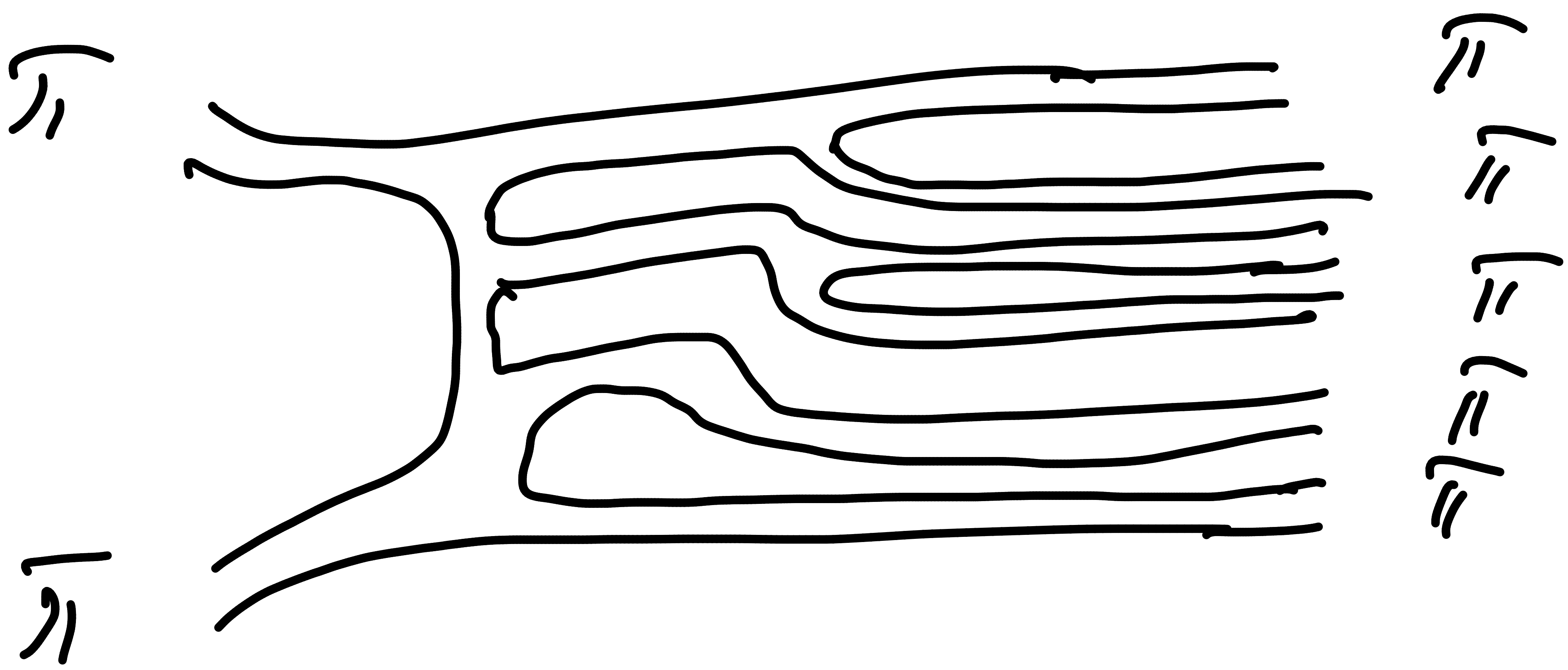
For

$$\lambda_F \simeq 0.54$$

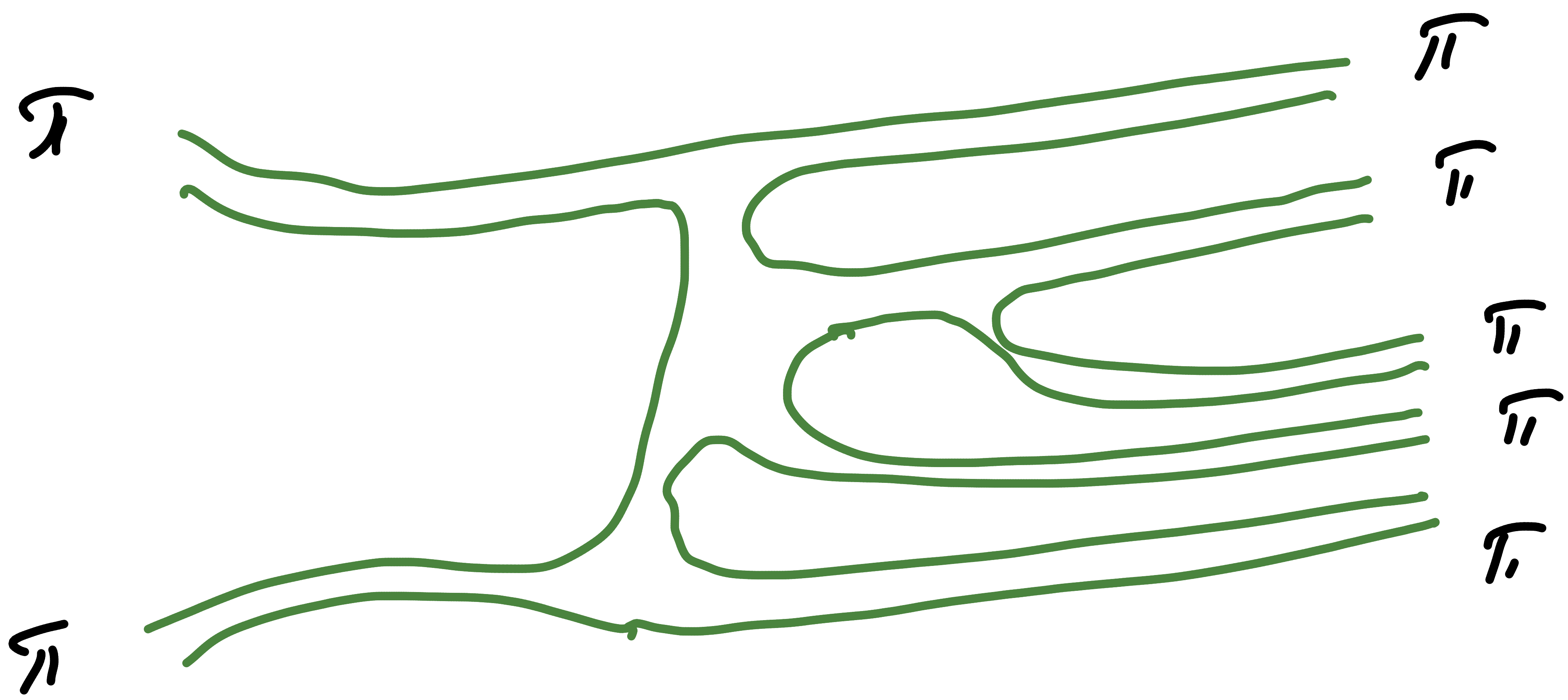
Baryon would violate entropy bound for $\lambda_F \gg 1$ but this is impossible:

* This violates asymptotic freedom which demands $\frac{\lambda_F}{\lambda_t} < \frac{11}{2}$;

* Simultaneously, $2 \to N$ Pion scattering would violate unitarity for momentum-transfer $q \ll R_{bar}$!



Pion scattering



$$N = \frac{1}{\alpha_\pi}$$

cross section

$$\sigma \simeq \left[(1+\lambda_F)\, e^{-1}\left(1+\frac{1}{\lambda_F}\right)^{\lambda_F}\right]^{\frac{1}{\alpha_\pi}}$$

$$\simeq e^{-\frac{1}{\alpha_\pi} + S_B} = e^{-N + S_B}$$

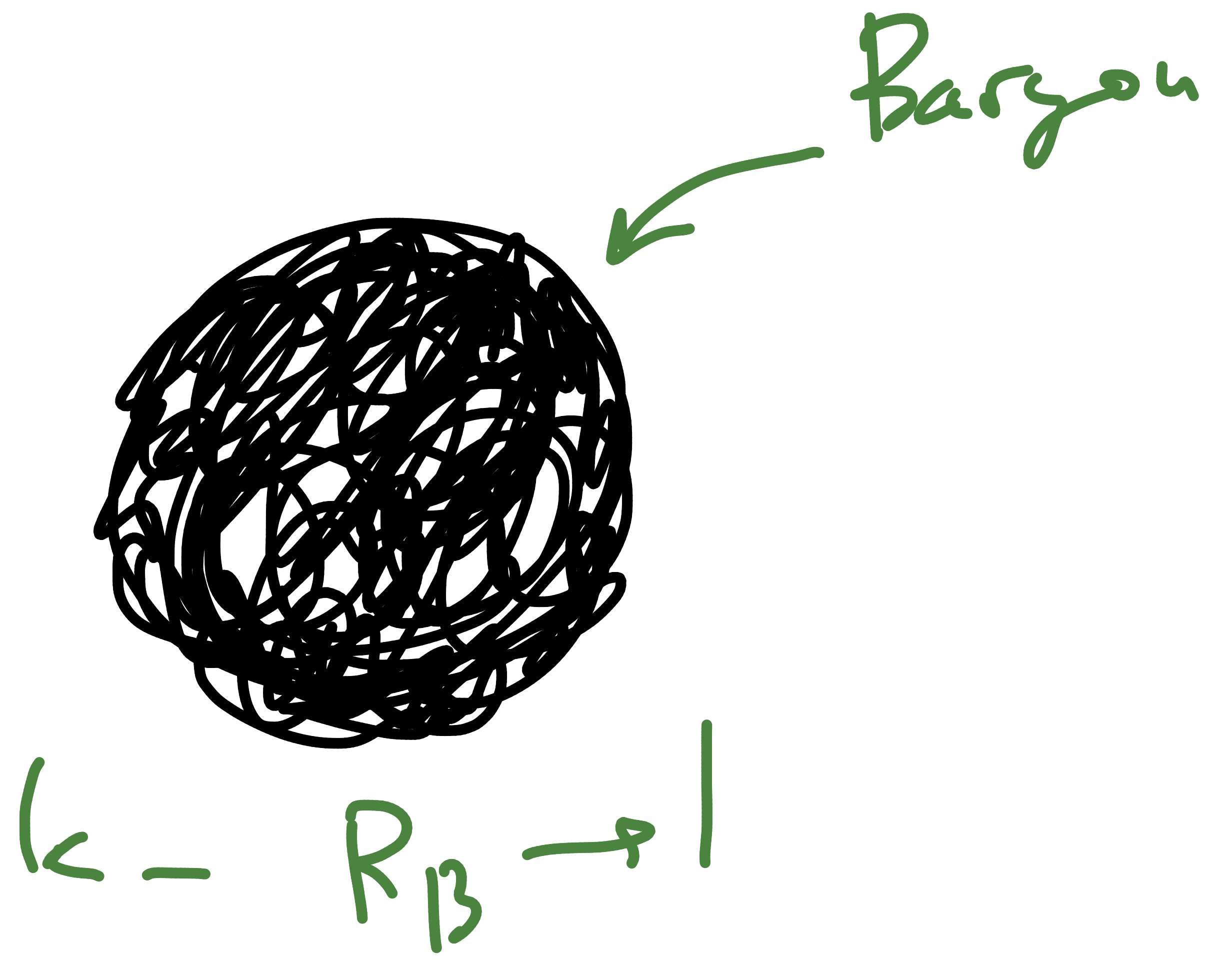
<u>Would</u> violate unitarity for $\lambda_F \gg 1$

For $\lambda_F \to \infty$

$$\hookrightarrow \quad S_B \sim \frac{1}{\alpha} \ln(\lambda_F)$$

$$\sigma \to (\lambda_F)^N$$

But, this is impossible as it would mean that baryon cannot be described as a state in theory of pions.



$$|\leftarrow \quad q^{-1} \quad \rightarrow|$$

Cutoff of the pion theory.

