Investigating three-nucleon forces with current and future Gravitational Wave detections

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A. Maselli, A. Sabatucci and O. Benhar, Phys. Rev. C 103, 065804, (2021)

Introduction

In this presentation I will report the details of a study aimed at *inferring direct information on the repulsive three-nucleon potential from multimessenger astronomy* [1].

- The baseline of our analysis is the dynamical model used to obtain the APR[2] equation of state.
- We have considered the coupling constant of the repulsive part of three-nucleon interaction -providing the dominant contribution at high density- as a free parameter.
- We have made bayesian inference employing the NICER and LIGO/Virgo datasets in order to constrain this coupling constant.
- We extended our analysis to the next generation of gravitational wave (GW) observatories, in particular we analyzed the potential of the Einstein Telescope (ET).

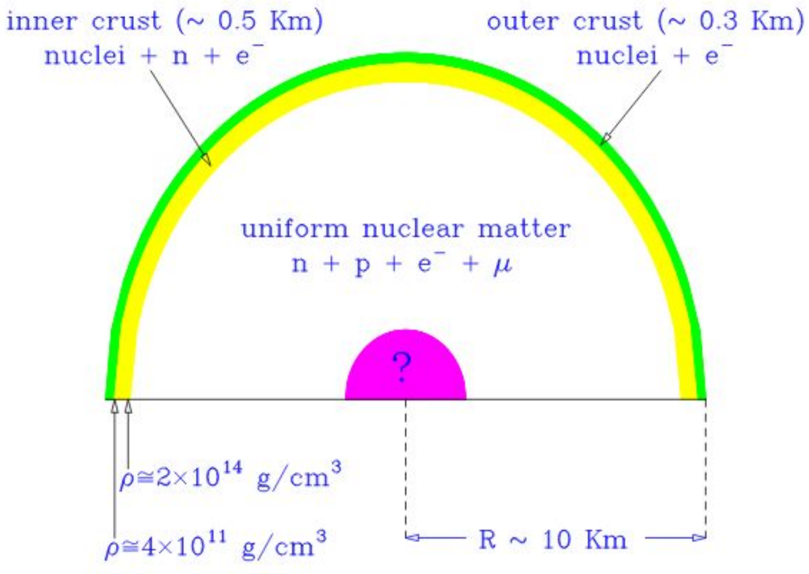
We are **fixing a microscopic model** and then trying to use **astrophysical observations to constrain** one parameter which is directly related to the underlying **microscopic dynamics**.

[1] A. Maselli, A. Sabatucci and O. Benhar, Phys. Rev. C 103, 065804, (2021)
[2] A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C 58, 1804 (1998)

Neutron Stars

Neutron Stars (NSs) are extremely compact objects, with masses as large as one or two solar masses and with radii of about ten kilometers.

In the NS interior matter can reach very extreme conditions, impossible for Earth-based experiments.



In the innermost region

$$\rho > \rho_0$$

$$\rho_0 = 2.67 \times 10^{14}\, g/cm^3\ (0.16\, fm^{-3})$$

$$T \sim 10^9\, K << T_F$$

$$T_F \sim 10^{12}\, K$$

NSs provide a unique opportunity to investigate the properties of nuclear matter at high density and low temperature.

Nuclear Dynamics

Non-relativistic nuclear many body theory (NMBT). We have point-like nucleons, interacting through nucleon-nucleon (NN) and three-nucleon (NNN) potentials.

$$\mathcal{H} = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

- For the APR EOS the authors employed the phenomenological **Argonne v18** NN potential and the **Urbana IX** NNN potential.
- The equation of state of cold nuclear matter is carried out by computing the ground state energy by means of variational approaches.

$$E_0 = \langle \psi_0 | \mathcal{H} | \psi_0 \rangle$$

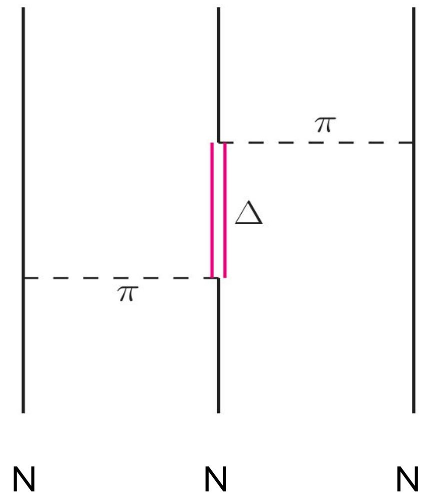
A variational approach is necessary because of the strong repulsive core of NN interaction which cannot be treated in perturbation theory.

Three-Nucleon Potential

Three-nucleon interactions must be introduced in order to account for processes involving the internal structure of nucleons.

The **Urbana IX** (UIX) model of three-nucleon potential comprises two terms.

$$V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{R}$$



This two terms bring **two free parameters** that are adjusted in order to reproduce the binding energy of tritium and the correct value of the nuclear saturation density respectively.

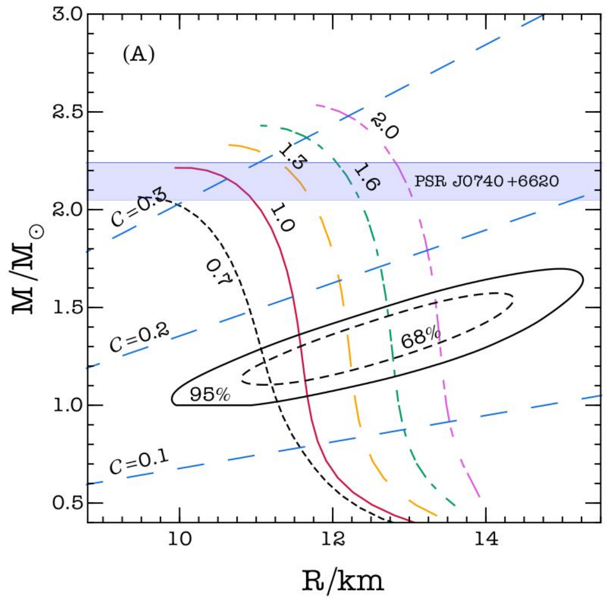
The repulsive term is purely phenomenological and it's only constrained to reproduce the correct value of the nuclear saturation density.

