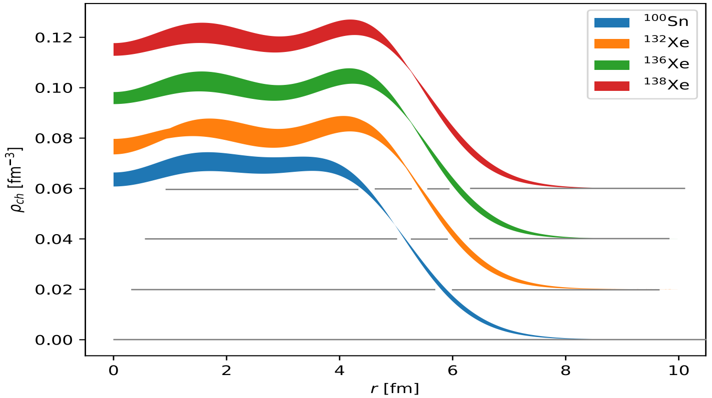
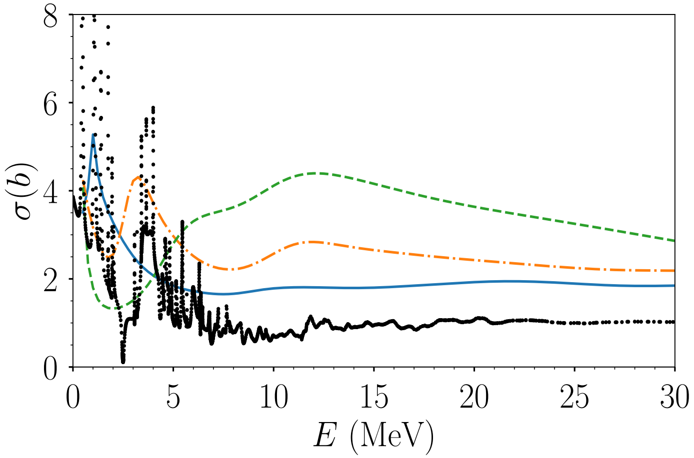
Quinto Incontro Nazionale di Fisica Nucleare
Laboratori Nazionali del Gran Sasso, 9-11 maggio 2022

UNIVERSITÀ DEGLI STUDI DI MILANO
DIPARTIMENTO DI FISICA

Ab Initio Computations of Ground States and Optical Potentials in Nuclei

Carlo Barbieri





Current Status of low-energy nuclear physics

Composite system of interacting fermions

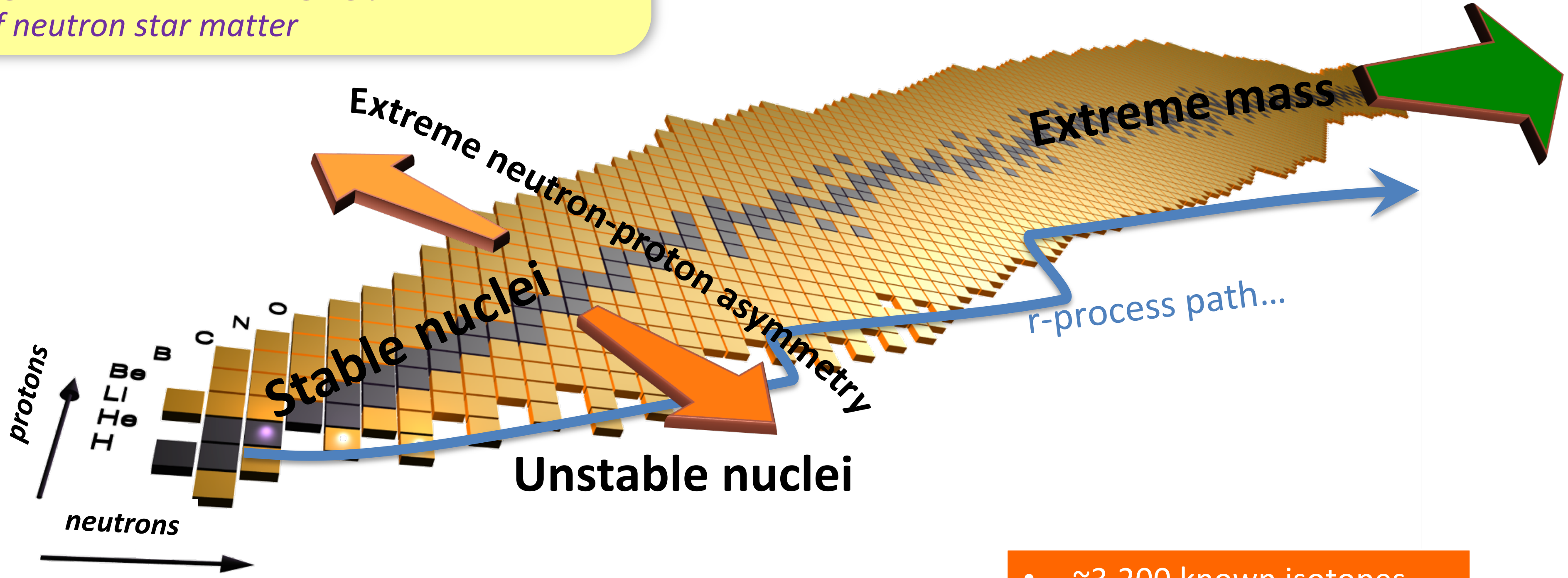
Binding and limits of stability

Coexistence of individual and collective behaviors

Self-organization and emerging phenomena

EOS of neutron star matter

Experimental programs RIKEN, FAIR, FRIB…



- ~3,200 known isotopes
- ~7,000 predicted to exist
- Correlation characterised in full for ~283 stable

Nature **473**, 25 (2011); **486**, 509 (2012)

Current Status of low-energy nuclear physics

Composite system of interacting fermions

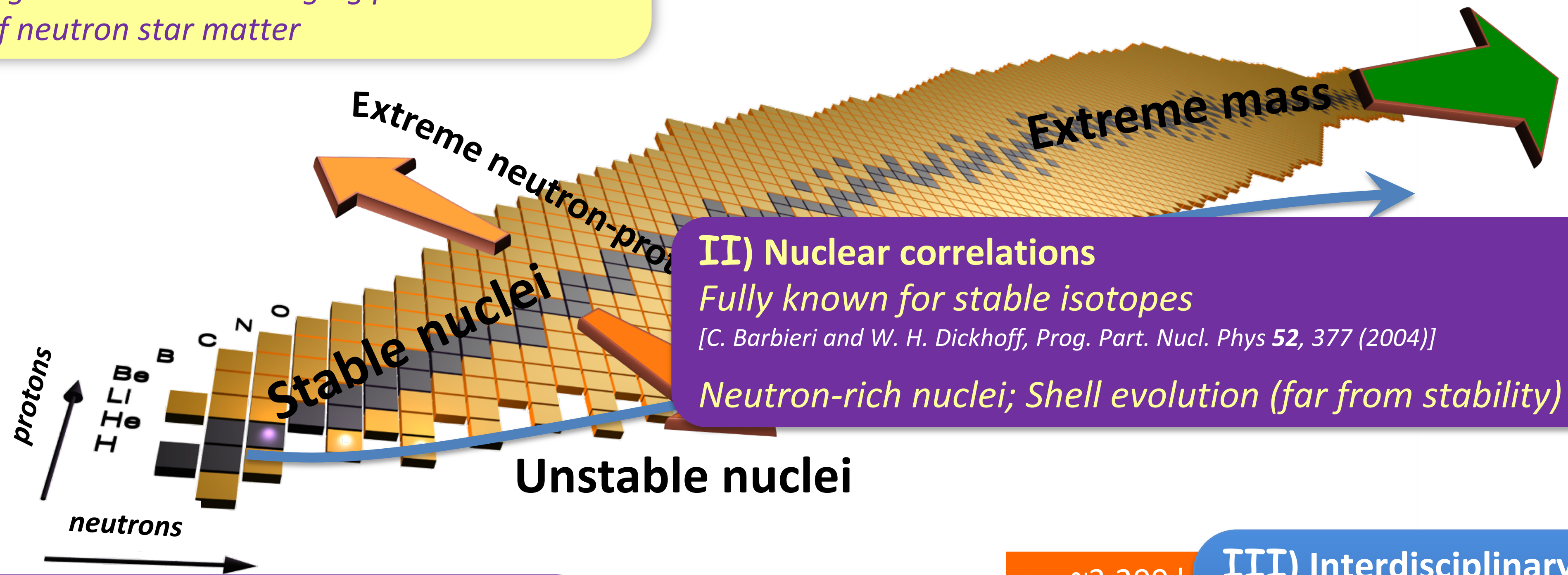
Binding and limits of stability

Coexistence of individual and collective behaviors

Self-organization and emerging phenomena

EOS of neutron star matter

Experimental programs
RIKEN, FAIR, FRIB, ISAC…



II) Nuclear correlations

Fully known for stable isotopes

[C. Barbieri and W. H. Dickhoff, Prog. Part. Nucl. Phys ***52****, 377 (2004)]*

Neutron-rich nuclei; Shell evolution (far from stability)

I) Understanding the nuclear force

QCD-derived; 3-nucleon forces (3NFs)

First principle (ab-initio) predictions

- ~3,200 k
- ~7,000 p
- Correlati
  in full fo

Nature **473**, 25 (

III) Interdisciplinary character

Astrophysics

Tests of the standard model

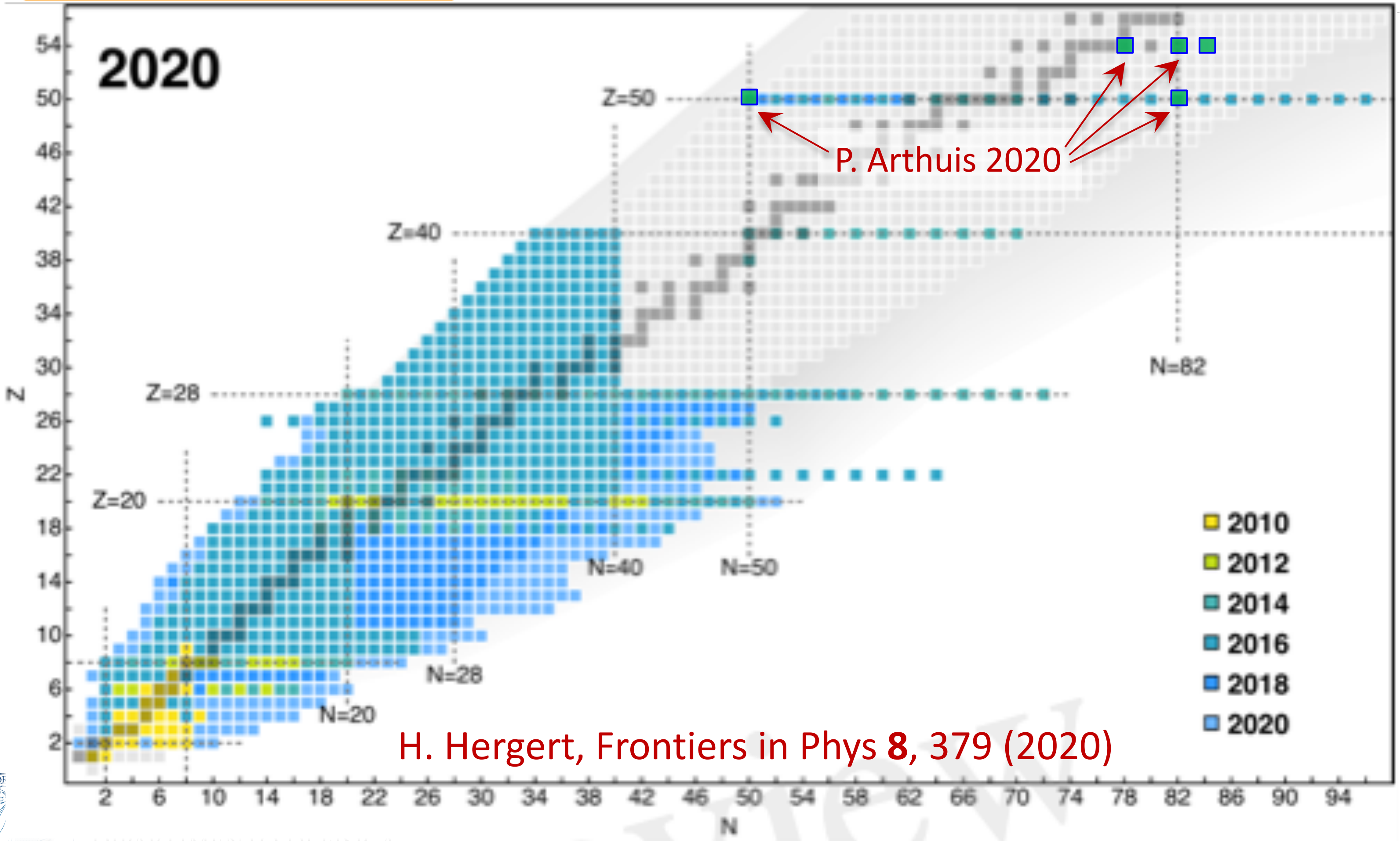
Other fermionic systems:
ultracold gasses; molecules;

Reach of ab initio methods across the nuclear chart

- ⦿ "Exact" approaches
  - ○ Since 1980's
  - ○ Monte Carlo, CI, …
  - ○ Factorial scaling

- ⦿ Approximate approaches for closed-shell nuclei
  - ○ Since 2000's
  - ○ SCGF, CC, IMSRG
  - ○ Polynomial scaling

- ⦿ Approximate approaches for open-shells
  - ○ Since 2010's
  - ○ GGF, BCC, MR-IMSRG
  - ○ Polynomial scaling



P. Arthuis 2020

H. Hergert, Frontiers in Phys **8**, 379 (2020)

Key developments in *SCGF*:

[V. Somà, Front. Phys. 8, 340 (2020)]

Dyson ADC(2-5)
Schirmer 1983 (formalism)

Particle-vibration coupling, FRPA(3)
CB 2000, 2007

Gorkov ADC(2): open shells!
Somà 2011, 2013

3-nucleon forces basic formalism
Carbone, Cipollone 2013
Raimondi 2018

Gorkov ADC(3) and higher orders (automatic)
Raimoindi, Arthuis 2019

Deformation, Symmetry restoration
???

THE FUTURE OF NUCLEAR STRUCTURE: CHALLENGES AND OPPORTUNITIES IN THE MICROSCOPIC DESCRIPTION OF NUCLEI

EDITED BY: Luigi Coraggio, Saori Pastore and Carlo Barbieri
PUBLISHED IN: Frontiers in Physics

frontiers Research Topics

Editors: L. Coraggio, S. Pastore, CB

*FRONTIERS topical review (*doi: 10.3389/fphy.2020.626976*) :*

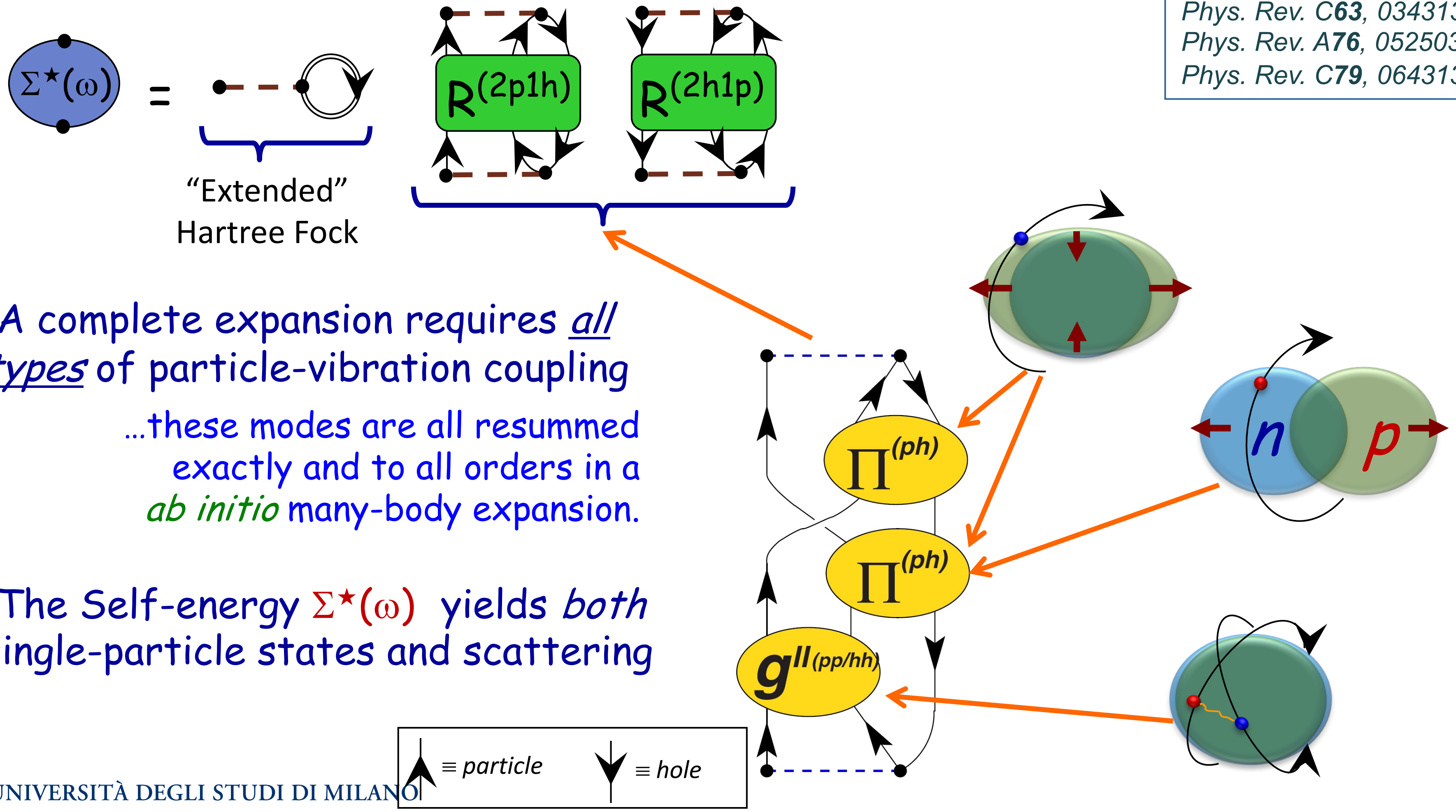
H. Hergert, Frontiers in Phys. **8**, 379 (2020)

V. Somà, Frontiers in Phys. **8**, 340 (2020)

The FRPA Method in Two Words

Particle vibration coupling is the main mechanism driving the redistribution and fragmentation of particle strength—expecially in the quasielastic regions around the Fermi surface…

CB et al.,
*Phys. Rev. C**63**, 034313 (2001)*
*Phys. Rev. A**76**, 052503 (2007)*
*Phys. Rev. C**79**, 064313 (2009)*



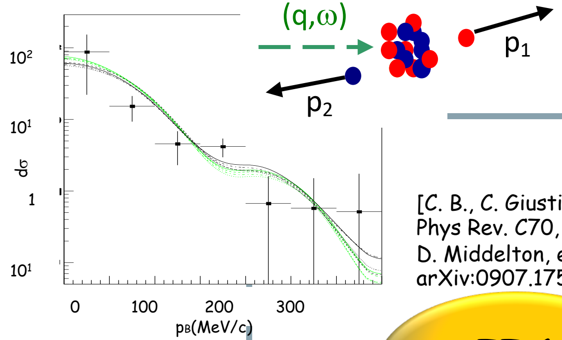
- A complete expansion requires *all types* of particle-vibration coupling

…these modes are all resummed exactly and to all orders in a *ab initio* many-body expansion.

