

# Coalescence plus fragmentation approach for heavy hadrons production: spectra and ratios in AA and pp collisions

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UNIVERSITÀ  
degli STUDI  
di CATANIA



DIPARTIMENTO DI  
FISICA E  
ASTRONOMIA  
“ETTORE MAJORANA”



Quinto Incontro Nazionale di Fisica Nucleare INFN 2022  
9-11 May 2022

# Outline

## Hadronization:

- Fragmentation
- Coalescence model

## Heavy hadrons in AA collisions:

- $\Lambda_c$ , D spectra and ratio: RHIC and LHC

## Heavy hadrons in small systems (pp @ 5.02 TeV):

- $\Lambda_c/D^0$
- $\Xi_c/D^0$ ,  $\Omega_c/D^0$

# Heavy flavour Hadronization: Fragmentation

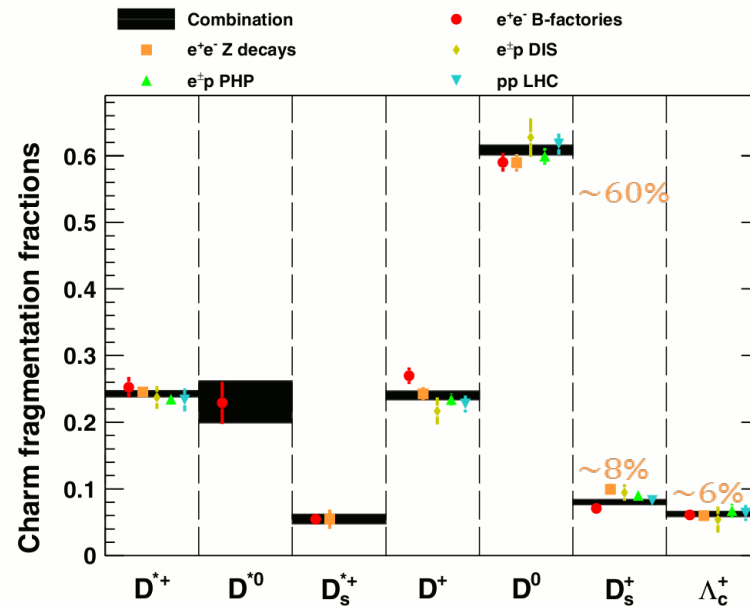
$$\frac{dN_h}{d^2p_h} = \sum_f \int dz \frac{dN_f}{d^2p_f} D_{f \rightarrow h}(z)$$

Fragmentation function

The distribution function is evaluated at the Fixed-Order plus Next-to-Leading-Log (FONLL)

M. Cacciari, P. Nason, R. Vogt, PRL 95 (2005) 122001

**In AA: bulk+charm evolution with Relativistic Transport Boltzmann Equation**



M. Lisovyi, et al. EPJ C76 (2016) no.7, 397

We use the Peterson fragmentation function

C. Peterson, D. Schalatter, I. Schmitt, P.M. Zerwas PRD 27 (1983) 105

$$D_{f \rightarrow h}(z) \propto \frac{1}{z \left[ 1 - \frac{1}{z} - \frac{\epsilon}{1-z} \right]^2}$$

**Charm Fragmentation Fraction (c→h)**  
Measurement in  $e^\pm p$ ,  $e^+ e^-$  and old  $pp$  data

$$\left( \frac{\Lambda_c^+}{D^0} \right)_{e^+ e^- pp} \simeq 0.1$$

$$\left( \frac{D_s^+}{D^0} \right)_{e^+ e^- pp} \simeq 0.13$$

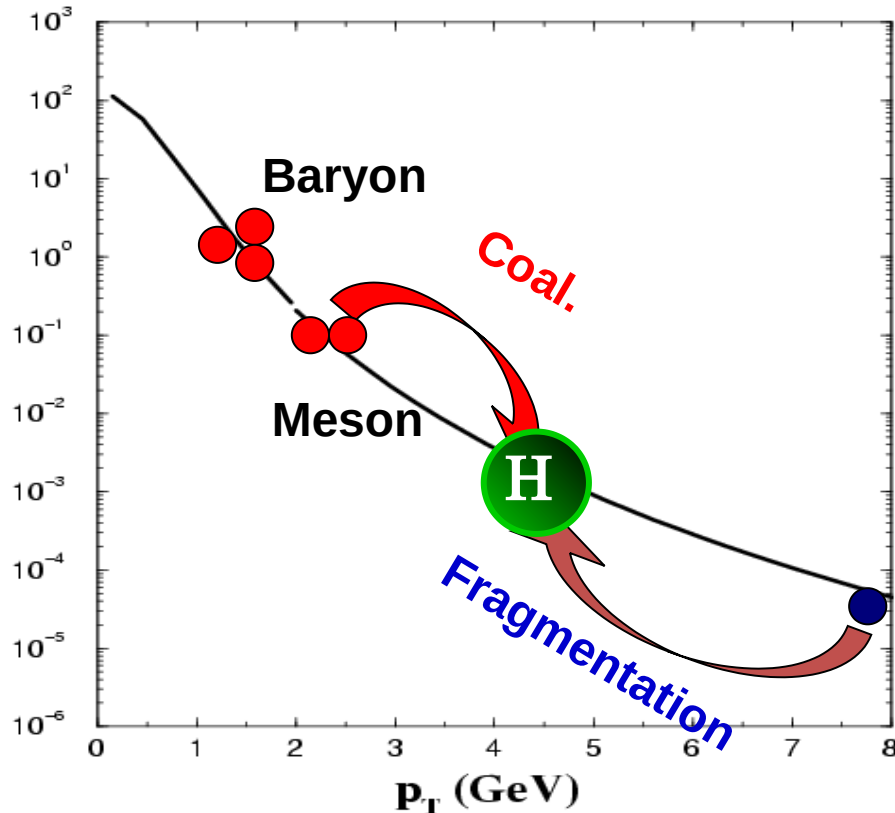
# Hadronization: Coalescence

*Statistical factor  
colour-spin-isospin*

*Parton Distribution  
function*

*Hadron Wigner  
function*

$$\frac{dN_{\text{Hadron}}}{d^2 p_T} = g_H \int \prod_{i=1}^n p_i \cdot d\sigma_i \frac{d^3 p_i}{(2\pi)^3} f_q(x_i, p_i) f_W(x_1, \dots, x_n; p_1, \dots, p_n) \delta(p_T - \sum_i p_{iT})$$



Used to describe first observations in light sector for the baryon/meson ratio and elliptic flow splitting at RHIC, more than a decade ago.

V. Greco, C.M. Ko, P. Levai PRL 90, 202302 (2003)

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LIGHT

Thermal+flow for **u,d,s** ( $p_T < 3$  GeV)

$$\frac{dN_{q,\bar{q}}}{d^2 p_T} \sim \exp\left(-\frac{\gamma_T - p_T \beta_T \bar{\mu}_q}{T}\right)$$

$$\beta(r) = \frac{r}{R} \beta_{max}$$

$$V = \pi R^2 \tau \cosh(y_z), R(\tau_f) = R_0(1 + \beta_{max} \tau_f)$$

$$\text{PbPb@5ATeV(0-10\%): } \tau_f = 8.4 \frac{fm}{c} \rightarrow V_{|y|<0.5} = 4500 fm^3$$

+quenched minijets for **u,d,s** ( $p_T > 3$  GeV)

CHARM

In AA collisions charm distribution from the studies of  $R_{AA}$  and  $v_2$  of **D-meson** to determine the Space Diffusion coefficient:

parton simulations solving relativistic Boltzmann transport equation

In pp collisions the charm distribution are the FONLL distribution

Coalescence simulation in a fireball with radial flow for light quarks  $\rightarrow$  dimension set by experimental constraints

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Wigner function – Wave function

Wigner function **width** fixed by root-mean-square charge radius from **quark model**

C.-W. Hwang, EPJ C23, 585 (2002), C. Albertus et al., NPA 740, 333 (2004)

$$\Phi_M^W(\mathbf{r}, \mathbf{q}) = \int d^3 r' e^{-i\mathbf{q}\mathbf{r}'} \phi_M(\mathbf{r} + \frac{\mathbf{r}'}{2}) \phi_M^*(\mathbf{r} - \frac{\mathbf{r}'}{2})$$

$\phi_M(\mathbf{r})$  meson wave function

$$\langle r^2 \rangle_{ch} = \frac{3}{2} \frac{m_2^2 Q_1 + m_1^2 Q_2}{(m_1 + m_2)^2} \sigma_{r1}^2 + \frac{3}{2} \frac{m_3^2 (Q_1 + Q_2) + (m_1 + m_2)^2 Q_3}{(m_1 + m_2 + m_3)^2} \sigma_{r2}^2$$

$$\sigma_{ri} = 1/\sqrt{(\mu_i \omega)} \quad \mu_1 = \frac{m_1 m_2}{m_1 + m_2} \quad \mu_2 = \frac{(m_1 + m_2) m_3}{m_1 + m_2 + m_3}$$

