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Finite temperature effects on nuclear excitations and weak interaction processes

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Nuclear Landscape

Ab initio Configuration interaction **Density Functional Theory**

Non-relativistic models: Skyrme, Gogny

Relativistic models: Nonlinear, meson exchange, point coupling

What can we learn using NEDFs?

- ✓ Static and dynamic properties of atomic nuclei across the nuclide map
- ✓ Nuclear processes and reactions
- ✓ Equation of state of (EOS)

Studying the changes in nuclear properties under extreme conditions is not only important for nuclear physics, but also for some astrophysical events, e.g, r-process and thermonuclear explosions of stars.

What do we need?

Masses, half-lives, decay modes, excitations...

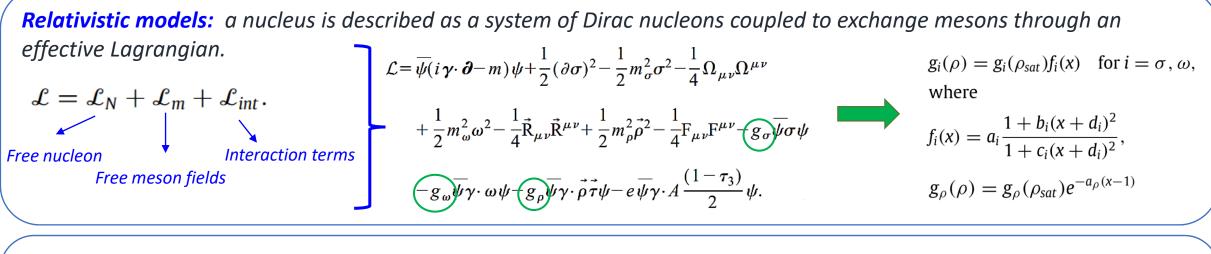
How do nuclear properties, excitations, and weak interaction processes evolve with increasing temperature?



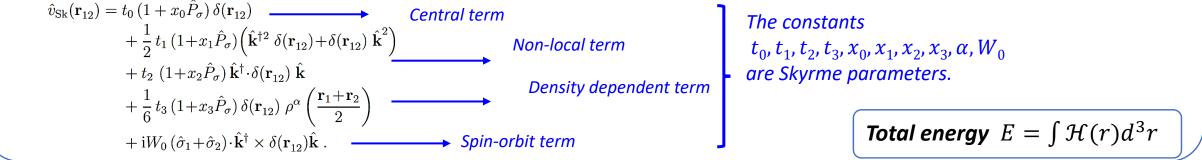
Credit for the figure: Colourful Nuclide Chart (anu.edu.au)

Nuclear energy density functionals (NEDF)

- > Nuclear Energy Density Functionals are successful in the description of static and dynamic properties of nuclei.
- > Interaction between nucleons is represented by a functional which depends on the densities and currents.



Non-Relativistic models: Based on Hamiltonians with effective interactions of nucleons.

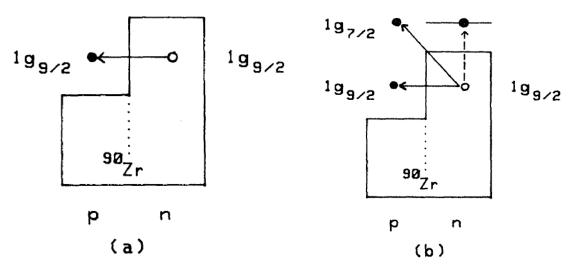


T. H. R. Skyrme, Nuclear Physics 9, 615–634 (1958), P. Ring and P. Schuck, the nuclear many-body problem (1984). T. Nikšic, N. Paar, D. Vretenar, and P. Ring, Comput. Phys. Commun. 185, 1808 (2014).

Spin-isospin excitations

✓ The spin-isospin resonances can be induced by isospin lowering (τ_{-}) or raising (τ_{+}) operators.

- Isobaric analog states (IAS): $\Delta L = \Delta J = \Delta S = 0;$
- Gamow-Teller resonance (GTR): $\Delta L = 0; \Delta J = \Delta S = 1;$
- Spin monopole $(J = 0^{-})$, dipole $(J = 1^{-})$ and quadrupole $(J = 2^{-})$ states.
- ✓ Their properties are important to understand the nuclear structure:
- Spin and isospin properties of the effective nuclear interaction M. Bender et.al., PRC 65, 054322 (2002), H. Liang et.al., PRL 101 (2008) 122502.,
- They can be used to predict neutron skin thickness of nuclei
 - D. Vretenar et.al., Phys.Rev.Lett. 91, 262502 (2003).