- The Self-energy $\Sigma^\star(\omega)$ yields *both* single-particle states and scattering

Self-Consistent Green's Function Approach

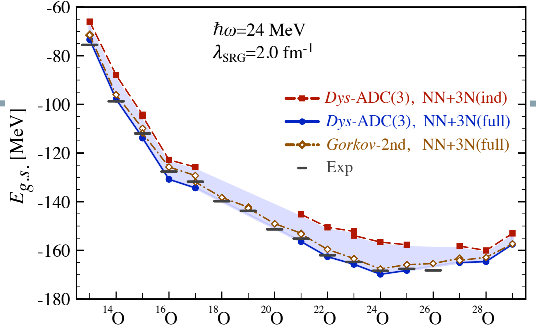
$^{16}O(e,e'pn)^{14}N$ @ MAINZ



[C. B., C. Giusti, et al.
Phys Rev. C70, 014606 (2004)
D. Middelton, et al.
arXiv:0907.1758; EPJA in print]

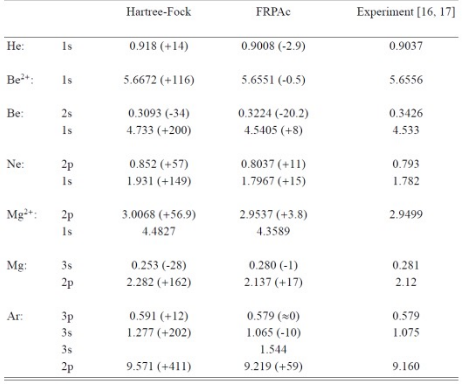
Binding energies

[PRL. 111, 062501 (2013),
PRC 92, 014306 (2015), PRC89, 061301R (2014)]

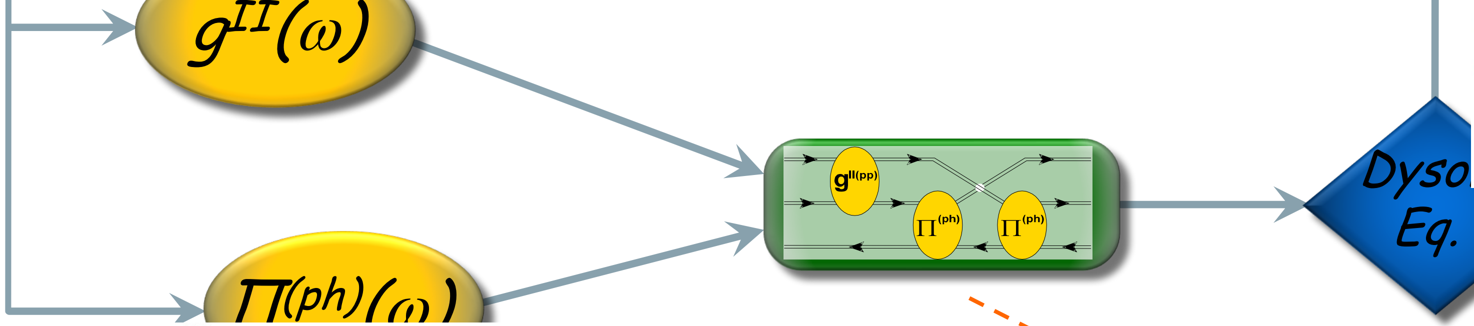


Ionization energies/ affinities, in atoms

[CB, D. Van Neck,
AIP Conf.Proc.1120,104 ('09) & in prep]

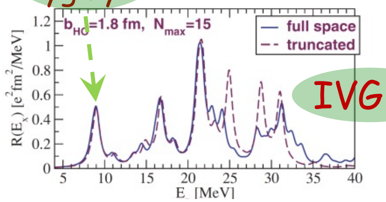


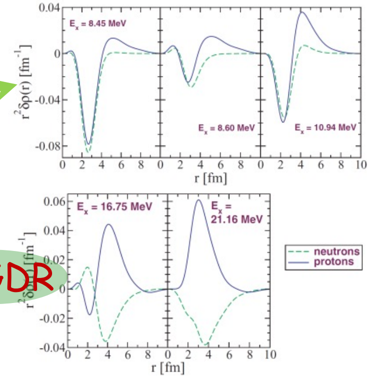
		Hartree-Fock	FRPAc	Experiment [16, 17]
He:	1s	0.918 (+14)	0.9008 (-2.9)	0.9037
Be^{2+}:	1s	5.6672 (+116)	5.6551 (-0.5)	5.6556
Be:	2s	0.3093 (-34)	0.3224 (-20.2)	0.3426
	1s	4.733 (+200)	4.5405 (+8)	4.533
Ne:	2p	0.852 (+57)	0.8037 (+11)	0.793
	1s	1.931 (+149)	1.7967 (+15)	1.782
Mg^{2+}:	2p	3.0068 (+56.9)	2.9537 (+3.8)	2.9499
	1s	4.4827	4.3589	
Mg:	3s	0.253 (-28)	0.280 (-1)	0.281
	2p	2.282 (+162)	2.137 (+17)	2.12
Ar:	3p	0.591 (+12)	0.579 (≈0)	0.579
	3s	1.277 (+202)	1.065 (-10)	1.075
	3s		1.544	
	2p	9.571 (+411)	9.219 (+59)	9.160



Isovector response for ^{32}Ar, ^{34}Ar

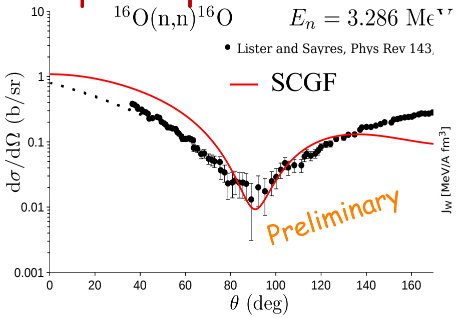
Proton Pygmy



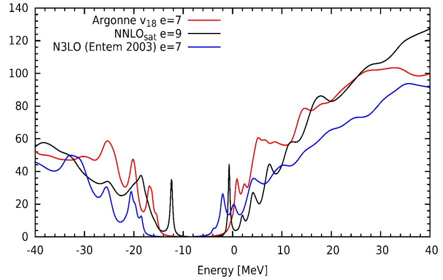


[C. B., K. Langanke, et al., Phys Rev. C77, 024304 (2008)]

Optical potential

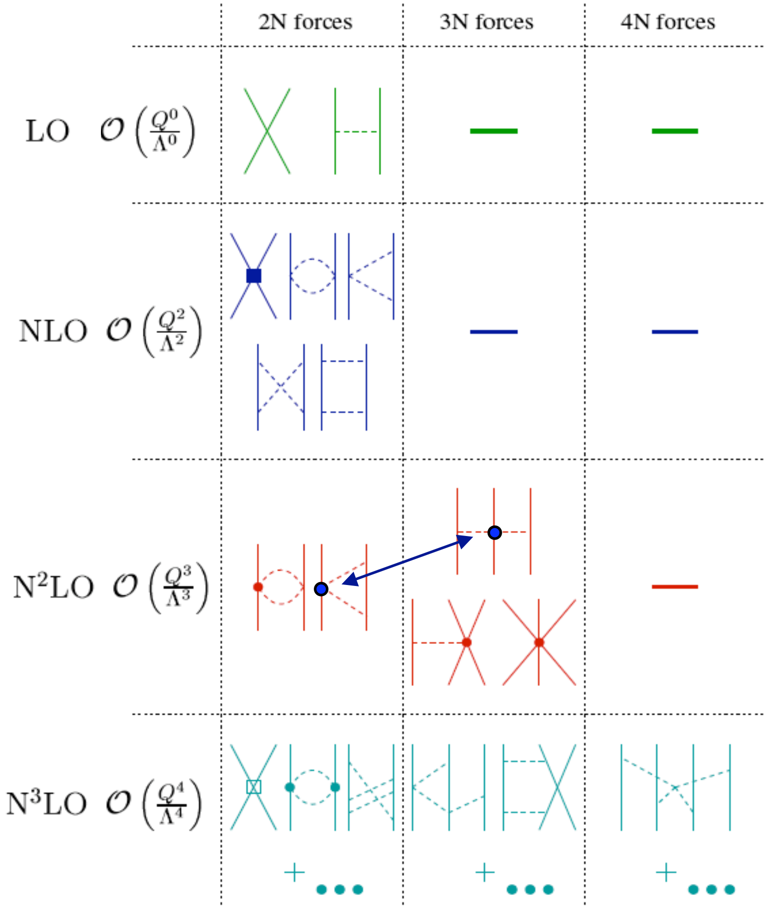


arXiv:1612.01478 [nucl-th]

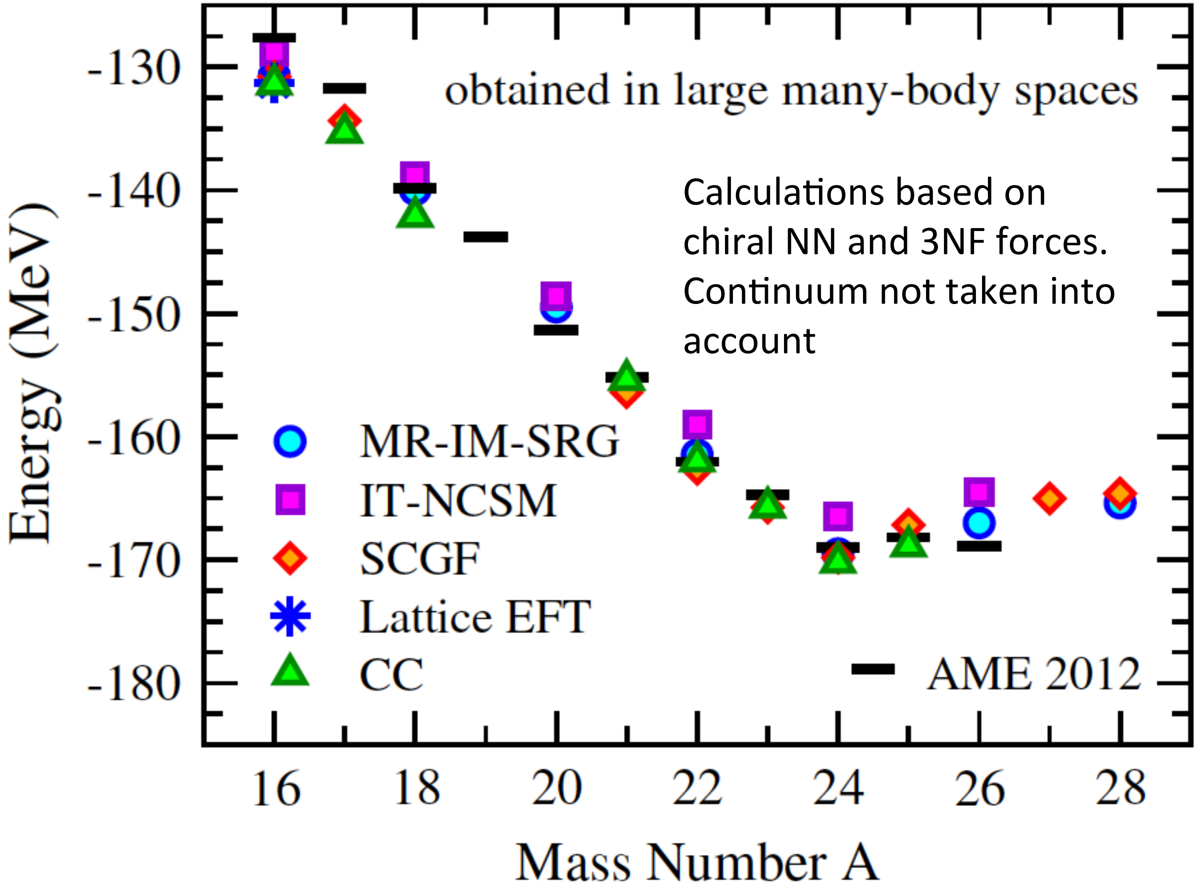


Realistic nuclear forces form Chiral EFT

Chiral EFT for nuclear forces:



(3NFs arise naturally at N2LO)



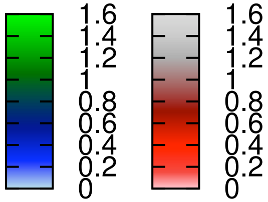
Calculations based on chiral NN and 3NF forces. Continuum not taken into account

K. Hebeler et al., Annu. Rev. Nucl. Part. Sci. **65**, 457 (2015)

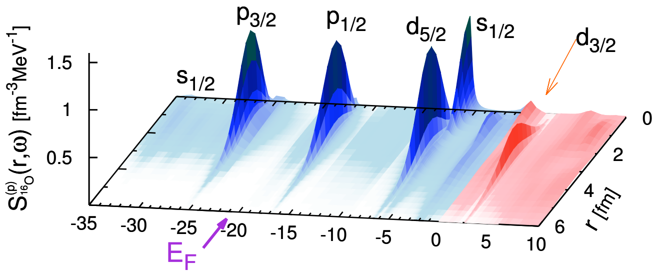
See also:
A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. 111, 062501 (2013)

Neutron spectral function of Oxygens

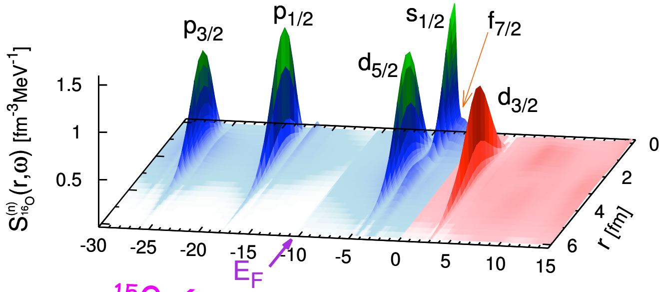
A. Cipollone, CB, P. Navrátil, *Phys. Rev. C* ***92***, *014306 (2015);*
Phys. Rev. Lett. 111, 062501 (2013)



^{14}O:

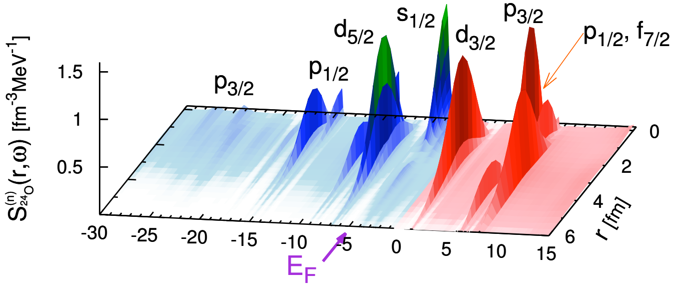


^{16}O:

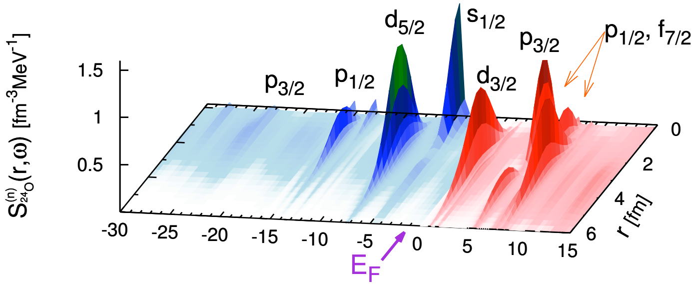


^{15}O ← → ^{17}O

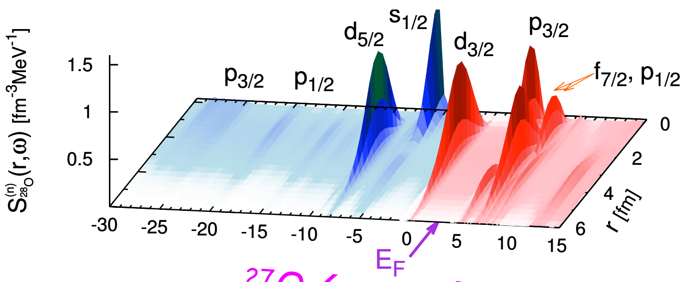
^{22}O:



^{24}O:

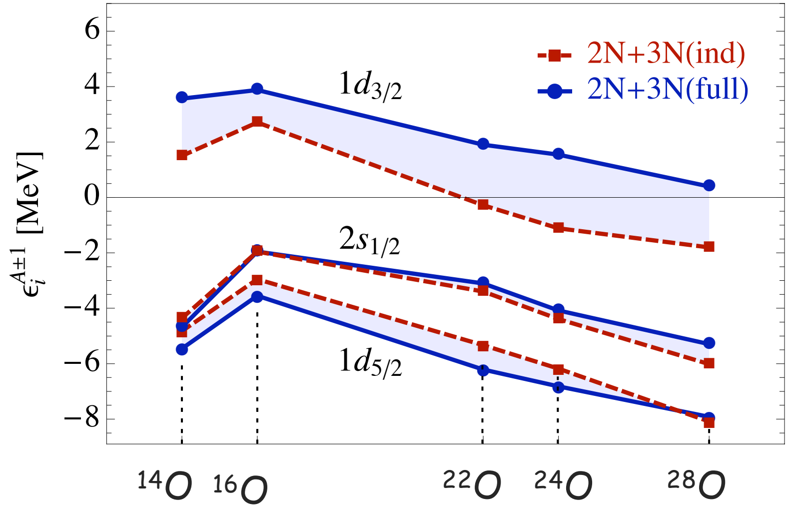


^{28}O:



^{27}O ← → ^{29}O

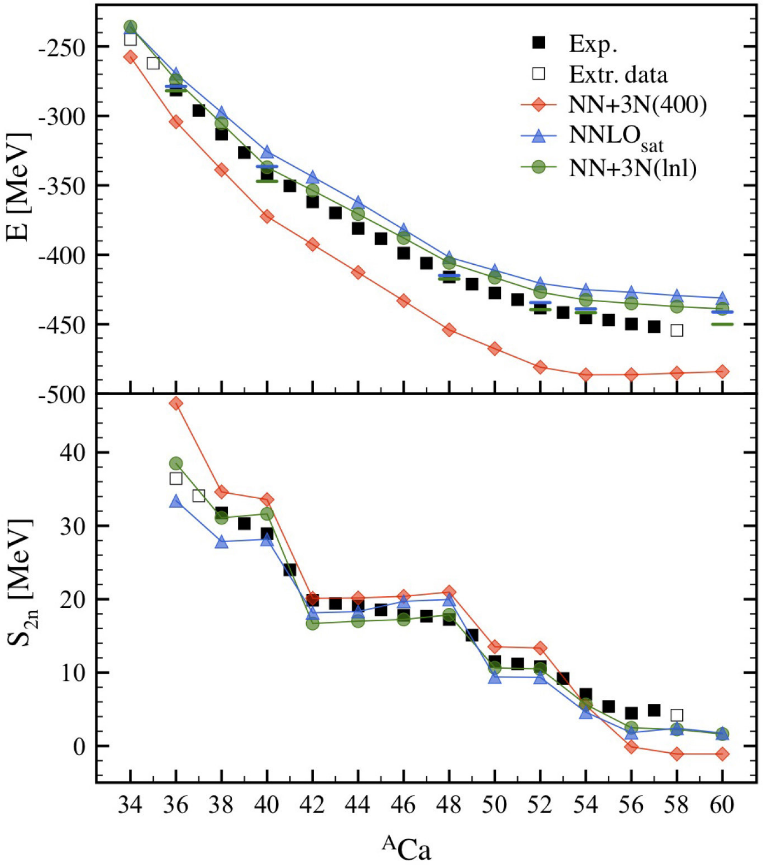
Neutron quasiparticle energies

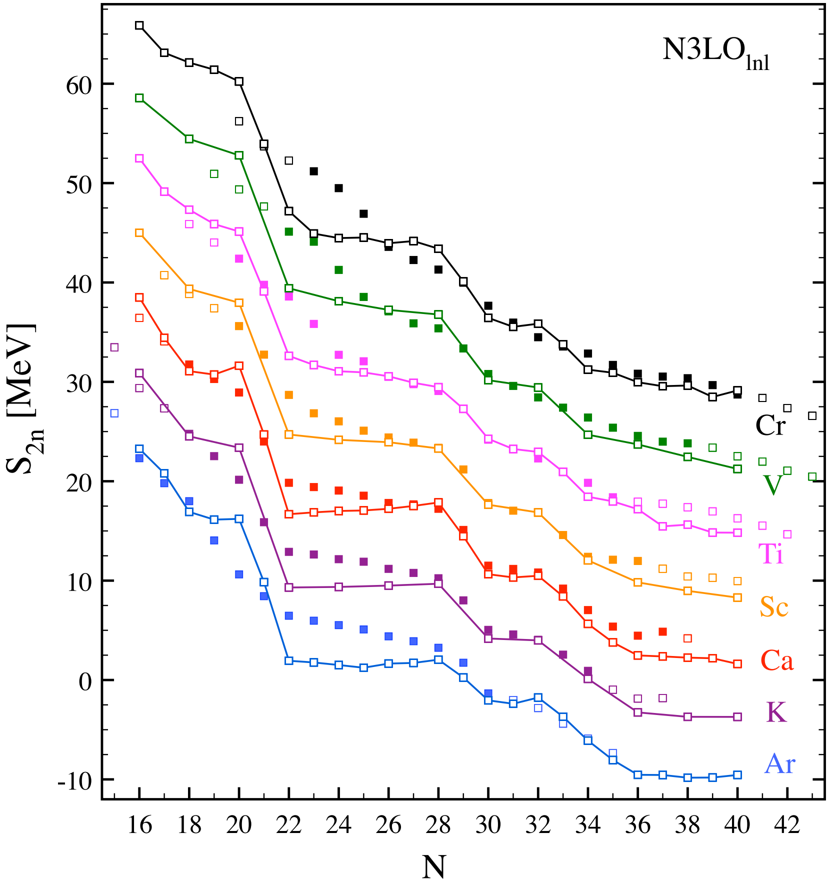


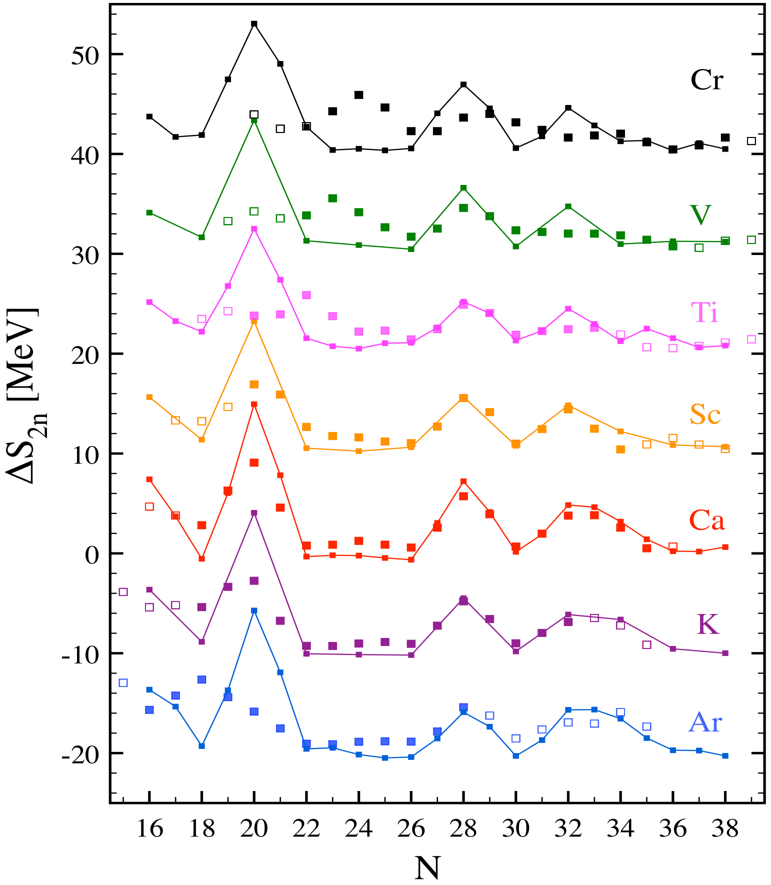
N3LO(500) + nln 3NF

N3LO-lnl: a ***second-generation*** Chiral EFT Hamiltonian

Computations w/ SCGF – Gorkov-ADC(2)

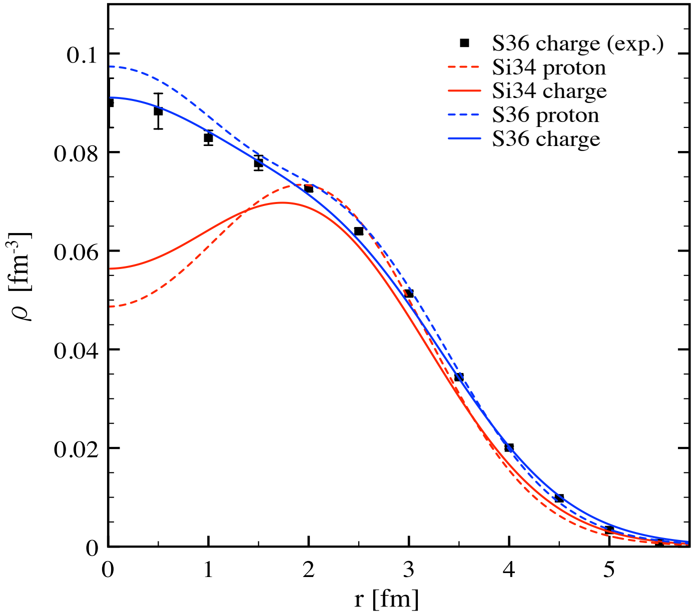






V. Somà, P. Navrátil, F. Raimondi, CB, T. Duguet, Phys Rev C**101**, 014318 (2020); Eur. Phys. J. A**57**, 135 (2021)

Bubble nuclei… ^{34}Si prediction



Duguet, Somà, Lecuse, CB, Navrátil,
Phys.Rev. C**95**, 034319 (2017)

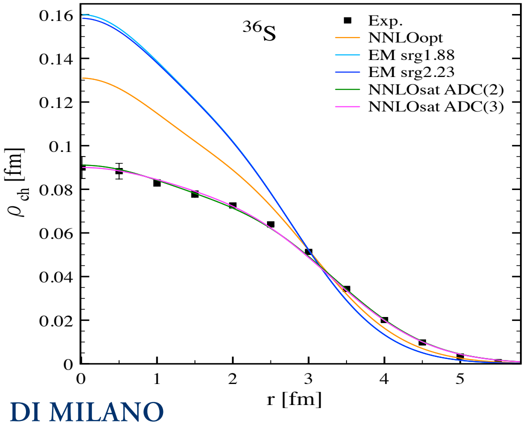
- ^{34}Si is unstable, charge distribution is still unknown

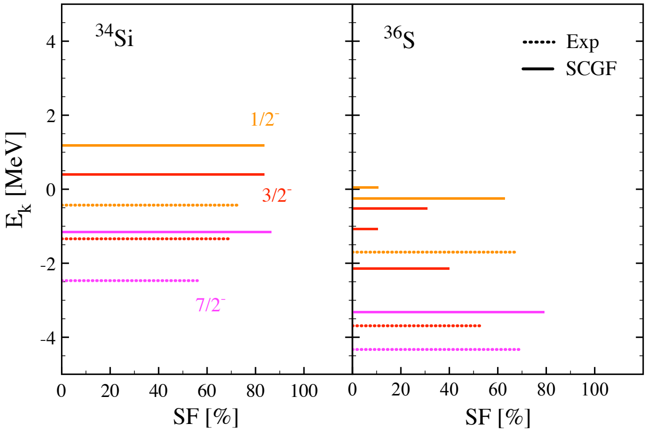
- Suggested central depletion from mean-field simulations

- *Ab-initio* theory confirms predictions

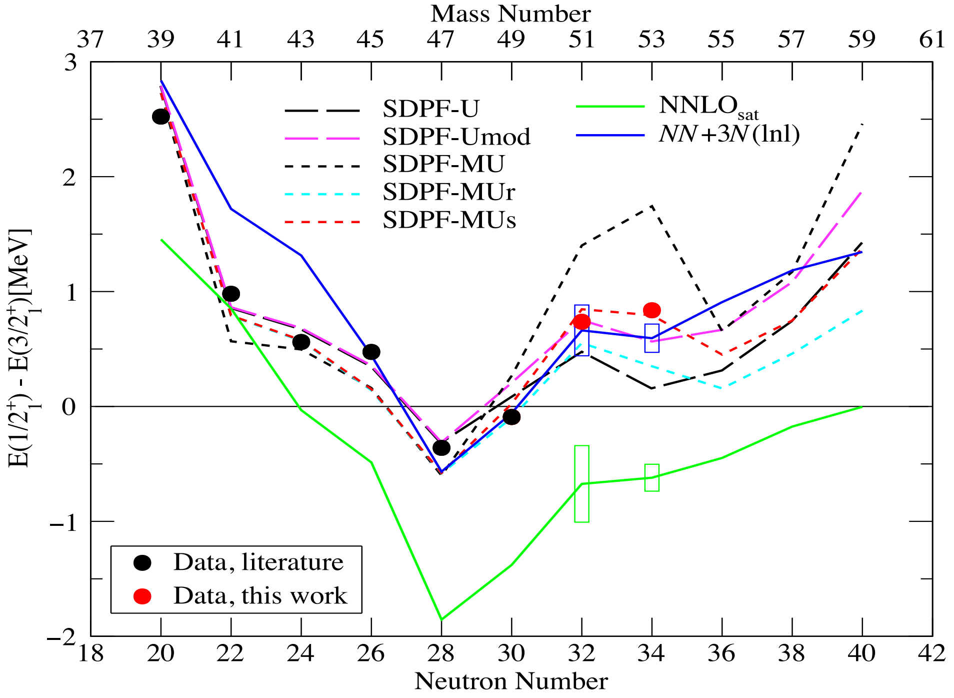
- Other theoretical and experimental evidence:
  Phys. Rev. ***C 79****, 034318 (2009),*
  Nature Physics ***13****, 152–156 (2017).*

Validated by charge distributions and neutron quasiparticle spectra:



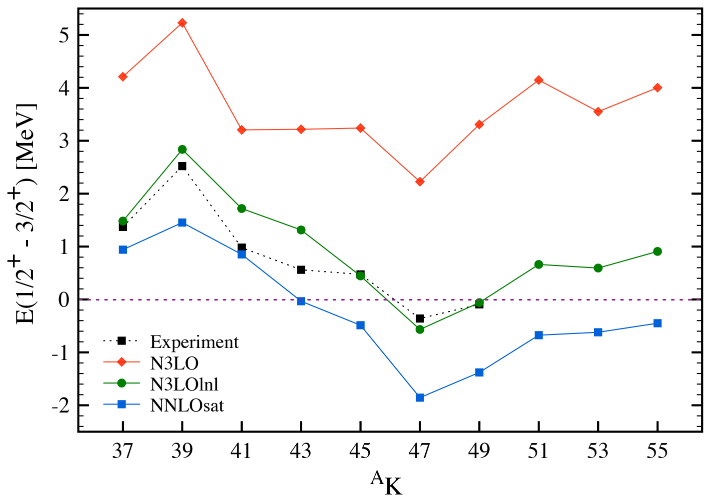


$d_{3/2} - s_{1/2}$ inversion in K isotopes and bubbles at N=28

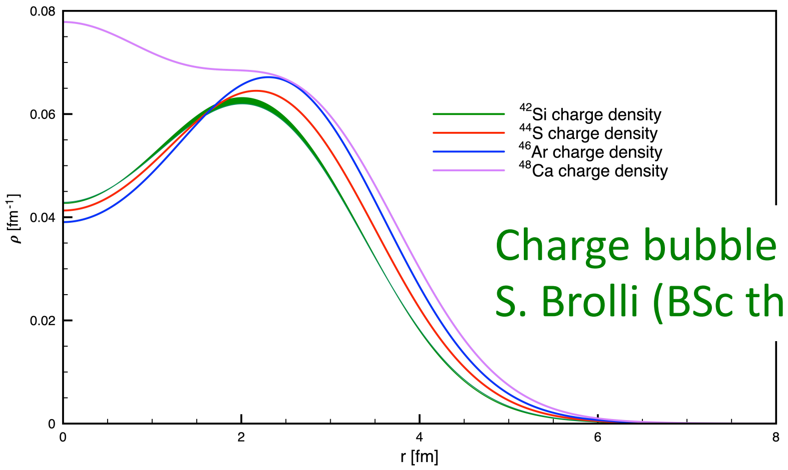


Papuga et al., PRL**110**, 172503 (2013); PRC**90**, 034321 (2014)

RIKEN, SEASTAR coll., Phys. Lett. B**802** 135215 (2020)



V. Somà, *et al.,* Phys. Rev. C**101**, 014318 (2020)



Charge bubble for 42Si-46Ar ??

S. Brolli (BSc thesis) in preparation

Electron-Ion Trap colliders…

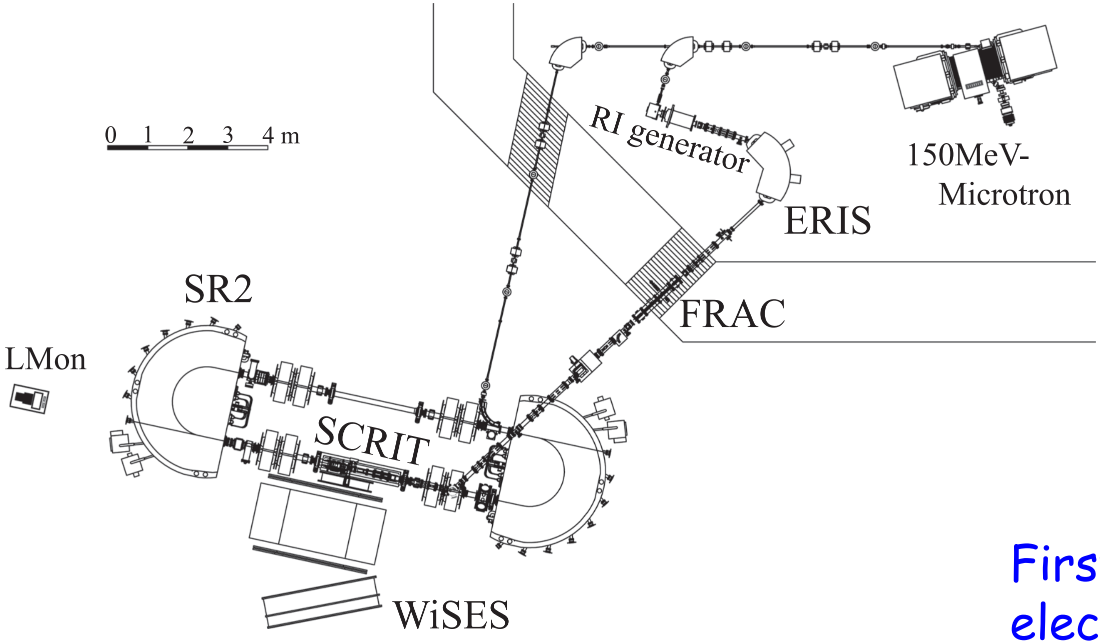


FIG. 1. Overview of the SCRIT electron scattering facility.

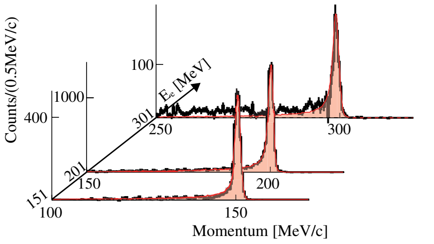
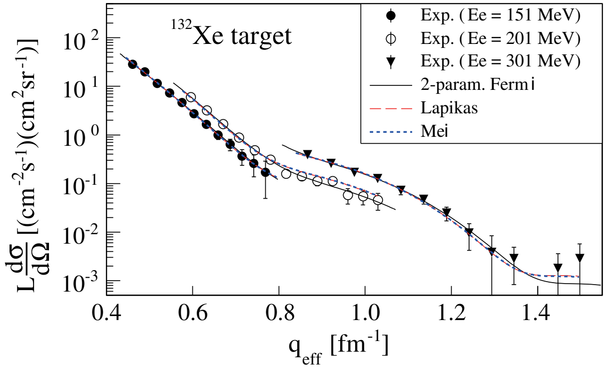


FIG. 3. Reconstructed momentum spectra of ^{132}Xe target after background subtraction. Red shaded lines are the simulated radiation tails following the elastic peaks.

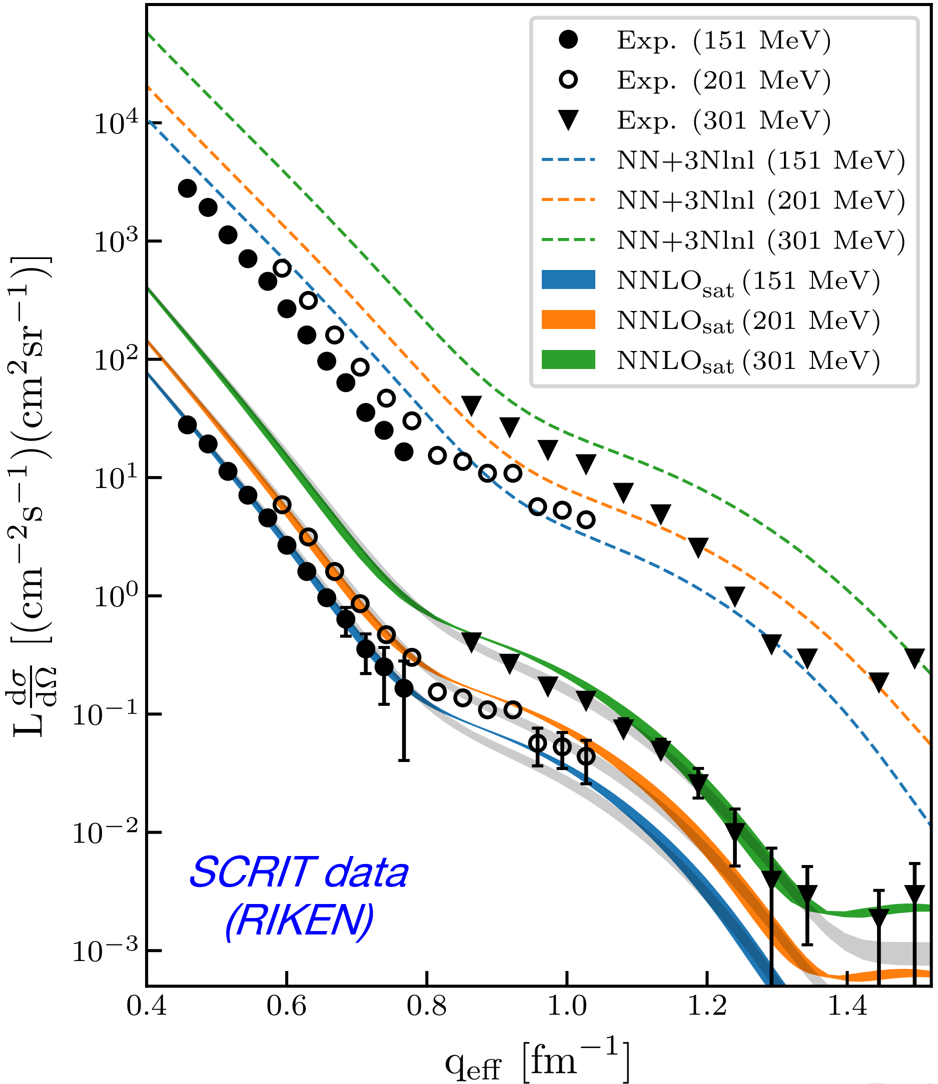


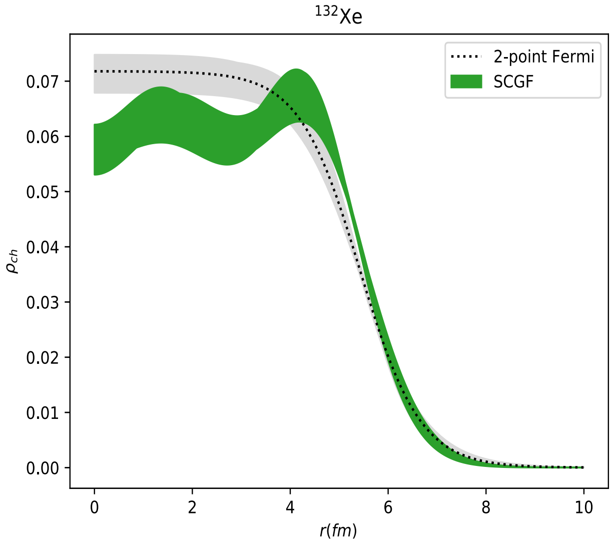
First ever measurement of charge radii through electron scattering with and ion trap setting that *can be used on radioactive isotopes* !!