Assuming gaussian wave function

$$f_H(\dots) = \prod_{i=1}^{N_q-1} A_W \exp\left(-\frac{x_{ri}^2}{\sigma_{ri}^2} - p_{ri}^2 \sigma_{ri}^2\right)$$

only one width coming from  $\phi_M(\mathbf{r})$ ,

constraint  $\sigma_r \sigma_p = 1$

Meson	$\langle r^2 \rangle_{ch}$	$\sigma_{p1}$	$\sigma_{p2}$
$D^+ = [c\bar{d}]$	0.184	0.282	—
$D_s^+ = [\bar{s}c]$	0.083	0.404	—
Baryon	$\langle r^2 \rangle_{ch}$	$\sigma_{p1}$	$\sigma_{p2}$
$\Lambda_c^+ = [udc]$	0.15	0.251	0.424
$\Xi_c^+ = [usc]$	0.2	0.242	0.406
$\Omega_c^0 = [ssc]$	-0.12	0.337	0.53

# Hadronization: Coalescence

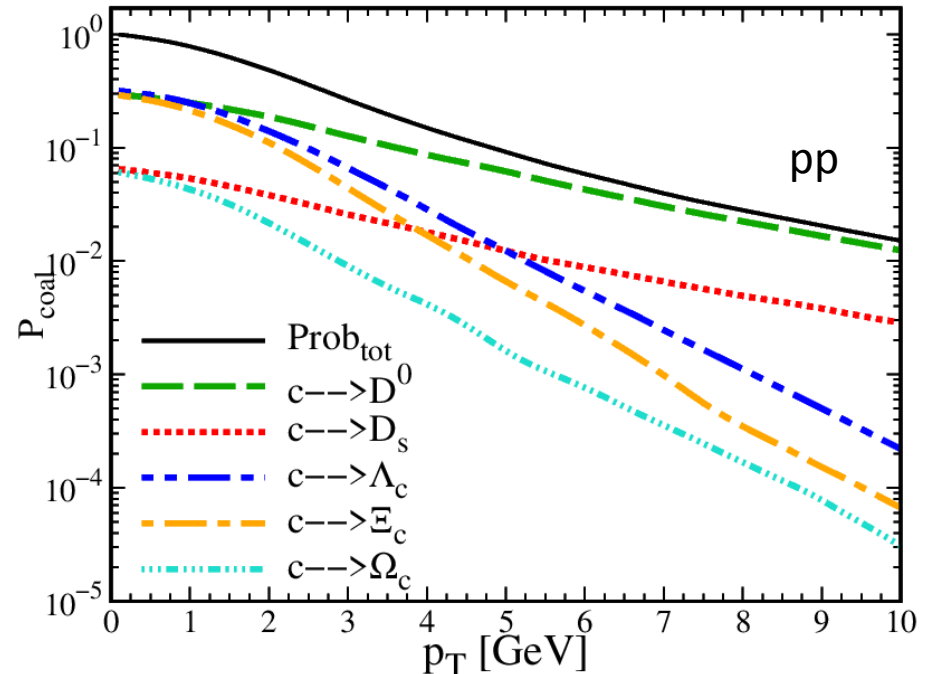
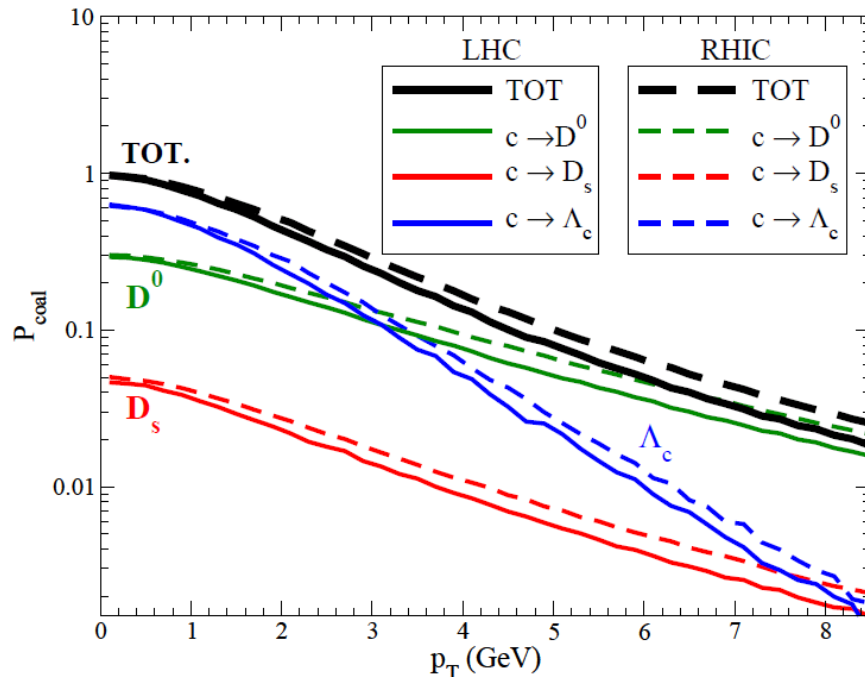
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- Normalization in  $f_W(\dots)$  requiring that  $P_{\text{coal}}=1$  at  $p=0$
- The charm that does not coalesce undergo fragmentation



# Heavy flavour: Resonance decay

Meson	Mass(MeV)	I (J)	Decay modes	B.R.
$D^+ = \bar{d}c$	1869	$\frac{1}{2}$ (0)		
$D^0 = \bar{u}c$	1865	$\frac{1}{2}$ (0)		
$D_s^+ = \bar{s}c$	2011	0 (0)		
<b>Resonances</b>				
$D^{*+}$	2010	$\frac{1}{2}$ (1)	$D^0\pi^+; D^+X$	68%,32%
$D^{*0}$	2007	$\frac{1}{2}$ (1)	$D^0\pi^0; D^0\gamma$	62%,38%
$D_s^{*+}$	2112	0 (1)	$D_s^+X$	100%
<b>Baryon</b>				
$\Lambda_c^+ = udc$	2286	0 ( $\frac{1}{2}$ )		
$\Xi_c^+ = usc$	2467	$\frac{1}{2}$ ( $\frac{1}{2}$ )		
$\Xi_c^0 = dsc$	2470	$\frac{1}{2}$ ( $\frac{1}{2}$ )		
$\Omega_c^0 = ssc$	2695	0 ( $\frac{1}{2}$ )		
<b>Resonances</b>				
$\Lambda_c^+$	2595	0 ( $\frac{1}{2}$ )	$\Lambda_c^+\pi^+\pi^-$	100%
$\Lambda_c^+$	2625	0 ( $\frac{3}{2}$ )	$\Lambda_c^+\pi^+\pi^-$	100%
$\Sigma_c^+$	2455	1 ( $\frac{1}{2}$ )	$\Lambda_c^+\pi$	100%
$\Sigma_c^+$	2520	1 ( $\frac{3}{2}$ )	$\Lambda_c^+\pi$	100%
$\Xi_c^{\prime+0}$	2578	$\frac{1}{2}$ ( $\frac{1}{2}$ )	$\Xi_c^{+0}\gamma$	100%
$\Xi_c^+$	2645	$\frac{1}{2}$ ( $\frac{3}{2}$ )	$\Xi_c^+\pi^-$ ,	100%
$\Xi_c^+$	2790	$\frac{1}{2}$ ( $\frac{1}{2}$ )	$\Xi_c^{\prime+}\pi$ ,	100%
$\Xi_c^+$	2815	$\frac{1}{2}$ ( $\frac{3}{2}$ )	$\Xi_c^{\prime+}\pi$ ,	100%
$\Omega_c^0$	2770	0 ( $\frac{3}{2}$ )	$\Omega_c^0\gamma$ ,	100%

In our calculations we take into account hadronic channels including the ground states + first excited states