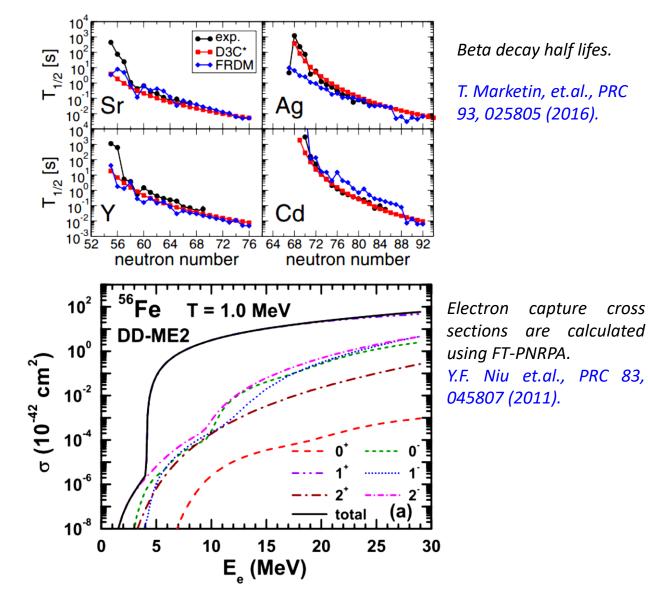
(a) Fermi (b) Gamow-Teller transitions.F. Osterfeld, Rev. Mod. Phys., 64, 491557 (1992).

Spin-isospin excitations

... and nuclear weak interaction processes in stellar \checkmark environments:

- Calculation of the β -decay rates of r-process nuclei J.Engel et.al., Phys. Rev. C 60, 014302 (1999); T. Marketin et.al., Phys. Rev. C 93, 025805 (2015),
- *Electron capture cross sections and rates* ٠ K. Langanke et.al., Phys. Rev. Lett. 90, 241102 (2003); A. L. Cole et.al., Phys. Rev. C86, 015809 (2012).
- and charged-current neutrino-nucleus reactions ٠ N. Paar et.al., Phys. Rev. C 77, 024608 (2008); N. Paar et. al., Phys. Rev. C 87, 025801 (2013).
- ... which take place at finite temperatures.
- ✓ Therefore, accurate calculations are quite *important for the astrophysical calculations.*

HFB(BCS) + *Q(RPA)* are standing as the prominent tools for calculations. *Finite temperature calculations needs extra* work!



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Finite temperature QRPA

The starting point in the Equation of Motion (EOM) method is the definition of a suitable excitation operator

$$\Gamma^{\dagger}_{\nu} = \sum_{a \ge b} X^{\nu}_{ab} a^{\dagger}_{a} a^{\dagger}_{b} - Y^{\nu}_{ab} a_{b} a_{a} + P^{\nu}_{ab} a^{\dagger}_{a} a_{b} - Q^{\nu}_{ab} a^{\dagger}_{b} a_{a}$$
(1)

two-quasiparticle creation/destruction operators and one-quasiparticle creation/ destruction operators. With $|BCS\rangle$ as the approximate thermal vacuum the equation of motion can be written as:

$$\langle BCS | \left[\delta \Gamma, H, \Gamma_{\nu}^{\dagger} \right] | BCS \rangle = E_{\nu} \langle BCS | \left[\delta \Gamma, \Gamma_{\nu}^{\dagger} \right] | BCS \rangle$$
⁽²⁾