Thus, at the saturation point the baryon entropy is

$$S_B = N = \frac{1}{\alpha} = \frac{1}{\alpha_\pi}$$

Recall: $f_\pi = \sqrt{N}\,\Lambda_{QCD} = \sqrt{N}\,R_B^{-1}$

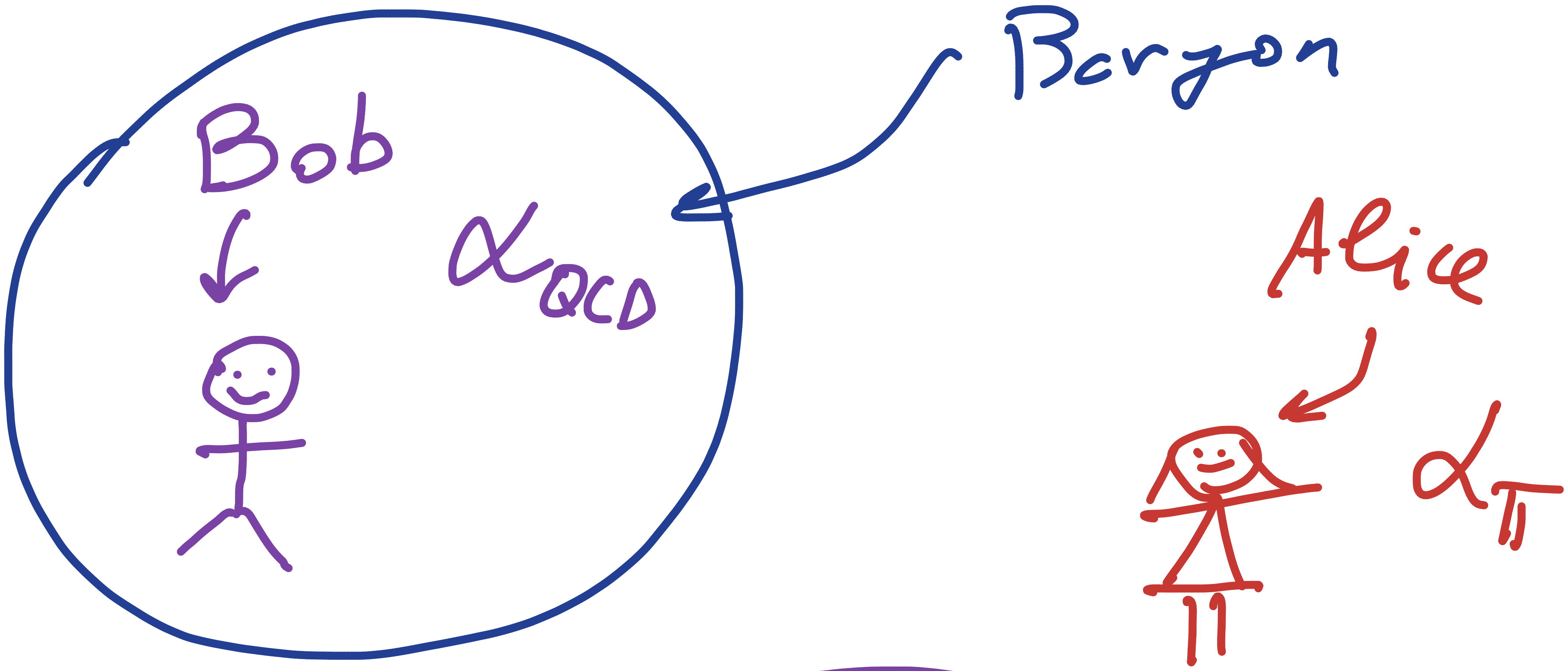
$$M_B = N\,\Lambda_{QCD} = N\,R_B^{-1}$$

Thus,

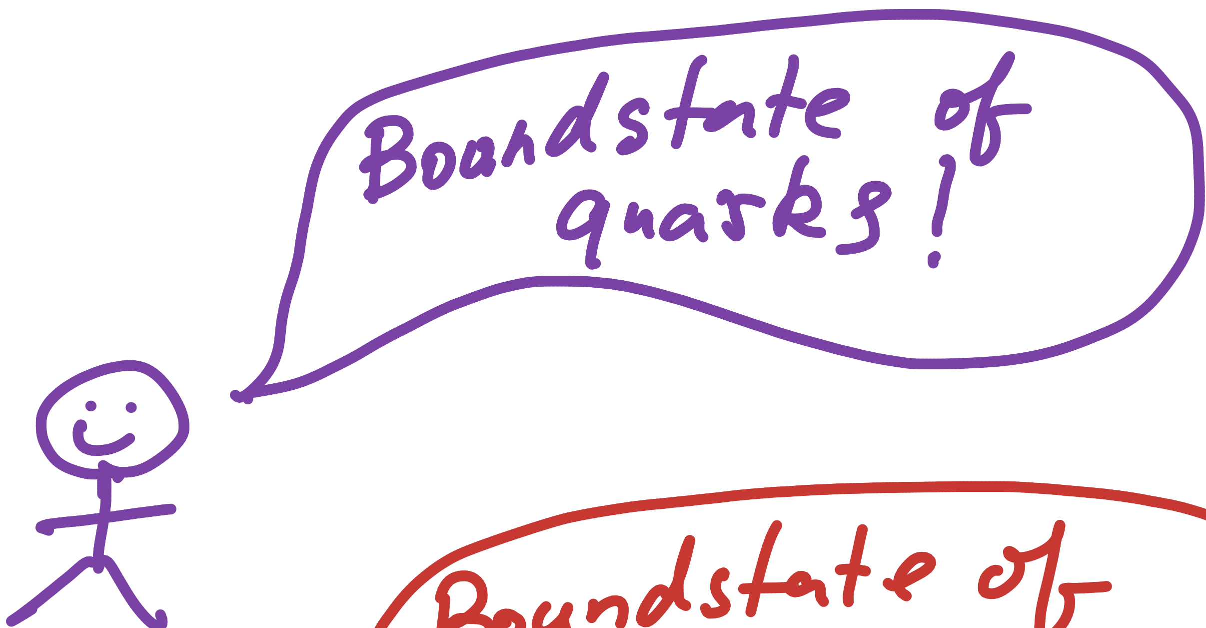
$$S_B = \frac{1}{\alpha} = \frac{1}{\alpha_\pi} = \left(R_B f_\pi\right)^2 = M_B R_B$$

Area $\cdot\, f_\pi^2$

Just like a Black Hole!