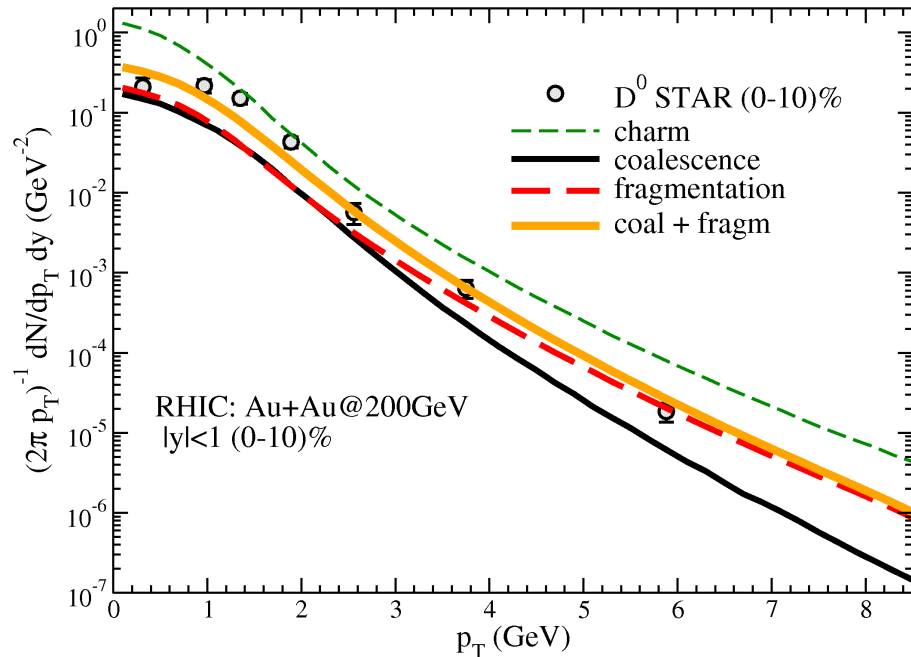
**Statistical factor suppression for resonances**

$$\frac{[(2J+1)(2I+1)]_{H^*}}{[(2J+1)(2I+1)]_H} \left(\frac{m_{H^*}}{m_H}\right)^{3/2} e^{-(m_{H^*}-m_H)/T}$$



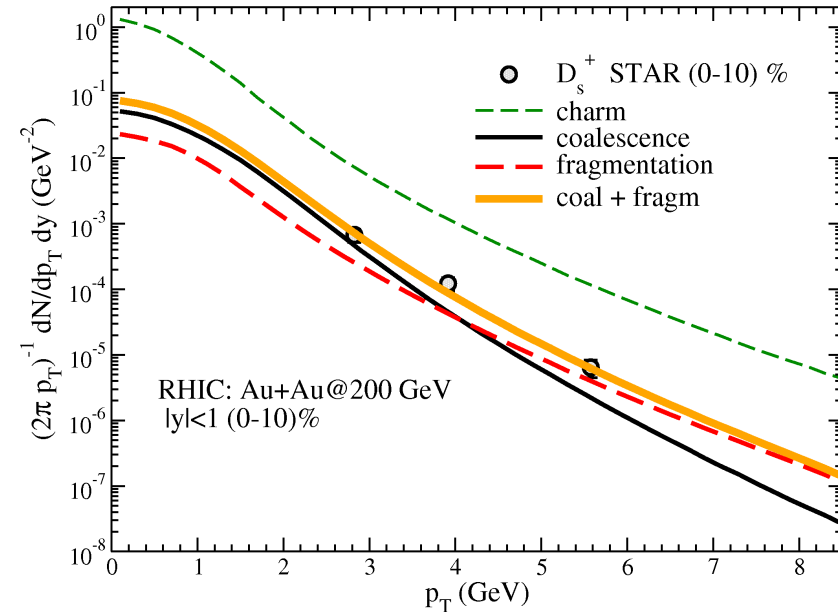
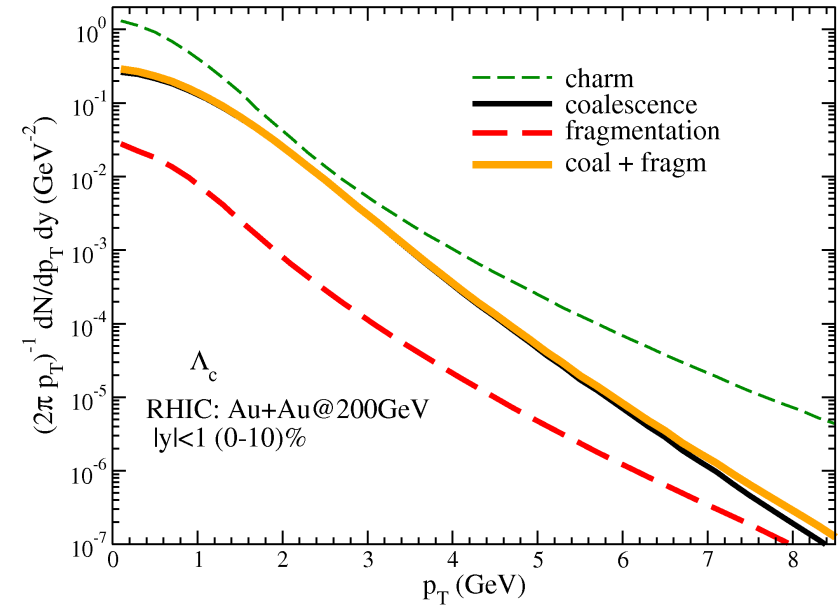
# RHIC: results

S. Plumari, V. Minissale et al., *Eur. Phys. J. C* **78** no. 4, (2018) 348



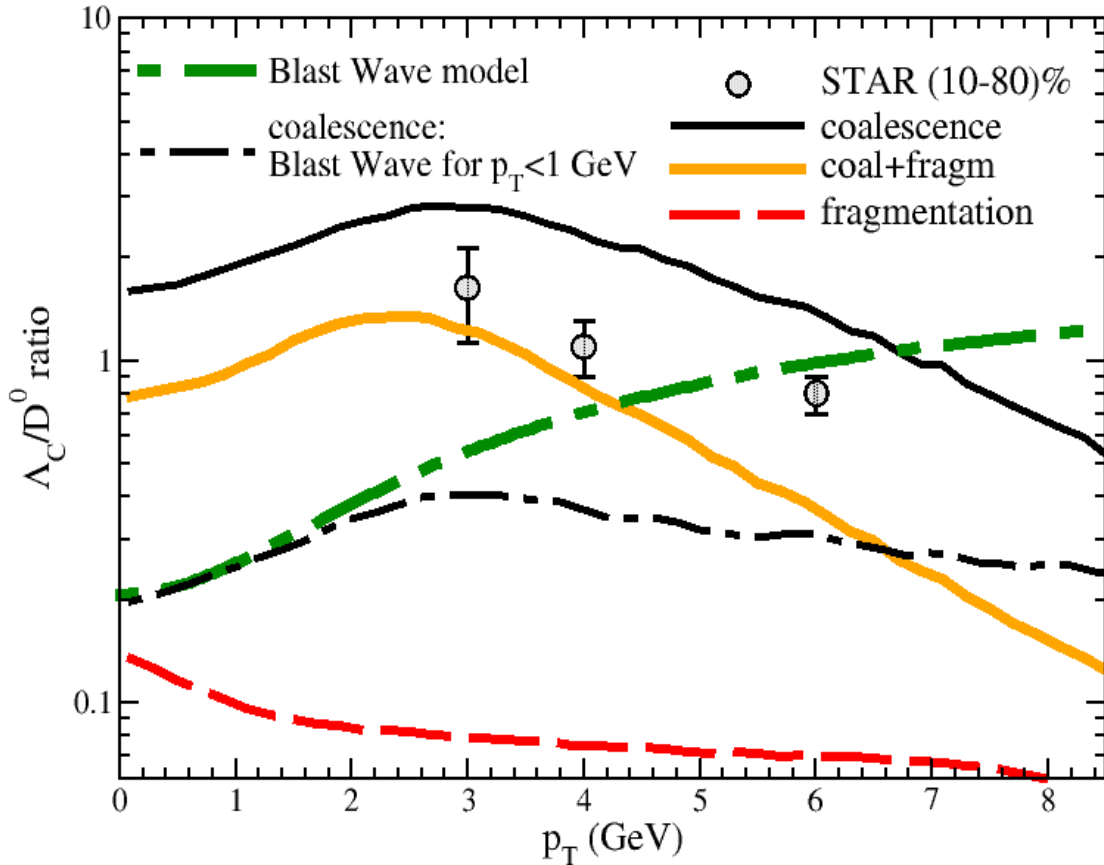
Data from STAR Coll. PRL **113** (2014) no.14, 142301

- For  $D^0$  coalescence and fragmentation comparable at 2 GeV
- fragmentation fraction for  $D_s^+$  are small and less than about 8% of produced total heavy hadrons
- $\Lambda_c^+$  fragmentation is even more smaller, coalescence gives the dominant contribution



# RHIC: Baryon/meson

STAR, Phys.Rev.Lett. 124 (2020) 17, 172301



Compared to light baryon/meson ratio the  $\Lambda_c/D^0$  ratio has a larger width (flatter)