The FT-QRPA equations are derived as:

$$\widetilde{A}_{abcd} = \sqrt{1 - f_a - f_b} \underline{A}'_{abcd} \sqrt{1 - f_c - f_d} + (\underline{E}_a + \underline{E}_b) \delta_{ac} \delta_{bd}$$

$$\widetilde{B}_{abcd} = \sqrt{1 - f_a - f_b} \underline{B}_{abcd} \sqrt{1 - f_c - f_d},$$

$$\widetilde{B}_{abcd} = \sqrt{f_b - f_a} \underline{C}'_{abcd} \sqrt{f_d - f_c} + (\underline{E}_a - \underline{E}_b) \delta_{ac} \delta_{bd},$$

$$\widetilde{D}_{abcd} = \sqrt{f_b - f_a} \underline{D}_{abcd} \sqrt{f_d - f_c},$$

$$\widetilde{a}_{abcd} = \sqrt{f_b - f_a} \underline{a}_{abcd} \sqrt{1 - f_c - f_d},$$

$$\widetilde{a}_{abcd} = \sqrt{f_b - f_a} \underline{a}_{abcd} \sqrt{1 - f_c - f_d},$$

$$\widetilde{b}_{abcd} = \sqrt{f_b - f_a} \underline{b}_{abcd} \sqrt{1 - f_c - f_d},$$

H. M. Sommermann, Ann. Phys. (NY) 151, 163 (1983).

E. Yüksel, G. Colò, E. Khan, Y.F. Niu, K. Bozkurt Phys. Rev. C. 96, 024303 (2017).

 $A'_{abcd} = (u_a u_b u_c u_d + v_a v_b v_c v_d) V_{abcd}$

 $C'_{abcd} = (u_a v_b u_c v_d + v_a u_b v_c u_d) V_{a\bar{b}c\bar{d}}$ $+ (u_a u_b u_c u_d + v_a v_b v_c v_d) V_{adbc}$

 $V_{acbd}^{\rm ph}$

+ $(u_a v_b u_c v_d + v_a u_b v_c u_d) V_{a\bar{d}\bar{b}c}$ - $(-1)^{j_c+j_d+J} (u_a v_b v_c u_d + v_a u_b u_c v_d) V_{a\bar{c}\bar{b}d}$

Finite temperature QRPA

The finite temperature QRPA equations can be combined into a single matrix as

$$\begin{pmatrix} \tilde{C} & \tilde{a} & \tilde{b} & \tilde{D} \\ \tilde{a}^{+} & \tilde{A} & \tilde{B} & \tilde{b}^{T} \\ -\tilde{b}^{+} & -\tilde{B}^{*} & -\tilde{A}^{*} & -\tilde{a}^{T} \\ -\tilde{D}^{*} & -\tilde{b}^{*} & -\tilde{a}^{*} & -\tilde{C}^{*} \end{pmatrix} \begin{pmatrix} \tilde{P} \\ \tilde{X} \\ \tilde{Y} \\ \tilde{Q} \end{pmatrix} = \hbar \omega \begin{pmatrix} \tilde{P} \\ \tilde{X} \\ \tilde{Y} \\ \tilde{Q} \end{pmatrix}$$
In the limit $T \rightarrow 0$
$$FT-QRPA \rightarrow QRPA$$

• \tilde{A} and \tilde{B} (and their complex conjugates) describe the effects of two-quasiparticle excitations ($a^{\dagger}a^{\dagger}$ and aa).