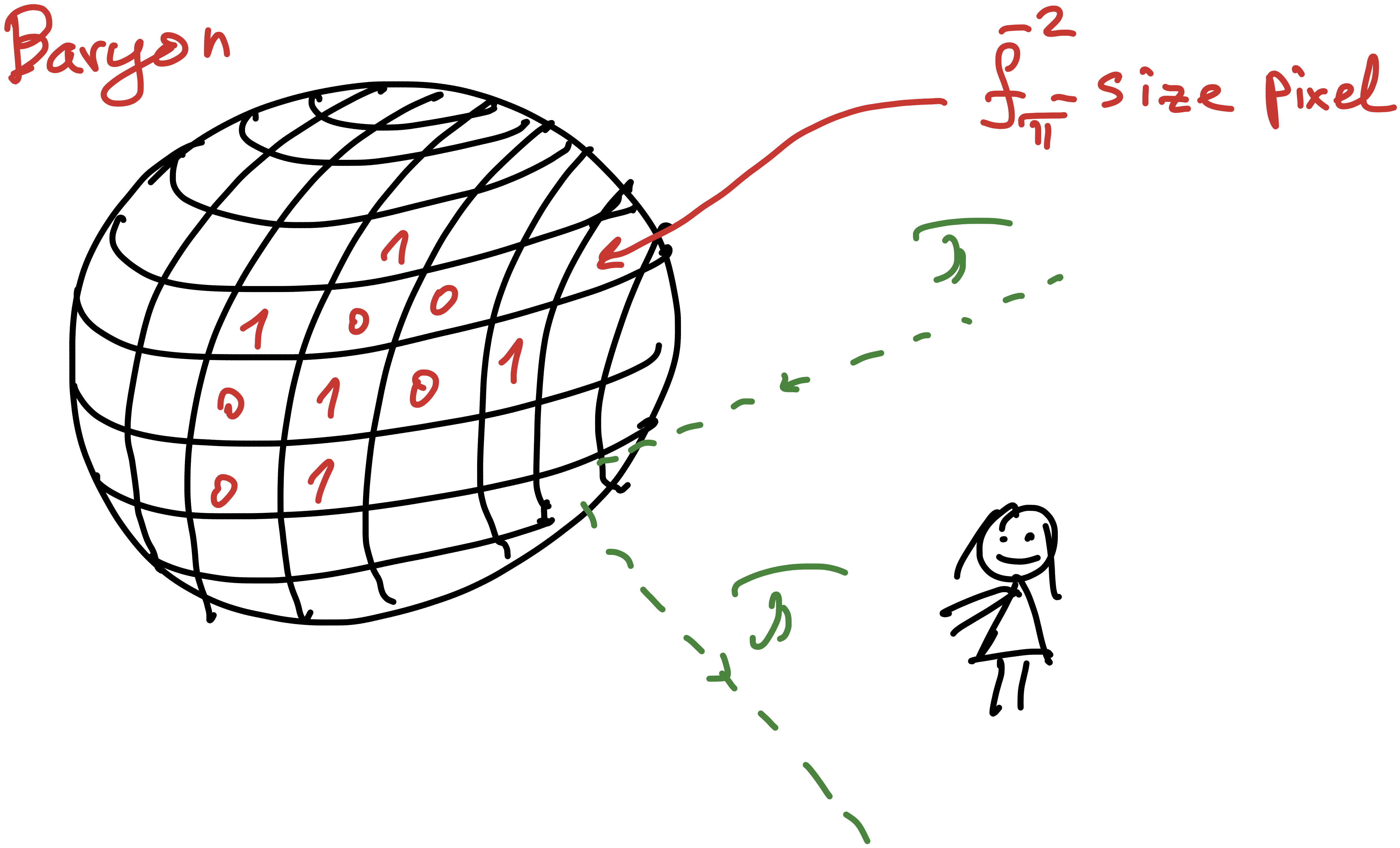
* Bob:



* Alice:

For both:

$$S = \frac{1}{\text{coupling}} = \text{Area}$$



Alice can scatter pions in order to decode information stored in a baryon.

It takes time

$$t \gtrsim S_{baryon} \cdot R_{baryon}$$

to start decoding this information.

Quantum information encoded in the flavor content of the baryon

Total number of possibilities:

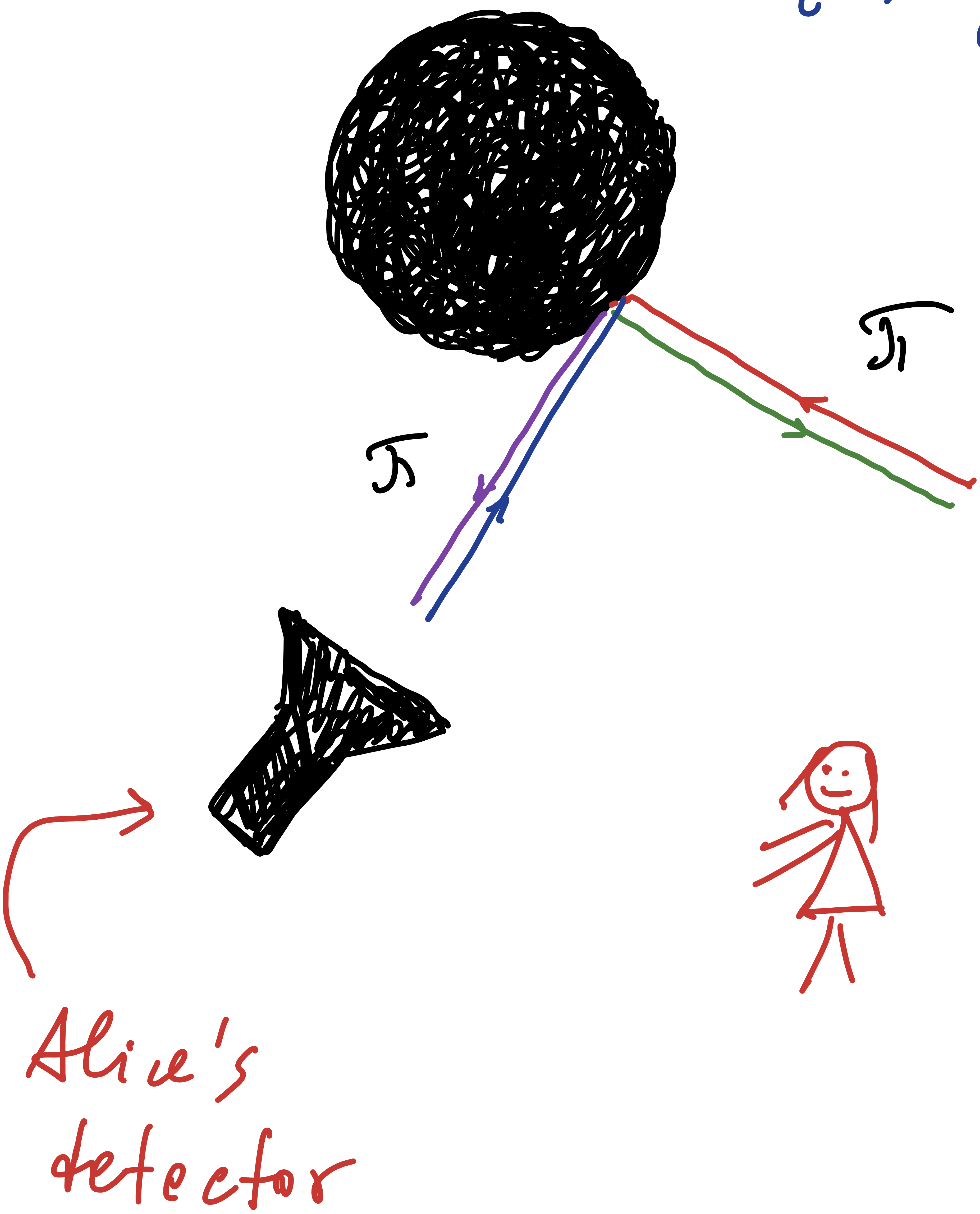
$$n_{st} = e^{S_B}$$

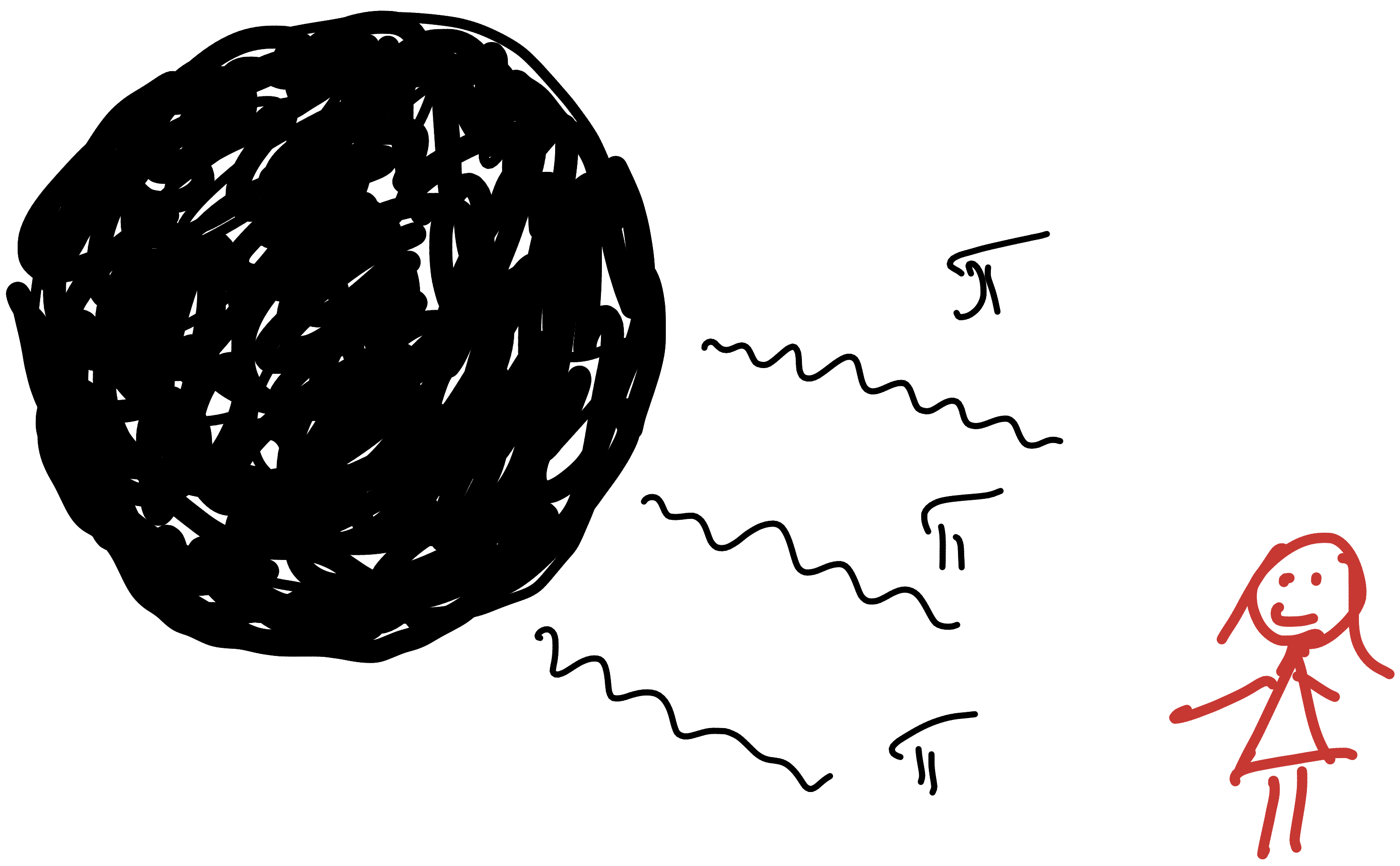
$$\overset{K \longleftarrow\ N\ \longrightarrow 1}{\text{message: } (uuu\,dddd\,ss\,\ldots\ldots)}$$