Resulting Equations of State

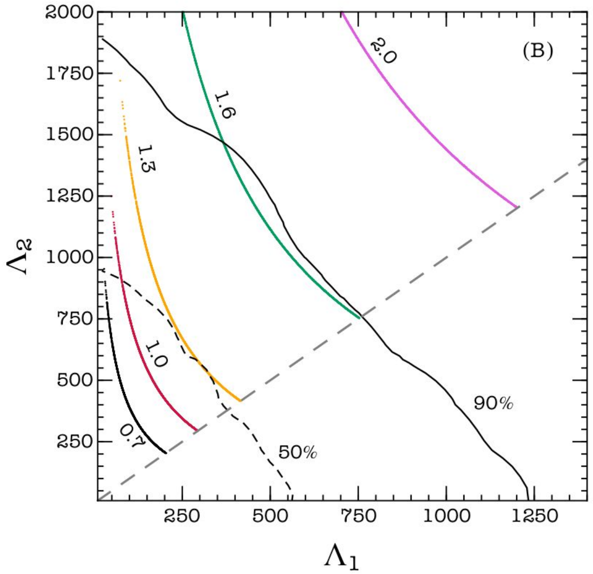
We have generated **a set of APR-like EOSs** computed replacing :

$$\left\langle V_{ijk}^{R} \right\rangle \to \alpha \left\langle V_{ijk}^{R} \right\rangle$$

NICER PSR J0030+0451



LIGO/Virgo GW170817



Bayesian Inference Framework

We have made Bayesian inference on α employing the following dataset:

- Gravitational Wave (GW) observation of the binary system GW170817 made by the LIGO-Virgo collaboration (LVC)
- The spectroscopic observation of the millisecond pulsars PSR J0030+0451 performed by the NICER satellite.
- The maximum mass constraint provided by the high-precision radio pulsars timing of the binary PSR J0740+6620

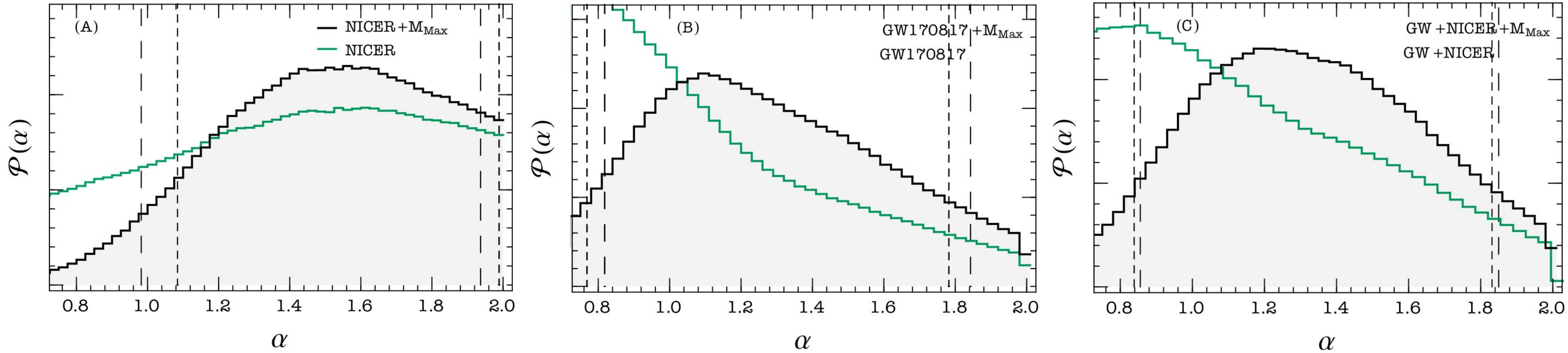
The posterior distribution defined through Bayes Theorem

$$\mathcal{P}(\theta|O) \propto \mathcal{P}_0(\theta) \prod_{i=1}^{n} \mathcal{L}(O^{(i)}|D(\theta))$$

Is sampled with Markov Chain Monte Carlo (MCMC) simulations with the *emcee* algorithm[3].

[3] D. Foreman-Mackey, D. W. Hogg, D. Lang, and J. Goodman, Astron. Soc. Pac. 125, 306 (2013)

Posterior Distributions



The GWs alone turn out to be not enough to extract relevant information about the strength of NNN repulsion and the shape of the PDF appears to be dominated by the maximum mass requirement.

However this analysis, yielding $\alpha_{\mathrm{GW+EM}} = 1.32^{+0.48}_{-0.51}$ has shown that there is sensitivity of NS observables with respect to the considered microscopic parameter.

A. Maselli, A. Sabatucci and O. Benhar, Phys. Rev. C 103, 065804, 2021

Extension to Future GW detections

We extended our study by repeating the **same analysis** but with a set of **simulated data** in order to investigate the following scenarios

- Increasing number of observations
- New generation detectors

We simulate **30 binary neutron star events** for two different observatories:

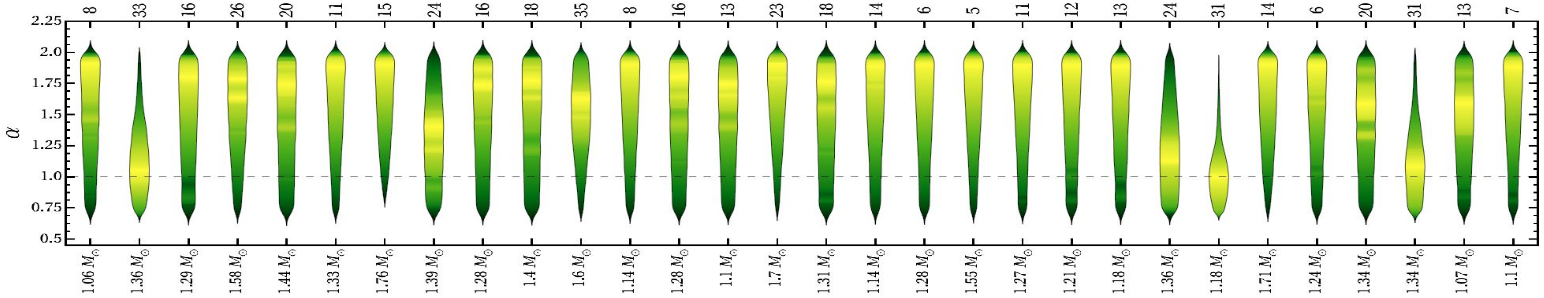
- **LIGO Hanford, LIGO Livingston, and Virgo detectors** at design sensitivity
- The future third-generation interferometer **Einstein Telescope**