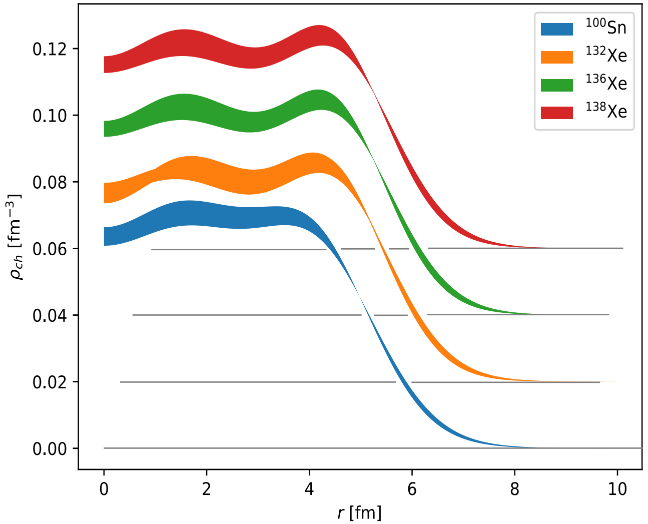
K. Tsukada *et al.*, Phy rev Lett **118**, 262501 (2017)

P. Arthuis, CB, M. Vorabbi, P. Finelli,
Phys. Rev. Lett. ***125****, 182501 (2020)*

Charge density for Sn and Xe isotopes





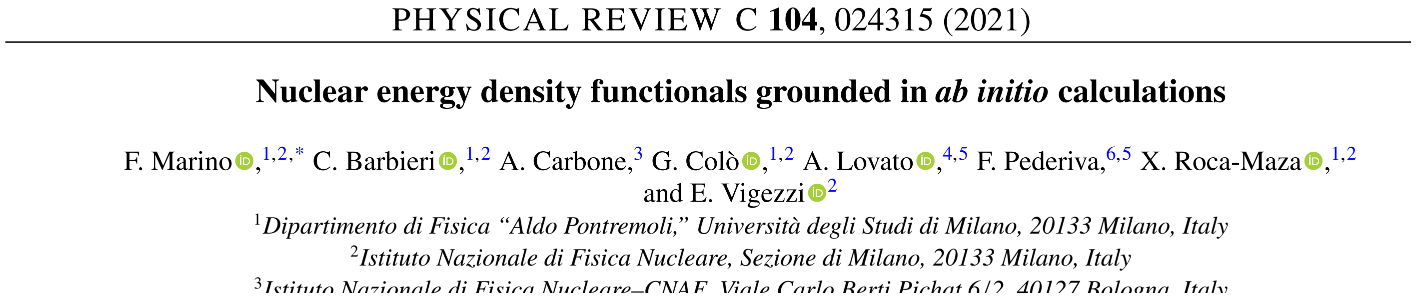


	SCGF	Exp.
^{100}Sn	4.525 – 4.707	
^{132}Sn	4.725 – 4.956	4.7093
^{132}Xe	4.700 – 4.948	4.7859
^{136}Xe	4.715 – 4.928	4.7964
^{138}Xe	4.724 – 4.941	4.8279

P. Arthuis, CB, M. Vorabbi, P. Finelli, Phys. Rev. Lett. ***125****, 182501 (2020)*

Gorkov ADC(2) and Dyson ADC(3) with N3LO-lnl and NNLOsat Hamiltonians

Nuclear Density Functional Theory



PHYSICAL REVIEW C **104**, 024315 (2021)

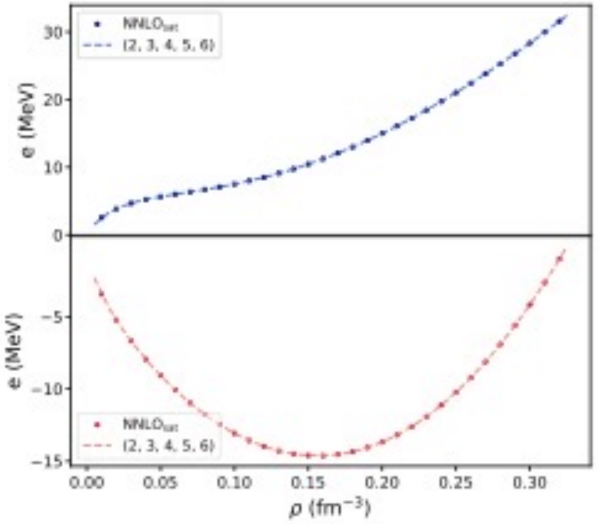
Nuclear energy density functionals grounded in *ab initio* calculations

F. Marino,[1,2,*] C. Barbieri,[1,2] A. Carbone,[3] G. Colò,[1,2] A. Lovato,[4,5] F. Pederiva,[6,5] X. Roca-Maza,[1,2]
and E. Vigezzi[2]
[1]*Dipartimento di Fisica "Aldo Pontremoli," Università degli Studi di Milano, 20133 Milano, Italy*
[2]*Istituto Nazionale di Fisica Nucleare, Sezione di Milano, 20133 Milano, Italy*
[3]*Istituto Nazionale di Fisica Nucleare–CNAF, Viale Carlo Berti Pichat 6/2, 40127 Bologna, Italy*

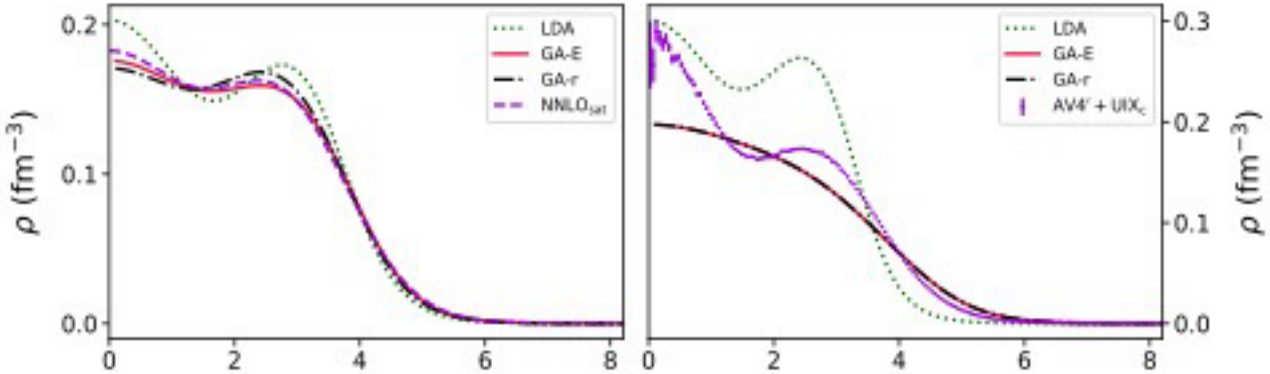
DFT is in principle exact – but the energy density functional (EDF) is not known

For nuclear physics this is even more demanding: need to link the EDF to theories rooted in QCD!

Machine-learn DFT functional
on the nuclear equation of state



Benchmark in finite systems

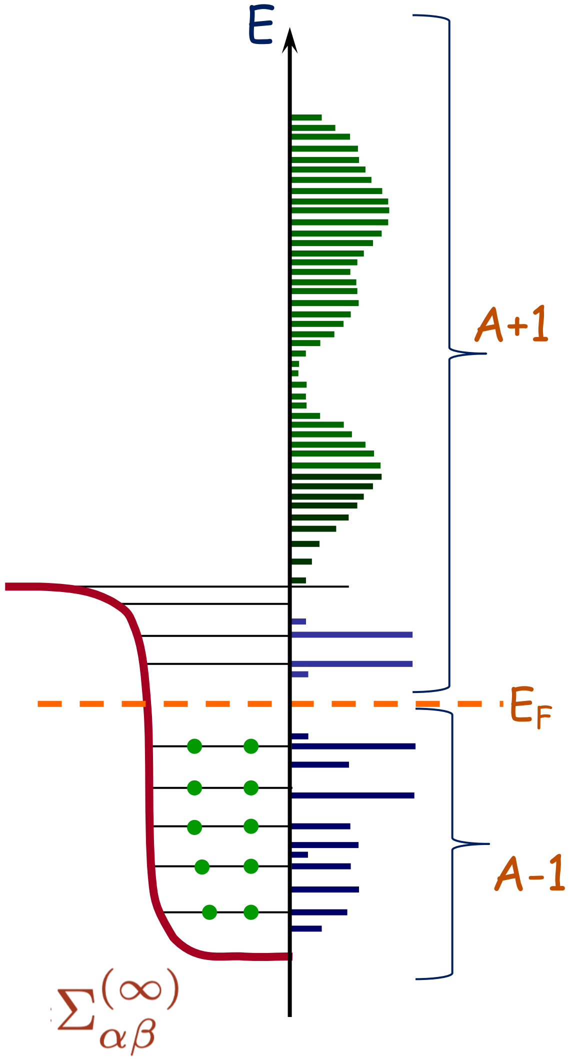


Ab initio optical potentials from propagator theory

Relation to Fesbach theory:
Mahaux & Sartor, Adv. Nucl. Phys. 20 (1991)
Escher & Jennings Phys. Rev. C**66**, 034313 (2002)

Previous SCGF work:
CB, B. Jennings, Phys. Rev. C**72**, 014613 (2005)
S. Waldecker, CB, W. Dickhoff, Phys. Rev. C**84**, 034616 (2011)
A. Idini, CB, P. Navrátil, Phys. Rv. Lett. **123**, 092501 (2019)
M. Vorabbi, CB, et al., in preparation

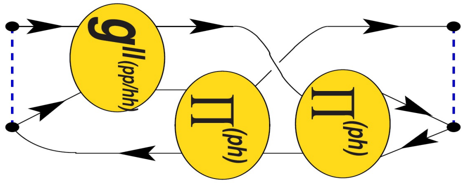
Microscopic optical potential



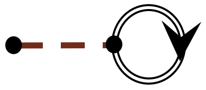
Nuclear self-energy $\Sigma^\star(\mathbf{r},\mathbf{r}';\varepsilon)$:

- contains *both particle* and *hole* props.
- it is proven to be a Feshbach opt. pot → in general it is *non-local !*

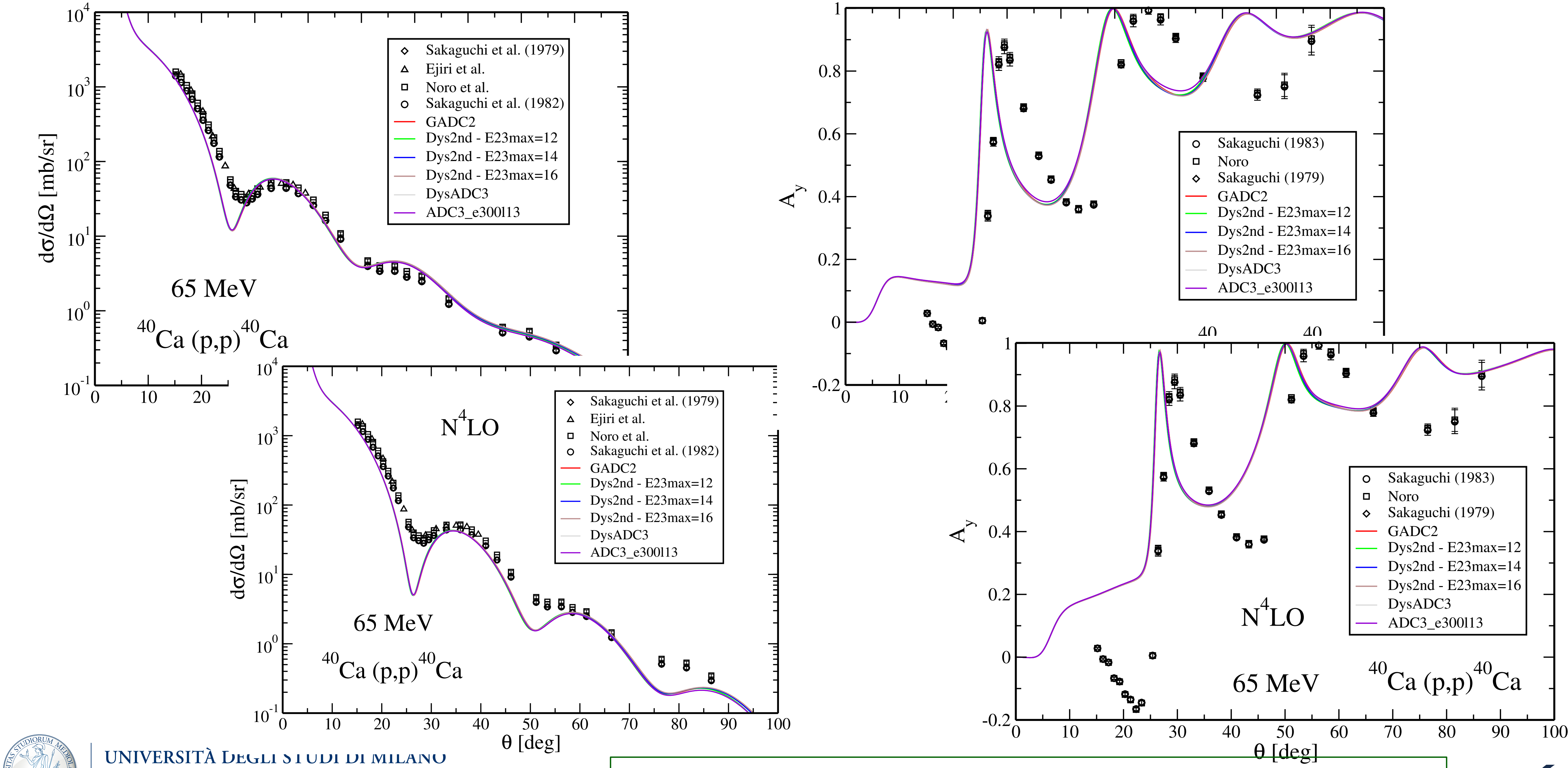
$$\Sigma^\star_{\alpha\beta}(\omega) = \underset{\text{mean-field}}{\Sigma^{(\infty)}_{\alpha\beta}} + \underbrace{\sum_{i,j} \mathbf{M}^\dagger_{\alpha,i}\left[\frac{1}{E-(\mathbf{K}^{>} + \mathbf{C}) + i\Gamma}\right]_{i,j}\mathbf{M}_{j,\beta} + \sum_{r,s}\mathbf{N}_{\alpha,r}\left[\frac{1}{E-(\mathbf{K}^{<}+\mathbf{D})-i\Gamma}\right]_{r,s}\mathbf{N}^\dagger_{s,\beta}}_{\text{Particle-vibration couplings:}}$$



$\Sigma^{(\infty)}_{\alpha\beta}$ =

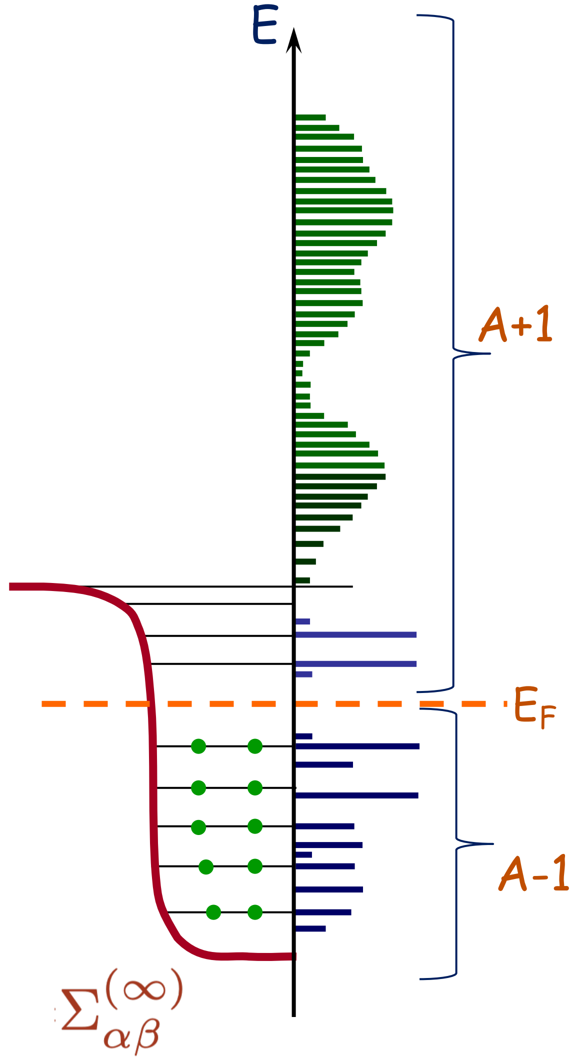


Elastic nucleon nucleus scattering



M. Vorabbi et al. – **in preparation**

Microscopic optical potential

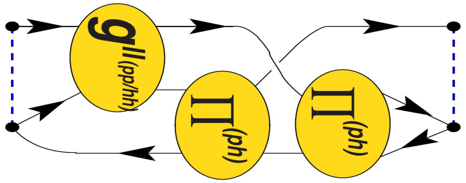


Nuclear self-energy $\Sigma^\star(\mathbf{r},\mathbf{r}';\varepsilon)$:

- contains *both particle* and *hole* props.
- it is proven to be a Feshbach opt. pot → in general it is *non-local !*

$$\Sigma^\star_{\alpha\beta}(\omega) = \Sigma^{(\infty)}_{\alpha\beta} + \underbrace{\sum_{i,j} \mathbf{M}^\dagger_{\alpha,i}\left[\frac{1}{E-(\mathbf{K}^{>}+\mathbf{C})+i\Gamma}\right]_{i,j}\mathbf{M}_{j,\beta} + \sum_{r,s} \mathbf{N}_{\alpha,r}\left[\frac{1}{E-(\mathbf{K}^{<}+\mathbf{D})-i\Gamma}\right]_{r,s}\mathbf{N}^\dagger_{s,\beta}}_{\text{Particle-vibration couplings}}$$

mean-field



Solve scattering and overlap functions directly in momentum space:

$$\Sigma^{\star\, l,j}(k,k';E) = \sum_{n,\,n'} R_{n\,l}(k)\,\Sigma^{\star\, l,j}_{n,\,n'}\,R_{n\,l}(k')$$

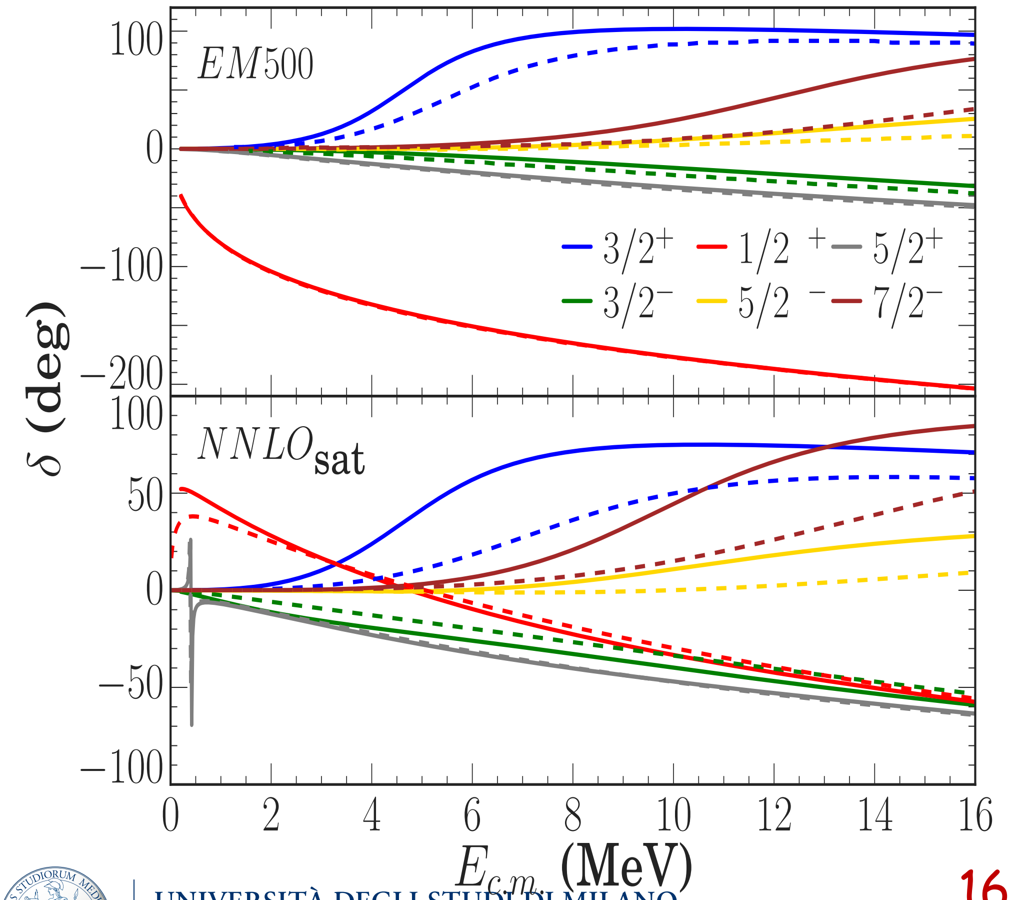
$$\frac{k^2}{2\mu}\psi_{l,j}(k) + \int \mathrm{d}k'\,k'^2\,\Sigma^{\star l,j}(k,k';E_{c.m.})\psi_{l,j}(k') = E_{c.m.}\psi_{l,j}(k)$$

Low energy scattering – from SCGF

[A. Idini, CB, Navratil,
Phys. Rev. Lett. **123**, 092501 (2019)]

Benchmark with NCSM-based scattering.