encoded in baryon

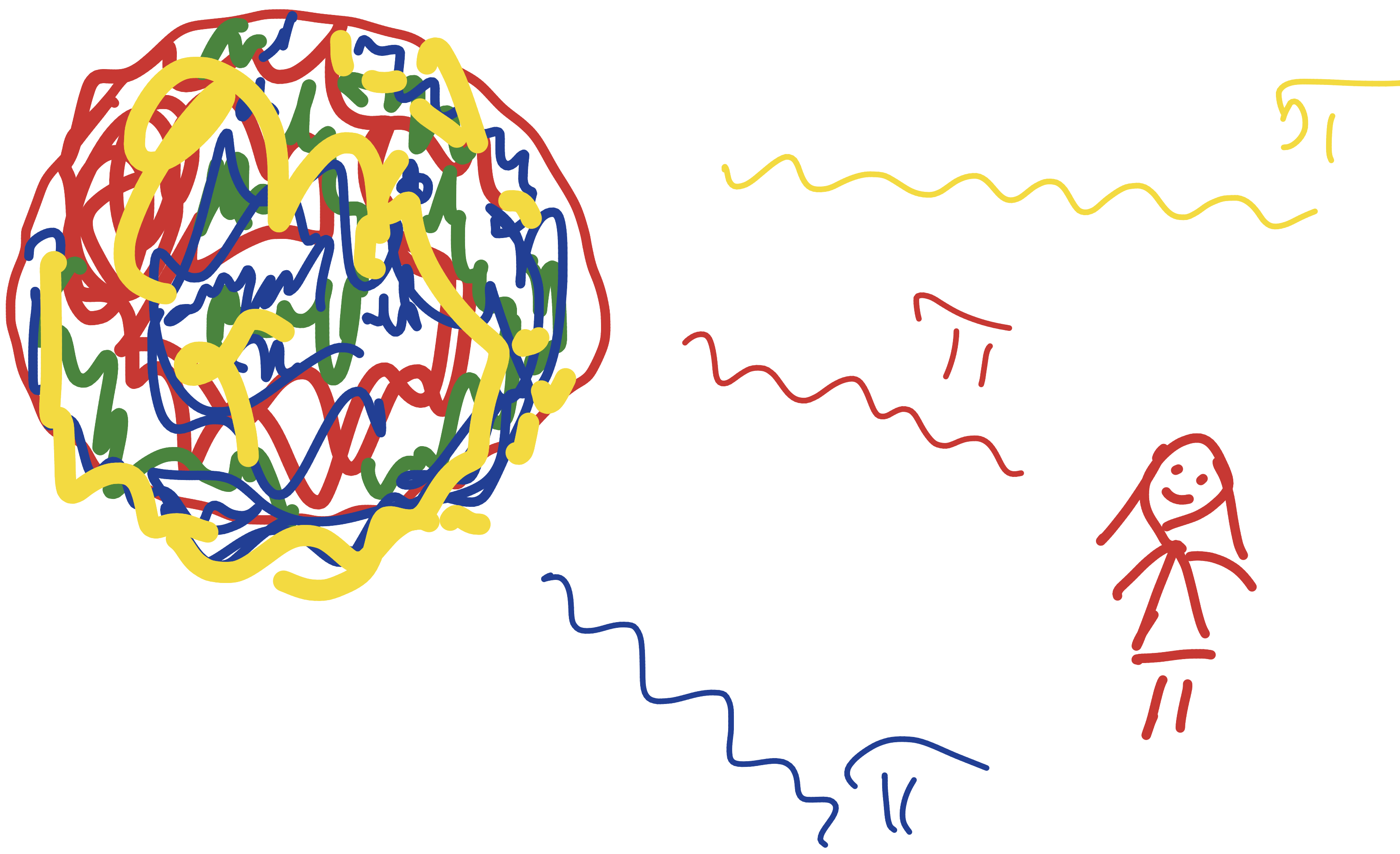
But Alice needs a very long time to decode the message

