



Analysis and design of X-ray optical systems applying hierarchical models including partial coherence

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ESRF
(including work done in Berkeley)

High Precision X-ray measurements 8-10 June 2021

- Motivation: optics for 4th generation synchrotron sources
- Hierarchical approach for beamline simulation (with examples)
- Ray tracing
- Wave optics
- Partial coherence
- Crystal optics



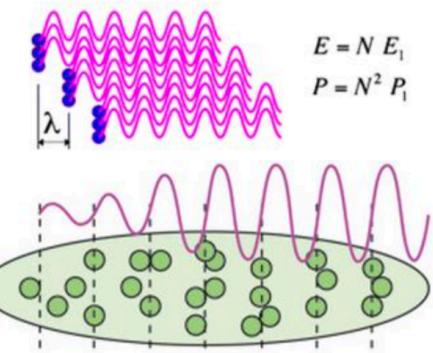
4TH GENERATION SYNCHROTRON SOURCES

X-FEL

Bunch length \sim wavelength emitted:

emission from different electrons add coherently:

almost full transversal (spatial) and longitudinal (time) coherence



STORAGE RINGS

Bunch length \gg wavelength:

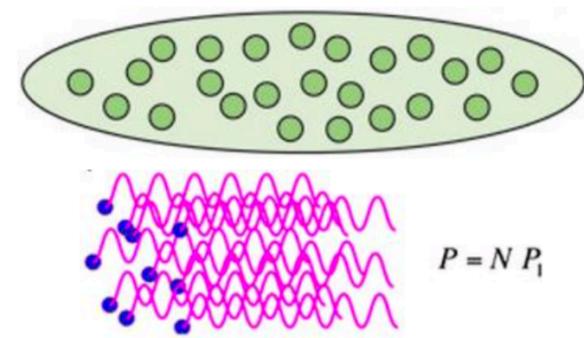
emission from electrons add incoherently:

low longitudinal (time) coherence (then improved with monochromators) and partial transversal (spatial) coherence

DLSR (diffraction limited storage ring) sources:
natural emission \gg electron size.

At present diffraction limit (total coherence) can be attained at low photon energies only ($< 1\text{keV}$), but hard X-rays present partial coherence.

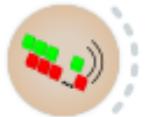
The beamline optics can improve the coherence fraction (at the price of lowering intensity).



Big increase of brightness and transverse coherence



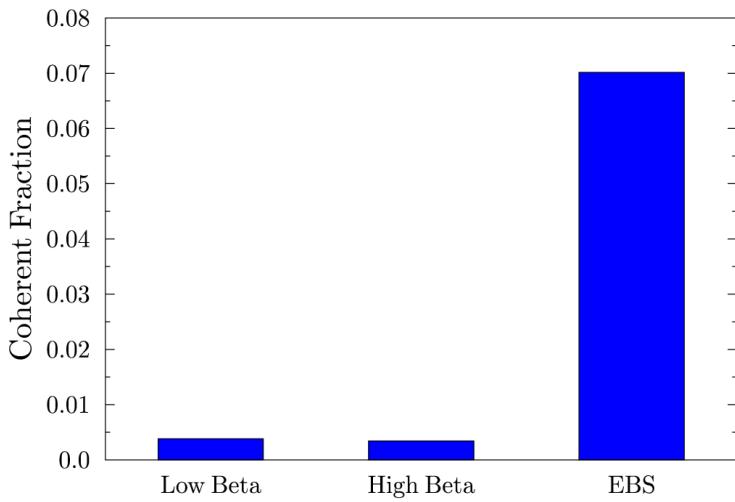
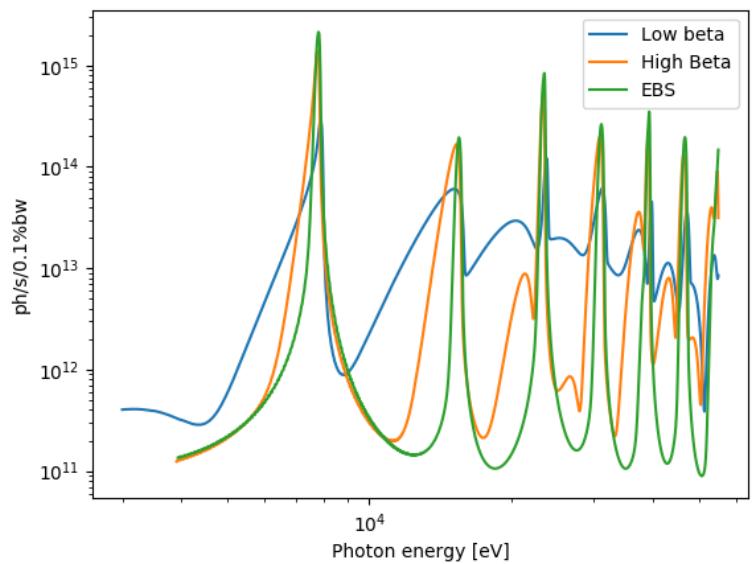
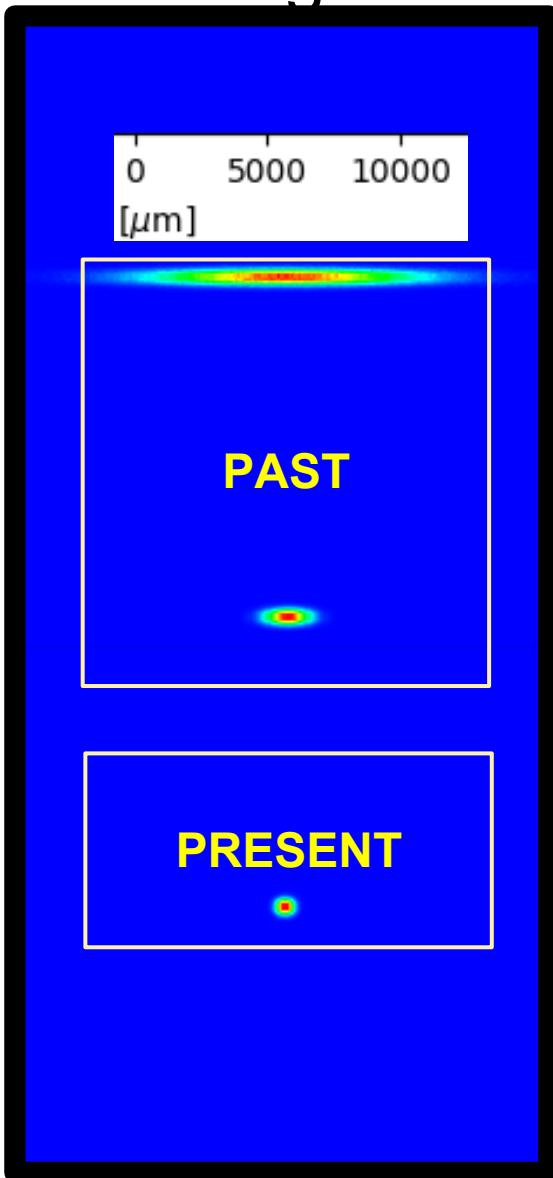
Low Beta U18