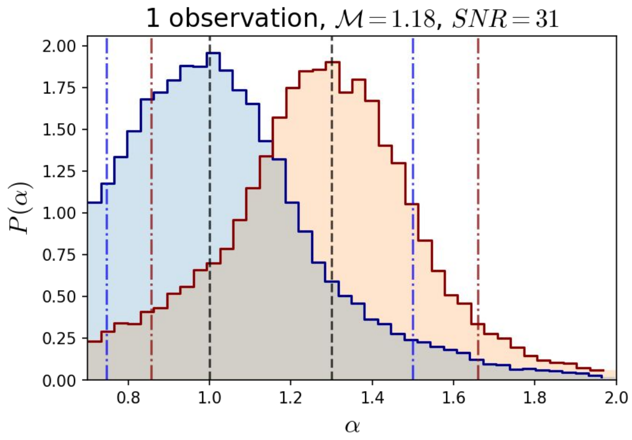
We have generated two different set of 30 binaries by using EOSs associated with two different values of α for each observatory.

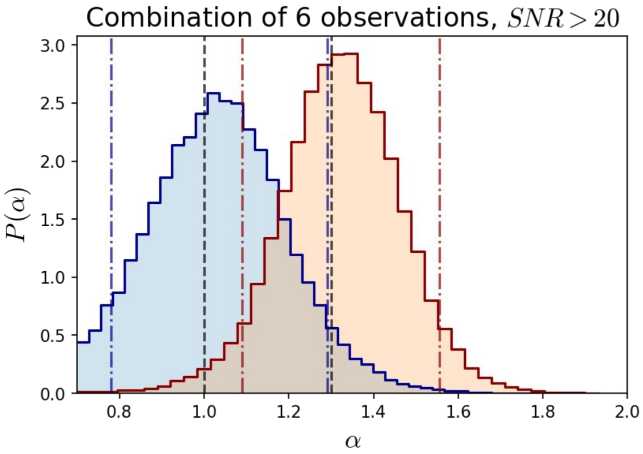
- The injected values of α are α=1.0 and α=1.3
- Sky location and inclination uniformly distributed over the sky.
- We assumeed the chirp mass of each event to be measured with infinitesimal precision.