$$t \gg \frac{1}{\alpha_\pi} R_B$$





$t = 0$

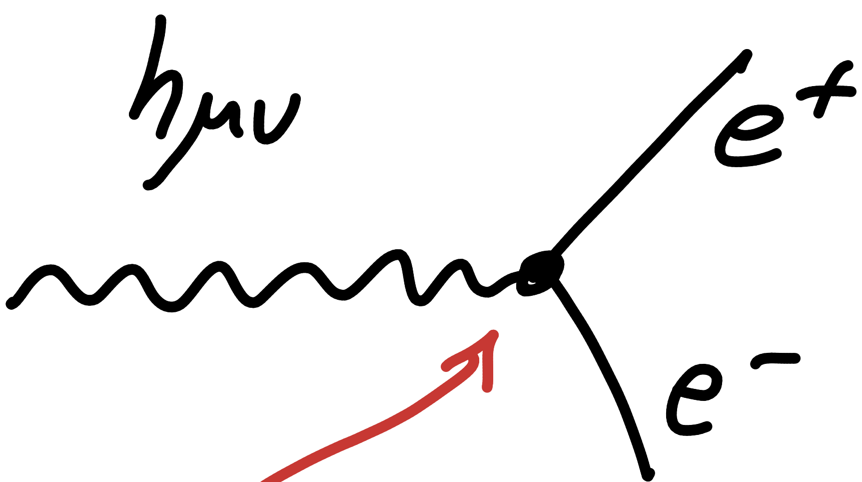


$t \gtrsim N R_B$

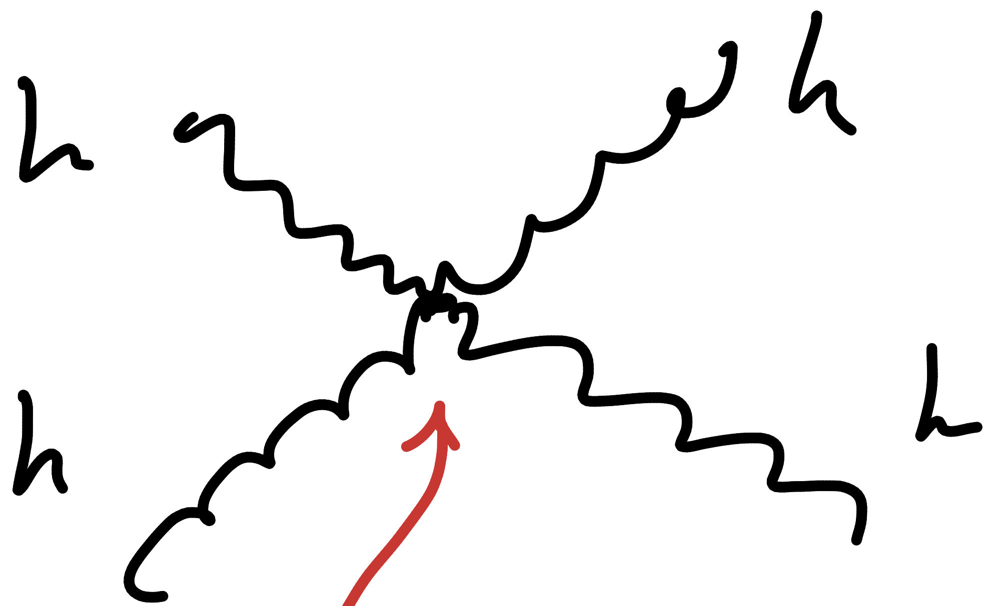
Exactly as Page's time for a black hol!

What can we learn about black holes?

In order to set the dictionary, first notice:



$$\frac{q}{M_P}$$



$$\alpha_{gr} = \frac{q^2}{M_P^2}$$

Graviton decay constant

$$f_{gr} = M_P$$

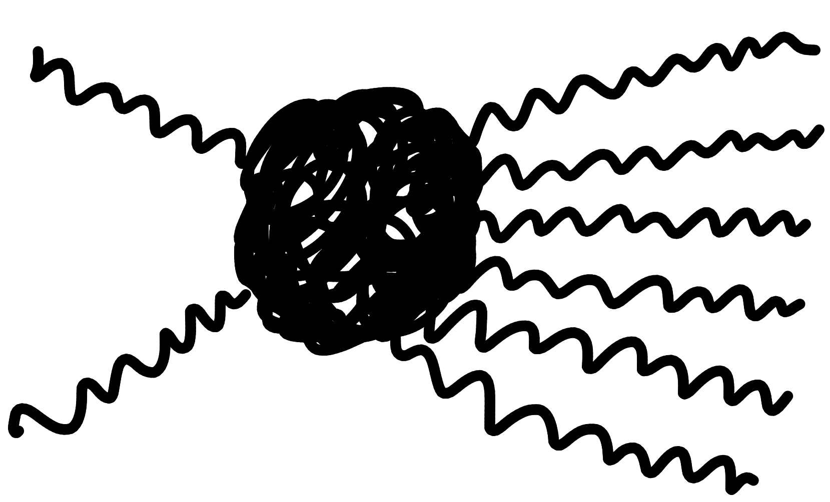
Now, it is obvious that black hole entropy satisfies relation

$$S_{BH} = \frac{1}{\alpha_{gr}(q)} = \left(R\, f_{gr}\right)^2$$

G.D., Gomez, '11

where $q = \frac{1}{R}$.

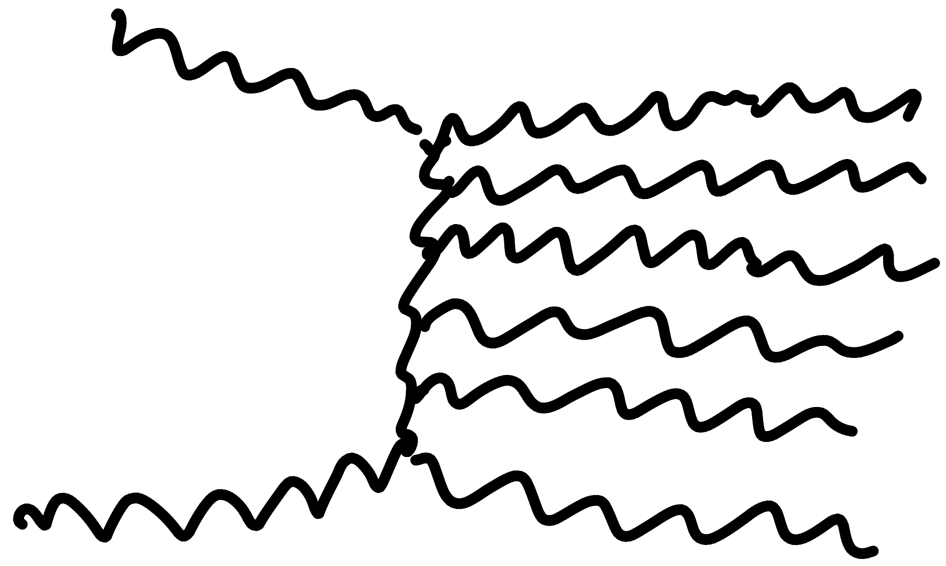
Similarly, $2 \to n$ graviton amplitudes saturate unitarity for $n = \frac{1}{\alpha}$ and $q = \frac{1}{R}$



$$\Big\} \; n = \frac{1}{\alpha_{gr}}$$

G.D., Gomez, Kehagias '11

Computation of $2 \to n$ graviton amplitudes at $E \gg M_P$.



$n = \frac{1}{\alpha_{gr}}$

$$\sigma = n! \, \alpha_{gr}^{n} \, e^{S} = e^{-\frac{1}{\alpha_{gr}} + S}$$

G.D., Gomez, Isermann, Lüst, Stieberger, '14

Addazi, Bianchi, Veneziano, '16

Evidence for black hole N-portrait ?