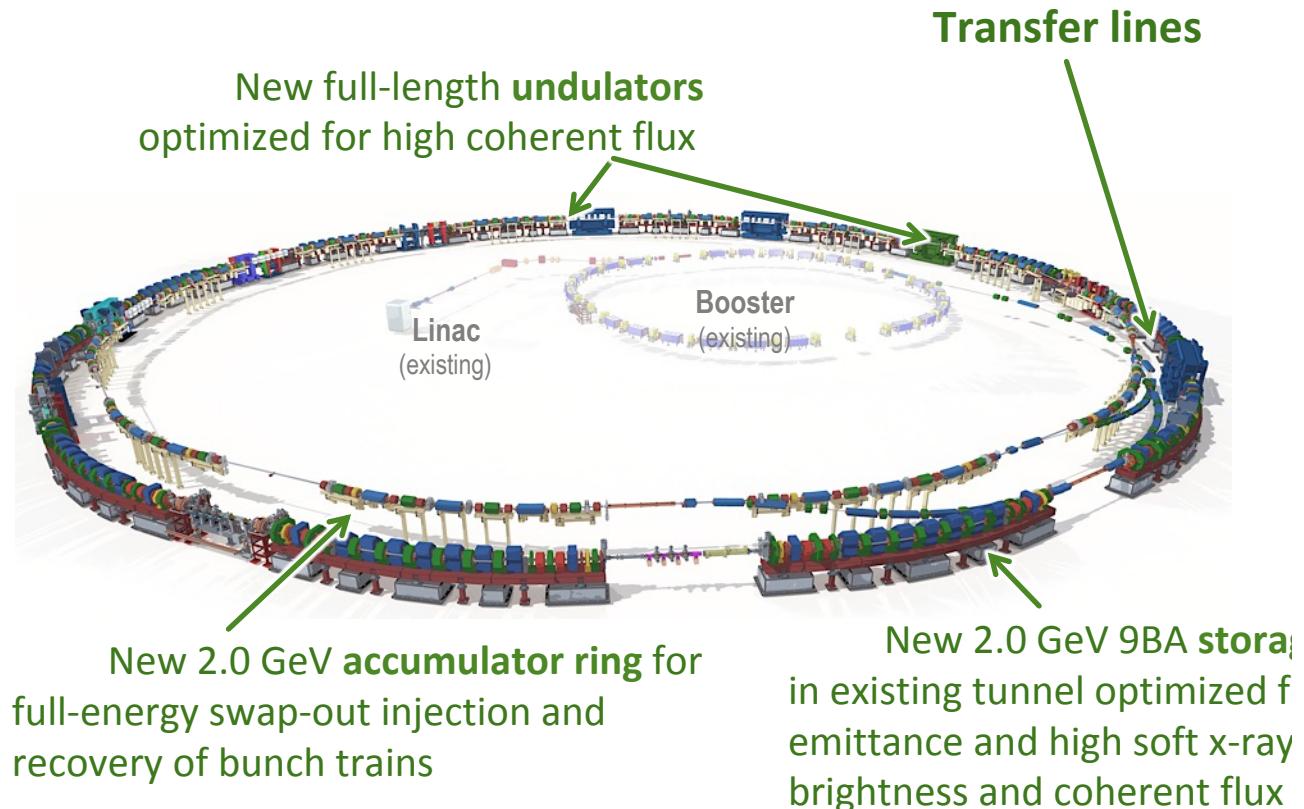


High Beta U18

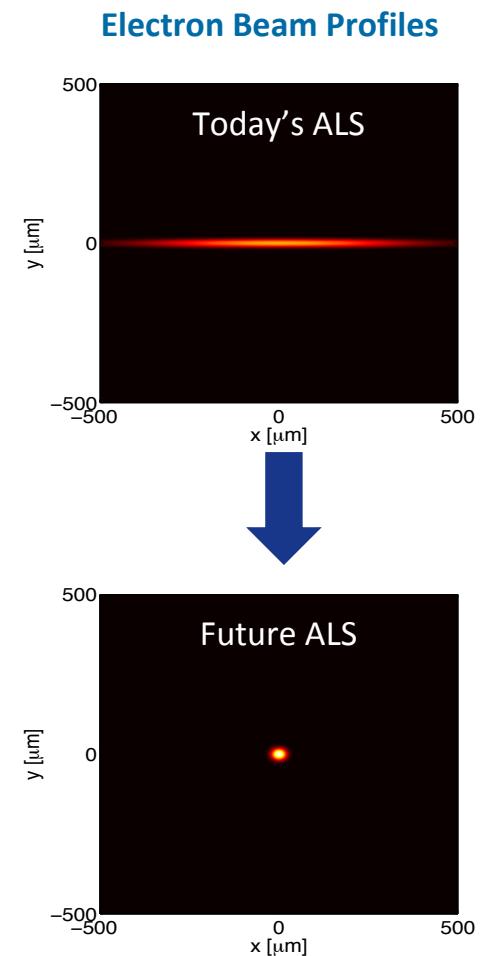


EBS U18





CF ~ 0.8 with a 4 m undulator at 230 eV

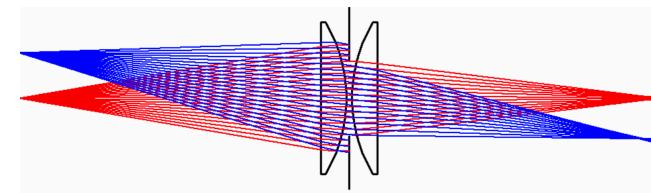


Analytical model (by hand)

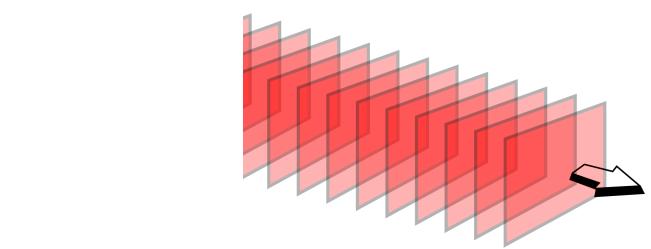


C
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Ray tracing (ShadowOui)



Hybrid model (ShadowOui)



Simplified wave optics (WOFRY)

Monte Carlo (multi e⁻) wave optics (SRW)

Coherent Mode Decomposition (COMSYL/WOFRY)

M. Sanchez del Rio et al, JSR 26 1887 (2019)

<https://doi.org/10.1107/S160057751901213X>

SRW

WOFRY

COMSYL

$$\nabla \cdot \vec{D} = \rho_{\text{free}}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}_{\text{free}} + \frac{\partial \vec{D}}{\partial t}$$



Maxwell Equations



Wave Equations



Helmholtz Equation

$$\frac{\partial^2 \mathbf{E}}{\partial t^2} - c_0^2 \cdot \nabla^2 \mathbf{E} = 0$$

$$\frac{\partial^2 \mathbf{B}}{\partial t^2} - c_0^2 \cdot \nabla^2 \mathbf{B} = 0$$

$$\vec{E} = \vec{e} e^{ik_0 S(r)}$$

$$\vec{H} = \vec{h} e^{ik_0 S(r)}$$

$$(\nabla S)^2 = n^2$$

$$\nabla S = n \vec{s}$$

$$\frac{d}{ds} \left(n \frac{d\vec{r}}{ds} \right) = \nabla n$$

$$\nabla n = 0 \Rightarrow \frac{d\vec{r}}{ds} = 0 \Rightarrow \vec{r} = s\vec{a} + \vec{b}$$

Wavefront

$$(\nabla^2 + k^2)\mathbf{E} = 0, \mathbf{B} = -\frac{i}{k} \nabla \times \mathbf{E},$$

Wave Optics

Ray



ShadowOui

XOPpy

Evolved from SHADOW

Accurate values of

Beam sizes including cropping,
and aberrations

Flux including o.e. physical models
(reflectivity transmittivity)

Monochromators/Analysers

Incoherent addition of rays (no interference/diffraction)