A. Sabatucci, O. Benhar, A. Maselli and C. Pacilio (paper in preparation)

Mocked Data: LIGO/Virgo

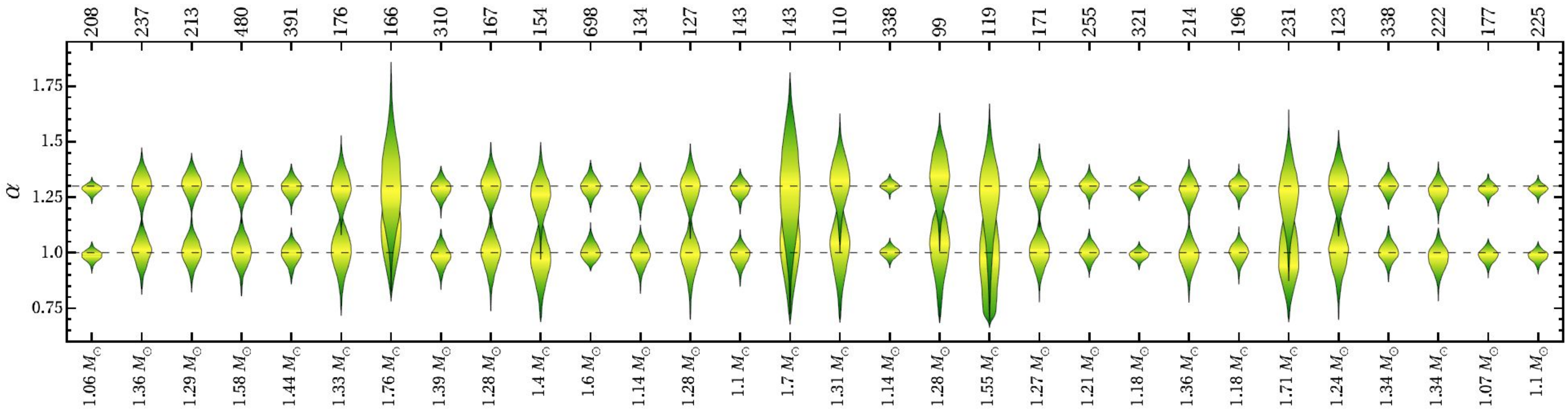




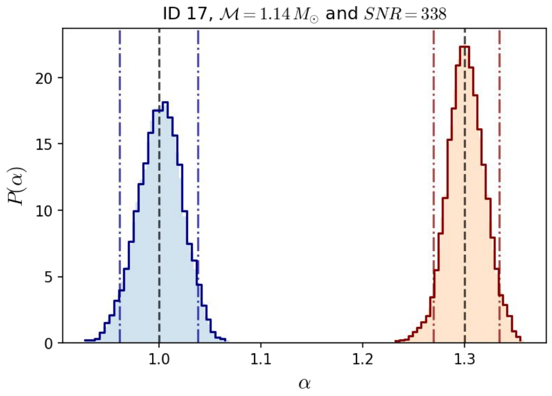


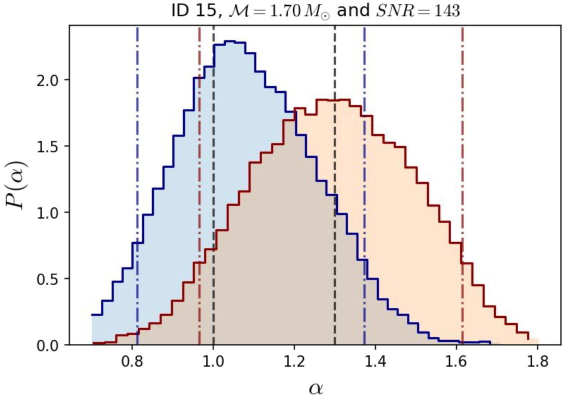
Mocked Data: Einstein Telescope

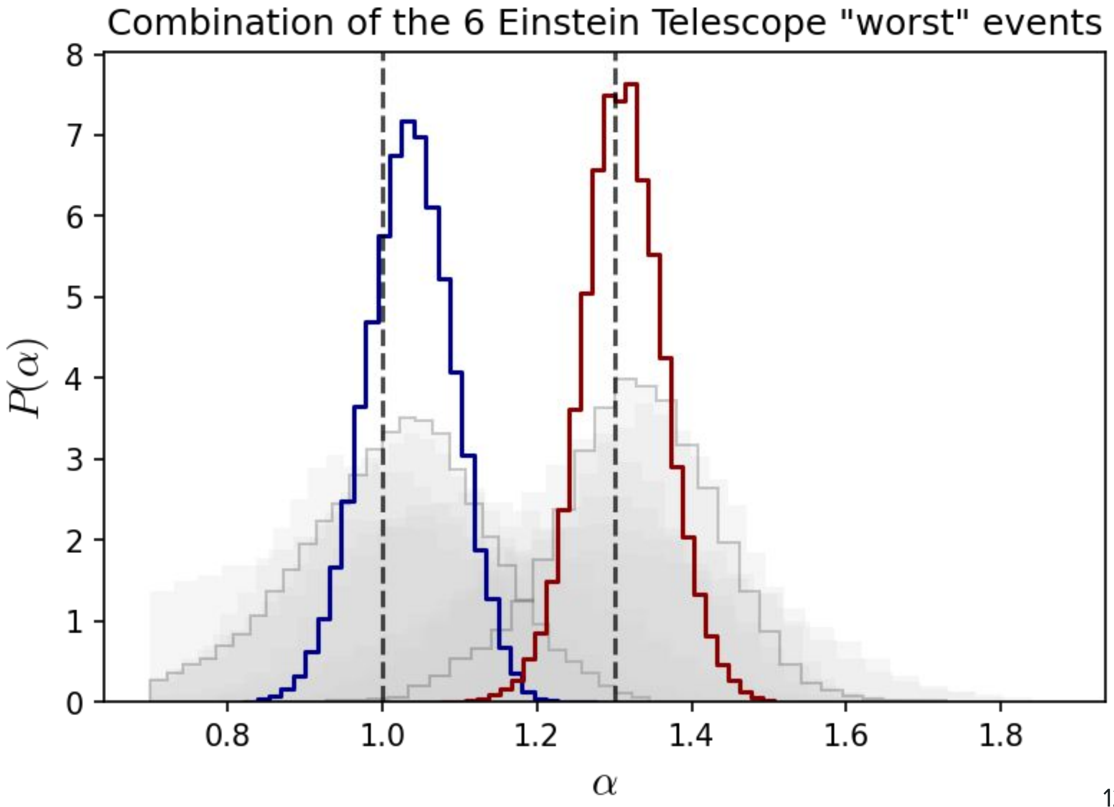
Violin plot of the marginal posterior of α for the 30 ET events. On the bottom and top axis are reported the chirp mass and the signal-to-noise ratio (SNR) for each event respectively.



Mocked Data: Einstein Telescope

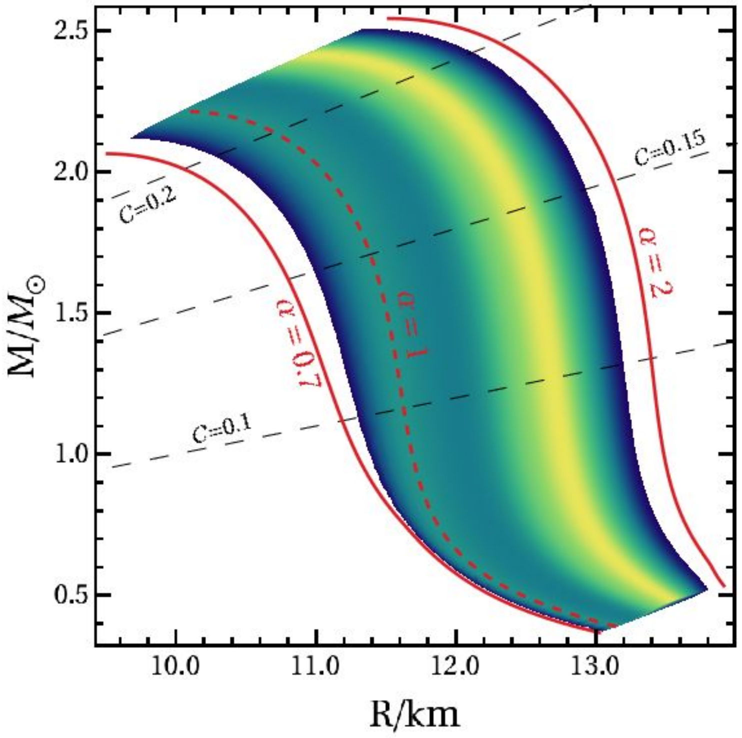




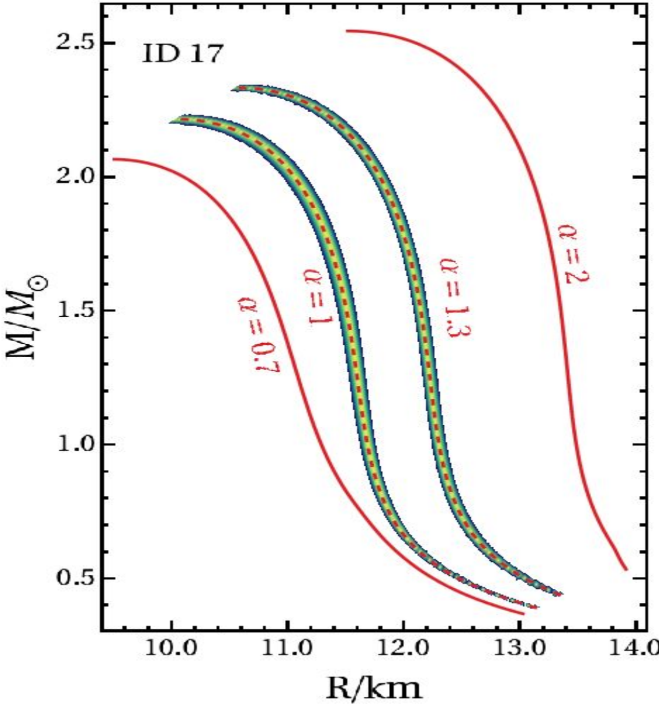


Comparison between current and future constraints

Current multimessenger data
(LIGO/Virgo + NICER)



Expected Einstein Telescope constraints



Summary and Outlook

- We have **investigated the constraints** that recent observations of GW170817 and the NICER pulsar PSR J0030+0451 can impose **on the NNN potential**.
- We have explored **both single and multimessenger constraints**, including also the bound on the maximum mass given by PSR J0740+6620.
- The results appears to be dominated by the maximum mass requirement, whereas the **GW170817** appears to be **not enough to infer relevant information**.
- However **there is sensitivity of neutron star observables with respect to α**, suggesting that future observations will definitely improve our understanding.
- The analysis with the **Einstein Telescope** appears to confirm this picture. For small values of the chirp mass we **can distinguish between two different values of α with just one observation**!