Scattering from mean-field only:



- - - - - NCSM/RGM [<u>**without**</u> core excitations]

EM500: NN-SRG λ_{SRG}= 2.66 fm^{-1}, Nmax=18 (IT)
[PRC**82**, 034609 (2010)]

NNLOsat: Nmax=8 (IT-NCSM)

—— SCGF [$\Sigma^{(\infty)}$ only], always Nmax=13

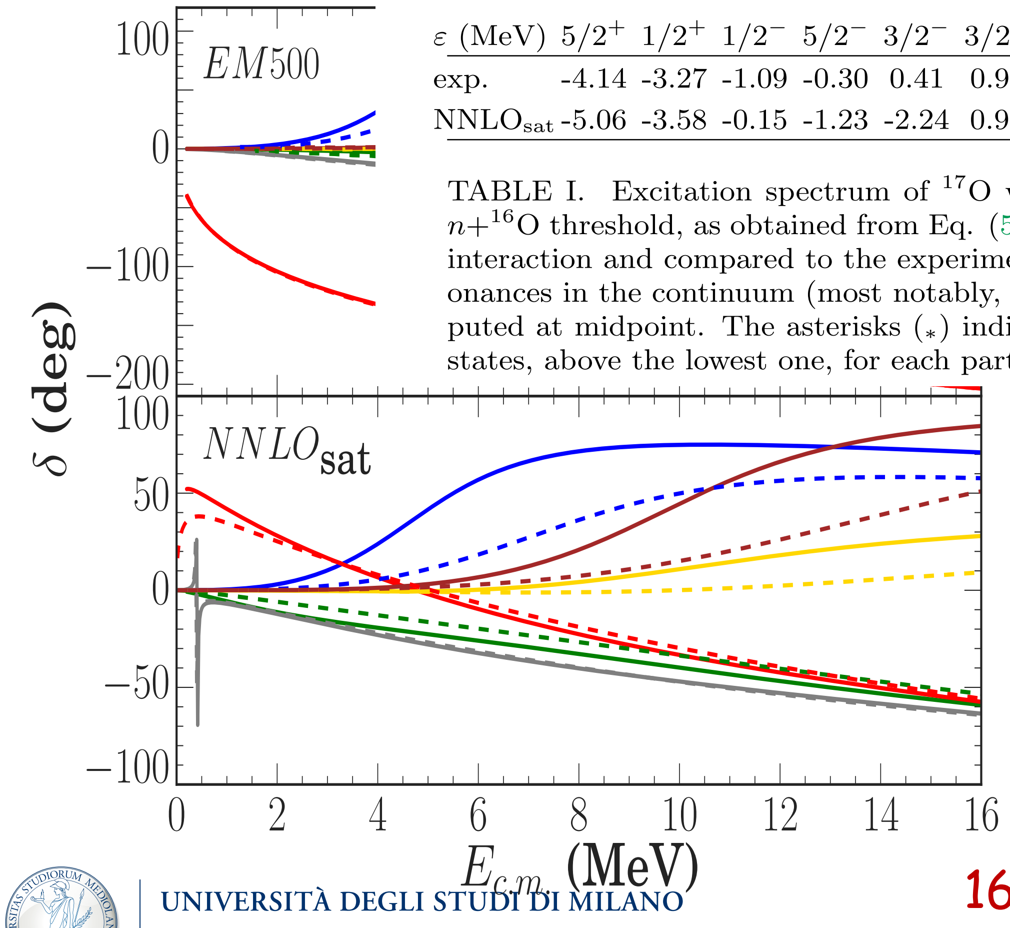
$^{16}O(n,n')^{16}O$

Low energy scattering – from SCGF

[A. Idini, CB, Navratil,
Phys. Rev. Lett. **123**, 092501 (2019)]

Benchmark with NCSM-based scattering.

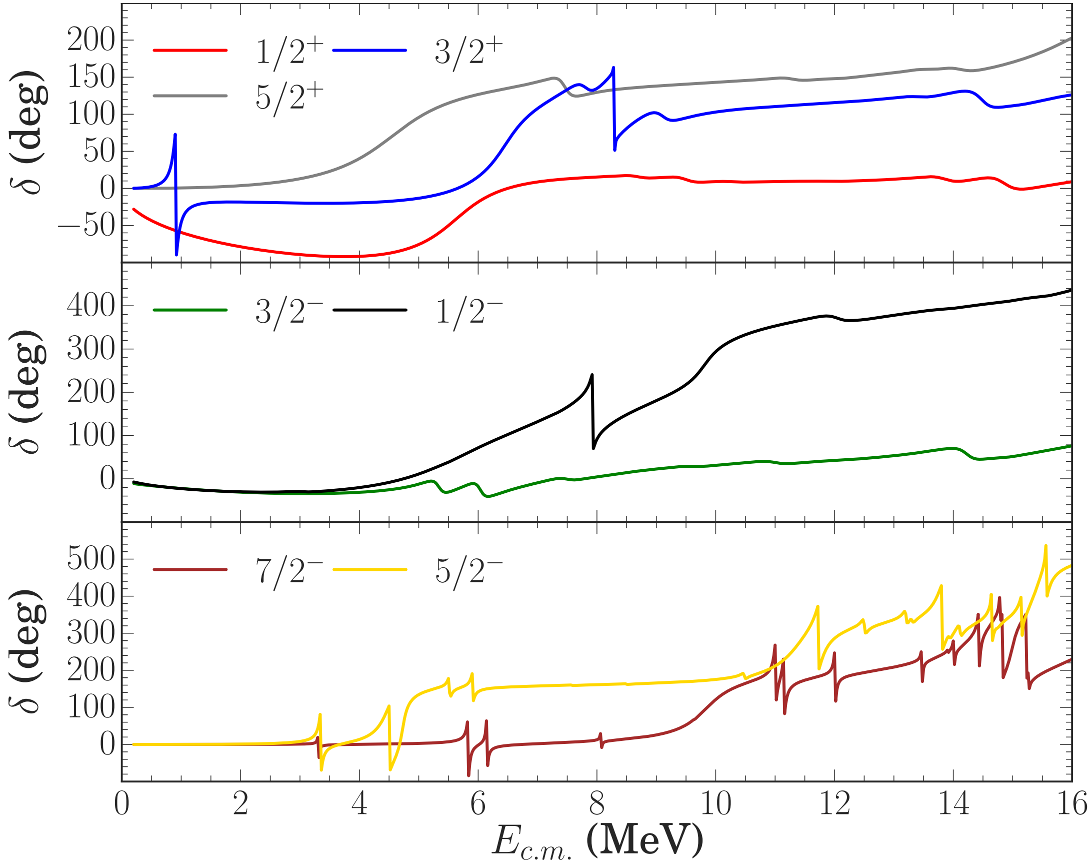
Scattering from mean-field only:



ε (MeV)	$5/2^+$	$1/2^+$	$1/2^-$	$5/2^-$	$3/2^-$	$3/2^+$	$5/2^+_*$	$5/2^-_*$	$7/2^-_*$
exp.	-4.14	-3.27	-1.09	-0.30	0.41	0.94	3.23	3.02	3.54
$\rm NNLO_{sat}$	-5.06	-3.58	-0.15	-1.23	-2.24	0.91	4.57	3.36	3.37

TABLE I. Excitation spectrum of ^{17}O with respect to the $n+^{16}$O threshold, as obtained from Eq. (5) and the $\rm NNLO_{sat}$ interaction and compared to the experiment [45]. Broad resonances in the continuum (most notably, the $5/2^+$) are computed at midpoint. The asterisks ($_*$) indicate higher excited states, above the lowest one, for each partial wave.

Full self-energy from SCGF:

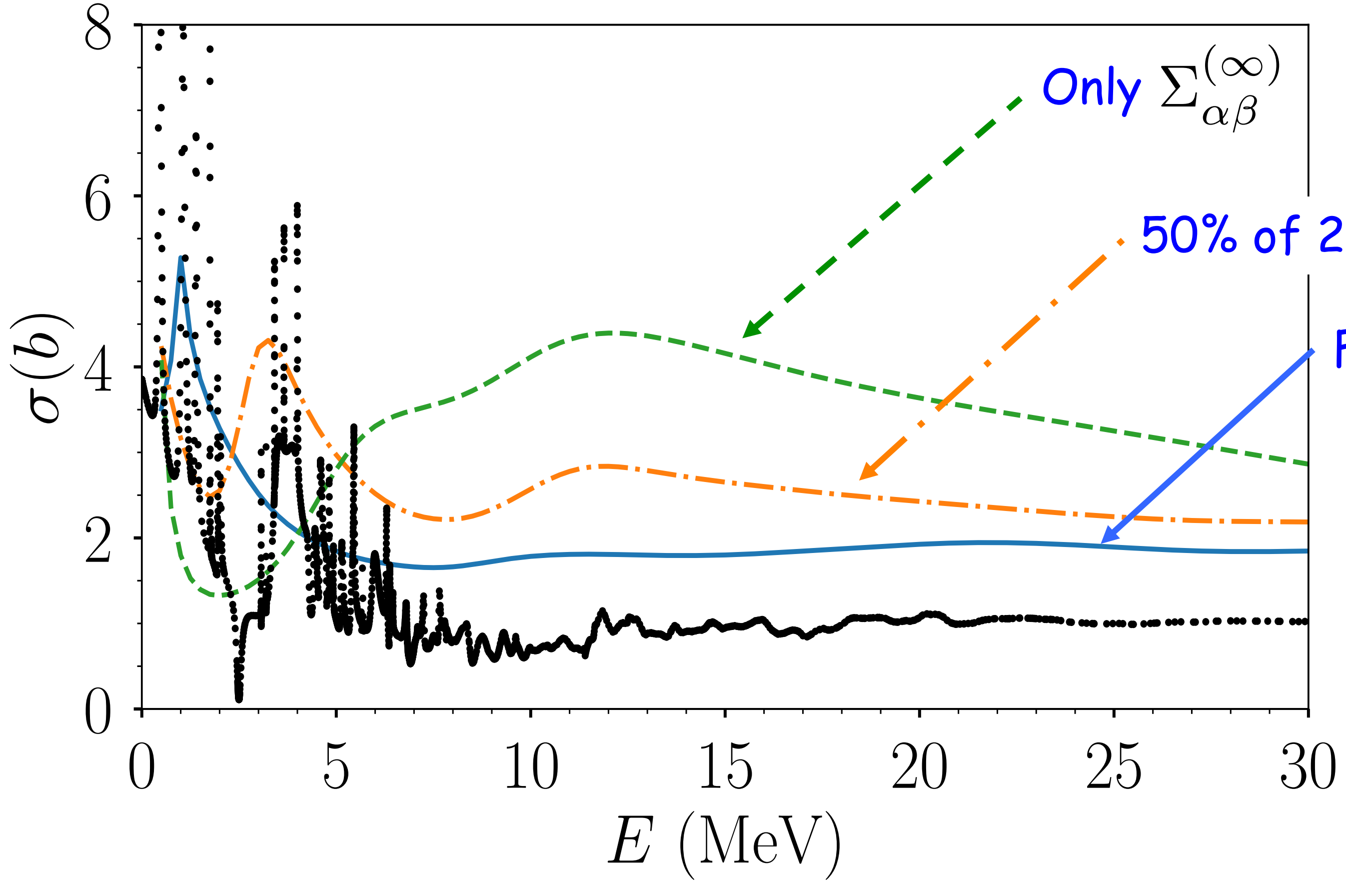


$^{16}O(n,n')^{16}O$

Role of intermediate state configurations (ISCs)

n-^{16}O, total elastic cross section

[A. Idini, CB, Navratil,
Phys. Rev. Lett. **123**, 092501 (2019)]



Only $\Sigma^{(\infty)}_{\alpha\beta}$

50% of 2p1h/2h1p poles suppressed

Full $\Sigma^{\star}_{\alpha\beta}(\omega)$ (all ISCs included)

High order configurations, or ADC(n>>3), to be critical for fully ab initio optical potentials

$$\Sigma^{\star}_{\alpha\beta}(\omega) = \Sigma^{(\infty)}_{\alpha\beta} + \sum_{i,j} \mathbf{M}^{\dagger}_{\alpha,i} \underbrace{\left[\frac{1}{E-(\mathbf{K}^{>}+\mathbf{C})+i\Gamma}\right]_{i,j}}_{\text{2p1h}} \mathbf{M}_{j,\beta} + \sum_{r,s} \mathbf{N}_{\alpha,r} \underbrace{\left[\frac{1}{E-(\mathbf{K}^{<}+\mathbf{D})-i\Gamma}\right]_{r,s}}_{\text{2h1p}} \mathbf{N}^{\dagger}_{s,\beta}$$

Current challenges:

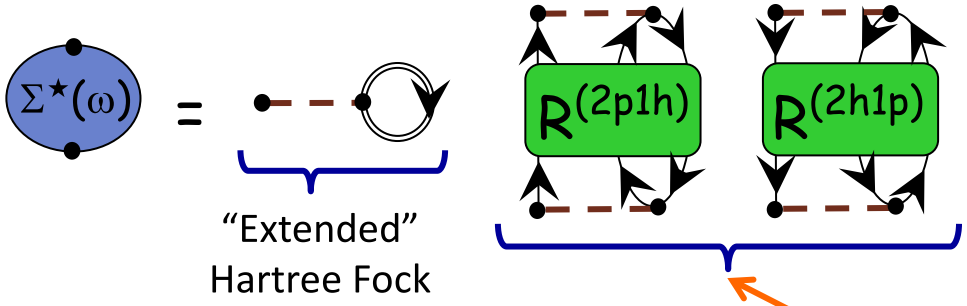
- Pushing ab-initio methods to medium energies – not just g.s.
- *Poor description of correlations at intermediate energies…*
- *C.O.M. problems …maybe not so critical at large A.*

➔ *Need for an efficient sampling of collective configurations and diagrammatic expansion.*

The FRPA Method in Two Words

Particle vibration coupling is the main mechanism driving the redistribution and fragmentation of particle strength—expecially in the quasielastic regions around the Fermi surface…

CB et al.,
*Phys. Rev. C**63**, 034313 (2001)*
*Phys. Rev. A**76**, 052503 (2007)*
*Phys. Rev. C**79**, 064313 (2009)*

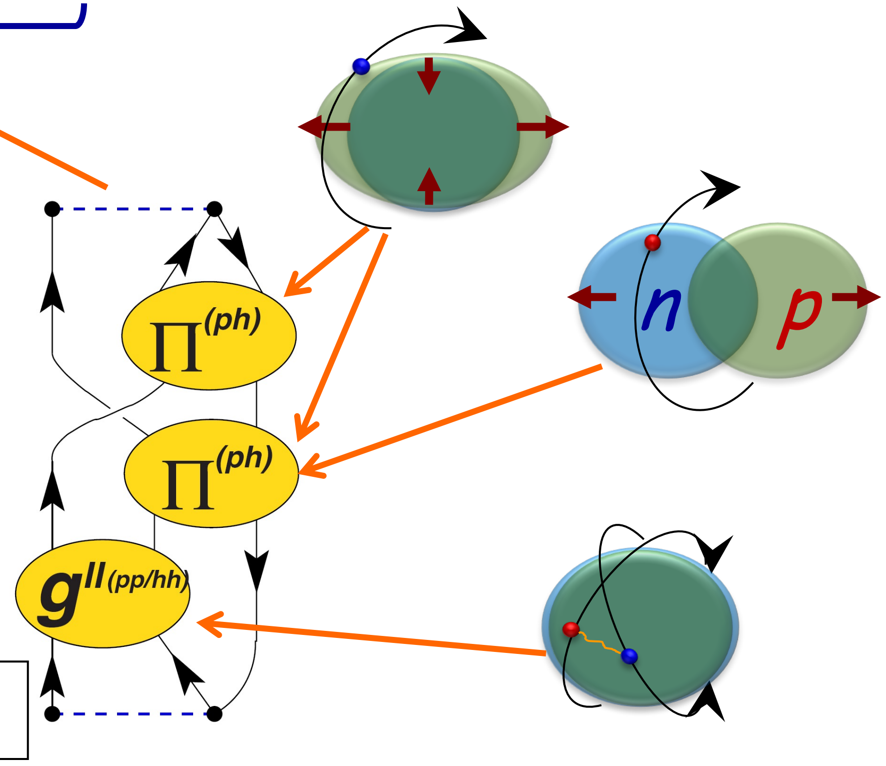


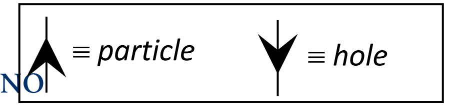
“Extended”
Hartree Fock

- A complete expansion requires *__all types__* of particle-vibration coupling

…these modes are all resummed exactly and to all orders in a *ab initio* many-body expansion.

