Probing Electron Properties in ECR Plasmas using X-ray Bremsstrahlung and Fluorescence Emission
Brief Overview

System of interest
Electron Cyclotron Resonance (ECR) Plasmas
Routinely used as ion sources to supply highly-charged beams to accelerators
Electron heating using microwaves and confinement using magnetic trap
Energy transfer not spatially homogenous, resultant plasma anisotropic in density and energy

Why study them?
Fundamental and Applied Research
Wave-plasma coupling in ECRIS not fully exploited
Beam intensity and emittance correlated with plasma instabilities [1], non-uniform density distribution [2] and variation of magnetic field [3]
New facilities like PANDORA require precise inputs on space-resolved properties

How to study them?
Direct and Indirect Methods
Various quantities of interest can be estimated using appropriate experiments
Direct methods: Precise determination of density, EEDF, energy etc. but perturb the plasma. Ex: Langmuir probes
Indirect methods: No plasma perturbation, but not sensitive to the full range of energies. Ex: Optical emission spectroscopy, ECR emission spectroscopy, X-ray spectroscopy

3D SPACE-RESOLVED ELECTRON DENSITY, EEDF AND TEMPERATURE
Confined electron density, temperature, EEDF estimate
Loss electron current density estimate

Space-resolved electron distribution simulations + Fluorescence reaction rate + Space-resolved geometrical efficiency
Theoretical emission model

Space-resolved X-ray maps obtained with CCD + pinhole setup
Volumetric X-ray spectra obtained with SDD + collimator setup

Warm electron distribution: Soft X-Ray Spectroscopy

Magnetic field $B$ applied longitudinally causing electrons to gyrate at frequency $\omega_c = \frac{eB}{m_e}$.

R-wave launched into the plasma at same frequency, leading to resonance heating.

From fluid model, adiabatic invariant defined as $\mu = \frac{1}{2} \frac{m v^2}{B} = \text{const}$

Min-B configuration to trap electrons longitudinally, prolonged heating for energetic reactions.

\[ \omega = \omega_c = \frac{eB}{m} \]

\[ P \propto E^2 \]

Schematic of ECRIS operation [4]

EM field in midplane at $f \approx 14$ GHz [5]


Energetic electrons (~ keV) and radiation interact with the ions through multiple processes (ionization, excitation etc.) to produce high charge states (NLTE population kinetics solved with CR model)

3D structure of plasma from simulations – the spatial anisotropy and non-homogeneity with respect to density and energy is visible [6]

Net particle density distribution (positive charge in blue, negative in red [7]

PANDORA: Plasmas for Astrophysics, Nuclear Decays Observation and Radiation for Archaeometry
A new ECR Ion Trap for in-plasma β-decay measurements

Q-value of bound state $^{187}\text{Re} \rightarrow ^{187}\text{Os}$ transitions as a function of degree of ionization [8]

Modification of half-life as a function of plasma parameters – effect of ion CSD

Precise measurement of space-resolved electron and ion properties

CSD and level population of plasma ions

EEDF of warm plasma electrons

Theory of Takahashi and Yokoi [8]

**ECR Plasma Characterisation Using X-Ray Spectroscopy**

Emission spectroscopy excellent indirect diagnostic tool

Different energy ranges of electrons can be probed by analysing different photon ranges

Novel plasma diagnostic technique developed by ATOMKI, Debrecen and LNS, Catania [6,11,12] – overall objective to study ECR plasma properties through multiple perspectives: space-resolved X-ray images, volumetric X-ray emission and ion current extraction

For high energy bremsstrahlung and contribution from extraction plate

For measuring ion currents (total and individual charge states)

Schematic of experimental setup showing various detectors used and distances [11]


2D Space-Resolved X-Ray Imaging – Experiment Details

Soft X-ray pin-hole camera tool

- **Sensitivity range ~ 2 ÷ 15 keV**
- **Sensor Size: 27.6 mm x 6.9 mm (1024 x 255 Pixels)**
- **Pixel size: 13 µm x 13 µm**
- **Max Energy Resolution ~ 150 eV**
- **Lead Pin-hole (diameters 100 µm)**

Exposure modes

- **Spectrally Integrated:**
  - Long exposure time (in tens of seconds)
  - Loss of info on individual photon energy
  - For study of shape, structure and local energy content of plasma

- **Spectrally Resolved (Photon Counting):**
  - Multiple frames with short exposure time (in ms)
  - Spatial and spectral resolution possible
  - Local electron energy distribution and fine structure of plasma studied

2D Space-Resolved X-Ray Imaging – Theoretical Model: General Outline

Spatially integrated spectrum for 30 W Ar plasma, and spectral filtering for Ar Kα fluorescence [6]

Photon counting mode, 2000 frames taken, exposure time = 150 ms/frame.

