Relativistic quantum theory and algorithms: A toolbox for modeling many-fermion systems in astrophysical scenarios

Francesca Triggiani [1,2], Stefano Simonucci [1,2], Simone Taioli [3,4]

[1] University of Camerino-Department of Physics,[2] INFN section of Perugia ,[3] ECT*-FBK ,[4] INFN section of Trento

Introduction

In this work we present a new computational relativistic quantum-mechanical method for calculating decays in astrophysical scenarios by considering both the temperature and density dependence, in addition to the electronic and nuclear excited states (ES) population dynamics. We apply it, for the first time, to ${}^{134}_{55}$Cs $\to$ ${}^{134}_{56}$Ba and ${}^{135}_{55}$Cs $\to$ ${}^{135}_{56}$Ba β^-decays, crucial production channels for Barium isotopes in Asymptotic Giant Branch (AGB) stars. In AGB stars, indeed, the Ba abundance depends solely on slow (s) process nucleosynthesis that, after the n-capture on stable ^{133}Cs, meets a branching point at ^{134}Cs, where further n-captures (when n-capture on ^{134}Cs feeds the longer-lived ^{135}Cs, the latter can β-decay to ^{135}Ba) compete with β-decay, e.g. the ${}^{134}_{55}$Cs $\to$ ${}^{134}_{56}$Ba . New scenarios concerning the heavy-element nucleosynthesis in stars were recently opened, offering new constraints on the isotopic abundances generated in stellar processes, whose understanding now depends also on improving the assessment of nuclear reactions.

Method

Our approach is based on the calculation of the total Hamiltonian

$$H = H_{nucl} + H_{e-e} + H_{weak}$$

- H_{nucl} contains the interactions between nucleons in the initial and final nuclear states (the nucleon-nucleon interaction is described by a semi-empirical scalar and vector relativistic Wood-Saxon (WS) spherical symmetric potential)

- H_{e-e} is the electron-electron Coulomb correlation modelled via a local density approximation (LDA) to the electron gas ($V_{ex} \propto \rho(r)^{1/3}$). Electrons populate the energy levels according to a Fermi-Dirac (FD) distribution

- H_{weak} is the weak interaction Hamiltonian that is defined as the product of leptonic (L^μ) and hadronic (H_μ) currents

$$H_{weak} = \frac{G_F}{\sqrt{2}} H_\mu L^\mu + h.c.$$

The leptonic current is factorized in the product of the electron and neutrino quantum field operators, while the hadronic one is separable into neutron and proton field operators (the decaying neutron acts as an independent particle correlated only geometrically to the *core* of the remaining nucleons). Both are reckoned by mean-field central potentials.

The main purpose is to compute the transition probability per unit time

$$N_{i\to f} = 2\pi Tr(\hat{\rho}_i H_{weak} P_f H_{weak})\delta(E_i - E_f) + h.c.$$

where $\hat{\rho}_i = p_i |i\rangle\langle i|$ is a statistical mixture of initial states $|i\rangle = |h_i\rangle \otimes |e_i\rangle$ and $P_f = \sum_f |f\rangle\langle f|$ a mixture of final states $|f\rangle = |h_f\rangle \otimes |e_f\rangle \otimes |\bar{\nu}_f\rangle$.