- The Self-energy $\Sigma^\star(\omega)$ yields *both* single-particle states and scattering



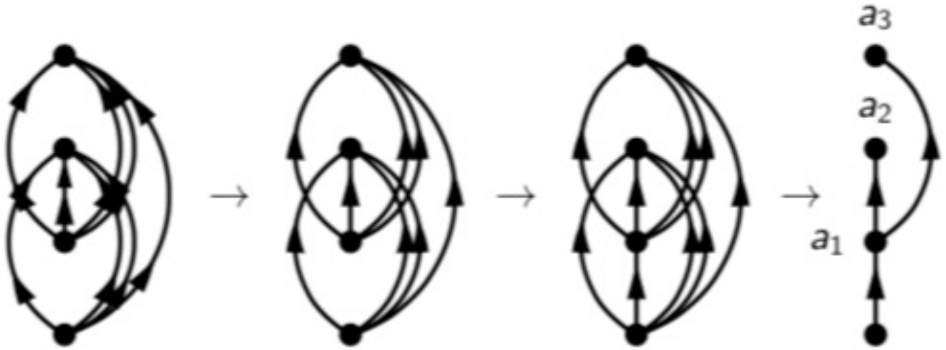


Automatic Diagrammatic Generation (ADG) of the self-energy

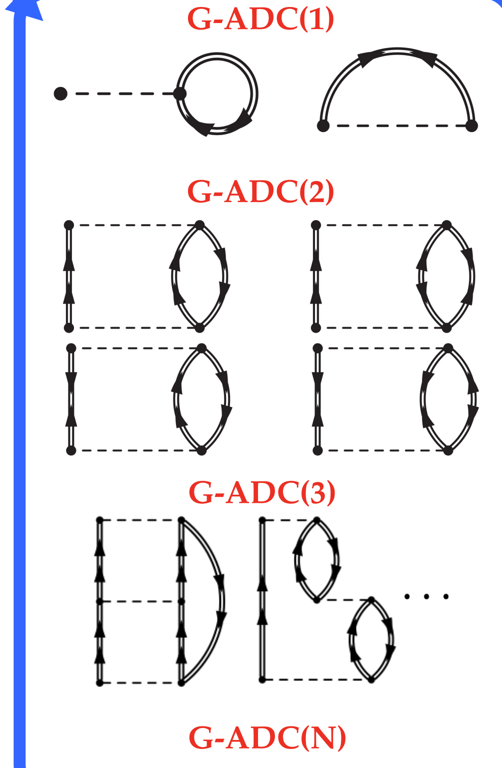
Goal: Drawing of self-energy Feynman diagrams and derivation of corresponding algebraic expressions are performed automatically

Background: ADG of the BMBPT expansion (P. Arthuis *et al* Comp. Phys. Comm. **240**, 202 (2019))

Tree structure of B-MBPT diagrams:



Order		0	1	2	3	4	5
0/2/4-leg vertex	General	1	2	8	59	568	6 805
	HFB vacuum	1	1	1	10	82	938
0/2/4/6-leg vertex	General	1	3	23	396	10716	+ 100 000
	HFB vacuum	1	2	8	77	5 055	+ 100 000

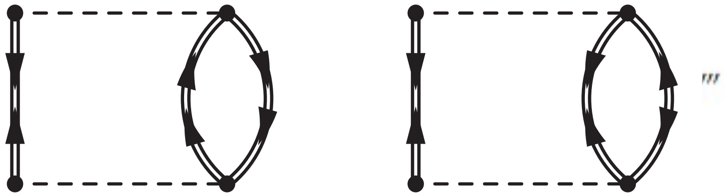


*Work in progress by **F. Raimondi**, CEA, Saclay*

Reaching (Gorkov - 3NF - higher ordes...) is a mess

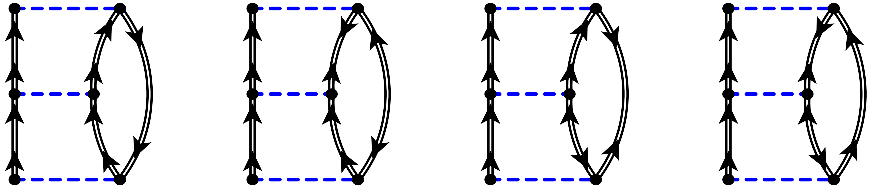
Gorkov at 2nd order and ONLY NN forces:

$$\Sigma^{11\,(2)}_{ab}(\omega) =$$

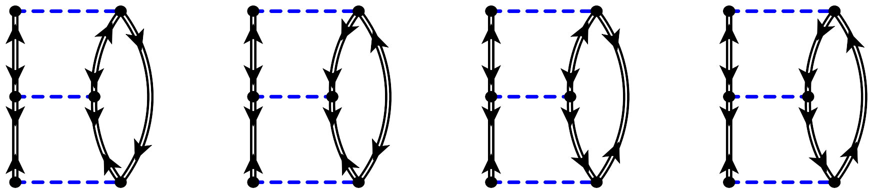


Gorkov at 3rd order and ONLY NN forces:

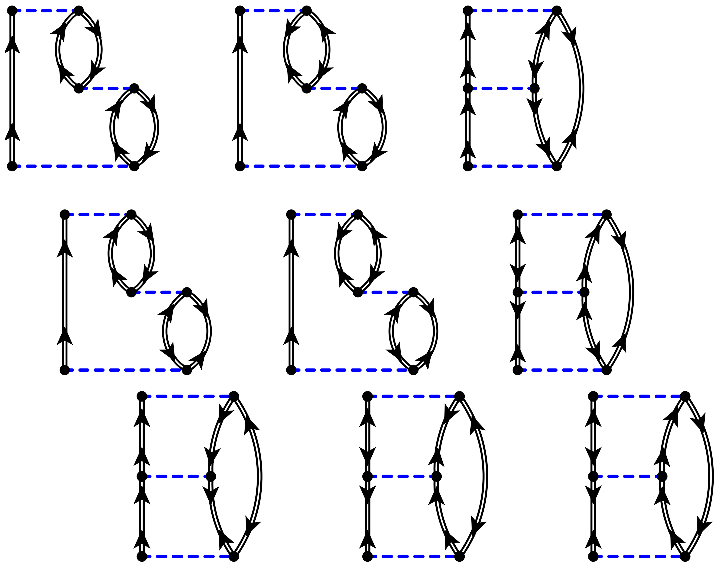
pp/hh-ladders:



hh-interactions (hh int. among pp ladders!!!)

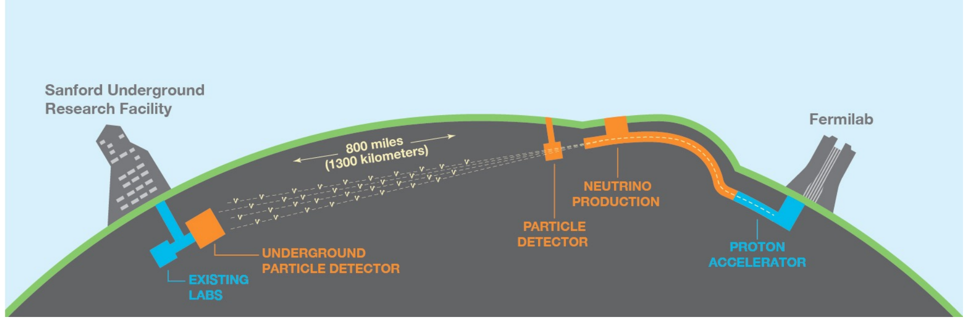


ph-rings:



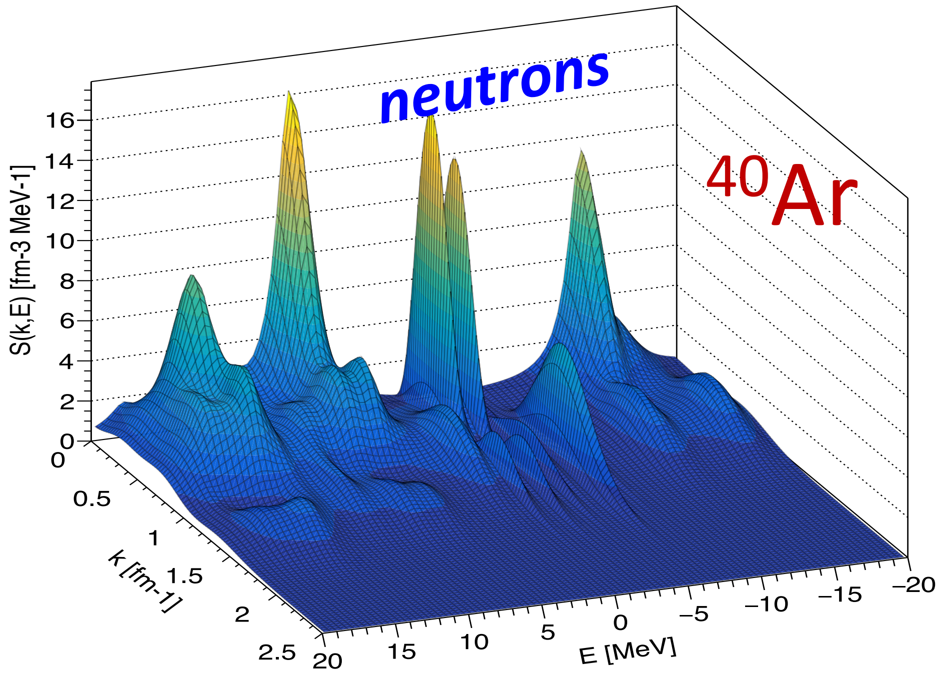
CB, V. Somà, T. Duguet, Phys Rev C 105, 044330 (2022)

Electron and ν scattering on ^{40}Ar and Ti

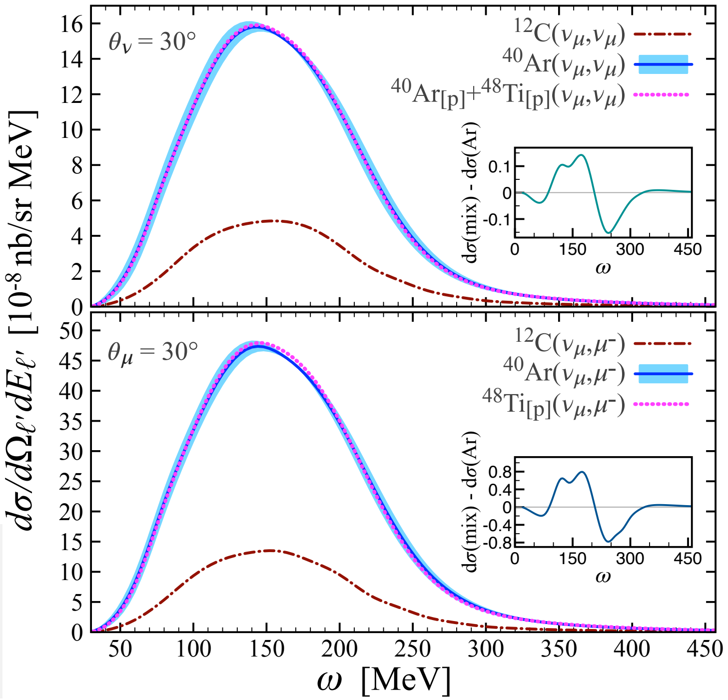


Liquid Argon projection chamber is being used. It will require one order of magnitude (20% → 2%) improvement in theoretical prediction for ν-^{40}Ar cross sections to achieve proper event reconstruction.

➔ **Need good knowledge of ^{40}Ar spectral functions and consistent structure-scattering theories.**



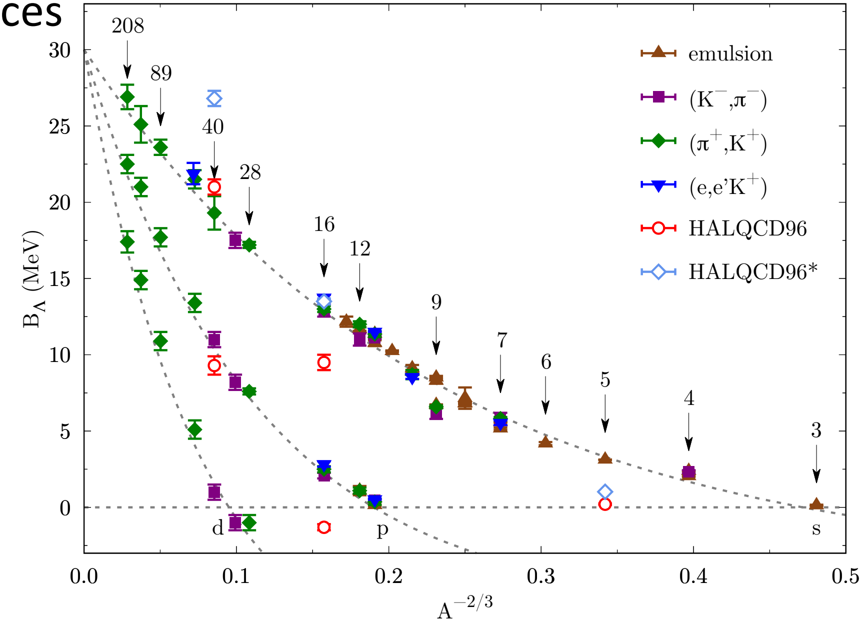
Ti protons contribution ('mix') is nearly identical to neutrons in ^{40}Ar.

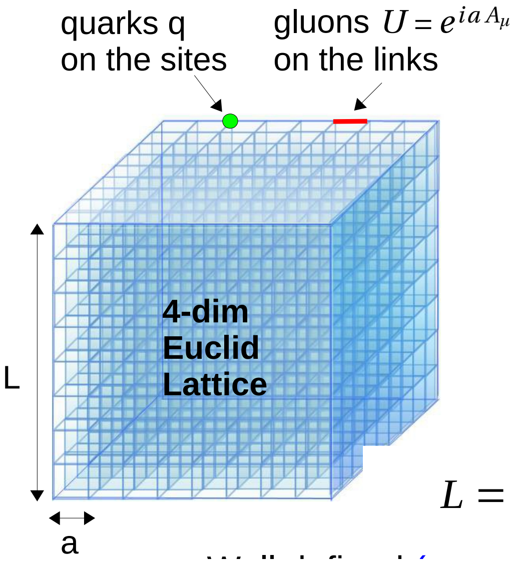


CB, N. Rocco, V. Somà, Phys. Rev. C**100**, 062501(R) (2019)

HAL-QCD and application for Ys in nuclei now possible

- AV4’ + UIX requires very large with phenomenological hypernuclear forces requires large ΛNN 3-baryon force
- Physical mass now under reach ($m_\pi \approx 145$ MeV) for hyperons
- HALQCD ΛN 3-baryon force is already very close to experiment





Nuclear force from Lattice QCD (HAL collaboration,n RIKEN-YITP Japan)

$$L = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \bar{q}\gamma^\mu \left(i\partial_\mu - g t^a A^a_\mu \right) q - m\bar{q}q$$

Table 1: Λ separation energies (in MeV) for different hypernuclei with the hyperon in different single-particle states. Second column reports the AFDMC results using the original HALQCD96 ΛN potential. Third column shows the results for the modified HALQCD96 ΛN potential (see text for details). In the last column, the available experimental data [] are reported.

${}^{A}_{\Lambda}Z$	J^π (state)	HALQCD96	HALQCD96*	Exp
${}^{5}_{\Lambda}$He	$1/2^+$ (s)	0.21(5)	1.02(3)	3.12(2)
${}^{16}_{\Lambda}$O	1^- (s)	9.5(5)	13.5(2)	13.4(4)
	2^+ (p)	−1.3(2)	0.5(1)	2.5(2)
${}^{40}_{\Lambda}$Ca	2^+ (s)	21.0(5)	26.8(5)	19.3(1.1)
	3^- (p)	9.3(6)	13.7(6)	11.0(5)

D. Lonardoni, A. Lovato, CB, T. Inoue, HALQCD coll

Summary and outlook

Thank you for your attention!!!

Ab initio applications to structure and reactions are becoming increasingly powerful:

→ *Nuclear forces being advanced (through EFT) and challenges on many-body theory*

→ *Systematic applications beyond testing forces and structure becoming available*

The Self-Consistent Green's Function method (SCGF):

- ADC(n) and FRPA diagrammatic expansions (particle-vibration coupling)
- *Automatization* of diagram generation and sampling

Applications:

- Mixed Local-Nonlocal cutoffs in chiral interactions (standard WPC)
  [Somà, Navratil, Raimondi, CB, Duguet, Phys Rev C101, 014318 (2020); EPJA in press (arXiv:2009.01829)]
- Optical potentials from ab initio
  [A. Idini, CB, P. Navratil, Phys. Rev. Lett. 123, 092501 (2019); **Vorabbi et al. in prep**]
- Reaching A≈132 mass
  [P. Arthuis, CB, M. Vorabbi, P. Finelli – Phys, Rev, Lett. 125, 182501 (2020)]
- (Hyper)nuclear forces from LQCD [Lonardoni et al. in prep]
- Neutrino Nucleus scattering (@ GeV energies)
  [CB, N. Rocco, V. Somà, Phys. Rev. C100, 062501(R) (2019)]

And thanks to my collaborators (over the years…)

Thank you for your attention!!!

E. Vigezzi, G. Colò, X. Roca-Maza,
F. Marino, A. Scalesi

UNIVERSITY OF SURREY

A. Cipollone, A. Rios,
A. Idini, ***P. Arthuis, M. Drissi***

cea
energie atomique • energies alternatives

V. Somà, T. Duguet, ***A. Scalesi***

Argonne NATIONAL LABORATORY

A. Lovato , N. Rocco

Los Alamos NATIONAL LABORATORY

D. Lonardoni

UNIVERSITÀ DI TRENTO

F. Pederiva, A. Roggero

TRIUMF

P. Navrártil

C. Giusti

P. Finelli

BROOKHAVEN NATIONAL LABORATORY

M. Vorabbi

Hadrons to Atomic nuclei HAL from Lattice QCD

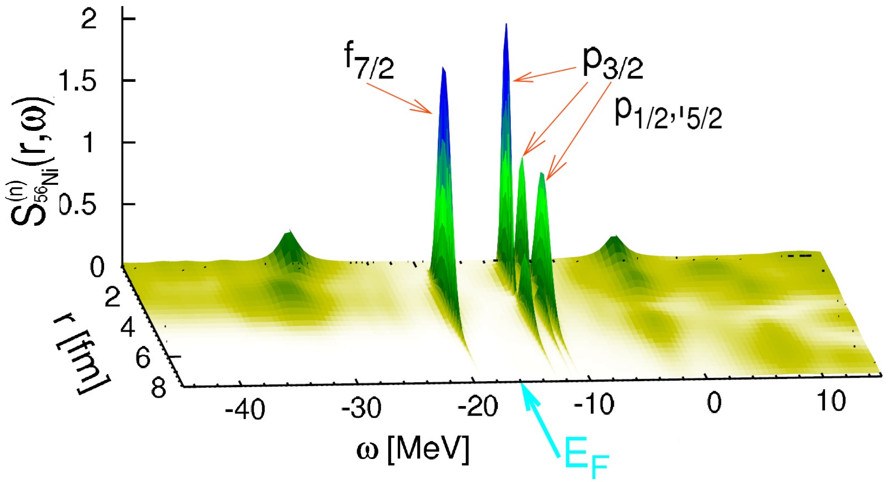
S. Aoki,
T. Hatsuda,
T. Doi,
T. Inoue, …

Lepton-nucleon cross section

$$\left(\frac{d\sigma}{dT'd\cos\theta'}\right)_{\nu/\bar{\nu}} = \frac{G^2}{2\pi}\frac{k'}{2E_\nu}\left[\hat{L}_{CC}R_{CC} + 2\hat{L}_{CL}R_{CL} + \hat{L}_{LL}R_{LL} + \hat{L}_T R_T \pm 2\hat{L}_{T'}R_{T'}\right],$$

Nuclear structure is in the hadronic tensor:

$$W^{\mu\nu}(\mathbf{q},\omega) = \int \frac{d^3k}{(2\pi)^3} dE\, P_h(\mathbf{k},E) \frac{m^2}{e(\mathbf{k})e(\mathbf{k}+\mathbf{q})} \times \sum_i \langle k|j_i^{\mu\,\dagger}|k+q\rangle\langle k+q|j_i^\nu|k\rangle \times \delta(\omega + E - e(\mathbf{k}+\mathbf{q}))\,,$$



$$R_{CC} = W^{00}$$

$$R_{CL} = -\frac{1}{2}(W^{03} + W^{30})$$

$$R_{LL} = W^{33}$$

$$R_T = W^{11} + W^{22}$$

$$R_{T'} = -\frac{i}{2}(W^{12} - W^{21})\,,$$

$$W^{\mu\nu} = \sum_f \langle 0|j^{\mu\,\dagger}|f\rangle\langle f|j^\nu|0\rangle\delta(E_0 + \omega - E_f)$$