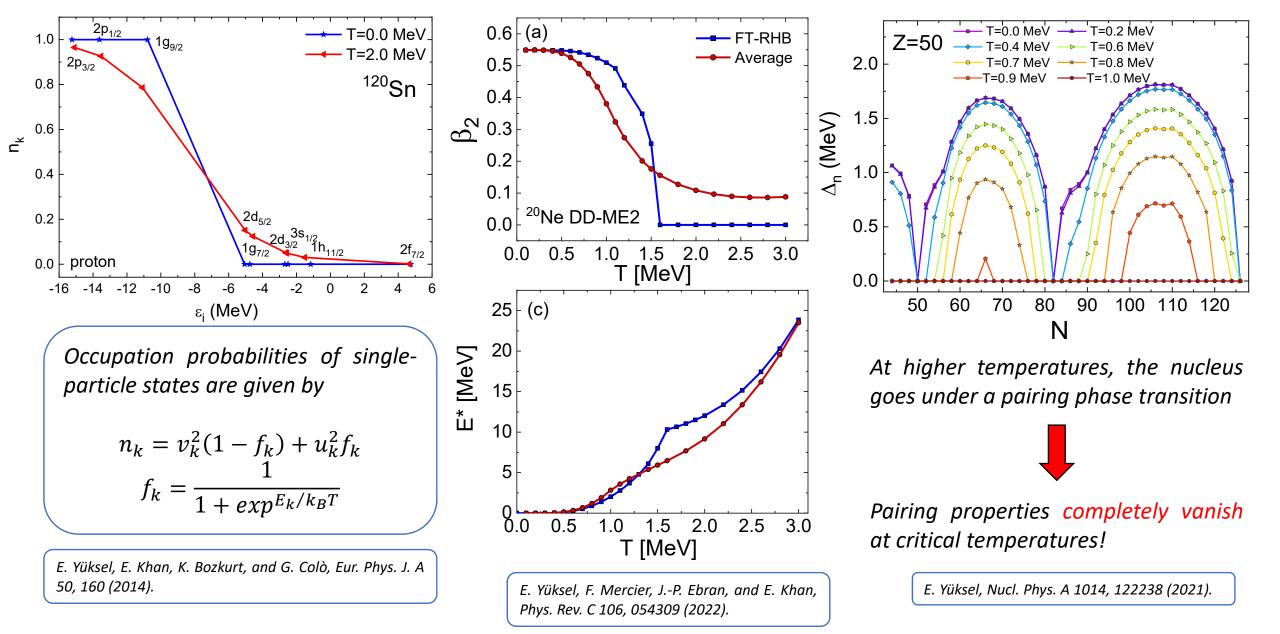
$$\sum_{a\geq b} \left\{ \left| \widetilde{X}_{ab}^{\nu} \right|^2 - \left| \widetilde{Y}_{ab}^{\nu} \right|^2 + \left| \widetilde{P}_{ab}^{\nu} \right|^2 - \left| \widetilde{Q}_{ab}^{\nu} \right|^2 \right\} = 1,$$
(18)

The reduced transition probability is given as

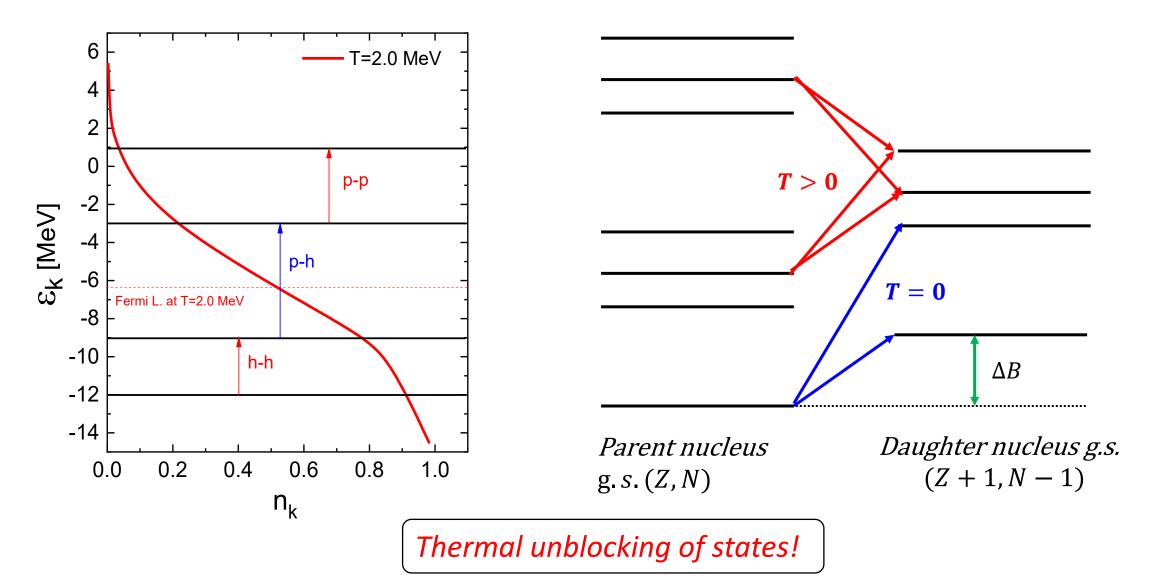
$$B(EJ,\widetilde{0} \to \nu) = \left| \langle \nu || \hat{F}_J || \widetilde{0} \rangle \right|^2$$