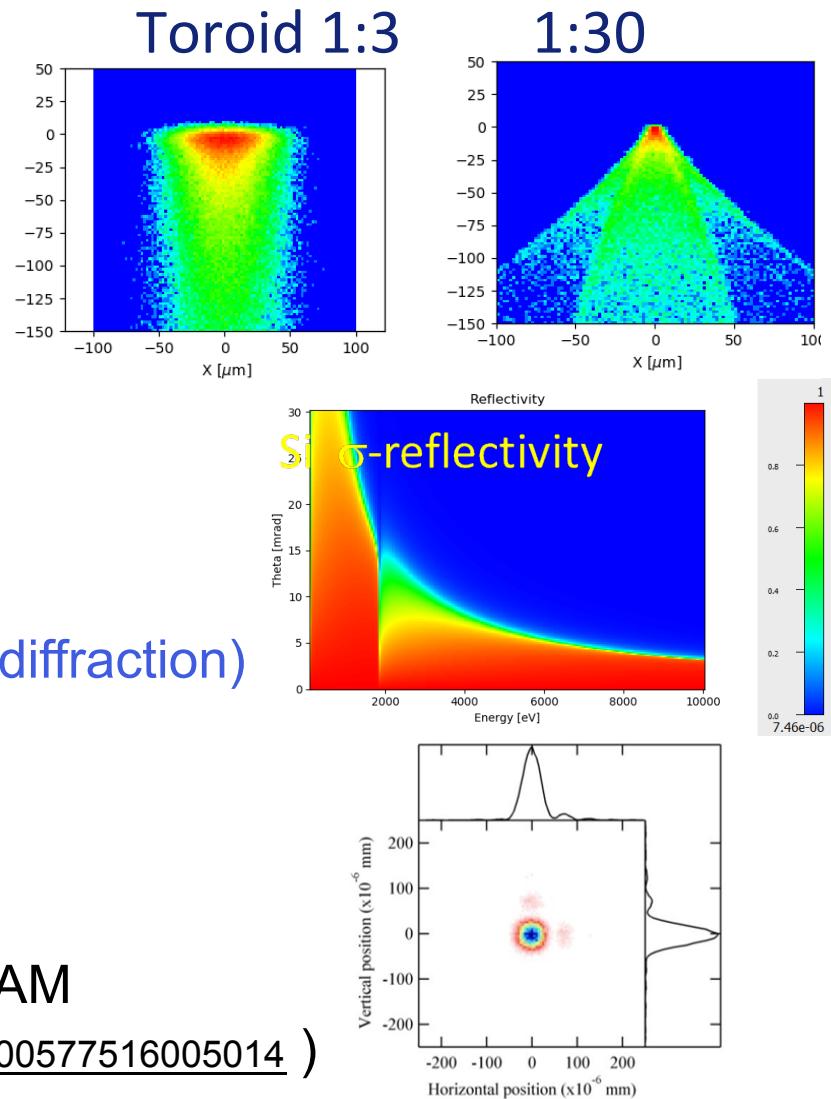
New features

Interoperability

Optical element deformation database DABAM
(Sanchez del Rio *et al.* <http://dx.doi.org/10.1107/S1600577516005014>)

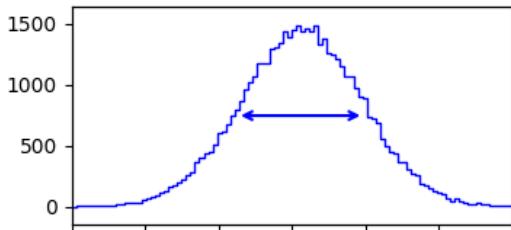
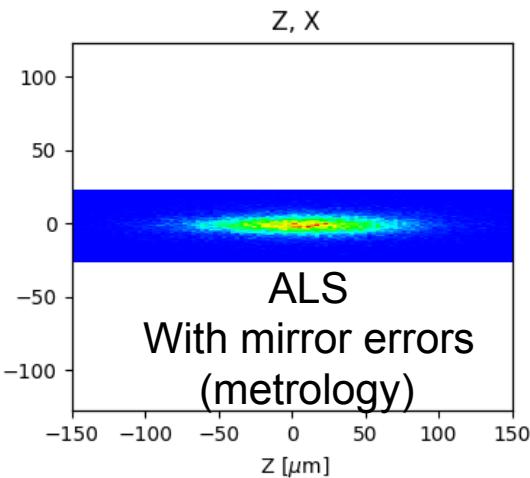
Corrections for coherence with Hybrid (Shi *et al.* <http://dx.doi.org/10.1107/S160057751400650X>)

Power transport (Rebuffi *et al.* <http://dx.doi.org/10.1107/S160057752000778X>)

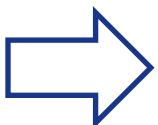


ALS->ALS-U WHAT HAPPENS IF WE USE THE NEW SOURCE WITH AN EXISTING BEAMLINE?

94 x 7 um (806eV)
RP~3000
source: 6.8e14 ph/s/0.1%bw

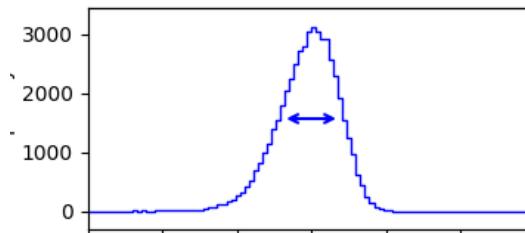
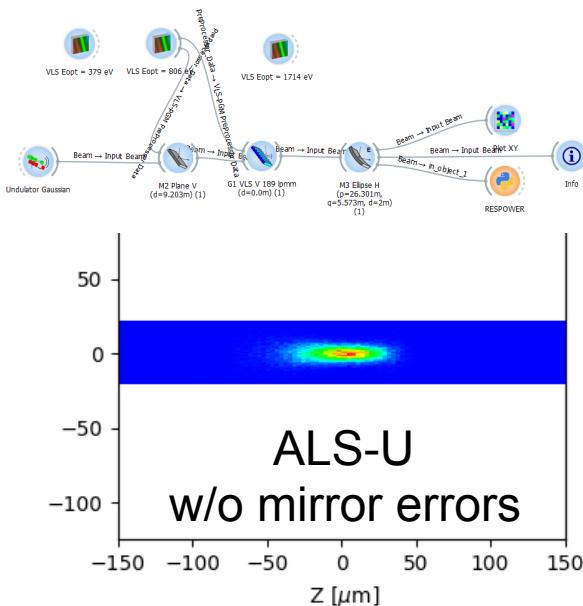


Current source w/slope

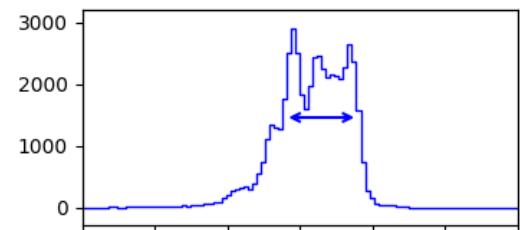
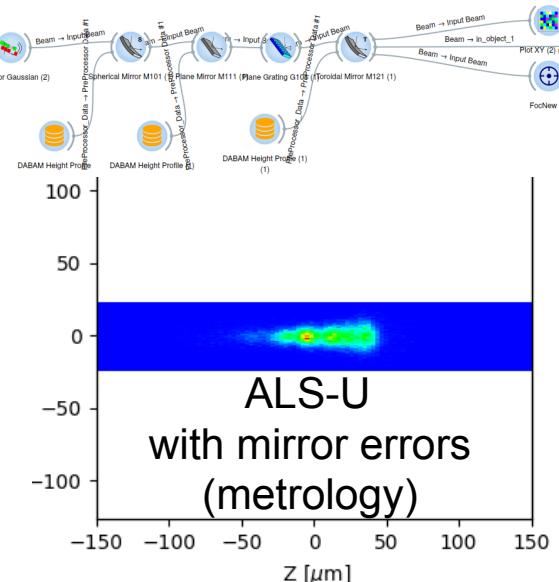


We need a new beamline!!