Thank you for the attention!

Backup

Nucleon-Nucleon Potential

In order to derive a realistic NN potential one has to rely upon **phenomenological models** constrained as much as possible from the large body of **data of coming from two-nucleon systems**, both in bound and scattering states.
A typical phenomenological NN potential is the **Argonne V18 (AV18)** potential

$$v_{ij} = \sum_{p=1}^{18} v^p(r_{ij}) O_{ij}^p$$

Where:

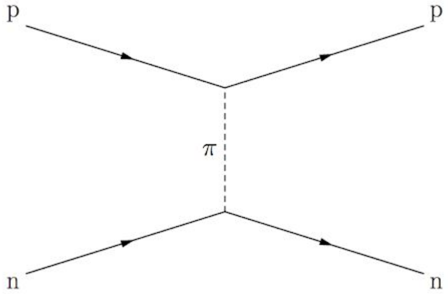
$$O_{ij}^{p\leq 6} = [1, \vec{\sigma}_i \cdot \vec{\sigma}_j, S_{ij}] \otimes [1, \vec{\tau}_i \cdot \vec{\tau}_j]$$

$$O_{ij}^{p=7,8} = (\vec{L} \cdot \vec{S}) \otimes [1, \vec{\tau}_i \cdot \vec{\tau}_j]$$

$$S_{ij} = \frac{3}{r_{ij}^2}(\vec{\sigma}_i \cdot \vec{r}_{ij})(\vec{\sigma}_j \cdot \vec{r}_{ij}) - (\vec{\sigma}_i \cdot \vec{\sigma}_j)$$

The operators with p=9, ... , 18 corresponds to small corrections.
The first six operators form an algebra. Therefore one can define the simplified **Argonne V6' (AV6P)** potential, which is a reprojection of the AV18 potential on the first six operators.

Derivation of the One-Pion-Exchange (OPE) Potential



$$\mathcal{L} = \bar{\psi}^N (i\gamma^\mu \partial_\mu - m)\psi^N + \mathcal{L}_\Pi + \mathcal{L}_I$$

$$\mathcal{L}_I = ig(\bar{\psi}_i^N \gamma^5 \psi_j^N)(T_{ij}^a \pi^a)$$

$$\downarrow$$

$$i\mathcal{M} = -g^2\, \bar{u}(p_{2'}, s_{2'})\gamma^5 u(p_2, s_2) \frac{1}{k^2 - m_\pi^2} \bar{u}(p_{1'}, s_{1'})\gamma^5 u(p_1, s_1)\langle T_1^a\rangle\langle T_2^a\rangle.$$

$$S_{fi} = \sqrt{\frac{m}{E_1}}\sqrt{\frac{m}{E_2}}\sqrt{\frac{m}{E_{1'}}}\sqrt{\frac{m}{E_{2'}}}(2\pi)^4\delta^{(4)}(p_{2'} + p_{1'} - p_2 - p_1)\ [\mathcal{M} - \mathcal{M}'],$$

$$S_{fi} \approx -i\frac{g^2}{4m^2}(2\pi)^4\delta^{(4)}(p_{1'} + p_{2'} - p_1 - p_2) \cdot \langle T_1^a T_2^a\rangle\, \chi_{1'}^\dagger \chi_{2'}^\dagger \frac{-(\vec{\sigma}_1 \cdot \vec{k})(\vec{\sigma}_2 \cdot \vec{k})}{|\vec{k}|^2 + m_\pi^2}\chi_2\chi_1$$

$$S_{fi} = -i(2\pi)^4\delta^{(4)}(p_{1'} + p_{2'} - p_1 - p_2)\langle v^\pi(\vec{k})\rangle$$

$$v^\pi(\vec{k}) = -\left(\frac{f_\pi}{m_\pi}\right)^2 \frac{(\vec{\sigma}_1 \cdot \vec{k})(\vec{\sigma}_2 \cdot \vec{k})}{\vec{k}^2 + m_\pi^2} T_1^a T_2^a \longrightarrow v_\pi(\vec{r}) = \frac{g^2}{4m^2} T_1^a T_2^a\, (\vec{\sigma}_1 \cdot \nabla)\, (\vec{\sigma}_2 \cdot \nabla)\, \frac{e^{-m_\pi r}}{r}$$

Nucleon-Nucleon Potential

- Observation of deuteron only in the state with S=1,T=0 ➔ strong spin-isospin dependence
- Non-central charge distribution in atomic nuclei ➔ non central interactions
- Saturation of central density ➔ short range repulsion
- Binding energy per nucleon nearly constant with increasing mass number ➔ short range interaction

$$v_{NN} = \sum_{S,T} [v_{TS}(r) + \delta_{S1} v_{tT}(r) S_{12}]\, P_S \Pi_T$$

$$P_0 = \frac{1}{4}(1 - \vec{\sigma}_1 \cdot \vec{\sigma}_2) \quad P_1 = \frac{1}{4}(3 + \vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

$$S_{12} = \frac{3}{r^2}(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

⇩

$$v_{ij} = \sum_{p=1}^{6} v^p(r_{ij}) O^p_{ij}$$

$$\{O^p_{ij}\} = (1, \vec{\sigma}_i \cdot \vec{\sigma}_j, S_{ij}) \otimes (1, \vec{\tau}_i \cdot \vec{\tau}_j).$$