N. Rocco, CB, O. Benhar, de Pace , A. Lovato, Phys. Rev. C**99**, 025502 (2019)

Lepton-nucleon cross section

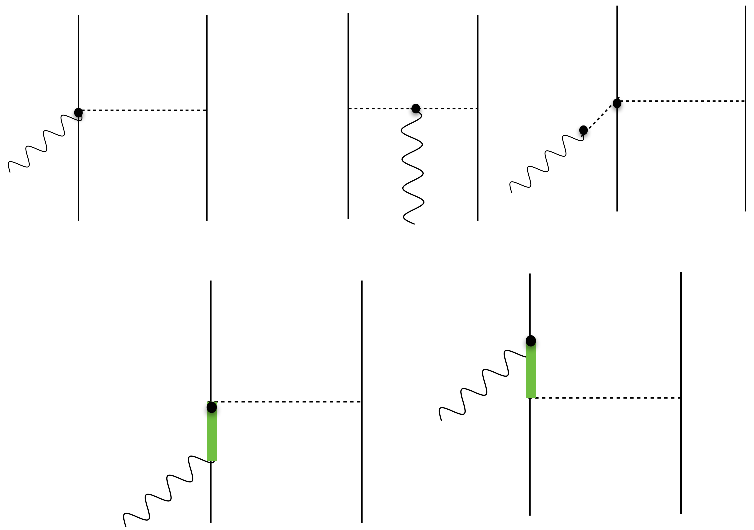
$$\left(\frac{d\sigma}{dT'd\cos\theta'}\right)_{\nu/\bar{\nu}} = \frac{G^2}{2\pi}\frac{k'}{2E_\nu}\left[\hat{L}_{CC}R_{CC} + 2\hat{L}_{CL}R_{CL} + \hat{L}_{LL}R_{LL} + \hat{L}_T R_T \pm 2\hat{L}_{T'}R_{T'}\right] ,$$

Nuclear structure is in the hadronic tensor:

$$\begin{aligned} W^{\mu\nu}(\mathbf{q},\omega) = \int & \frac{d^3k}{(2\pi)^3} dE\, P_h(\mathbf{k},E) \frac{m^2}{e(\mathbf{k})e(\mathbf{k}+\mathbf{q})} \\ & \times \sum_i \langle k|j_i^{\mu\,\dagger}|k+q\rangle\langle k+q|j_i^\nu|k\rangle \\ & \times \delta(\omega + E - e(\mathbf{k}+\mathbf{q}))\,, \end{aligned}$$

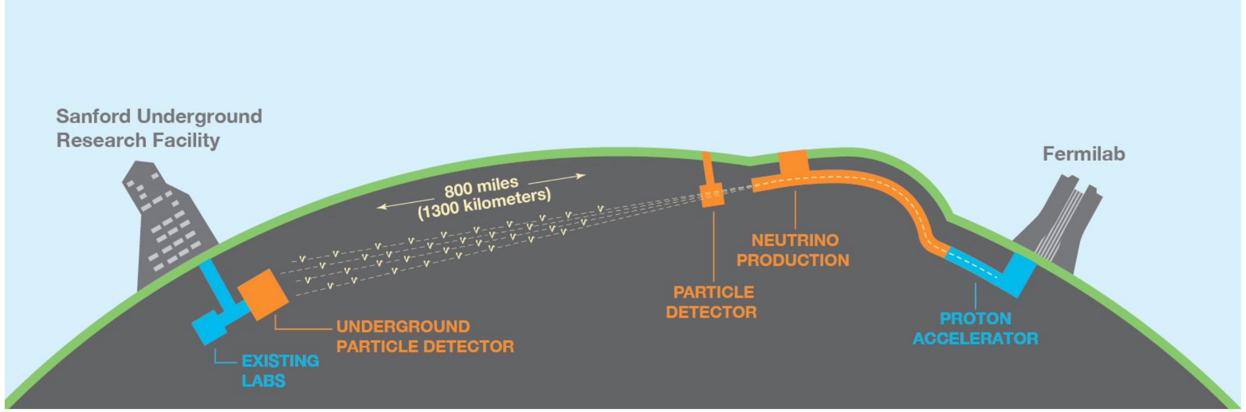
$$\begin{aligned} W^{\mu\nu}_{\rm 2b}(\mathbf{q},\omega) = \frac{V}{2}\int & d\tilde{E}\frac{d^3k}{(2\pi)^3} d\tilde{E}'\frac{d^3k'}{(2\pi)^3}\frac{d^3p}{(2\pi)^3} \\ & \times \frac{m^4}{e(\mathbf{k})e(\mathbf{k}')e(\mathbf{p})e(\mathbf{p}')} P_h^{\rm NM}(\mathbf{k},\tilde{E})P_h^{\rm NM}(\mathbf{k}',\tilde{E}') \\ & \times \sum_{ij} \langle k\,k'|j_{ij}^{\mu\,\dagger}|p\,p'\rangle\langle p\,p'|j_{ij}^\nu|k\,k'\rangle \\ & \times \delta(\omega + \tilde{E} + \tilde{E}' - e(\mathbf{p}) - e(\mathbf{p}'))\,. \end{aligned} \qquad (41)$$

Two-body diagrams contributing to the axial and vector responses



N. Rocco, CB, O. Benhar, de Pace , A. Lovato, Phys. Rev. C**99**, 025502 (2019)

Neutrino Oscillations – next generation experiments



DUNE experiment will measure long base line neutrino oscillations to:

- Resolve neutrino mass hierarchy
- Search for CP violation in weak interaction
- Search for other physics beyond SM

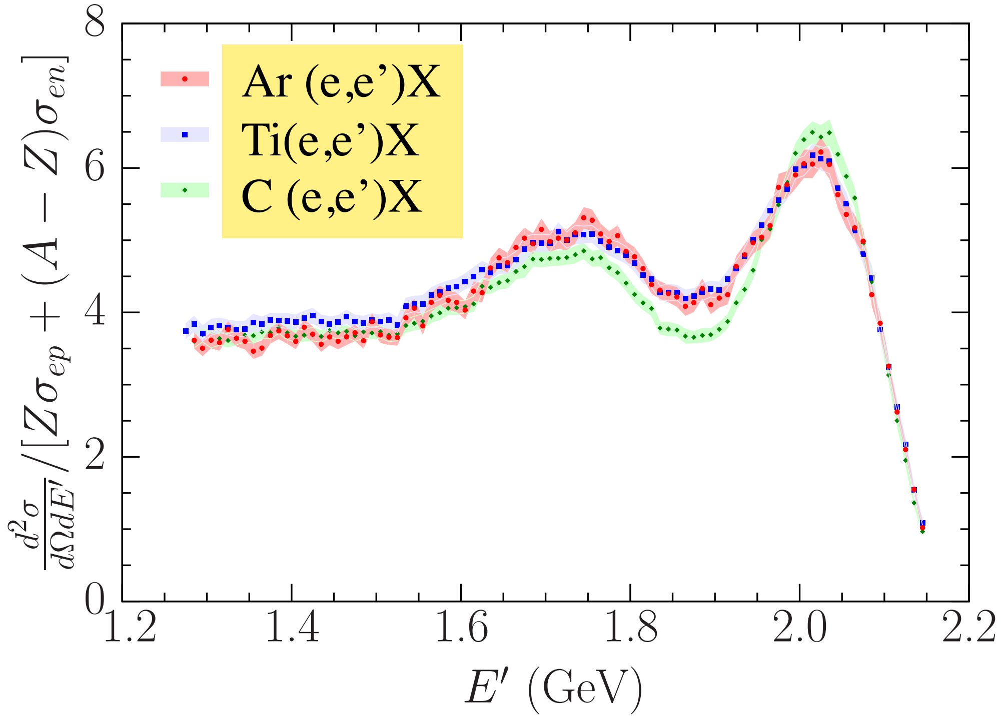
Liquid Argon projection chamber is being used. It will require one order of magnitude (20% → 2%) improvement in theoretical prediction for ν-^{40}Ar cross sections to achieve proper event reconstruction.

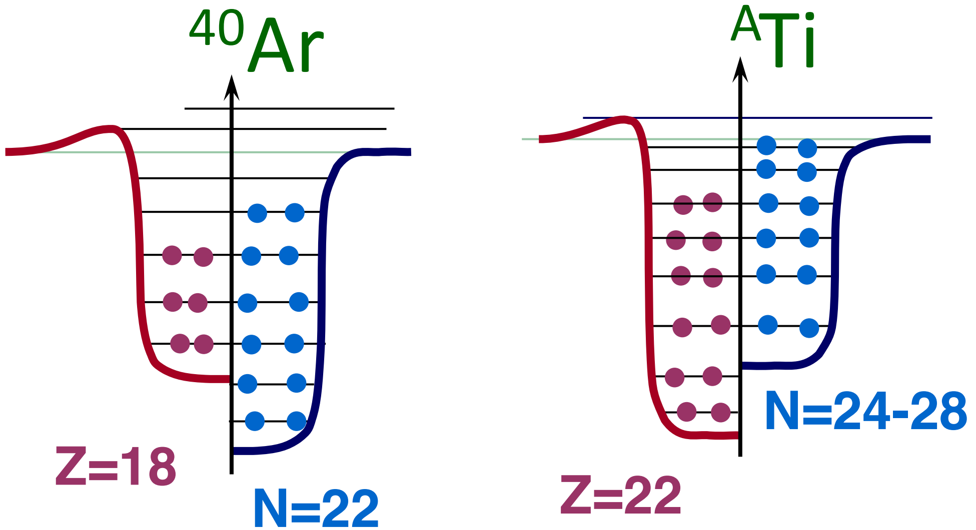
➔ Need good knowledge of ^{40}Ar spectral functions and consistent structure-scattering theories.

Spectral function for ^{40}Ar and Ti

Jlab experiment E12-14-012 (Hall A)

Phys. Rev. C 98, 014617 (2018); arXiv:1810.10575

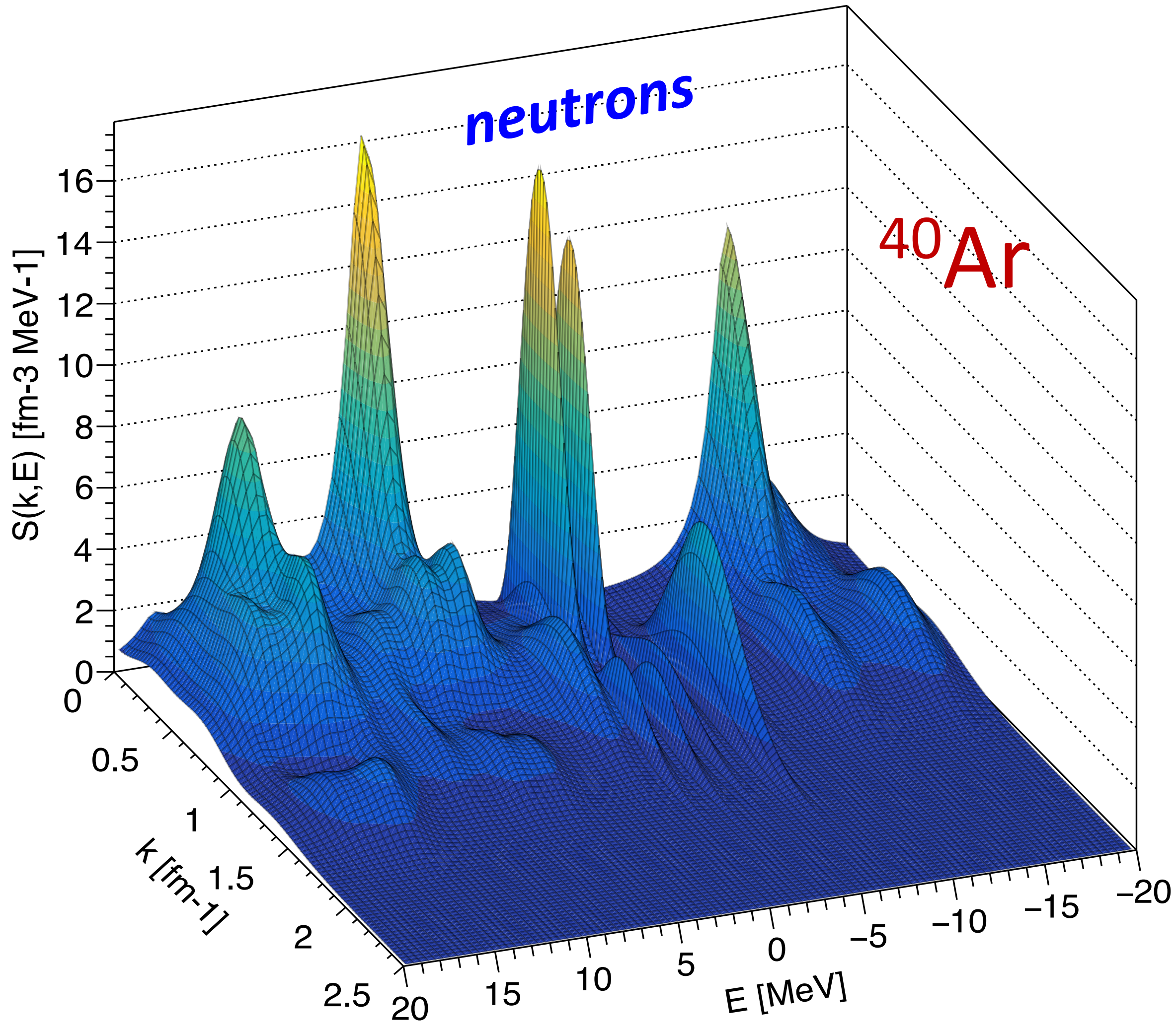




Proton distribution in Ti similar to neutron in ^{40}Ar ??

^{40}Ar(e,e'p) and Ti(e,e'p) data being analyzed

Spectral function for ^{40}Ar



- Experimental datat now available from Jlab:
  H. Dai et al., arXiv:1803.01910/ 1810.10575

- Ab initio simulations based on the ADC(2) truncation of the N2LO-sat Hamiltoninan

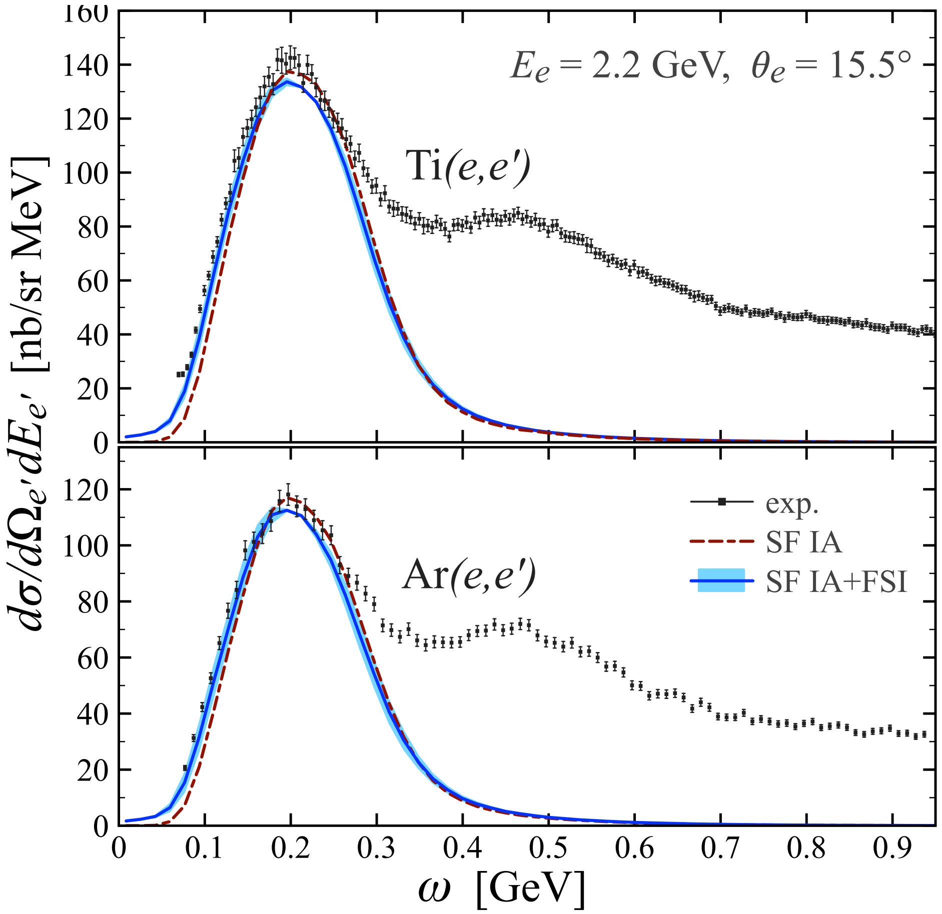
➔ Want validation of initial state correlation before they are implementer in neutrino-^{40}Ar simulations

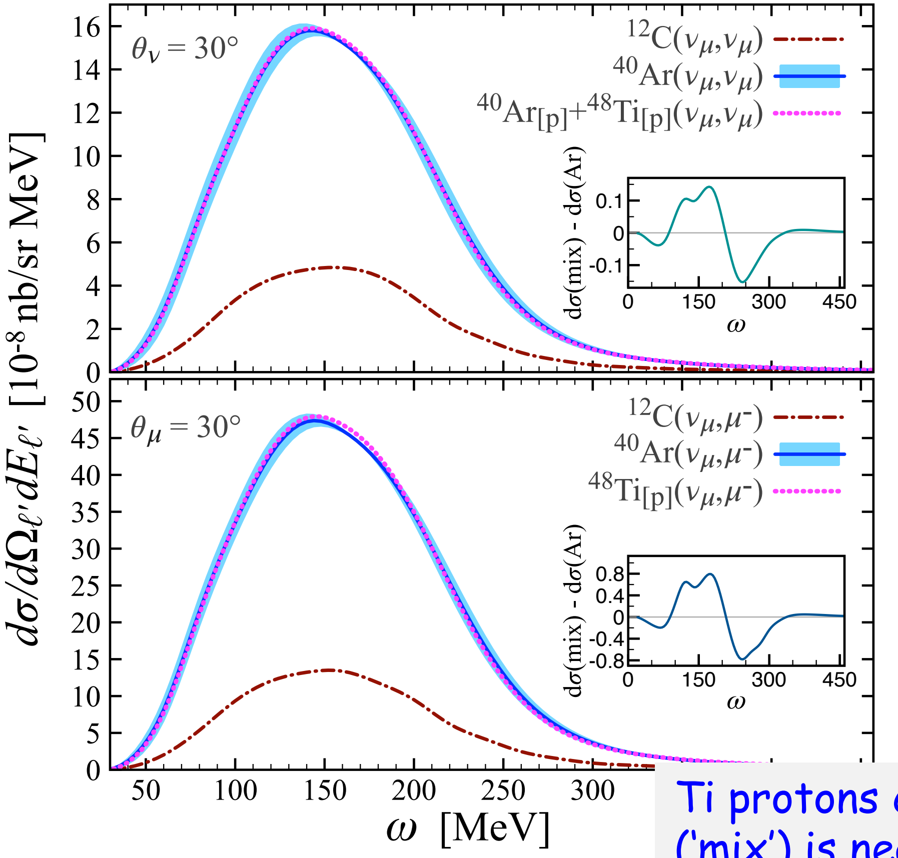
CB, N. Rocco, V. Somà, arXiv:1907.01122

Electron and ν scattering on ^{40}Ar and Ti

Jlab experiment E12-14-012 (Hall A)

[Phys. Rev. C 98, 014617 (2018)]





Ti protons contribution ('mix') is nearly identical to neutrons in ^{40}Ar.