8.5 x 7 um (806eV)
RP~7000
source: 1.2e15 ph/s/0.1%bw

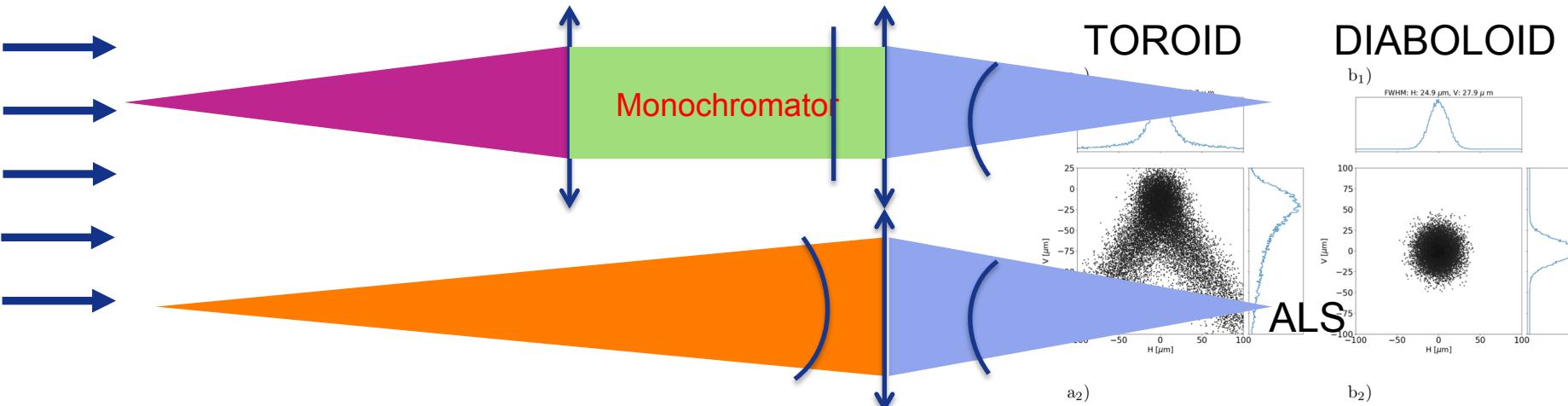


new source no/slope



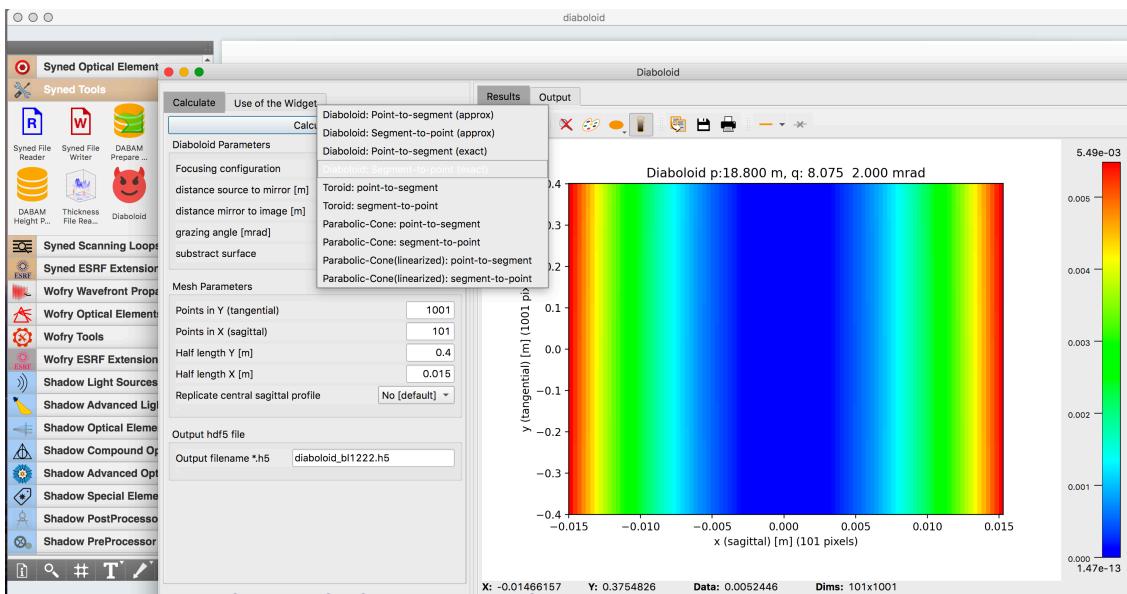
new source w/slope

EXAMPLE OF RAY TRACING: DIABOLOID



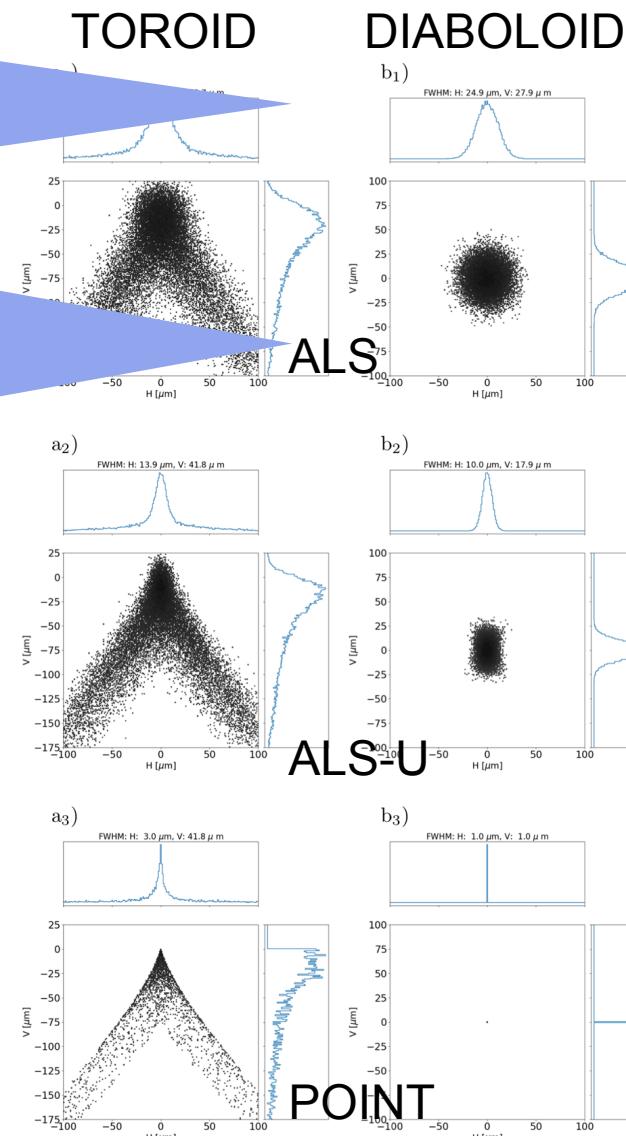
Approximated surface: Toroid (aberrations, good for 3rd SR, not for 4th)

Exact surface: Diaboloid (converts a wavefront from cylindrical to spherical)



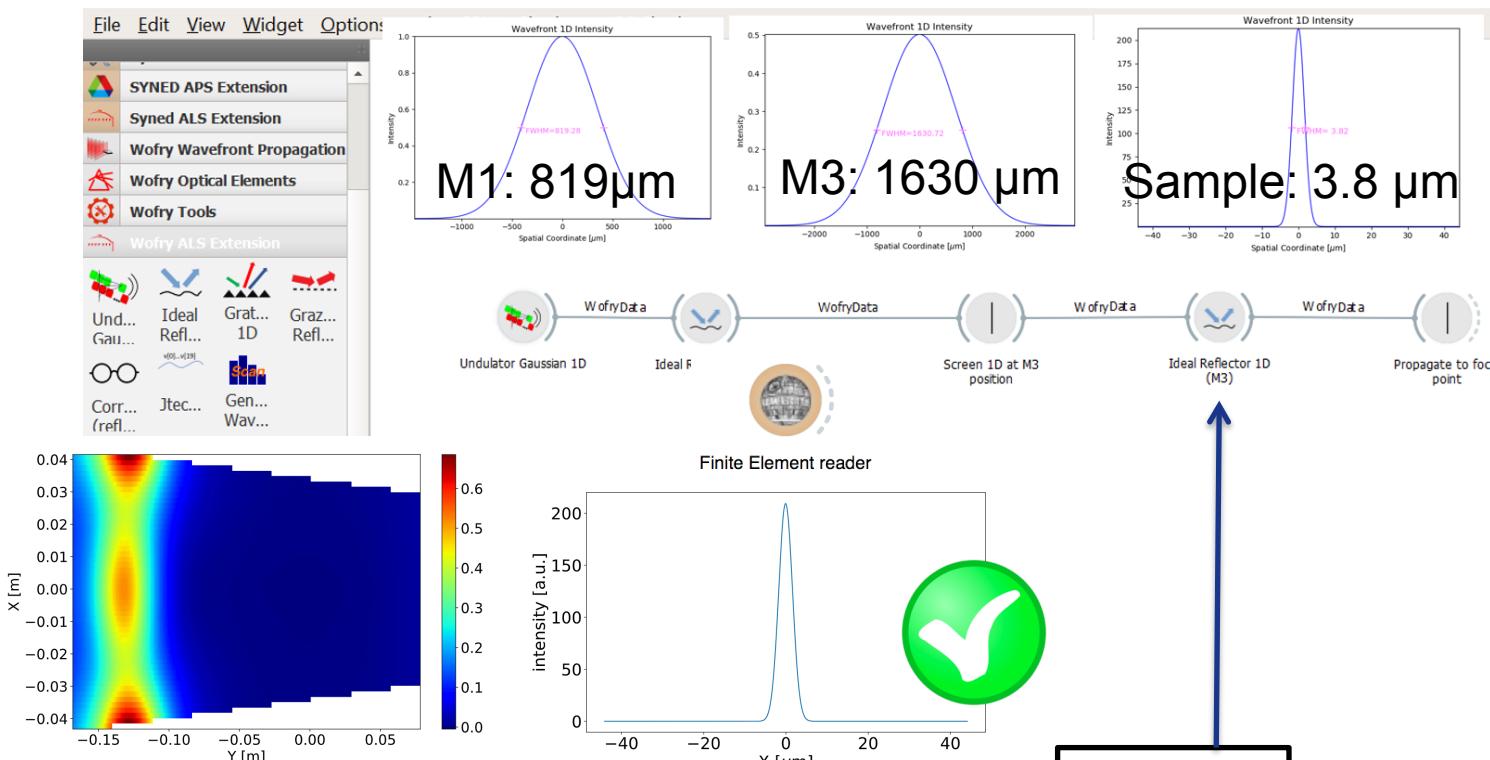
M. Sanchez del Rio et al, JSR 2021 <https://doi.org/10.1107/S160057752100401X>