More flatter  $\rightarrow$  should coalescence extend to higher  $p_T$ ? Indication also in light sector

V. Minissale, F. Scardina, V. Greco **PRC 92**,054904 (2015)

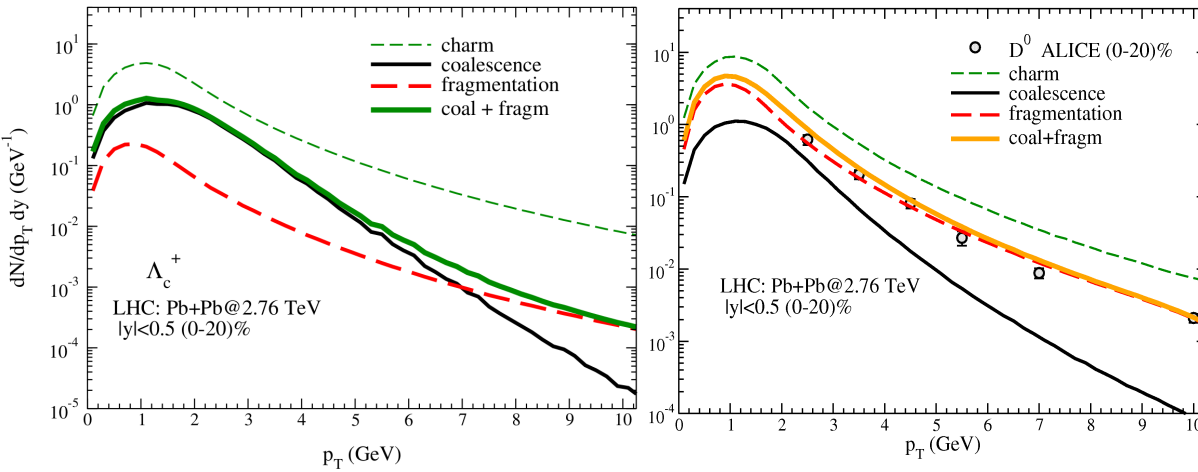
Cho, Sun, Ko et al., **PRC 101 (2020)** 2, 024909

Needed data at low  $p_T$

# LHC: results

wave function widths  $\sigma_p$  of baryon and mesons are the same at RHIC and LHC!

Data from ALICE Coll. JHEP 1209 (2012) 112



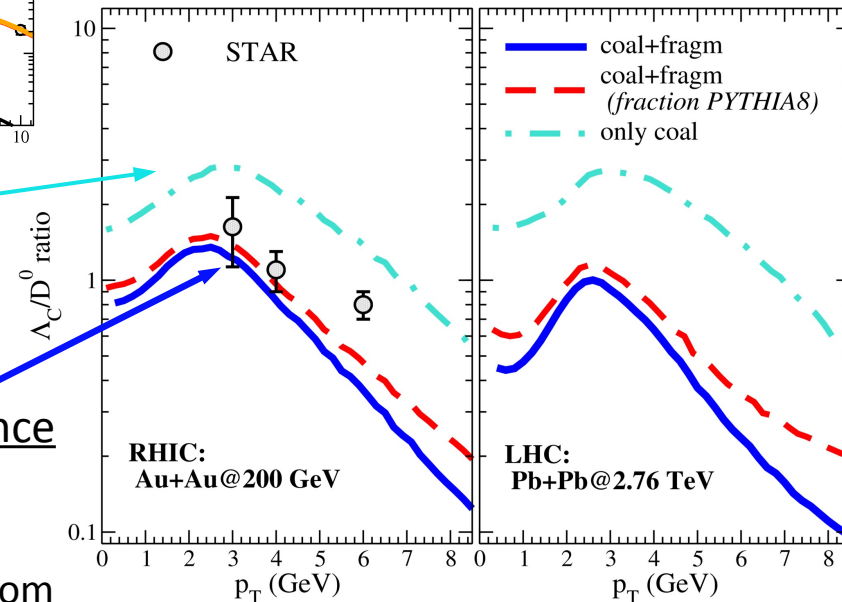
Coalescence lower than at RHIC  $\rightarrow$  main contribution from Fragmentation

Only **Coalescence** ratio is similar at both energies.

Fragmentation  $\sim 0.1$  at both energies.

the **combined ratio is different** because the coalescence over fragmentation ratio at LHC is smaller than at RHIC

Therefore at LHC the larger contribution in particle production from fragmentation leads to a final ratio that is smaller than at RHIC.



*S. Plumari, V. Minissale et al., Eur. Phys. J. C78 no. 4, (2018) 348*

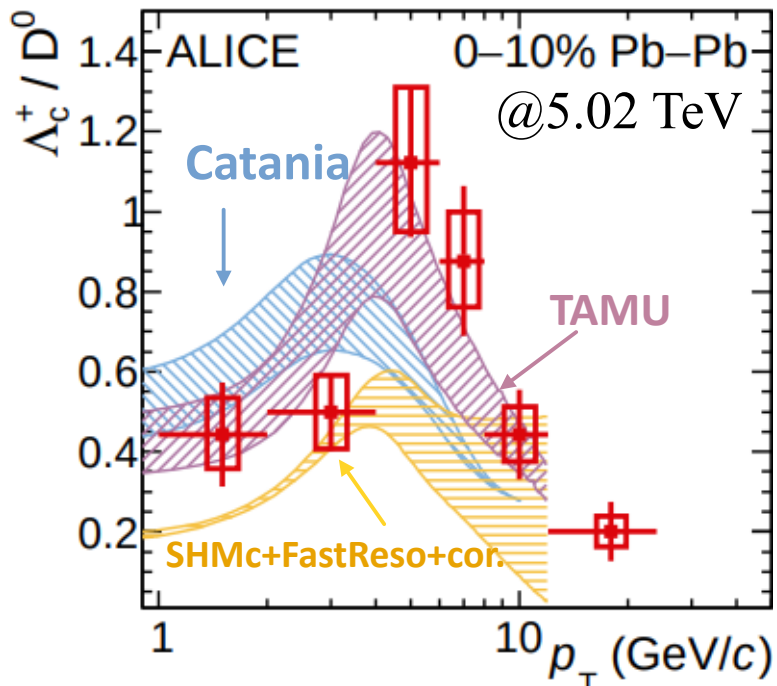
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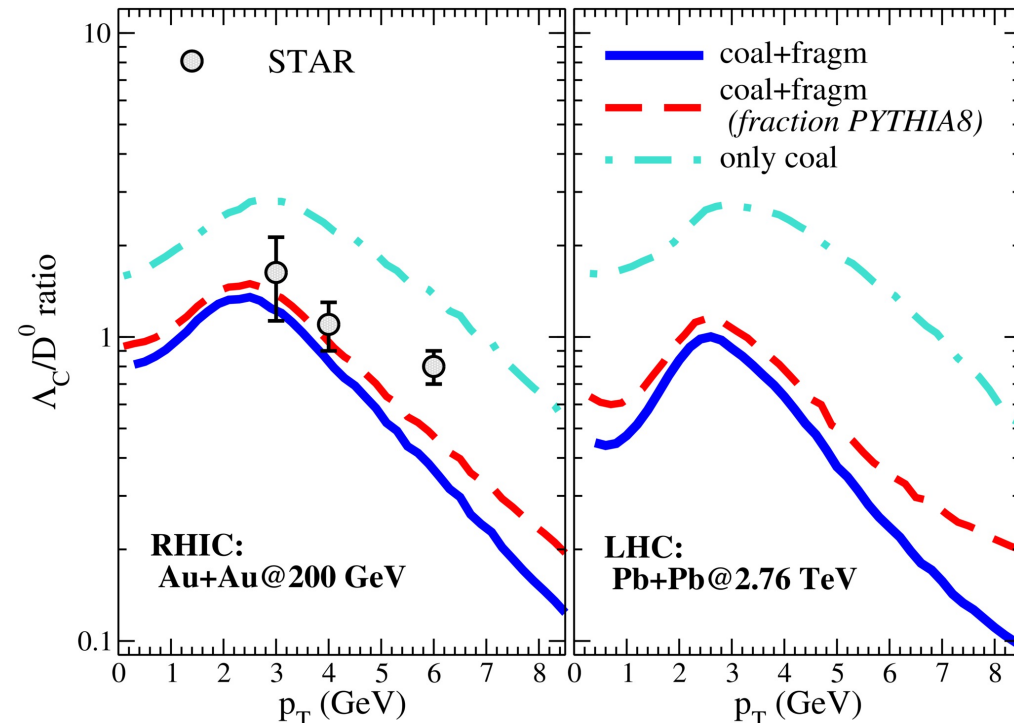
Results for 0-10% in PbPb @5.02TeV:

Consistent with the trend shown at RHIC and LHC @2.76TeV

Available data at low  $p_T \rightarrow$  differences recombination vs SHM



ALICE Coll. arXiv:2112.08156v1



S. Plumari, V. Minissale et al., Eur. Phys. J. **C78** no. 4, (2018) 348

# Small systems: Coalescence in pp?

Common consensus of possible presence of QGP in smaller system.

