Quantum Computing simulation for Collective Neutrino Oscillations

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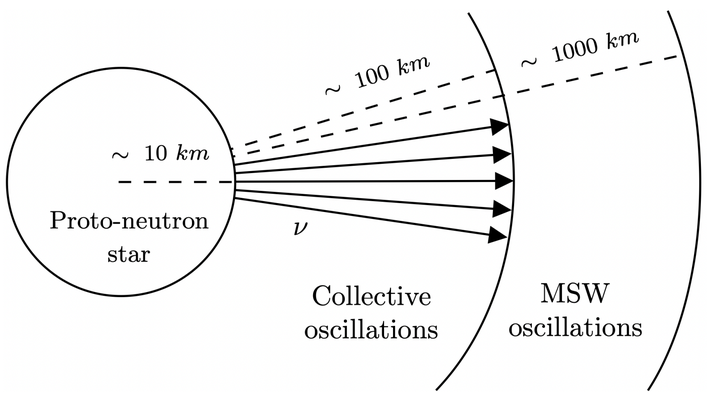
Motivations

- **Core-collapse supernovae** of massive stars $M \gtrapprox 8M_\odot$ emit a huge number of neutrinos ($\sim 10^{58}$).
- The physics of matter under extreme conditions is strongly **flavor-dependent** (nucleosynthesis, neutron-proton ratio, spectrum splits...)
- Interesting quantum many-body problem governed by weak interaction.
- Describing the full dynamic is very complicated due to the **collective neutrino oscillations** that make the equation non linear.



- We want to simulate the real time evolution:

$$|\Psi(t)\rangle = U(t)\,|\Psi_0\rangle\,, \text{ where } U(t) = e^{-iHt}\,. \tag{1}$$

Physical description

- Two-flavors approximation ($SU(2)$ model) to encode the flavor state in a qubit state:

$$|\nu_e\rangle \longmapsto |0\rangle\,; \quad |\nu_x\rangle \longmapsto |1\rangle \tag{2}$$

- N neutrinos encoded into N qubits.
- The flavor Hamiltonian of N neutrinos is:

$$H = \sum_{i=1}^{N} \boldsymbol{b}\cdot\boldsymbol{\sigma}_i + \sum_{i<j}^{N} J_{ij}\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j \tag{3}$$

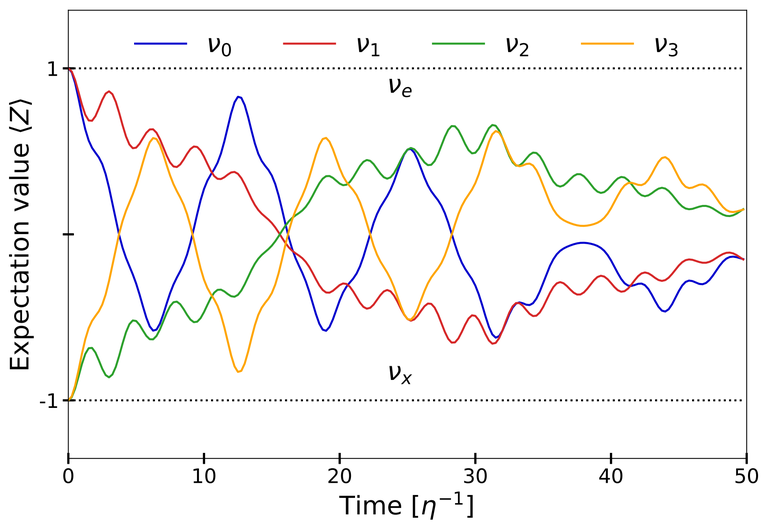
  - **1-body term**: vacuum mixing where

$$\boldsymbol{b} = \frac{\delta m^2}{4E_\nu}(\sin(2\theta_\nu), 0, -\cos(2\theta_\nu))\,.$$