V. Yashchuk et al. JSR 2021 <https://doi.org/10.1107/S1600577521004860>

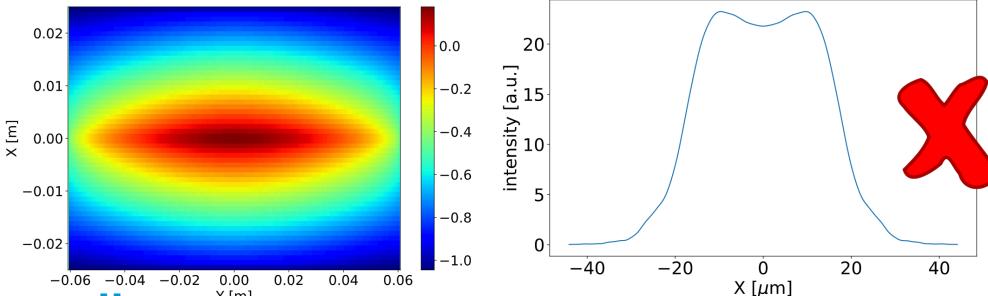


EXAMPLE OF WAVE OPTICS: CORRECTION OF THERMAL DEFORMATION WITH ADAPTIVE OPTICS

Flexon wofry



LN cooling Cutler et al. <http://dx.doi.org/10.1107/S1600577520009522>



Water cooling

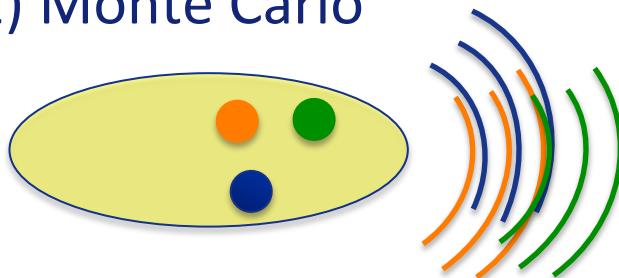
(CF ~ 0.8 with a 4 m undulator at 230 eV)

M. Sanchez del Rio et al, JSR 2020
<http://dx.doi.org/10.1107/S1600577520009522>

Partial coherence (work with several wavefronts)

$$\langle E^*(x_1, y_1) E(x_2, y_2) \rangle = W(x_1, y_1, x_2, y_2)$$

1) Monte Carlo

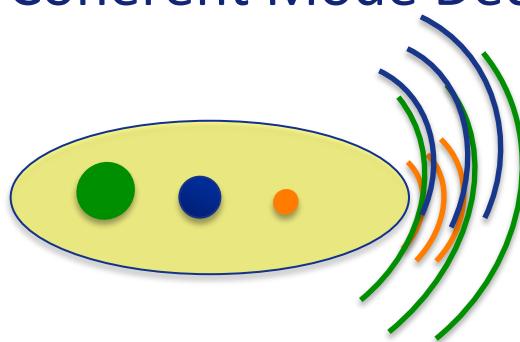


$N_x, N_y \in [100, 1000]$.

$W \sim 10^8 - 10^{12}$ (Gb-Tb)

- 4D W too big to calculate/store
- Not possible to calculate Coherent Fraction

2) Coherent Mode Decomposition



COMSYL (COherent Modes for SYnchrotron Light)
<https://github.com/oasys-kit/comsyl>

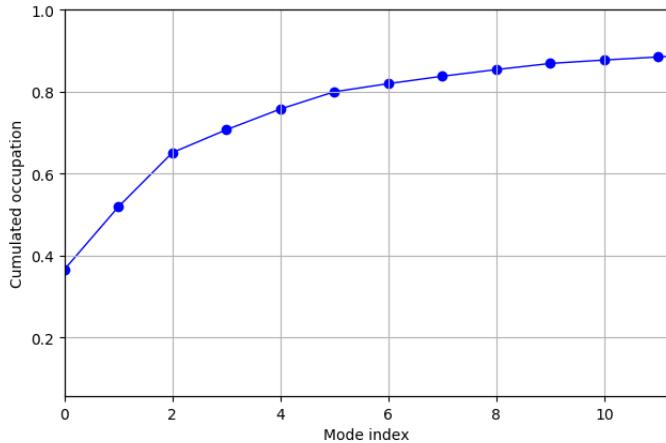
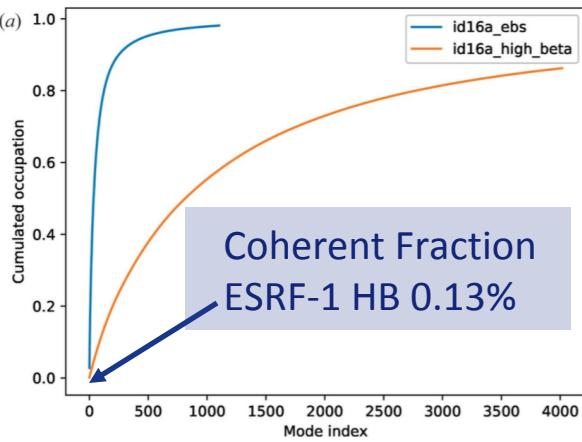
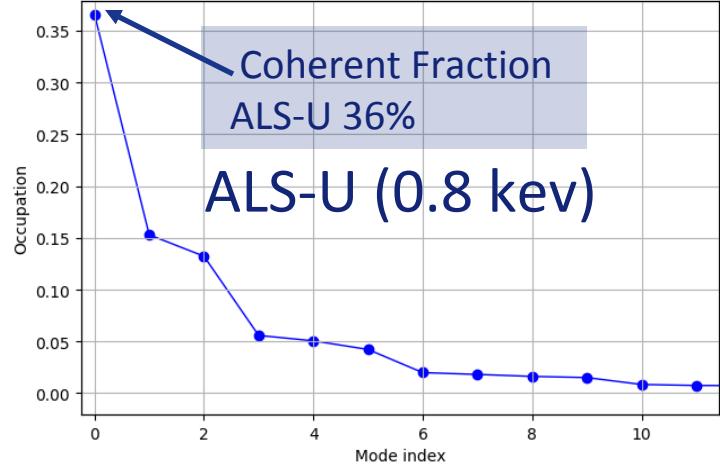
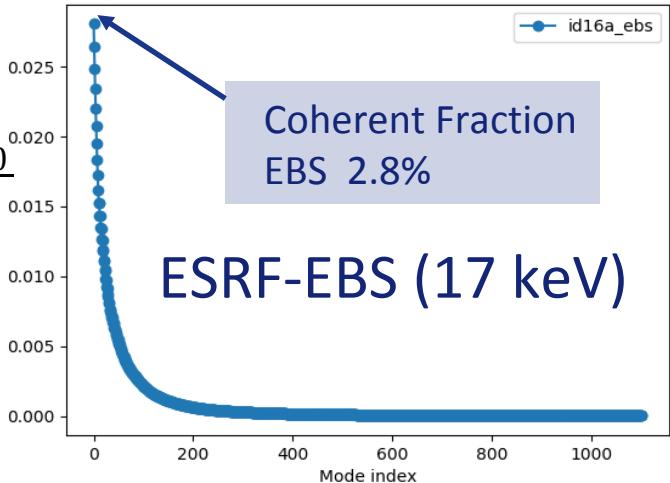
$$\langle E^*(x_1, y_1) E(x_2, y_2) \rangle = W(x_1, y_1, x_2, y_2) = \sum_{i=0}^{\infty} \varphi_i \Phi_i^*(x_1, y_1) \Phi_i(x_2, y_2)$$