*If we assume in p+p @ 5 TeV a medium similar to the one simulated in hydro:*

What if:

- Assuming QGP formation also in pp?
- What coalescence+fragmentation predicts in this case?

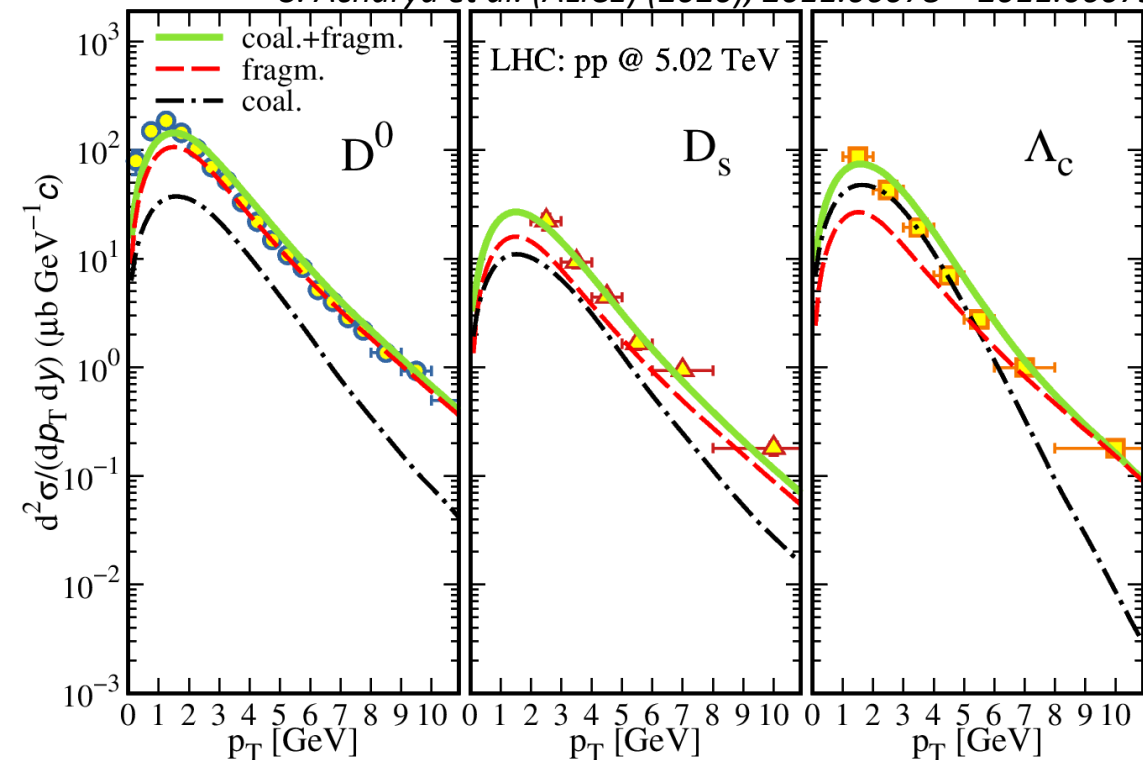
**p+p @ 5 TeV**

- $\tau_{pp} = 2$  fm/c
- $\beta_0 = 0.4$
- $R = 2.5$  fm
- $V \sim 30$  fm<sup>3</sup>

Data from:

S. Acharya et al. (ALICE), Eur. Phys. J. C 79, 388 (2019)

S. Acharya et al. (ALICE) (2020), 2011.06078 - 2011.06079



- Thermal Distribution ( $p_T < 2$  GeV)

LIGHT

$$\frac{dN_q}{d^2 r_T d^2 p_T} = \frac{g_q \tau m_T}{(2\pi)^3} \exp\left(-\frac{y_T(m_T - p_T \beta_T)}{T}\right)$$

- Minijet Distribution ( $p_T > 2$  GeV)  
NO QUENCHING

CHARM

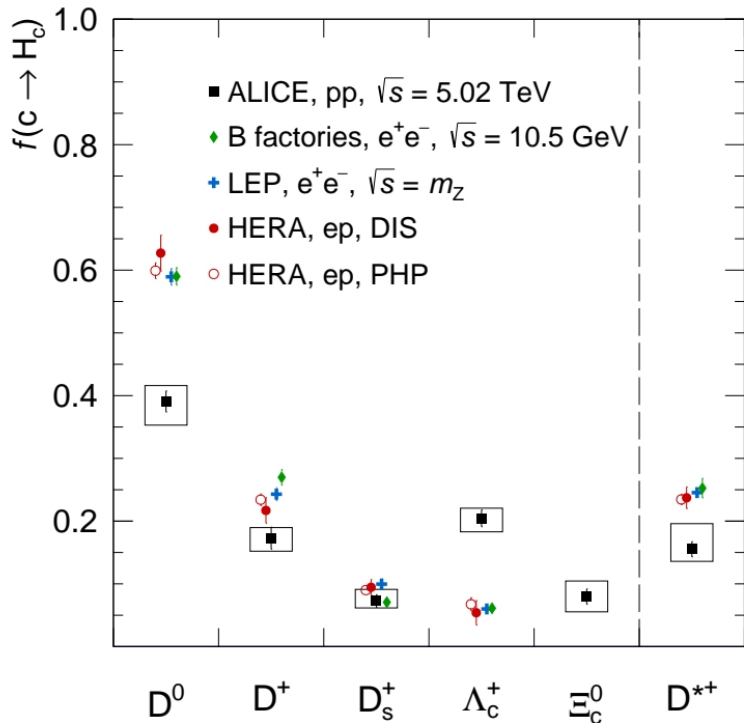
FONLL Distribution

*wave function widths  $\sigma_p$  of baryon and mesons kept the same from AA to pp*

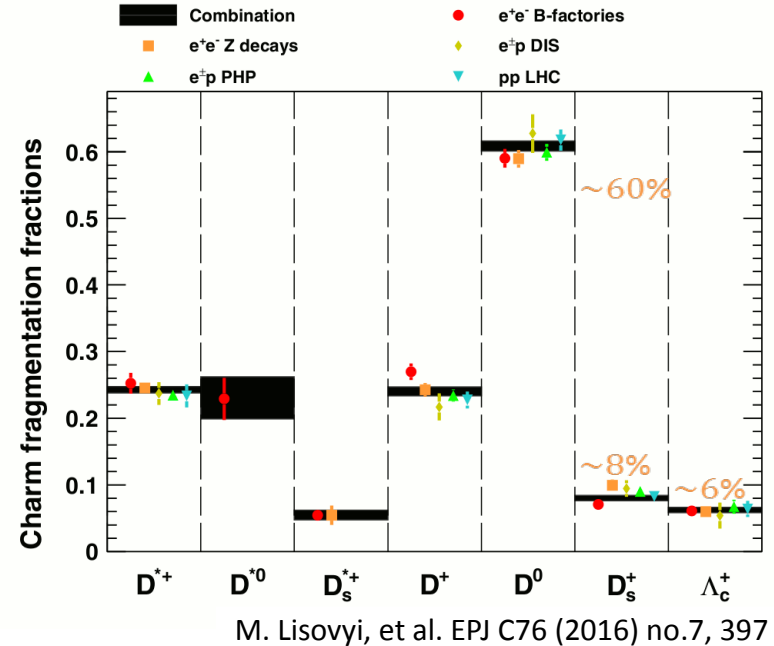
# Heavy flavour Hadronization: Fragmentation

$$\frac{dN_h}{d^2p_h} = \sum_f \int dz \frac{dN_f}{d^2p_f} D_{f \rightarrow h}(z)$$

*Fragmentation function*



ALICE Coll. ; Phys.Rev.D 105 (2022)



M. Lisovyi, et al. EPJ C76 (2016) no.7, 397

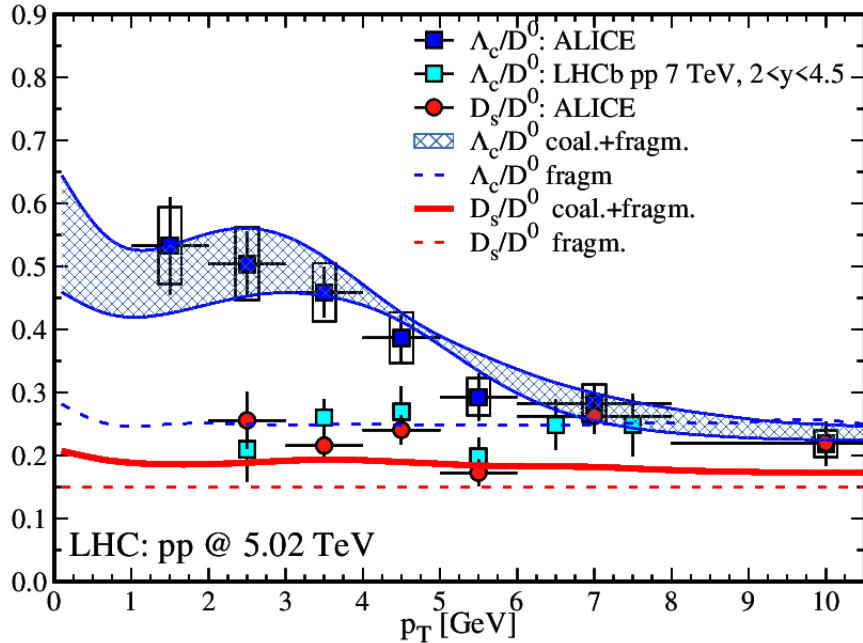
Updated fractions after experimental evidence in pp@5TeV:

Fragmentation fractions ( $c \rightarrow h$ ) depends on collision system...and QGP presence?