Anisotropic and non-homogenous emission source, space-resolved geometrical efficiency

\[
R = \rho_c \rho_i \omega_{K\alpha} \int_{3.205}^{\infty} \sigma_{K,ion}(E) v_e(E)f(E) \, dE
\]

\[
T = 300 \rho_c \rho_i \omega_{K\alpha} \int_{3.205}^{\infty} \sigma_{K,ion}(E) v_e(E)f(E) \, dE
\]

2D Space-Resolved X-Ray Imaging – Theoretical Model: Electron Kinetics

4 choices of EEDF – mix of Maxwell and Druyvesteyn distribution functions

<table>
<thead>
<tr>
<th>Ref. case name</th>
<th>Type</th>
</tr>
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<tbody>
<tr>
<td>EEDF1</td>
<td>Low-E $f_{\text{EA}} + \text{High-E } f_{\text{EA}}$</td>
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<tr>
<td>EEDF2</td>
<td>Low-E $f_{\text{EA}} + \text{High-E } f_{\text{D}}$</td>
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<td>EEDF3</td>
<td>Low-E $f_{\text{EA}} + \text{Medium-E } f_{\text{EA}} + \text{High-E } f_{\text{EA}}$</td>
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<td>EEDF4</td>
<td>Low-E $f_{\text{EA}} + \text{Medium-E } f_{\text{D}} + \text{High-E } f_{\text{EA}}$</td>
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Each EEDF tested in each ROI – better and more physical analysis

MSE and $r^2$ calculated for each cell of the ROI, then mean and SD of both quantities evaluated

Mean – average value of the statistic in the ROI
SD – variation of actual value from the mean within the ROI

Thus, low mean MSE, high mean $r^2$, and low SD for both implies best performance

EEDF2 = Cold Maxwell + Hot Druyvesteyn works in each ROI

But simulations give relative electron density distribution, absolute $n_e$ unknown still


[14] B. Mishra, accepted to be published, Il Nuovo Cimeno 2021
Lotz formula for K-shell ionisation cross-section

Lotz cross-section for Ar superposed on Maxwell EEDFs at different temperatures [15]

Strong indication that warm electrons 2-30 keV cannot reproduce experimental map with physically valid $n_e$

Schematic of the detection system

Spatially-resolved geometrical efficiency for anisotropic and non-homogeneous emitting source

Ray-Tracing Monte Carlo code to evaluate a 3D map of geometrical efficiency, a value for each plasma cell

Step 1: Determination of emission space for each plasma cell as defined by pinhole configuration

\[ \Delta \Omega = \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \int_{\phi_{\text{min}}}^{\phi_{\text{max}}} \sin \theta \, d\theta \, d\phi = (\cos \theta_{\text{min}} - \cos \theta_{\text{max}})(\phi_{\text{max}} - \phi_{\text{min}}) \]

Step 2: Calculation of photon detection probability through ray-tracing

Corrective factor to bare emission \( \varepsilon_g \) – simulate N photons within emission space and calculate LGE

LGE calculation still needs improvement – simulation of mesh, scattering effects, plasma self-absorption and contribution from plasma electrode
Current calculations have reproduced overall photon counts and structure BUT $n_e$ taken $10^{18}$ m$^3$, unphysically high

Direct consequence of considering only electrons between 2-30 keV – very small contribution

Verification needed for $n_e$ from volumetric spectrum analysis

$T = 300 \rho_e \rho_0 \sigma_{K,\alpha} g \int_{3.205}^{\infty} \sigma_{K,\alpha,\beta}(E) v_e(E) f(E) dE$
Volumetric X-Ray Spectroscopy – Experiment Details

Soft X-ray collimator setup

1. **Sensitivity range ~ 2 ÷ 30 keV**
2. **Max Energy Resolution ~ 150 eV**
3. **Collimator length**

**SDD detector used for soft X-ray spectroscopy with energy range 2-30 keV**

Long collimator configuration for measuring plasma emission in near-axis region.

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**Contribution from bremsstrahlung, Ar fluorescence (confined electrons) and Cr/Fe fluorescence (escaping electrons)**

**Calibration with Fe lines**
+ QE renormalisation
+ Dead time correction
+ Conversion to emissivity density

Emissivity density of Ar Plasma [15]

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**Bremsstrahlung Emissivity Density**

\[ J_{\text{theo, brem}}(h\nu) = \rho_e \rho_i \int_{h\nu}^{\infty} \frac{d\sigma_K(h\nu)}{dh\nu} v_c(E) f(E) dE \]

Kramer's formula for differential cross-section

\[ \frac{d\sigma_K(h\nu)}{dh\nu} = \frac{16\pi}{3\sqrt{3}} \alpha^2 \left( \frac{\hbar}{m_e c} \right) \left( \frac{c}{v_e} \right)^2 \left( \frac{Z^2}{h\nu} \right) \]

Single-component Maxwell EEDF

\[ f_M(E; k_B T_e) = \frac{2}{\sqrt{\pi}} \left( \frac{\sqrt{E}}{k_B T_e} \right)^3 e^{-E/k_B T_e} \]