Store: $m \times N \times N$; Propagate: 2D integrals

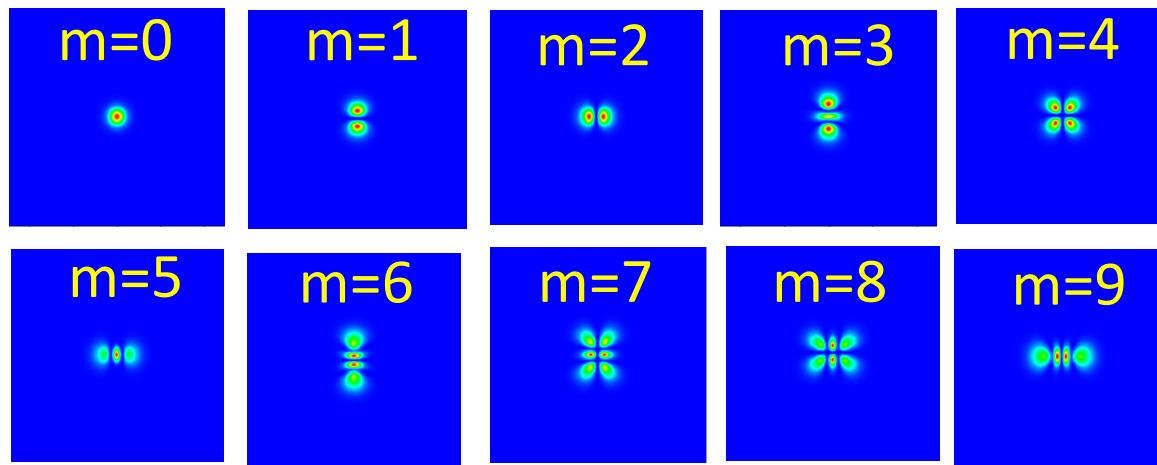
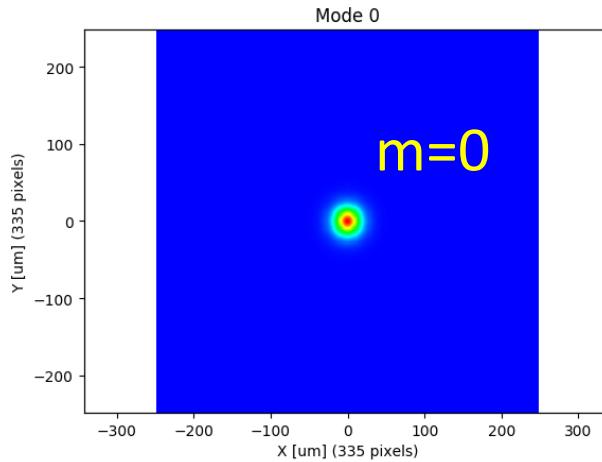
COHERENT MODE DECOMPOSITION: SPECTRUM OF MODES (OCCUPANCY)

$$\langle E^*(x_1, y_1) E(x_2, y_2) \rangle = W(x_1, y_1, x_2, y_2) = \sum_{i=0}^{\infty} \varphi_i \Phi_i^*(x_1, y_1) \Phi_i(x_2, y_2)$$

$$CF = \frac{\varphi_0}{\sum_{j=0}^{\infty} \varphi_j} = \frac{\varphi_0}{I}$$



$$\langle E^*(x_1, y_1) E(x_2, y_2) \rangle = W(x_1, y_1, x_2, y_2) = \sum_{i=0}^{\infty} \varphi_i \Phi_i^*(x_1, y_1) \Phi_i(x_2, y_2)$$



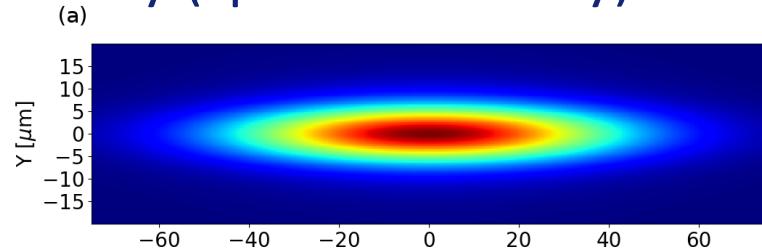
SINGULARITIES IN THE PHASE OF THE CROSS SPECTRAL DENSITY

COMSYL 1.4-m-long U18 EBS (6

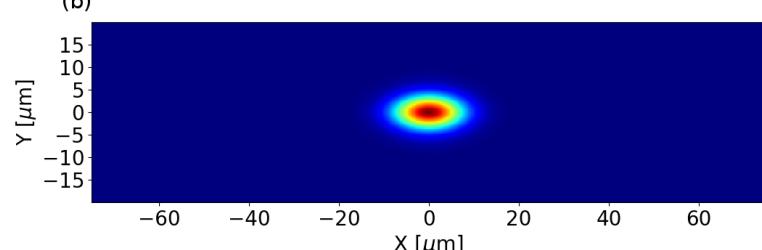
GeV, 147 pm rad emittance)

$E_0 = 17.226\text{keV}$ ($K = 0.411$)

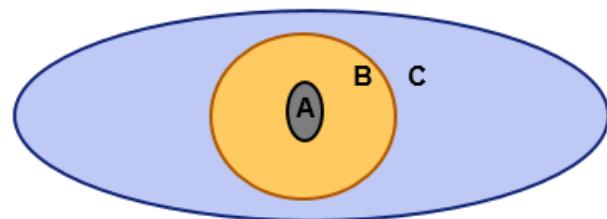
Intensity (spectral density)



First coherence mode



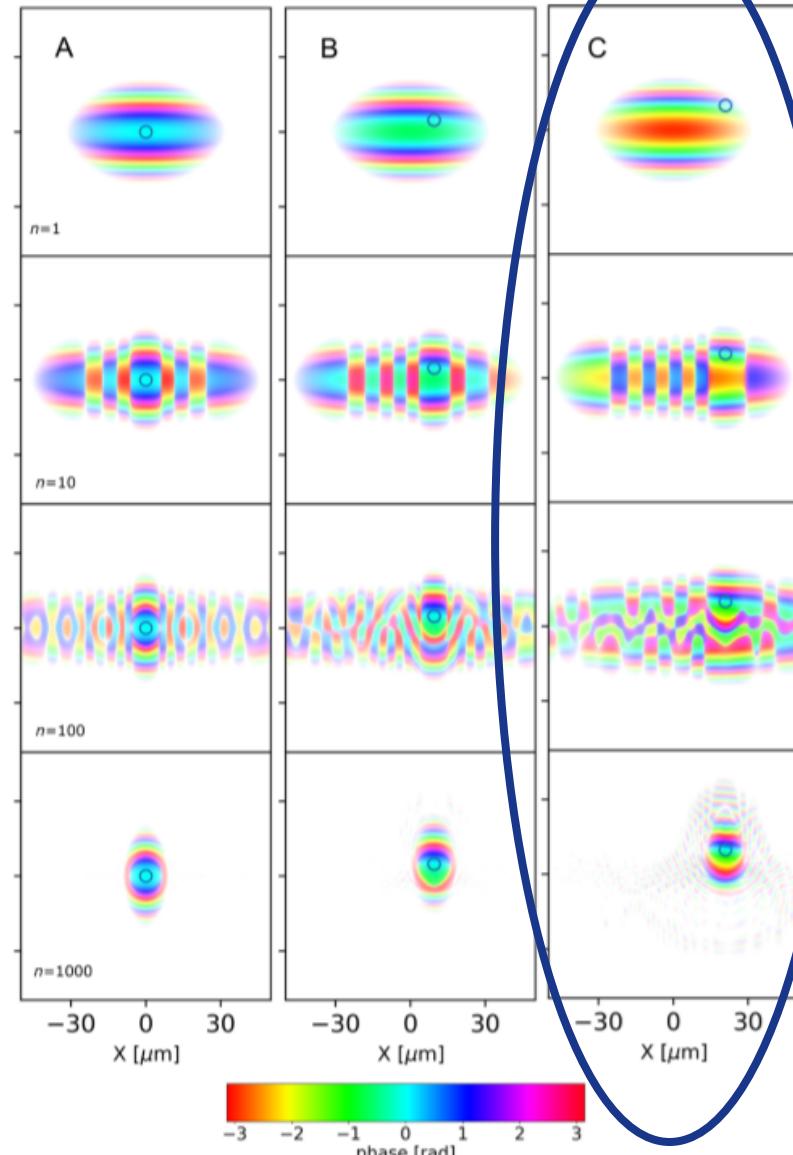
Fix a point



D Paganin, M Sanchez del Rio, Phys Rev A (2019)