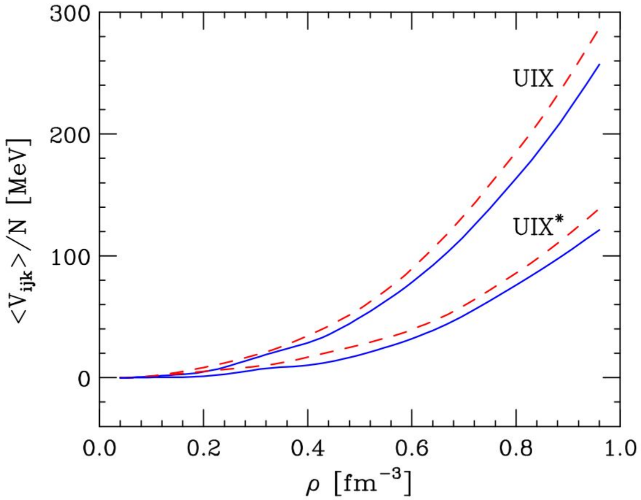
Boost Corrections to NN Potential

Being largely determined by a fit to NN scattering data, the NN potential is defined in the rest frame of two interacting nucleons. As a consequence in order to consistently describe NN interactions in locally inertial frame associated with a NS, the NN potential must be boosted to a frame in which the total momentum of the interacting pair is different from zero.
This consideration leads to the appearance of the so called **boost-correction** to the NN potential, which is an attempt to estimate relativistic effects in the framework of Quantum Mechanics.

The Hamiltonian becomes:

$$\mathcal{H}^* = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} (v_{ij} + \delta v(\vec{P}_{ij})) + \sum_{i<j<k} V^*_{ijk}$$

Perturbative calculations have shown that **the presence of the boost interaction accounts for the 37% of the the repulsive contribution of the NNN interaction.**



Boost Corrections

[3] Leslie L. Foldy.Phys. Rev., 122:275–288, 1961;
J. L. Forest et al Phys. Rev. C 52, 568 – 1995.

Boost corrections could be derived by imposing **relativistic covariance** on our system.

In the framework of Quantum Mechanics, relativistic covariance is implemented by requiring the Hilbert space of our theory to be a representation of the Poincaré group. By *imposing the commutation relations of the* ***Poincaré algebra***, and performing an *expansion in powers of 1/m* we can carry out the explicit expression of the boost interaction[3].

$$\begin{aligned}
&[P^i, P^j] = [H, P^i] = [J^i, H] = 0, \\
&[K^i, H] = iP^i, \qquad [J^i, J^j] = i\epsilon_{ijk}J^k, \\
&[K^i, P^j] = i\delta_{ij}H, \quad [J^i, K^i] = i\epsilon_{ijk}K^k, \\
&[J^i, P^j] = i\epsilon_{ijk}P^k, \quad [K^i, K^j] = -i\epsilon_{ijk}J^k.
\end{aligned}$$

↓

$$\delta v_{ij}(\mathbf{P}_{ij}, \mathbf{r}_{ij}) = -\frac{\mathbf{P}_{ij}^2}{8m^2}v_{ij} + \frac{1}{8m^2}\left[(\mathbf{P}_{ij}\cdot\mathbf{r}_{ij})(\mathbf{P}_{ij}\cdot\boldsymbol{\nabla}), v_{ij}\right] + \frac{1}{8m^2}\left[(\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j)\times(\mathbf{P}_{ij}\cdot\boldsymbol{\nabla}), v_{ij}\right]$$

Neutron Stars

Because of the high compactness Neutron Stars must be studied in the framework of *General Relativity*. By solving the Einstein equations for a static-spherically symmetric metric tensor one can carry out the stellar structure equations, referred to as **Tolman-Oppenheimer and Volkov (TOV)** equations.

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

$$G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

$$T^{\mu\nu} = u^\mu u^\nu(\epsilon + P) + g_{\mu\nu}P$$

TOV →

$$\begin{cases} \dfrac{dM(r)}{dr} = 4\pi r^2 \epsilon(r) \\ \dfrac{dP}{dr} = -\dfrac{[\epsilon(r) + P(r)]\,[M(r) + 4\pi r^3 P(r)]}{r\,[r - 2M(r)]} \end{cases}$$

In order to solve TOV equations we have to specify the equation of state (EOS) of neutron star matter, i.e. the relation:

$$P = P(\epsilon)$$

Details of the Parametrization

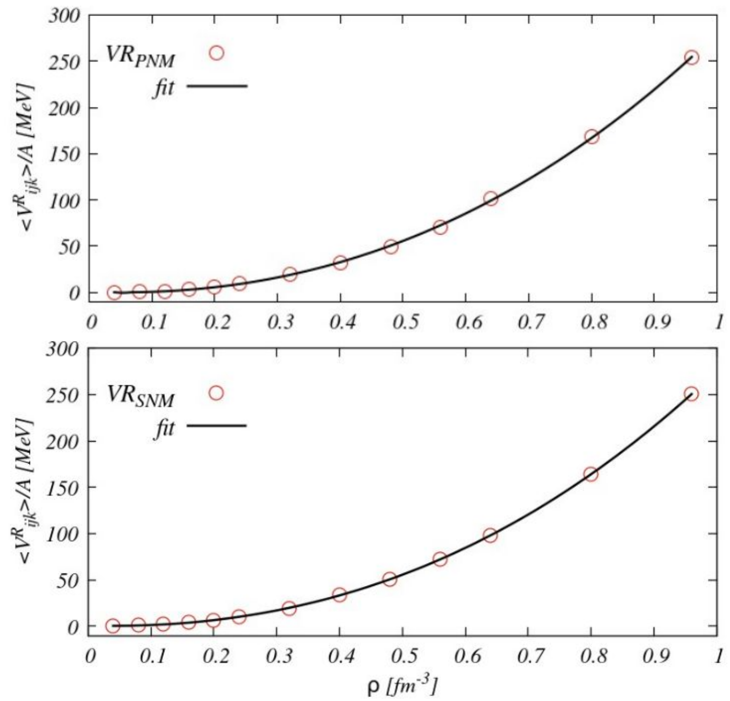
$$\epsilon(\varrho,x)=\left[\frac{1}{2m}+f(\varrho,x)\right]\tau_p+\left[\frac{1}{2m}+f(\varrho,1-x)\right]\tau_n+g(\varrho,x)$$