No more Universality?

# Small systems: Coalescence in pp?

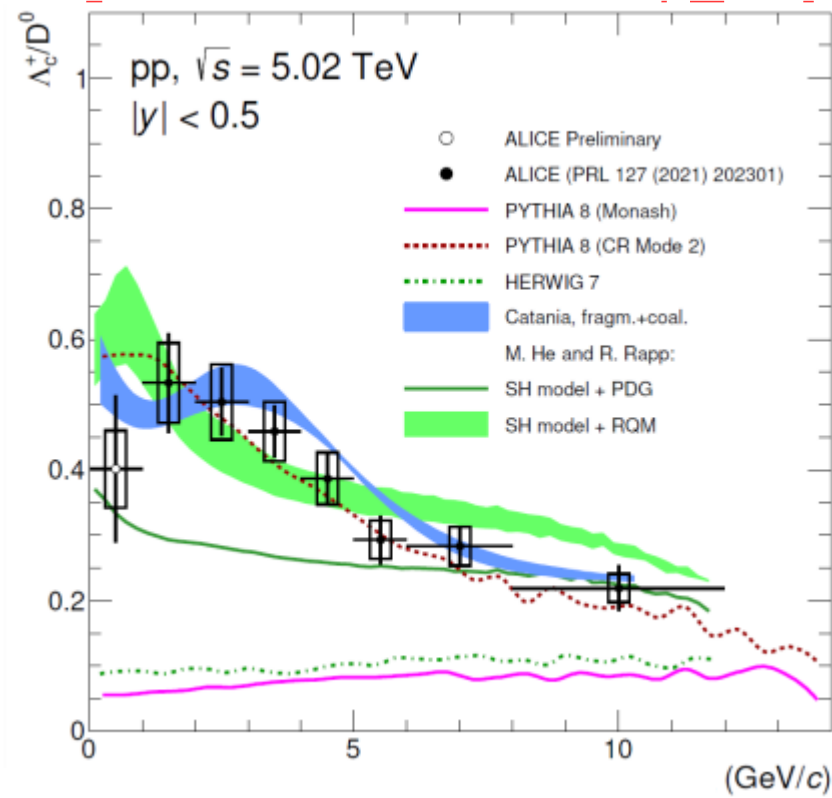
V. Minissale, S. Plumari, V. Greco, Phys.Lett.B821 (2021) 136622



Reduction of rise-and-fall behaviour in  $\Lambda_c / D^0$  ratio:

- Confronting with AA: Coal. contribution smaller w.r.t. Fragm.
- FONLL distribution flatter w/o evolution trough QGP
- Volume size effect

The increase of  $\Lambda_c$  production in pp have effect on  $R_{AA}$  of  $\Lambda_c$



ALICE, Phys.Rev.Lett. 127 (2021) 20, 202301

Error band correspond to  $\langle r^2 \rangle$  uncertainty in quark model

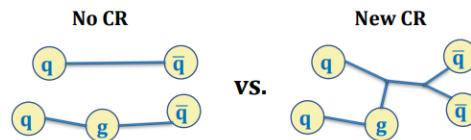
Other models:

He-Rapp, PLB795(2019): Increase  $\approx 2$  to  $\Lambda_c$  production:

SHM with resonance not present in PDG

PYTHIA8 + color reconnection

CR with SU(3) weights and string length minimization

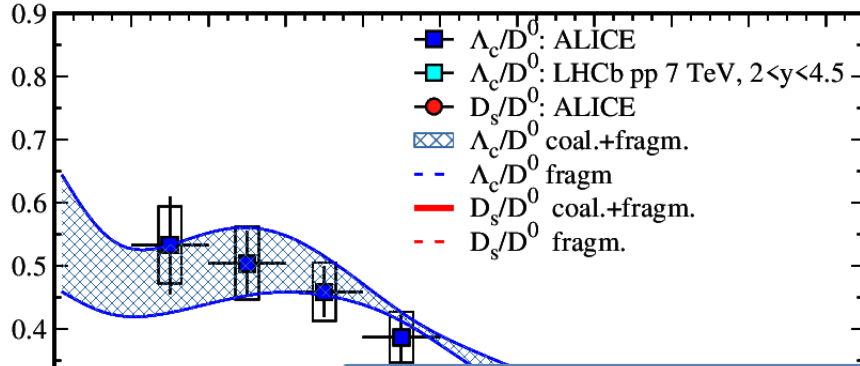




# Small systems: Coalescence in pp?

V. Minissale, S. Plumari, V. Greco, Phys.Lett.B821 (2021)

136622



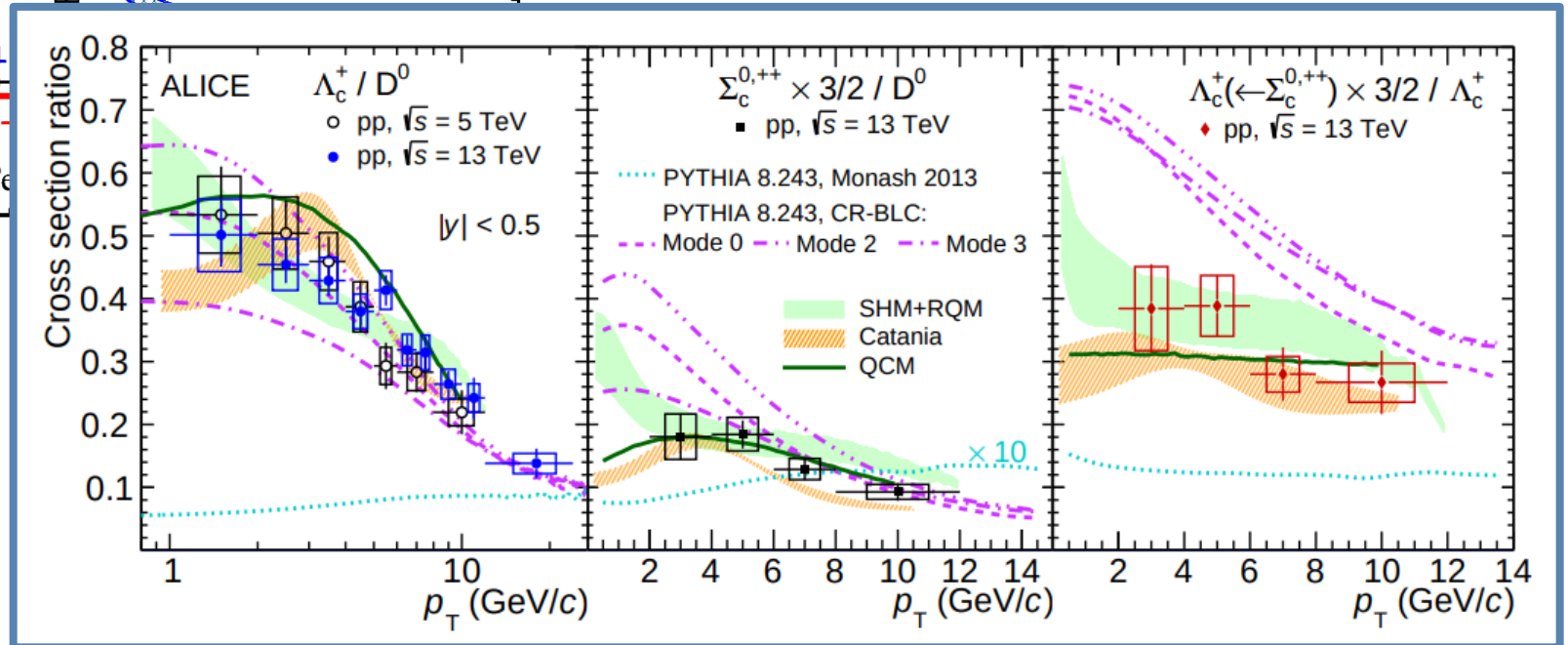
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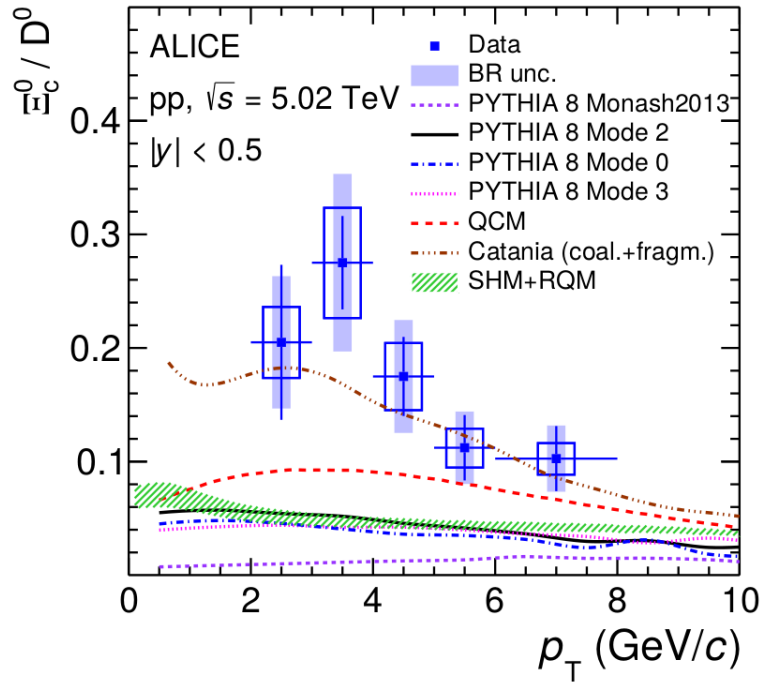
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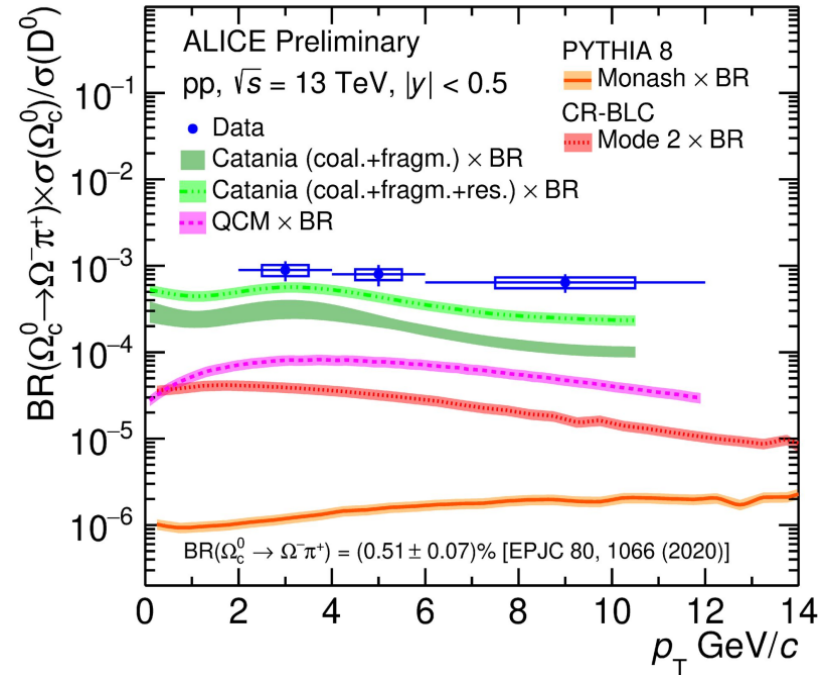
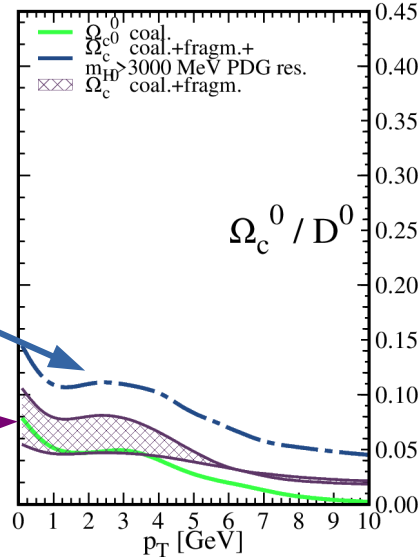


## New measurements of heavy hadrons at ALICE:

- $\Xi_c/D^0$  ratio, same order of  $\Lambda_c/D^0$ : coalescence gives enhancement
- very large  $\Omega_c/D^0$  ratio, our model does not get the big enhancement

Assuming additional PDG resonances with  $J=3/2$  and decay to  $\Omega_c$  additional to  $\Omega_c^0(2770)$

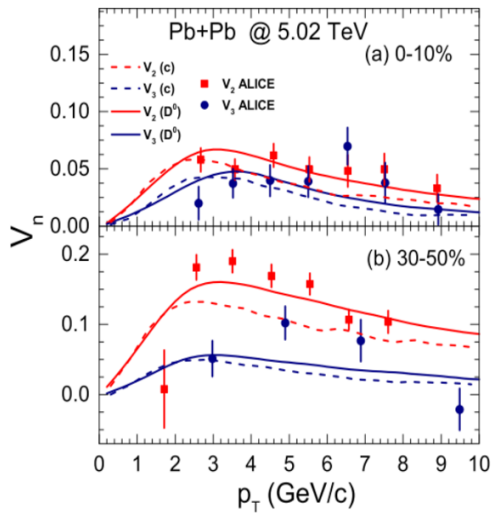
$\Omega_c^0(3000), \Omega_c^0(3005), \Omega_c^0(3065), \Omega_c^0(3090), \Omega_c^0(3120)$  supply an idea of how these states may affect the ratio



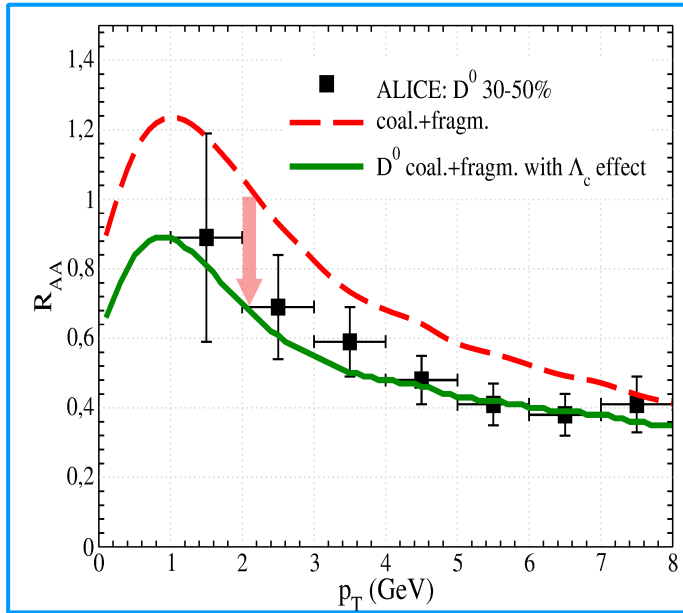
ALICE Coll. JHEP 10 (2021) 159

V. Minissale, S. Plumari, V. Greco, Phys.Lett.B821 (2021) 136622

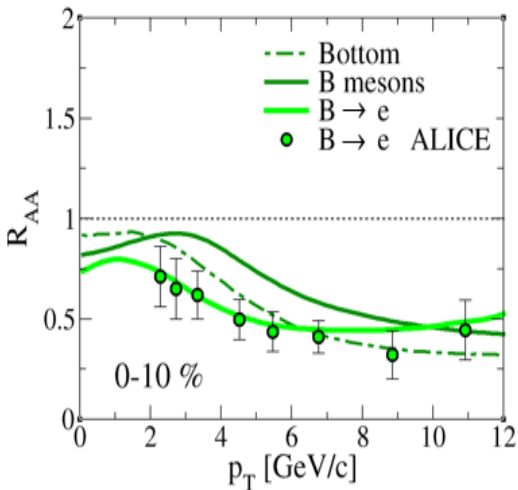
# Implications, developments and outlooks:



Coalescence give an enhancement to the  $v_n(p_T)$  of final hadrons compared to the charm  $v_n(p_T)$ .  
Sambataro,Sun,Minissale,Plumari,Greco(in preparation)