<http://dx.doi.org/10.1103/PhysRevA.100.043813>

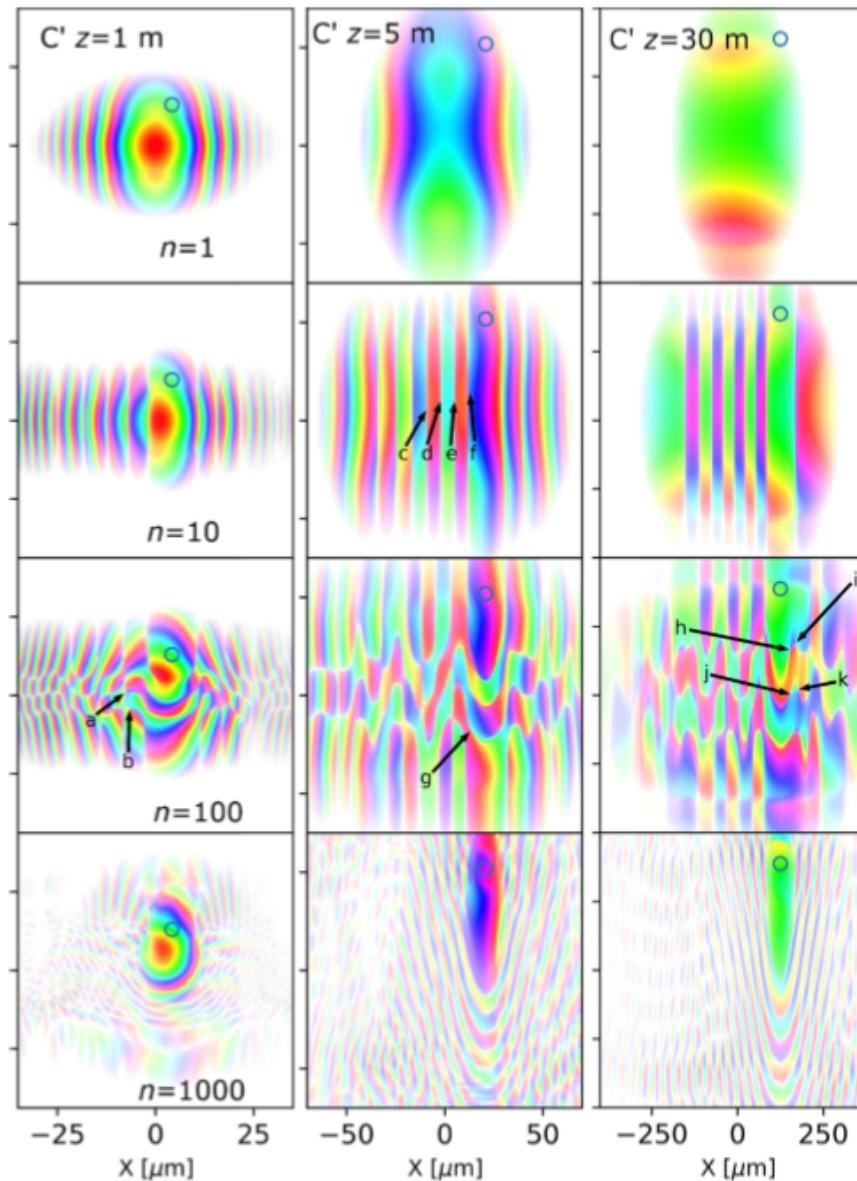
$$\arg[W(x, y, x_P, y_P)]$$



PROPAGATED BEAM: DIFFERENT TYPES OF SINGULARITIES APPEAR

$$\arg[W(x, y, x_P, y_P)]$$

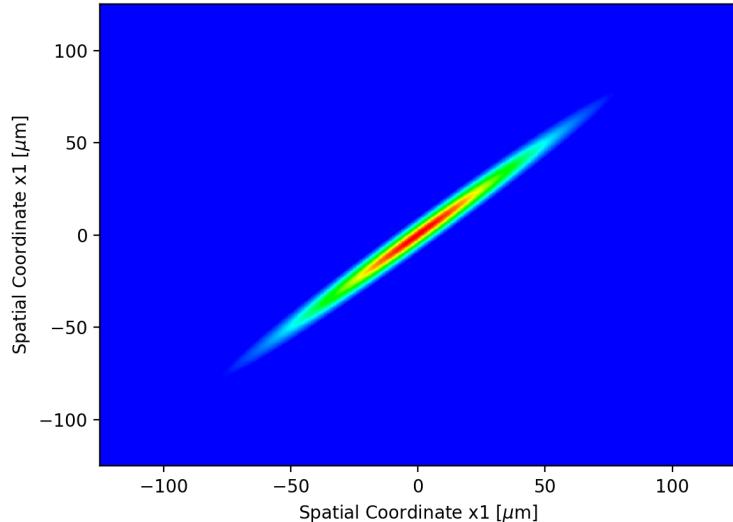
Domain walls



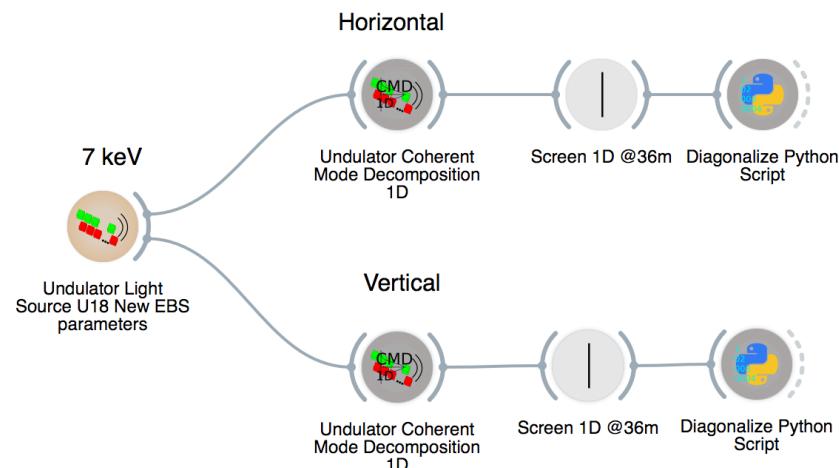
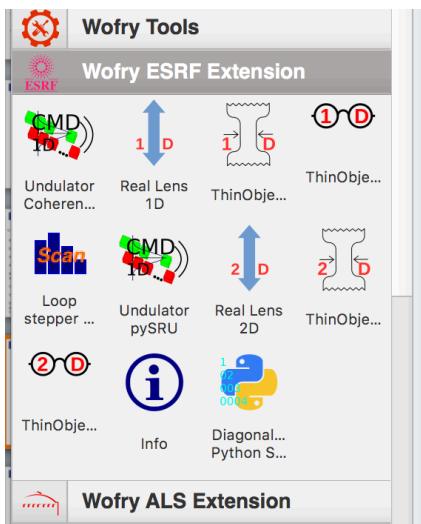
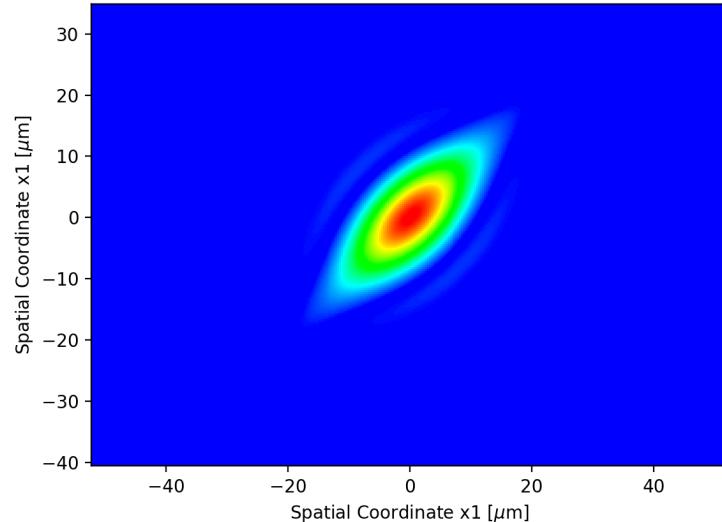
SIMPLER 1D CMD WITH WOFRY (EBS U18 2.5M 7 KEV)

$$\langle E^*(x_1)E(x_2) \rangle = W(x_1, x_2) = \sum_{i=0}^{\infty} \varphi_i \Phi_i^*(x_1) \Phi_i(x_2)$$