$$g(\varrho,x)=g(\varrho,1/2)[1-(1-2x)^2]+g(\varrho,0)(1-2x)^2,$$

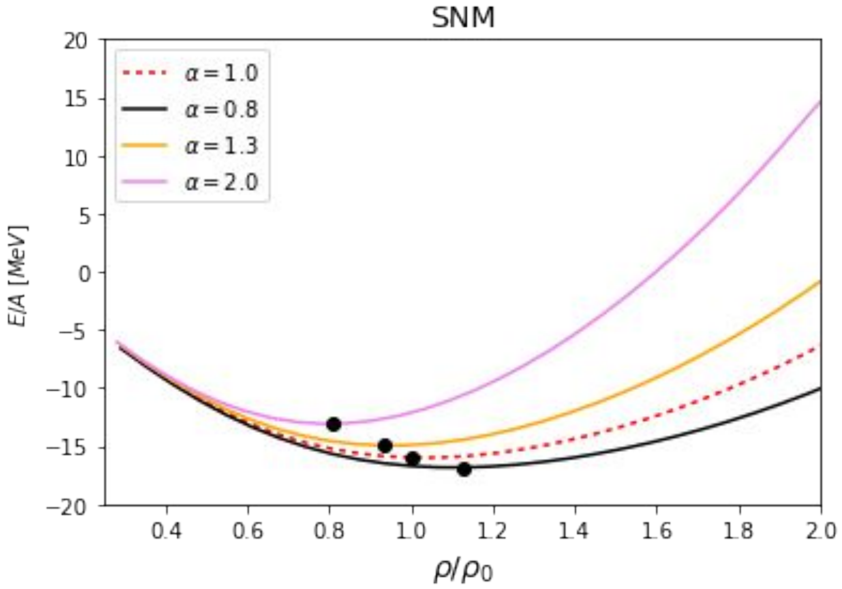
$$\langle V^R\rangle(\rho)=a_1+a_2\,\rho+a_3\,\rho^2+a_4\,\rho^3.$$

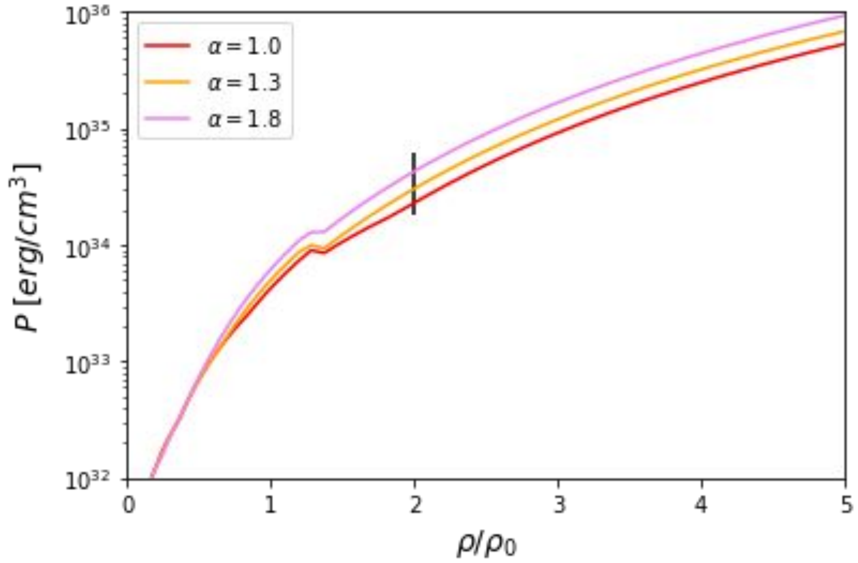
$$g'_A(\rho)=g_A(\rho)+(\alpha-1)v^R_A(\rho)$$

with $A=SNM,PNM$ and $v_R=\rho\langle V^R\rangle$



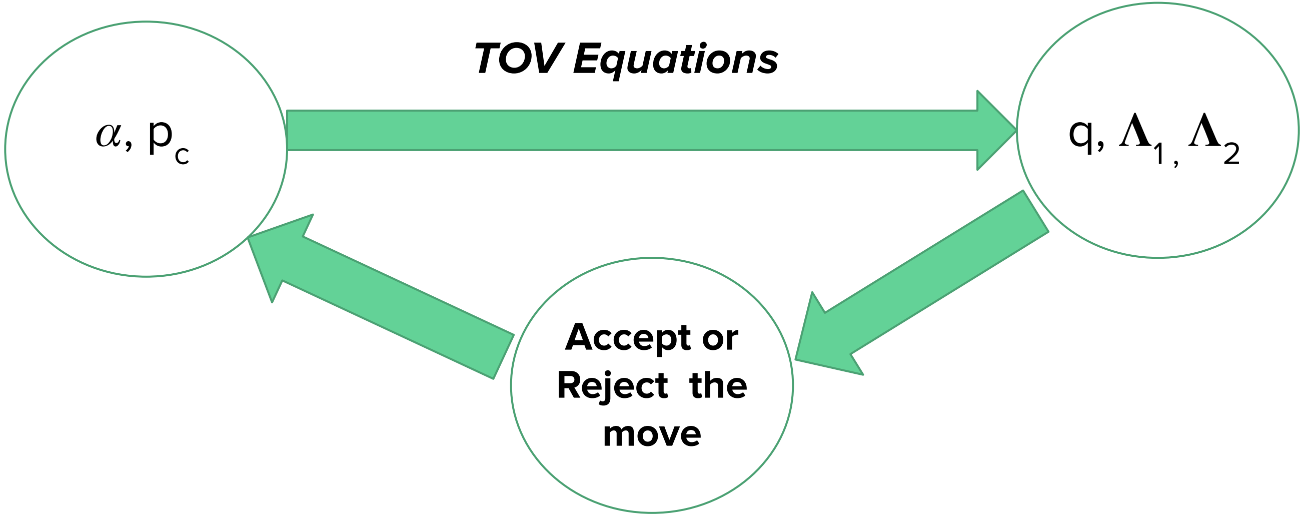
Resulting EOSs





Sampling The Posterior

$$\mathcal{P}(\alpha, p_c^{(1)} | O_{\mathrm{GW}}) \propto \mathcal{P}_0(\alpha, p_c^{(1)}, p_c^{(2)}) \mathcal{L}_{\mathrm{GW}}(q, \Lambda_1, \Lambda_2)$$