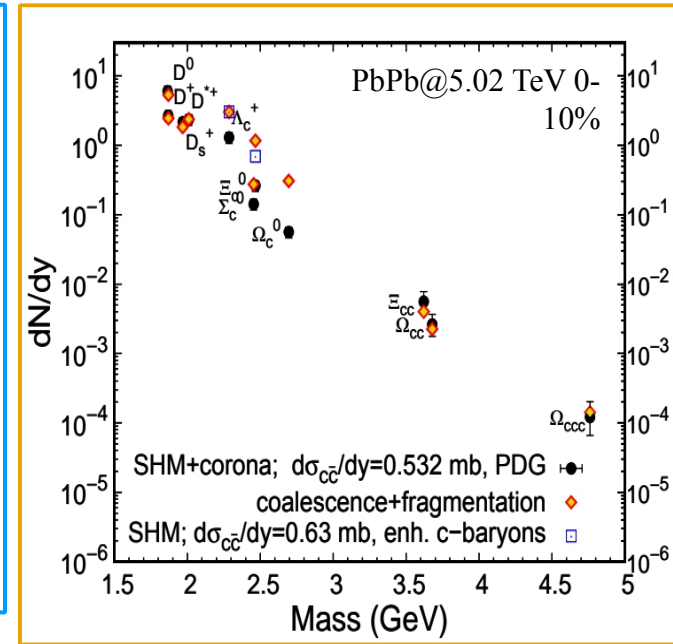


The large  $\Lambda_c$  production has effects on the  $R_{AA}$  of  $D^0$ , because of the charm conservation



Electrons from semileptonic B meson decay with a coal + fragm model for B meson production  
Sambataro, Minissale et al.(in preparation)

## MULTICHARM



Yields of multicharm hadrons similar to SHM in PbPb, possible effects with system size and on the distribution vs  $p_T$  are under investigation.

A baryon like  $\Omega_{ccc}$  formed only by heavy quarks can give insights on the baryon wave function and the potential  $V_{cc}(r,T)$  between quarks.

Enhancement for single charmed baryons as seen in pp collisions.

Minissale, Plumari, Greco (in preparation)

# Conclusions

- *Good agreement with experimental data of hadrons spectra in AA collisions from RHIC to LHC*
- *Extension to pp: description of D mesons and  $\Lambda_c$  spectra*
- *Coalescence plus fragmentation gives peculiar enhancement in baryon/ meson ratio for all heavy hadrons  $\Lambda_c, \Xi_c, \Omega_c$*
- *Outlook: multicharm hadrons production*

Backup Slides

# Elliptic Flow – Quark Number Scaling

Fourier expansion of the azimuthal distribution

$$f(\varphi, p_T) = 1 + 2 \sum_{n=1}^{\infty} v_n(p_T) \cos n\varphi$$

momentum anisotropy in the transverse plane

n=2 Elliptic flow

coalescence brings to

$$v_{2,M}(p_T) \approx 2v_{2,q}(p_T/2)$$
$$v_{2,B}(p_T) \approx 3v_{2,q}(p_T/3)$$

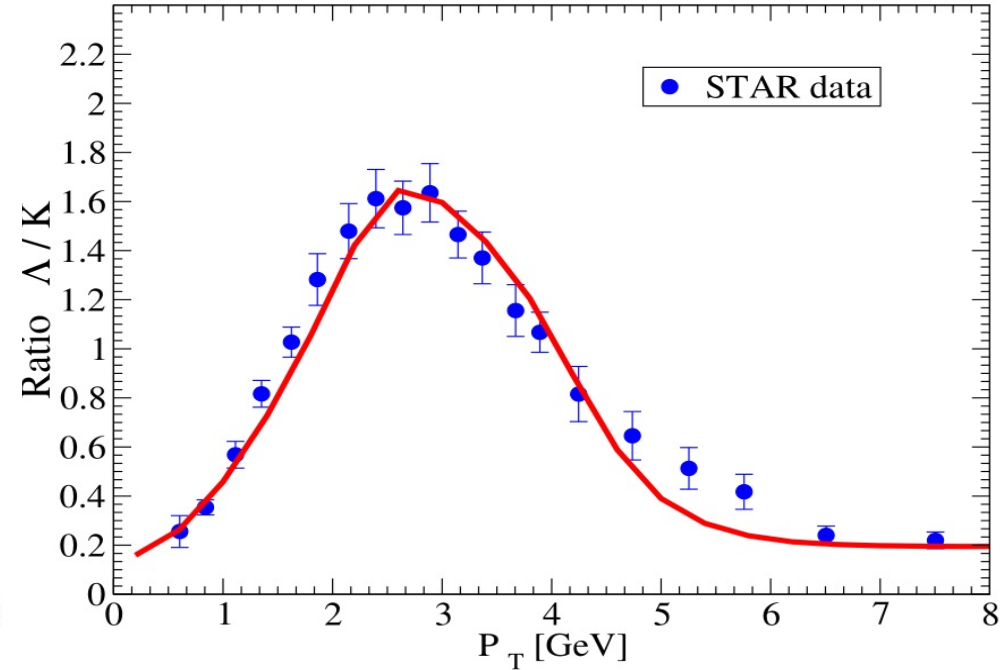
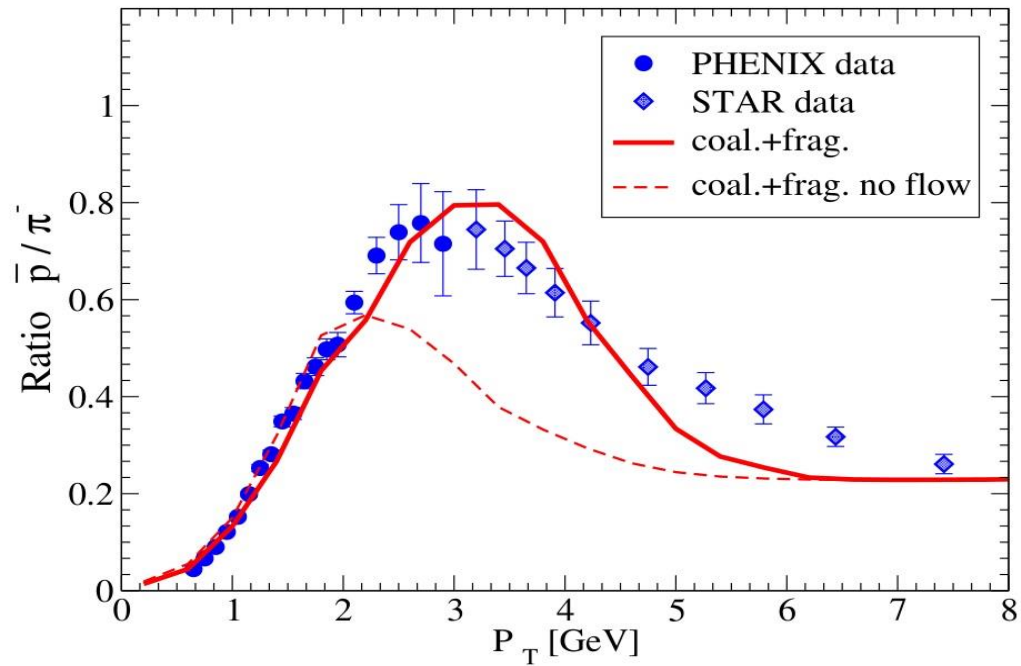
Partonic  
elliptic flow

Hadronic  
elliptic flow

Assumption

- one dimensional
- Dirac delta for Wigner function
- isotropic radial flow
- not including resonance effect

# Baryon to meson ratio at RHIC



- coalescence naturally predict a baryon/meson enhancement in the region  $p_T \approx 2-4\text{GeV}$  with respect to  $pp$  collisions
- Lack of baryon yield in the region  $p_T \approx 5-7\text{GeV}$

# Relativistic Boltzmann transport at finite $\eta/s$

## Bulk evolution

$$\underbrace{p^\mu \partial_\mu f_{q,g}(x,p)}_{\text{free-streaming}} + \underbrace{M(x) \partial_\mu^x M(x) \partial_p^\mu f_{q,g}(x,p)}_{\text{field interaction } \varepsilon-3p \neq 0} = \underbrace{C_{22}[f_{q,g}]}_{\text{collisions } \eta \neq 0}$$

## Heavy quark evolution

$$p^\mu \partial_\mu f_Q(x,p) = C[f_q, f_g, f_Q]$$

- Describes the evolution of the one body distribution function  $f(x,p)$
- It is valid to study the evolution of both bulk and Heavy quarks
- Possible to include  $f(x,p)$  out of equilibrium