  - **2-body term**: $\nu\nu$-interaction where

$$J_{ij} := \eta(1-\cos(\theta_{ij}))\,.$$

- Initial state for $N = 4$: $|\Psi_0\rangle = |0011\rangle$.
- We can look at the expectation value $\langle Z_i\rangle$



- Symmetry under particle exchange:

$$0 \longleftrightarrow 3; \quad 1 \longleftrightarrow 2 \tag{4}$$

Unitary implementation

- To perform the quantum simulation we need a **quantum gate decomposition** of the $U(t)$ operator ($2^N \times 2^N$ unitary matrix of the flavor basis):
- Divide 1-body and 2-body parts that commute:

$$U(t) = U_2(t)U_1(t)\,. \tag{5}$$

- Approximate the 2-body part as a product of pair interactions so each term is a 2-qubit gate:

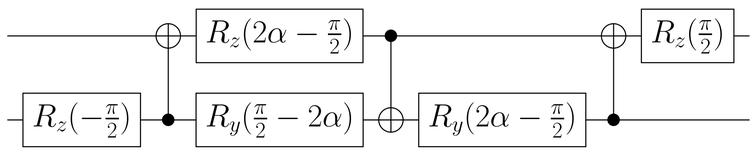
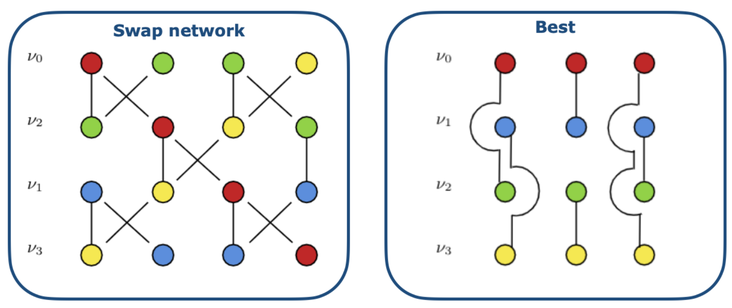


Fig. 3: Circuit implementing $e^{i\alpha((X\otimes X+Y\otimes Y+Z\otimes Z))}$, Ref. [2].

Qubit connectivity

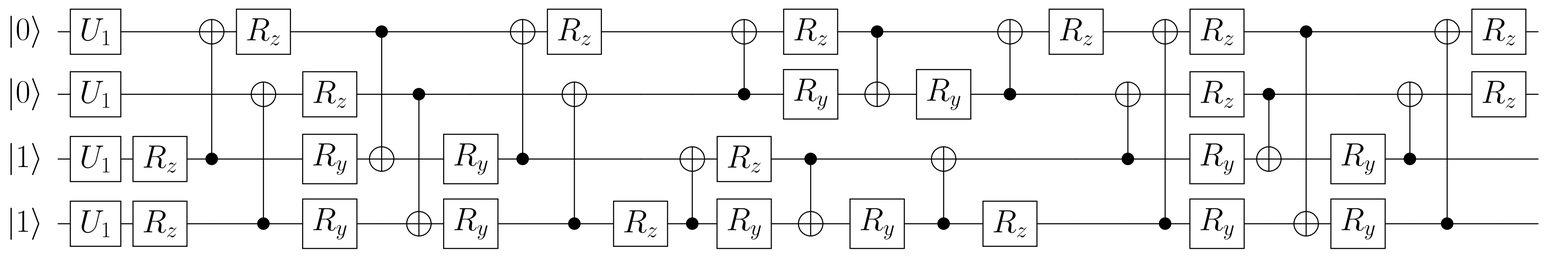
- All-to-all Hamiltonian means that each particle have to interact with all the others at each time step during the evolution.
- The order in which the pairs interact changes the error due to the commutators.



- The **swap network** proposed in Ref.[1] implements the interaction on a chain of linearly connected qubits but swap gates increase the circuit complexity.
- **All-to-all connectivity** allows for best ordering and lower circuit complexity.