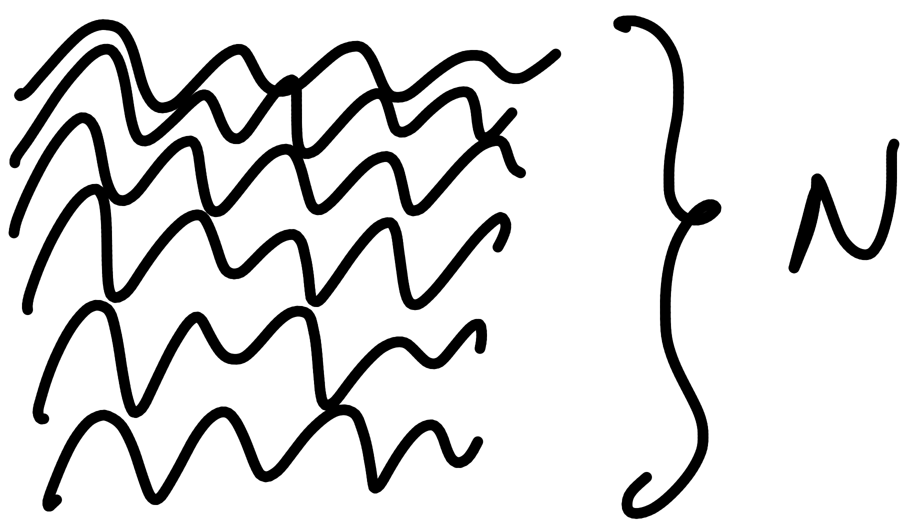
G.D., Gomez '11

Microscopic Theory: Black Hole's Quantum N-portrait

G.D., Gomez '10

Black hole represents a saturated state of soft gravitons:

$$|BH\rangle = |N\rangle$$



Can be viewed as "condensate" or a coherent state.

Wavelengths $= R$

Number $=$ $N = \frac{R^2}{L_P^2} = \frac{1}{\alpha_{gr}}$

Once we assume that black hole is a saturated state, no other assumption is needed!

This is the power of saturation

$$N = \frac{1}{\alpha_{gr}} = \frac{R^2}{L_P^2}$$

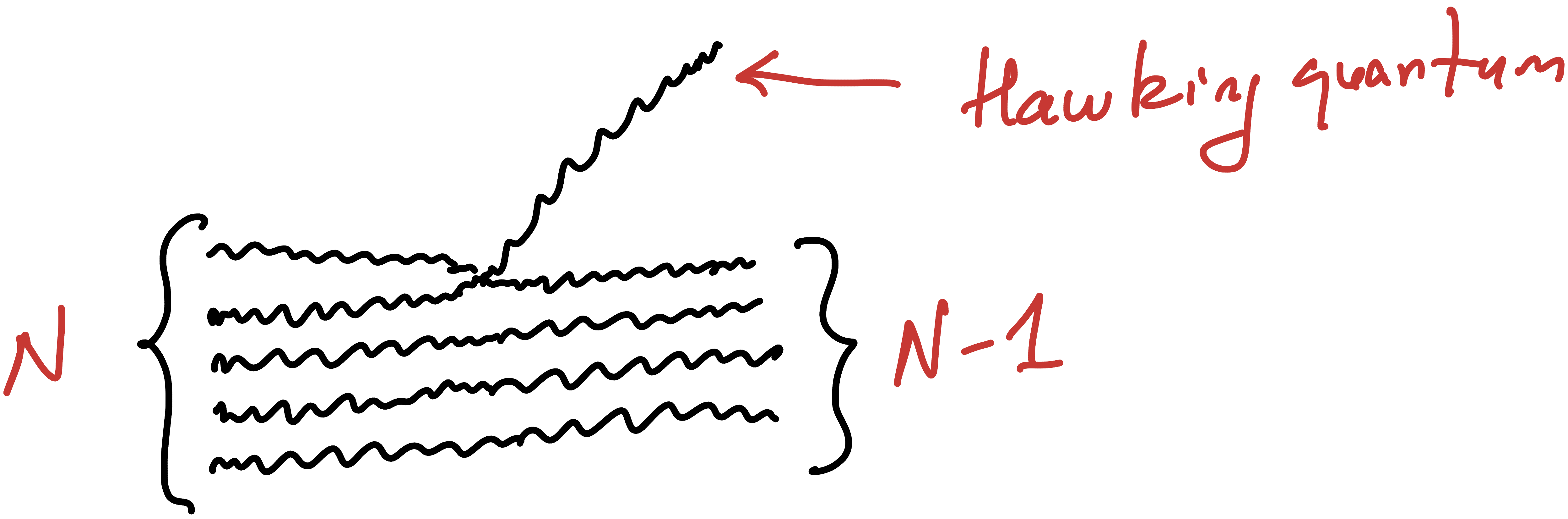
Black hole mass: $M = N\frac{1}{R} = \sqrt{N}\, M_P$

Black hole entropy: $S_{BH} = \frac{1}{\alpha_{gr}} = N = \frac{R^2}{L_P^2}$

Black hole temperature:

$$T = \frac{1}{R}$$

Hawking radiation = quantum depletion of graviton boundstate (condensate)



Quantum information starts to become readable after

$$t_* \sim N R$$

Prediction: for $t > t_*$ the black hole quantum-breaks!

Conclusions and outlook

- We have discovered that unitarity imposes universal bounds on entropy of any QFT object

$$S_{max} = \frac{1}{\alpha} = \text{Area}\, f^2_{Gold} \qquad (B1)$$

- Their saturation is in one-to-one correspondence with saturation of unitarity by $2 \to n = \frac{1}{\alpha}$ amplitudes.

- The objects that saturate the bound (B1) (saturons) exhibit universal black-hole-like properties.

Saturons can be: Solitons, baryons, instantons, ...

* This provides a strong evidence that black holes obey the same universal rules imposed by (B1) and unitarity as any other saturated object.

* This supports the idea of black hole N-portrait: Black hole is a saturated state of soft gravitons.

* Various non-perturbative phenomena (such as confinement in $SO(N)$) can be understood through the prism of entropy bound (B1) and unitarity.