**Fluorescence Emissivity Density (Ar ions)**

\[ J_{\text{theo},2.96} = \frac{h\nu_{2.96}}{\Delta E} \rho_e \rho_i \int_{3.205}^{\infty} \sigma_{K,\text{ion}}(E) v_c(E) f(E) dE \]

\[ J_{\text{theo},3.19} = \frac{h\nu_{3.19}}{\Delta E} \rho_e \rho_i \int_{3.205}^{\infty} \sigma_{K,\text{ion}}(E) v_c(E) f(E) dE \]

Lotz formula for K-shell ionisation cross-section

\[ \sigma_{K,\text{ion}} = a_K q_K \left\{ \ln \frac{\epsilon}{I} \right\} \left\{ 1 - b_K \exp \left[ -c_K (\epsilon/\epsilon - 1) \right] \right\} \]

Pseudo-Voigt profile for line-broadening

\[ D_{PV}(x - x_0, \tilde{f}) = \eta L(x - x_0, \tau_L) + (1 - \eta) G(x - x_0, \sigma_G) \]

**Bremsstrahlung Emissivity Density**

\[ J_{\text{theo, brem}}(h\nu) = \rho_e \rho_i \left( Z \hbar \right)^3 \left( \frac{4\alpha}{\sqrt{6m_e}} \right) \left( \frac{\pi}{k_B T_e} \right)^{1/2} e^{h\nu/k_B T_e} \]
Fluorescence Emissivity Density (Cr/Fe atoms)

Active emission area = $\frac{4\pi \epsilon_g^2 - \pi d^2}{4} \Delta d$

$J_{\text{theo}, \nu} = \frac{h\nu}{V_P \Delta E} \rho_{\text{e,loss}} N_t \int_I^{\infty} \sigma_{K,\text{ion}}(E) v_\epsilon(E) f(E) dE$

Deutsch-Mark formalism for K-shell ionisation cross-section

$\sigma_{1s,\text{ion}} = g_{1s} \pi (r_{1s})^2 \xi_{1s} f(U) F(U)$

Estimation of number of target atoms $N_t$

$N_t(E) = n_t \left(4\pi \epsilon_g^2 - \pi \frac{d^2}{4}\right) \Delta d(E)$

Pseudo-Voigt profile for line-broadening

$J_{\text{theo, line, loss}}(\nu) = \sum J_{\text{theo, line}} D_{PV}(\nu - \nu_0, f_{\text{line}}) \Delta E$

Parametrisation of penetration depth as a function of energy

$J_{\text{theo, } \nu} = \frac{h\nu}{V_P \Delta E} \rho_{\text{e,loss}} n_t \left(4\pi \epsilon_g^2 - \pi \frac{d^2}{4}\right) \int_I^{\infty} v_\epsilon(E) f(E) \int_E^{E'} \frac{1}{S(E')} \sigma_{K,\text{ion}}(E') dE' dE$
Fluorescence from plasma interior – Ar ions and confined electrons

Lotz formula for K-shell ionisation cross-section

Lotz cross-section for Ar superposed on Maxwell EEDFs at different temperatures [15]

Ineffective overlap for low temperatures, major contribution from $k_B T_e \sim 10 \text{ keV}$

Fluorescence from extraction plate – Cr/Fe atoms and escaping electrons

Deutsch-Mark formalism for K-shell ionisation cross-section

Lotz cross-section for Cr/Fe superposed on Maxwell EEDFs at different temperatures [15]

Product of confined plasma electron and ion density $\rho_e \rho_i$ obtained $\sim 1.32 \times 10^{32} \text{m}^3$, expected $n_e \sim 10^{16} \text{m}^{-3}$

EEDF is single Maxwellian – $T_e \sim 21.15 \text{keV}$

Escaping electron density $\rho_{e,\text{loss}} \sim 10^{12} \text{m}^3$, estimated $j_{av} \sim 2\text{-}3 \text{mA/cm}^2$

Matches ion current density to order of magnitude

Chosen model fits experimental data well and gives valuable information about confined and escaping electrons
Conclusions

**Space-resolved X-ray Imaging**

Theoretical model developed based on self-consistent numerical simulations on warm electrons + LGE evaluation

Preliminary results show match with respect to overall structure (correlated with simulations/relative distribution) and counts (correlated with absolute particle density)

Uncertainties in both experimental and theoretical counts – $n_e$ used too high owing to small contribution from warm electrons, corroborated with volumetric analysis

**Volumetric X-Ray Spectroscopy**

Theoretical model developed based on bremsstrahlung and fluorescence from trapped particles and fluorescence from escaping electrons impinging on extraction plate

Excellent match between model and experiment, fit procedure furnishes particle density in expected range, predicts contribution from hotter electrons towards Kα emissions

UPDATE SIMULATIONS
CONSIDER HOTTER ELECTRONS
IMPROVE LGE + OVERALL MODEL
THANK YOU FOR YOUR ATTENTION!

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