Circuit Optimization

- Machine-aware compilation.
- Optimal CNOT-based circuit for a single Trotter step and using the all-to-all connectivity:

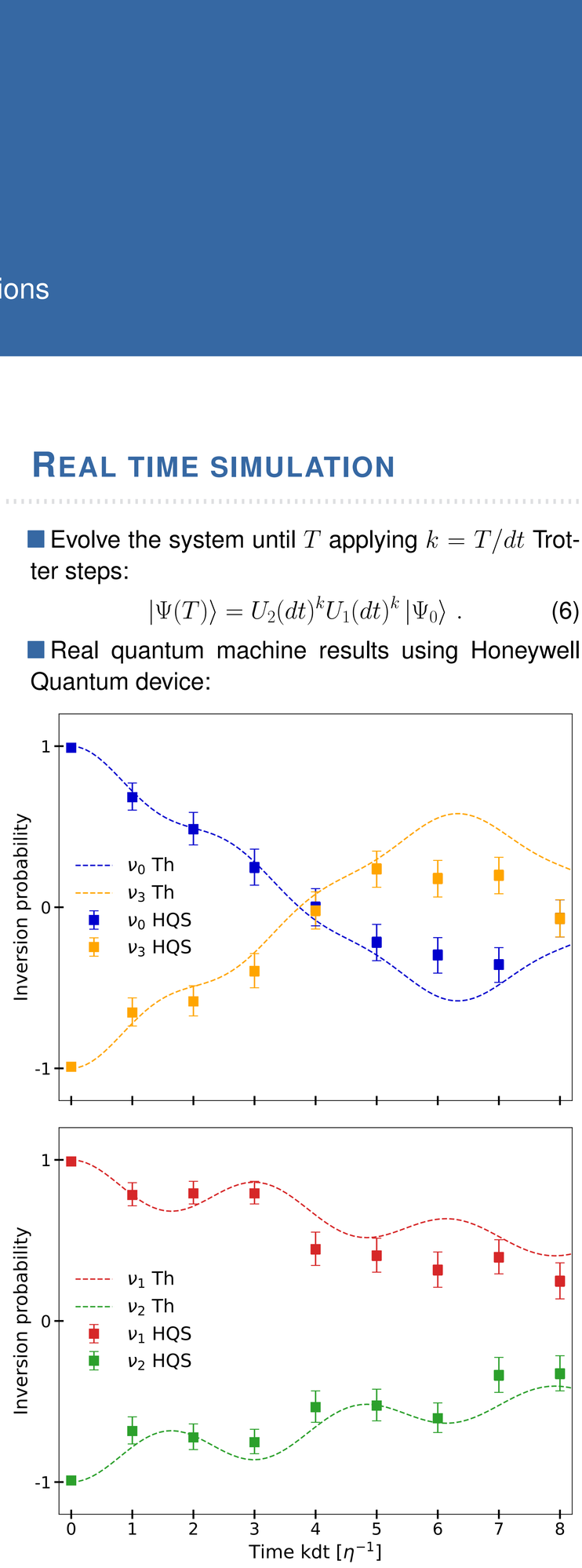


Real time simulation

- Evolve the system until T applying $k = T/dt$ Trotter steps:

$$|\Psi(T)\rangle = U_2(dt)^k U_1(dt)^k\,|\Psi_0\rangle\,. \tag{6}$$

- Real quantum machine results using Honeywell Quantum device:



# CNOT	18	36	54	72	90	108	126	144

- Very long circuits with a huge number of 2-qubit gates

Conclusion

- Full-connected qubits allows for more freedom in gate decomposition.
- The error scales linearly with the number of qubits.
- Quantum circuit optimization is a crucial step in order to perform simulations on a near-term quantum device.

References

[1] B. Hall et al. "Simulation of Collective Neutrino Oscillations on a Quantum Computer". In: *Phys. Rev. D 104, 063009* (2021).

[2] F. Vatan et al. "Optimal Quantum Circuits for General Two-Qubit Gates". In: *arXiv:quant-ph/0308006, Phys. Rev. A 69, 032315* (2004).

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