Horizontal $W(x_1, x_2)$
Cross Spectral Density

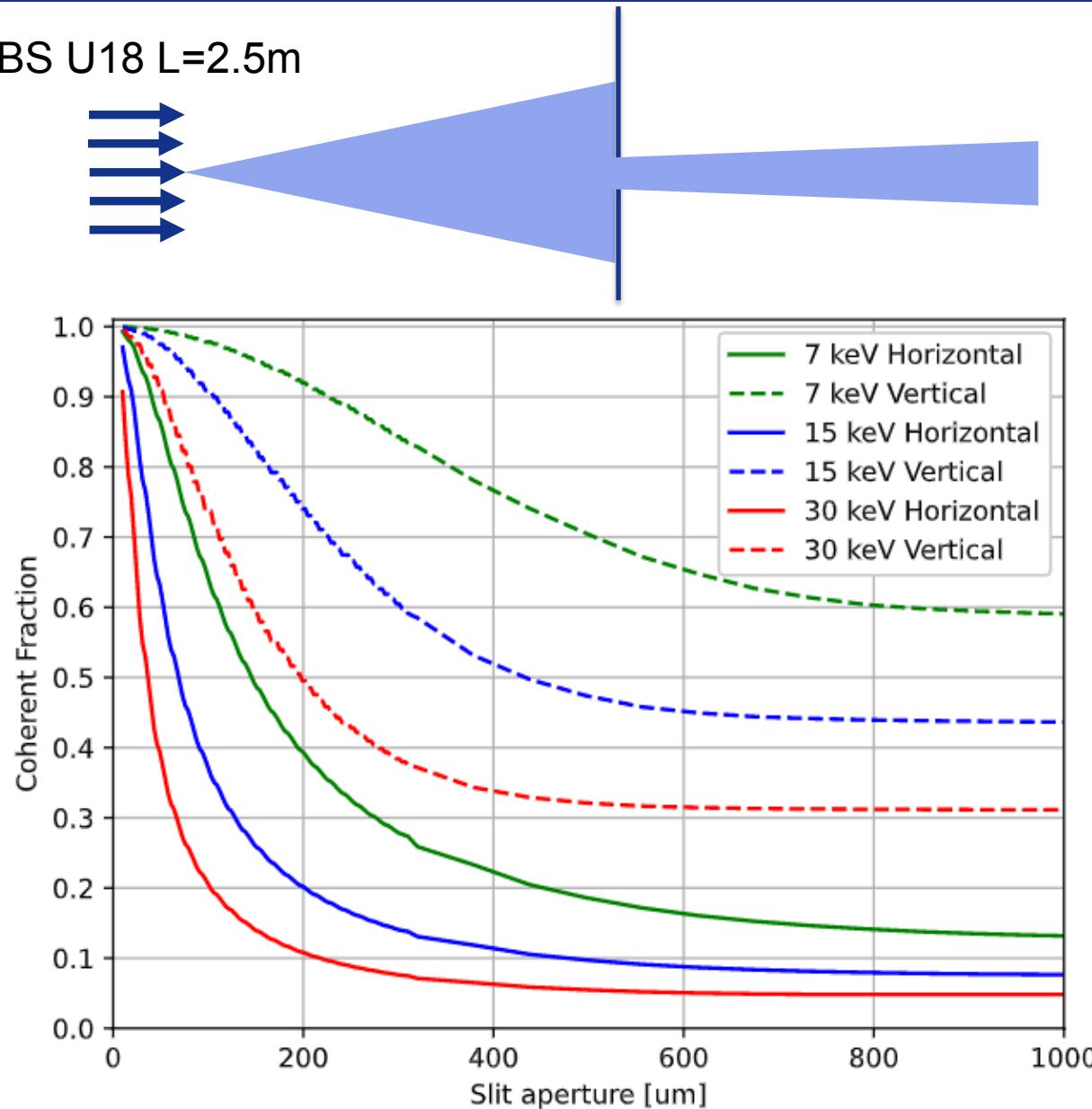


Vertical $W(y_1, y_2)$
Cross Spectral Density



COHERENCE FRACTION MODIFICATION BY SLITS

EBS U18 L=2.5m



P
E
R
F
E
C
T
N
E
S
S

Dynamical
theory of
diffraction

Darwin/
Laue/Ewald/
James/
Zachariasen

Takagi-
Taupin

Zachariasen/
Sears

**Perfect plane crystals (Si, Ge,
Diamond) 3-30 keV**

Laue (transmission) > 30 keV

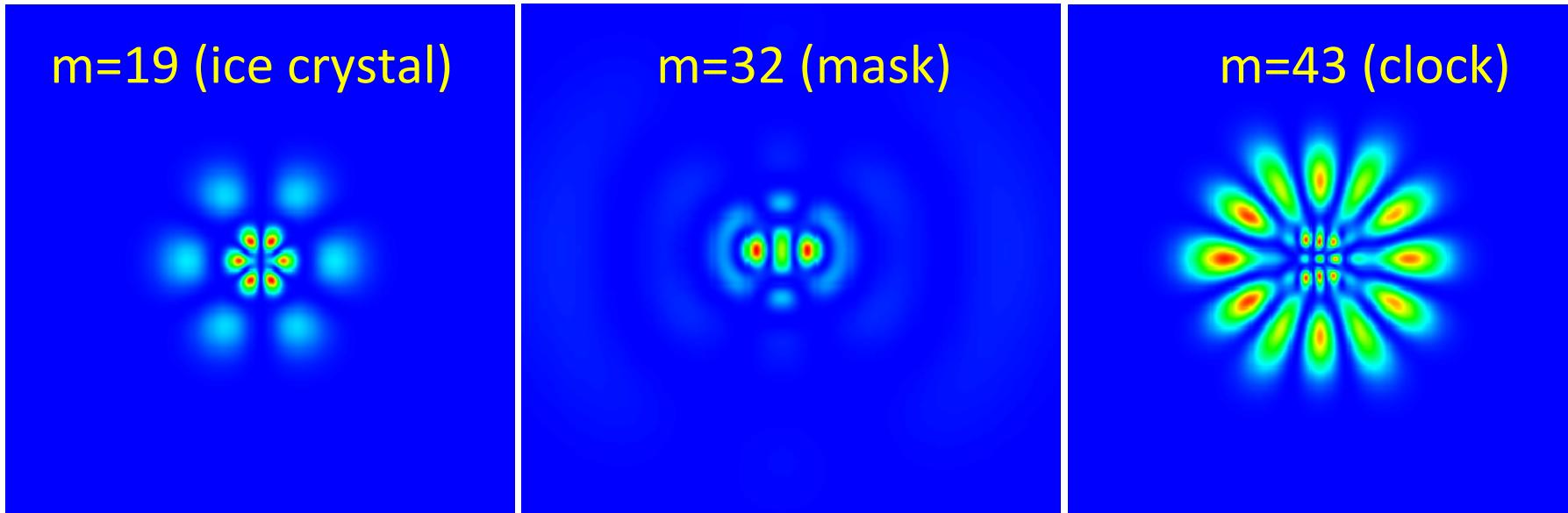
Bent (focusing)

High d-space (Quartz, etc) 2-5 keV

Ideally imperfect crystals (mosaic)

Mosaic crystals

Thank you very much



Many thanks to my colleagues at:

ESRF: J. Reyes-Herrera, R. Celestre, P. Brumund, M Glass
LBNL/ALS: A Wojdyla, H Padmore, K Goldberg, D Cocco, G Cutler
ANL/APS: L Rebuffi, X Shi, R. Reininger, Y Shvyd'ko
Monash Univ.: D Paganin