Chiral Potentials

Chiral EFT is a **low-energy** effective theory of QCD, in which nucleons and pions are chosen as the relevant degrees of freedom. This effective field theory is constrained to be symmetric under the group

$$SU(2)_L \otimes SU(2)_R \,.$$

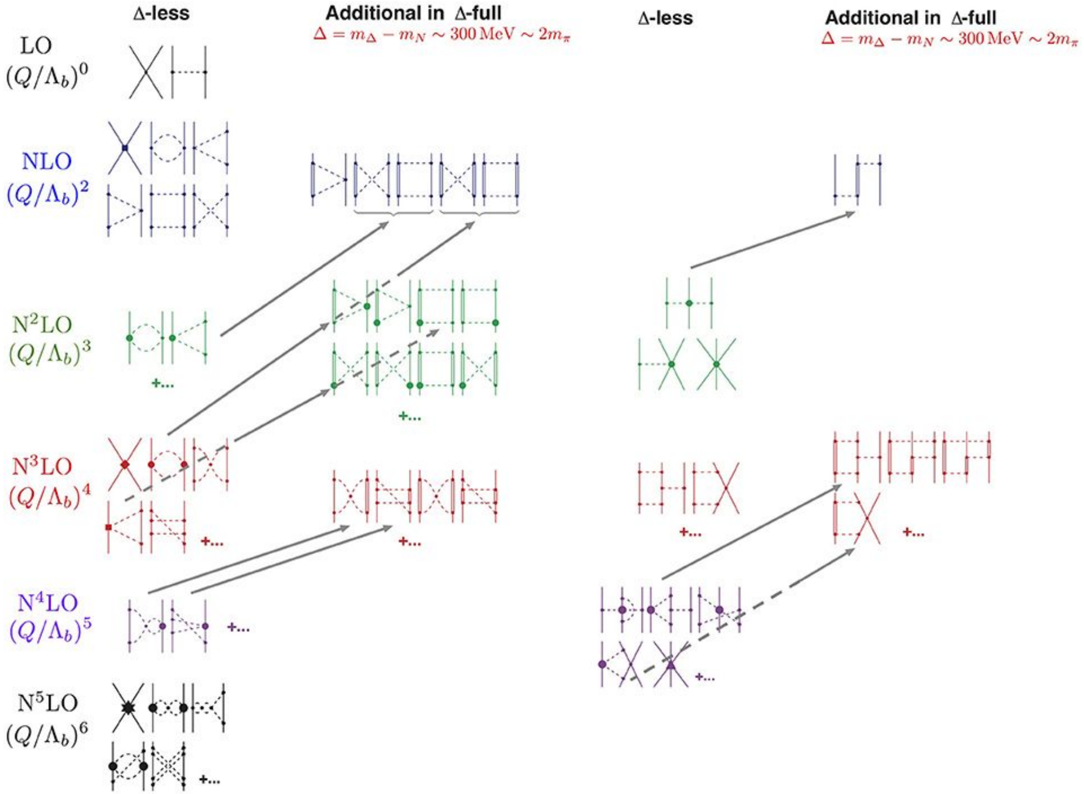
This is an approximated symmetry of the QCD lagrangian, that turns out to be a good approximation of the real theory in the light quark sector. This symmetry is **spontaneously broken** and the pions are its Goldstone bosons.

The starting point in chiral EFT is to write the most general Lagrangian in terms of the chosen degrees of freedom. This Lagrangian contains an infinite number of terms and must be truncated using a given power-counting scheme.

This approach was first proposed by Weinberg in 1990. The ***Interaction is expanded in powers*** of the typical p over the breakdown scale, p/Λ_b .

$$\mathcal{L} = \mathcal{L}^{(0)} + \left(\frac{p}{\Lambda_b}\right)^2 \mathcal{L}^{(2)} + \left(\frac{p}{\Lambda_b}\right)^3 \mathcal{L}^{(3)} + \ldots$$

Chiral Potentials

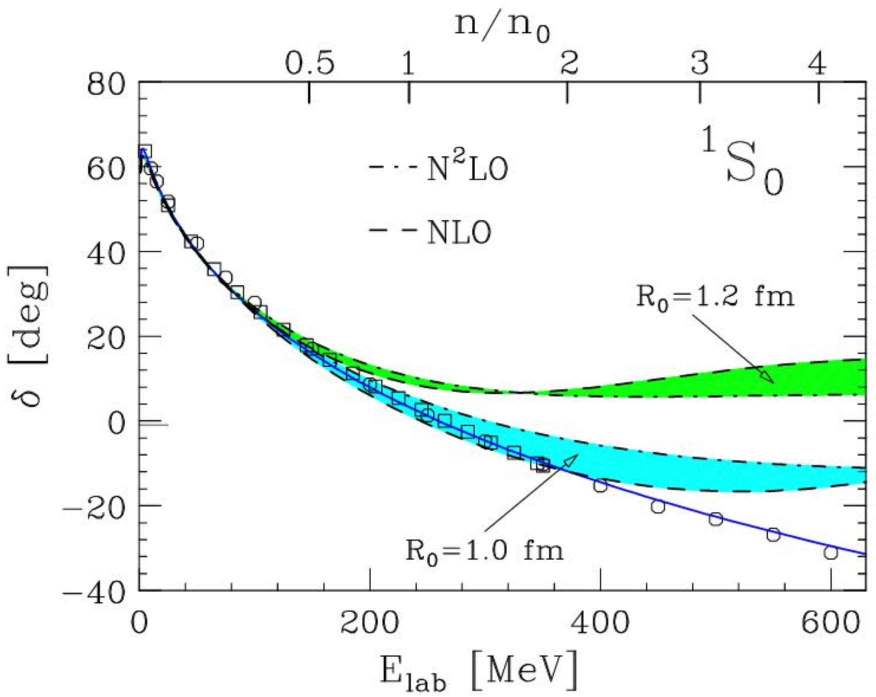


Chiral contributions to NN and NNN interactions based on Weinberg power counting.

The main advantage of the chiral EFTs is that they give a way to **systematically derive two- and many body-interactions in a consistent fashion**.

Maria Piarulli and Indigo Tews, Front. Phys., 30 January 2020

Comparison between Chiral and AV18 potentials



Neutron-proton scattering phase shifts as a function of the kinetic energy of the beam particle in the laboratory frame (bottom axis). The corresponding density is given in the top axis.

The kinetic energy in the lab frame is related to the particle density through:

$$E_{lab} = 2E_{cm} = \frac{2}{m}(3\pi^2\rho)^{2/3}$$

From this plot clearly appears that the AV18 potential yields an accurate description of the data up to energies of about 600 MeV corresponding to 4 times the nuclear saturation density. Conversely *chiral potentials seem to be limited up to twice the nuclear saturation density.*

O. Benhar, arXiv:1903.11353 [nucl-th] (2019).