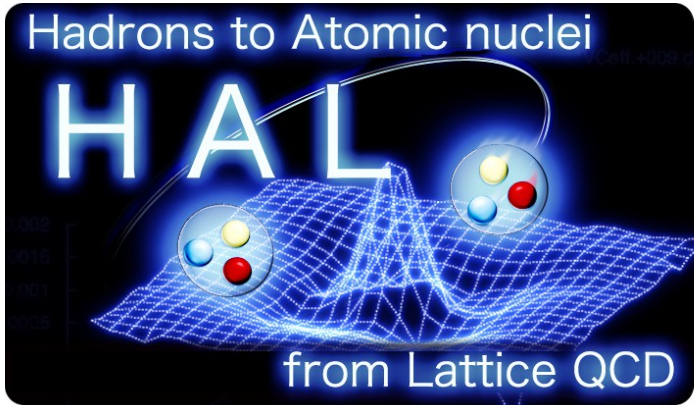
^{40}Ar(e,e′p) and Ti(e,e′p) data being analyzed

CB, N. Rocco, V. Somà, Phys. Rev. C**100**, 062501(R) (2019)

Study of nuclear interactions from Lattice QCD

*C. McIlroy, CB et al. Phys. Rev. C**97**, 021303(R) (2018)*
D. Lonardoni et al. - in preparation

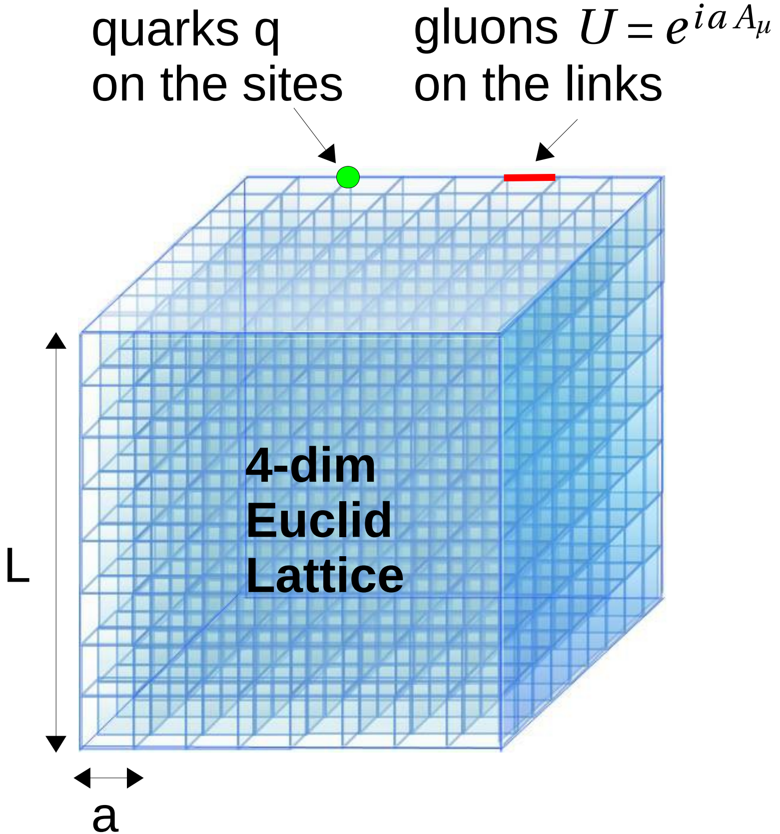
In collaboration with:



UNIVERSITÀ DEGLI STUDI DI MILANO
DIPARTIMENTO DI FISICA

Lattice QCD

$$L = -\frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a} + \bar{q}\,\gamma^{\mu}\left(i\partial_{\mu} - g\,t^{a} A^{a}_{\mu}\right) q - m\,\bar{q}\,q$$



Vacuum expectation value

$$\langle O(\bar{q},q,U)\rangle$$

path integral

$$= \int dU\, d\bar{q}\, dq\; e^{-S(\bar{q},q,U)}\, O(\bar{q},q,U)$$

$$= \int dU \det D(U)\, e^{-S_U(U)}\, O(D^{-1}(U))$$

quark propagator

$$= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i))$$

$\{ U_i \}$: ensemble of gauge conf. U
generated w/ probability $\det D(U)\, e^{-S_U(U)}$

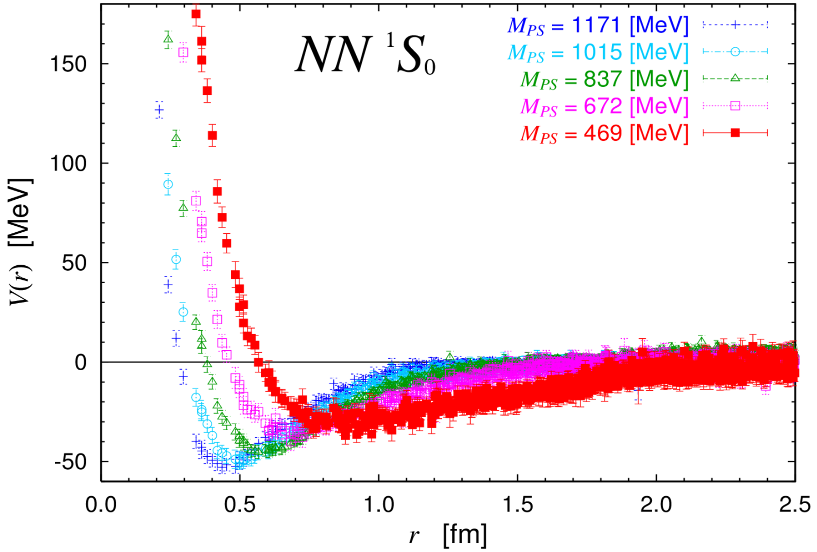
- Well defined (regularized)
- Manifest gauge invariance
- Fully non-perturbative
- Highly predictive

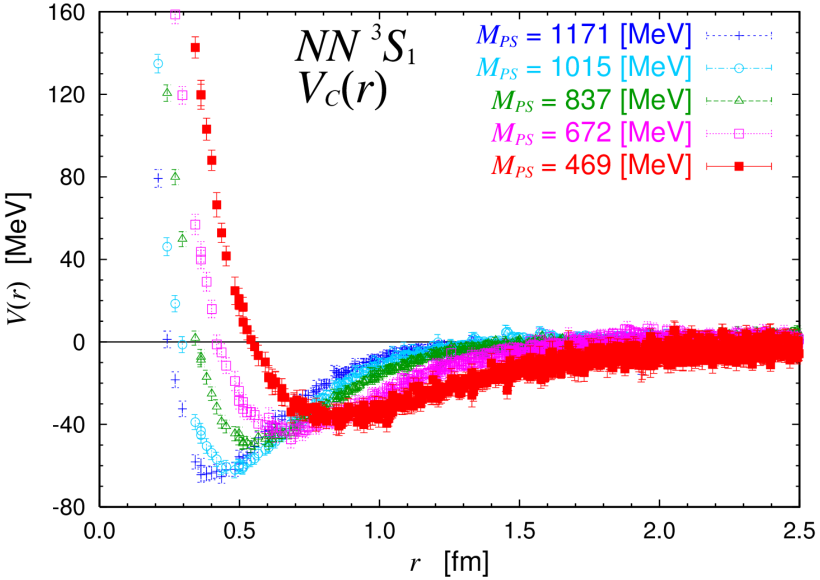
Slide, courtesy of T. Inoue (YITP talk, Oct. 8th 2015)

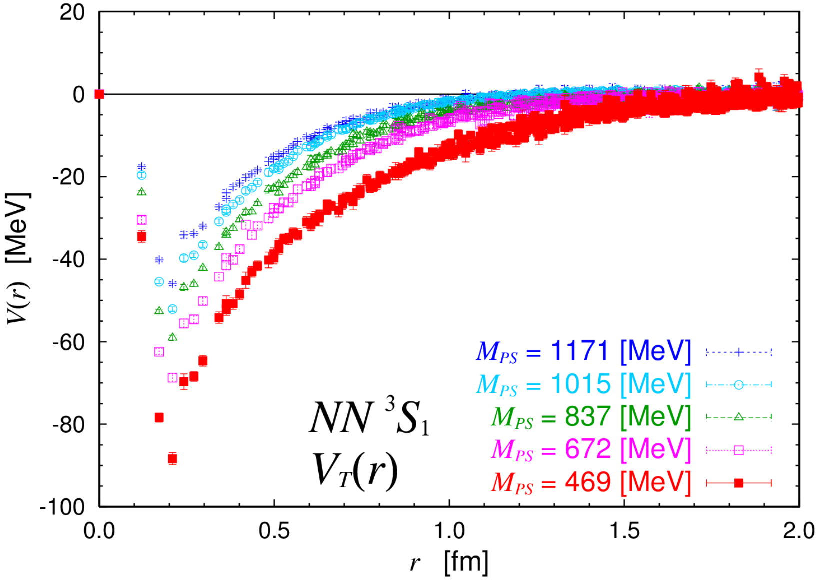
Two-Nucleon HAL potentials in flavour SU(3) symm.

Quark mass dependence of *V(r)* for NN partial wave (*$^1S_0$, $^3S_1$, $^3S_1$-$^3D_1$*)

➔ Potentials become stronger m_π as decreases.



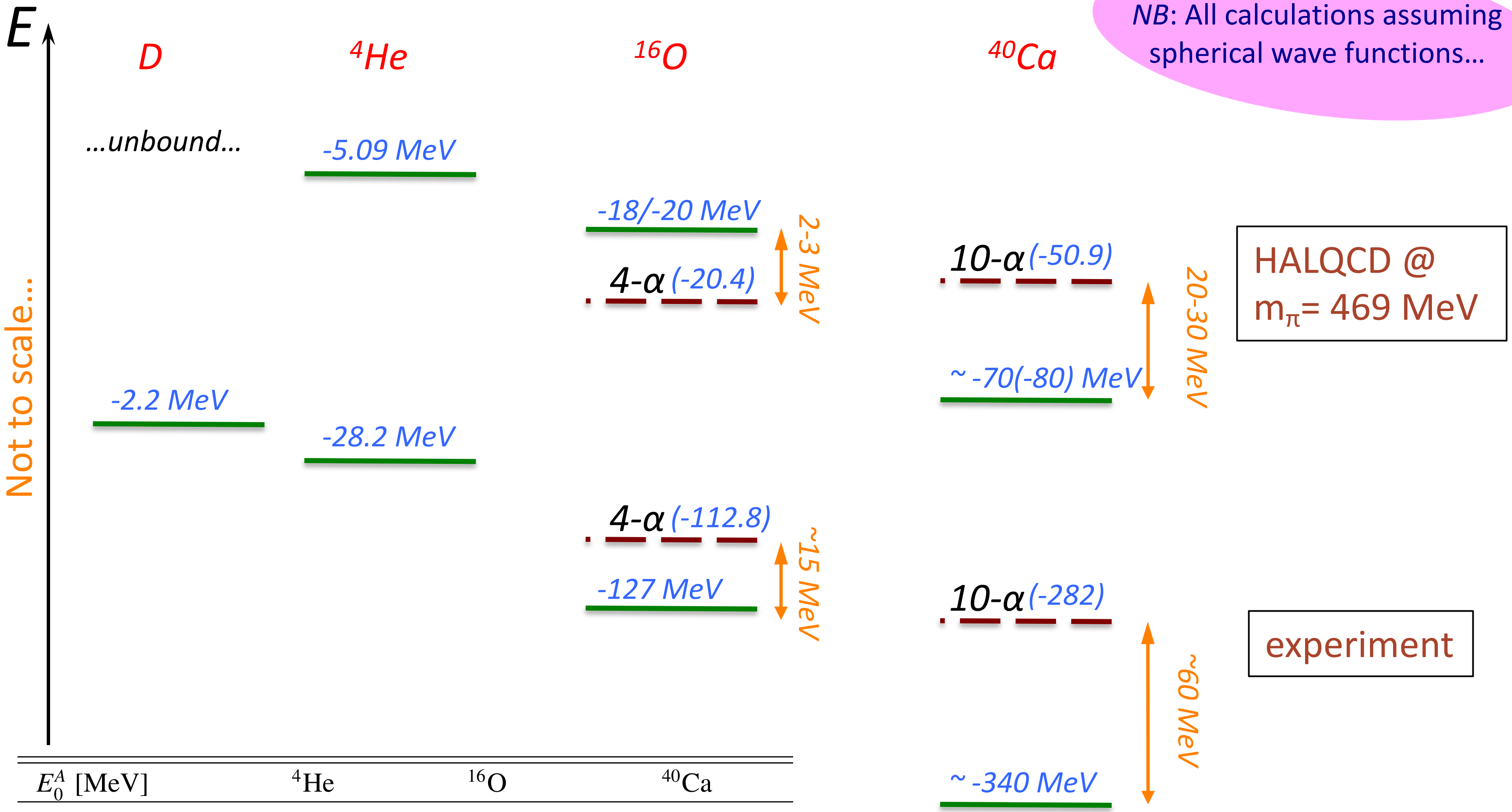




Prog. Theor. Exp. Phys. 01A105 (2012)

Results for binding

NB: All calculations assuming spherical wave functions…



E_0^A [MeV]	^{4}He	^{16}O	^{40}Ca
BHF [22]	-8.1	-34.7	-112.7
G(ω) + ADC(3)	-4.80(0.03)	-17.9 (0.3) (1.8)	-75.4 (6.7) (7.5)
Exact Result [51]	-5.09	–	–
Separation into ^{4}He clusters:		-2.46 (0.3) (1.8)	24.5 (6.7) (7.5)

C. McIlroy, CB, et al., Phys. Rev. C**97**, 021303(R) (2018)

Quantum MC calculations for Ys

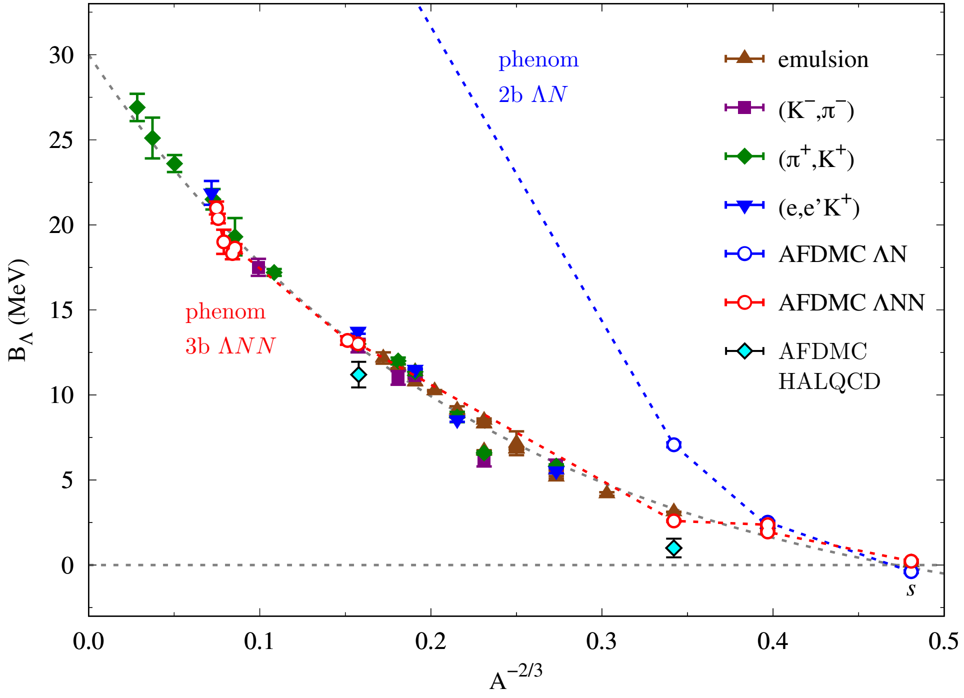
- AV4’ + UIX with phenomenological hypernuclear forces requires large ΛNN 3-baryon force
- Physical mass now under reach ($m_\pi \approx 145$ MeV) for hyperons
- HALQCD ΛN 3-baryon force is already very close to experiment

$$H = -\frac{\hbar^2}{2m_N}\sum_i \nabla_i^2 + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} - \frac{\hbar^2}{2m_\Lambda}\nabla_\Lambda^2 + \sum_i v_{i\Lambda}$$

Argonne v_4' (AV4’) nucleon-nucleon (NN) interaction $v_{ij} = \sum_{p=1,4} v^p(r_{ij})\, O_{ij}^p$

central component of the Urbana IX (UIX$_c$) $V_{ijk} = A_R \sum_{cyc} T^2(m_\pi r_{ij})\, T^2(m_\pi r_{ik})$

The hyperon-nucleon (YN) potential $v_{i\Lambda} = \sum_{p=c,\sigma,t} v^p(r_{i\Lambda})\, O_{i\Lambda}^p$



--○-- : phenomenological NΛ potential

--○-- : phenomenological NΛ + NNΛ potential

◆ : HALQCD NΛ potential

Diffusion Monte Carlo:

$$\langle X|\Psi_T\rangle = \langle X|\left(\prod_{i<j<k} U_{ijk}\right)\left(\prod_{i<j} F_{ij}\right)\left(\prod_i F_{i\Lambda}\right)|\Phi_{J^\pi,J_z,T_z}\rangle, \qquad |\Psi_0\rangle = e^{-(H-E_0)\tau}\,|\Psi_T\rangle$$

AFDMC: $$e^{-\lambda O^2\delta\tau/2} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dx\, e^{-x^2/2}\, e^{x\sqrt{-\lambda\delta\tau}\,O}$$

D. Lonardoni, A. Lovato, et al, Phys. Rev. Lett. 114, 092301 (2015) & arXiv:1506.04042

Future application for Ys in nuclei now possible

- AV4’ + UIX requires very large with phenomenological hypernuclear forces requires large ΛNN 3-baryon force
- Physical mass now under reach ($m_\pi \approx 145$ MeV) for hyperons
- HALQCD ΛN 3-baryon force is already very close to experiment

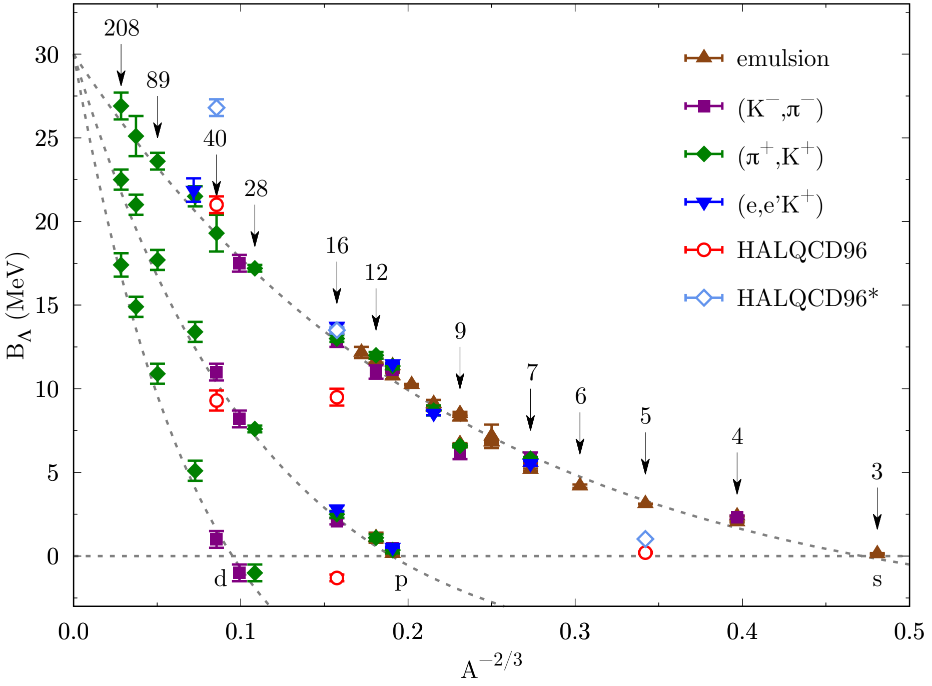


Table 1: Λ separation energies (in MeV) for different hypernuclei with the hyperon in different single-particle states. Second column reports the AFDMC results using the original HALQCD96 ΛN potential. Third column shows the results for the modified HALQCD96 ΛN potential (see text for details). In the last column, the available experimental data [] are reported.

${}^{A}_{\Lambda}Z$	J^π (state)	HALQCD96	HALQCD96*	Exp
${}^{5}_{\Lambda}\mathrm{He}$	$1/2^+$ (s)	0.21(5)	1.02(3)	3.12(2)
${}^{16}_{\Lambda}\mathrm{O}$	1^- (s)	9.5(5)	13.5(2)	13.4(4)
	2^+ (p)	−1.3(2)	0.5(1)	2.5(2)
${}^{40}_{\Lambda}\mathrm{Ca}$	2^+ (s)	21.0(5)	26.8(5)	19.3(1.1)
	3^- (p)	9.3(6)	13.7(6)	11.0(5)

D. Lonardoni, A. Lovato, CB, T. Inoue, HALQCD coll