Results

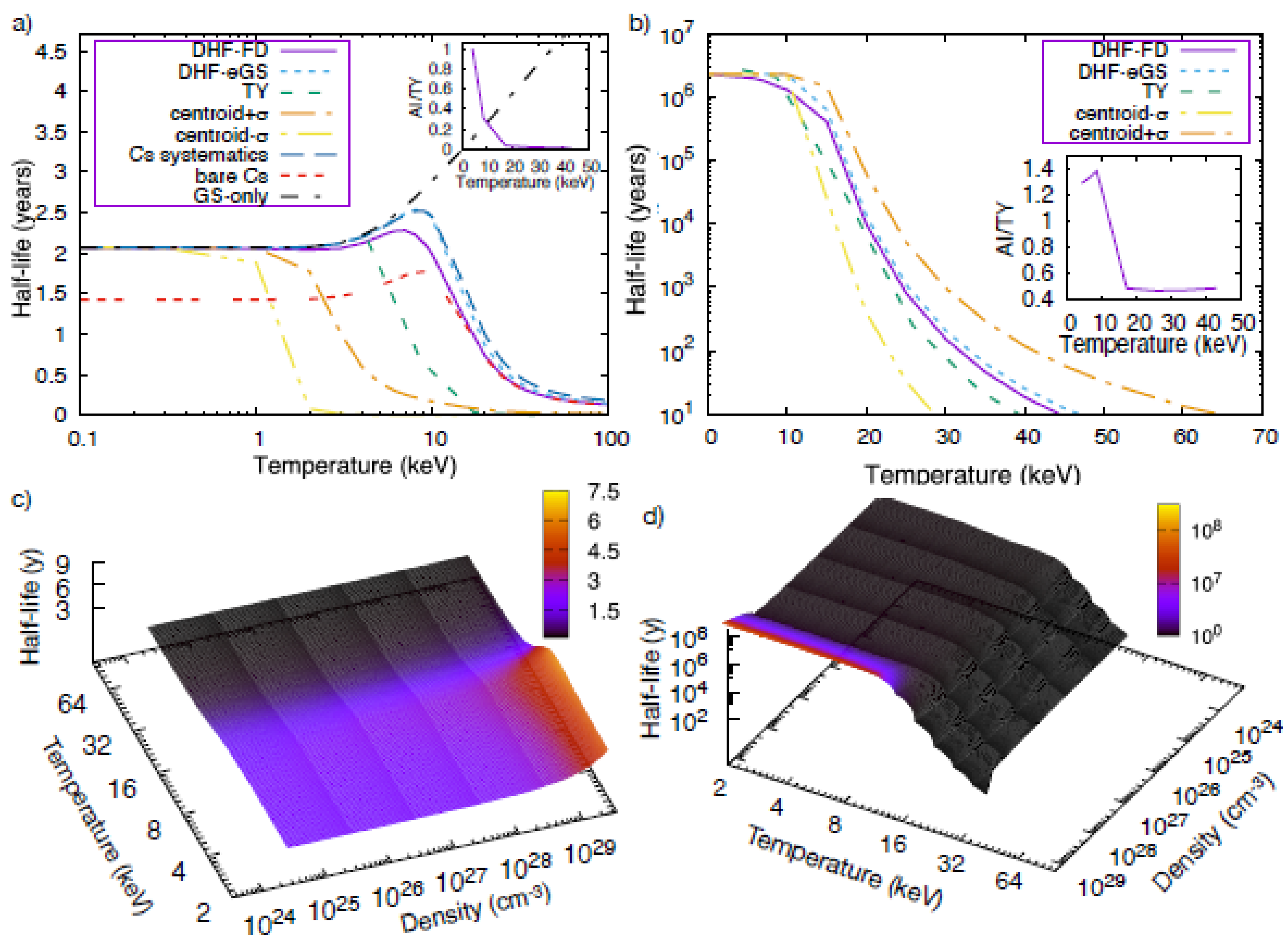

Figure 1: Half-life of ^{134}Cs (a) and of ^{135}Cs (b) vs. temperature (violet line) for $n_p = 10^{26}$ cm^{-3} using our DHF calculations. They include: (a) the nuclear states at 4^+, 5^+, 3^+ of ^{134}Cs and (b) the nuclear states of ^{135}Cs at $5/2^+$, $5/2^+$, as well as the electronic DOF. In cyan (DHF-eGS) the half-life computed when electrons are clamped in their ground state (GS). Orange and yellow lines show the maximum (centroid $+\sigma$) and minimum (centroid $-\sigma$) values obtained by taking the $log(ft)$ from the general systematics, while in the blue line we used the centroid of the specific ^{134}Cs systematics (this does not exist for ^{135}Cs). Green lines show the estimates by TY (Takahashi & Yokoi 1983, 1987). In panel a) we also plot (red line) the half-life for a completely ionized ^{134}Cs (bare Cs) and the nuclear GS to GS decay half-life (black line, GS-only). The trend is similar for ^{135}Cs (not shown). Insets of a) and b) give the ratios between our ab-initio rates (AI) and TY values for $n_p = 10^{26}$ cm^{-3}. Figures c) and d) display ^{134}Cs and ^{135}Cs half-lives according to our model, including both nuclear and electronic DOFs, vs. temperature and proton density as varying in the interior of stars.

Conclusion

We find increases in the half-lives of Cs isotopes for T$> 10^8$ K as compared to previous works based on systematics.
The observed changes are attributable to two factors in particular: the inclusion of both the nuclear and the electronic ESs of parent and daughter nuclei, up to complete ionization. The presence of fast-decaying nuclear ESs can in fact increase the rate by a factor of 15 at 100 keV and up to 23 at 1000 keV with respect to room temperature conditions.
Specifically, we note that:
- the 60 keV nuclear ES of ^{134}Cs is the fastest to $\beta-$decay, with a rate $\simeq$ 80 times higher than the GS-to-GS one;
- the rate increases with respect to GS decay only, close to a factor of $\simeq 3$ at 20 keV, $\simeq 6$ at 30 keV and $\simeq 8$ at 40 keV, which are the typical energies characterizing AGB stars. Similar effects for ^{135}Cs.

Our results are within the standard deviation of the general systematics and the use of the modified rates to s-process computations in AGB stars reconciliates models and observations.

References

- S. Taioli, D. Vescovi, M. Busso, S. Palmerini, S. Cristallo, A. Mengoni, S. Simonucci, 2021, Physical Review Letters.
- Morresi, T., Taioli, S., & Simonucci, S. 2018, Advanced Theory and Simulations, 1, 1870030.
- Simonucci, S., Taioli, S., Palmerini, S., & Busso, M. 2013, The Astrophysical Journal, 764, 118.
- Busso, M., Vescovi, D., Palmerini, S., Cristallo, S., & Antonuccio-Delogu, V. 2021, ApJ, 908, 55.
- Palmerini, S., Busso, M., Simonucci, S., & Taioli, S. 2016, Journal of Physics: Conference Series, 665, 012014.
- Takahashi, K., & Yokoi, K. 1983, Nuclear Physics A, 404, 578. 1987, Atomic Data and Nuclear Data Tables,36, 375.
- Salvat, F., Martìnez, J. D., Mayol, R., & Parellada, J. 1987, Phys. Rev. A, 36, 467.
- Slater, J. C. 1951, Phys. Rev., 81, 385.
- Schwierz, N., Wiedenhover, I., & Volya, A. 2007.

Acknowledgements

- PANDORA-INFN