=
$$\left| \sum_{c \ge d} \left\{ (\widetilde{X}_{cd}^{\nu} + \widetilde{Y}_{cd}^{\nu}) (v_c u_d + u_c v_d) \sqrt{1 - f_c - f_d} + (\widetilde{P}_{cd}^{\nu} + \widetilde{Q}_{cd}^{\nu}) (u_c u_d - v_c v_d) \sqrt{f_d - f_c} \right\} \langle c || \hat{F}_J || d \rangle \right|^2.$$
(19)

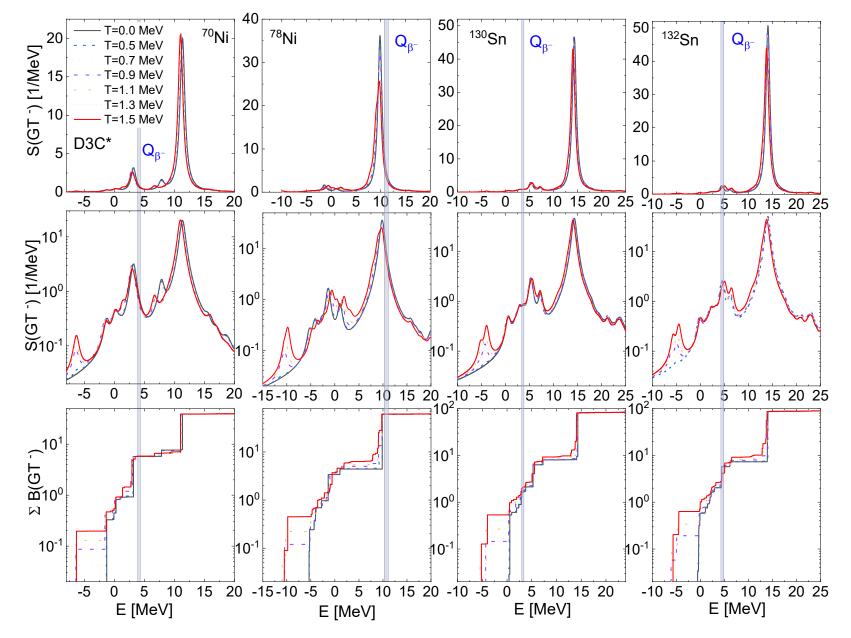
How do nuclei behave at finite temperature?



How do nuclei behave at finite temperature?



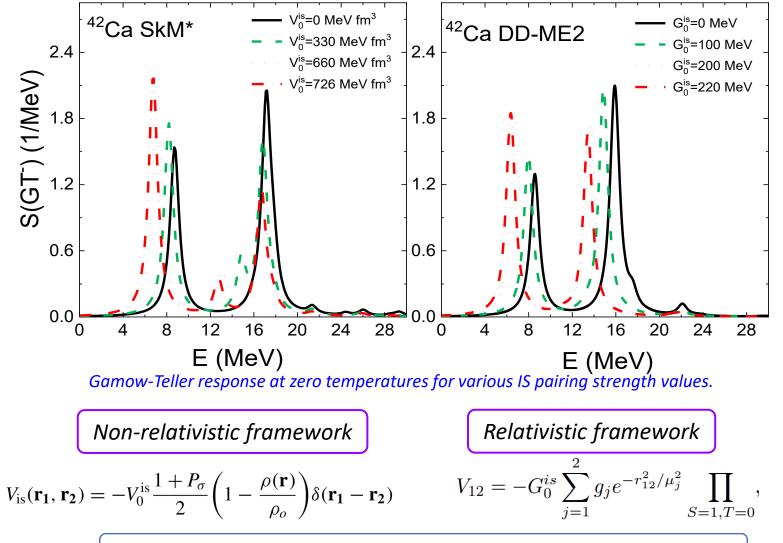
Gamow-Teller response at finite temperature



- ✓ The new excitation channels become possible due to the smearing of the Fermi surface.
- ✓ The changes occur due to the temperature induced effects on the nuclear properties of nuclei and the residual particle-hole interaction.
- ✓ Continuum plays an important role → transitions from thermally unblocked states give rise to an increase in the very low-energy region of the GT strength at finite temperatures.

Impact of Isoscalar pairing in Gamow-Teller states

Gamow-Teller response and isoscalar pairing



E. Yüksel, N. Paar, G. Colò, E. Khan, and Y. F. Niu, Phys. Rev. C 101, 044305, (2020).

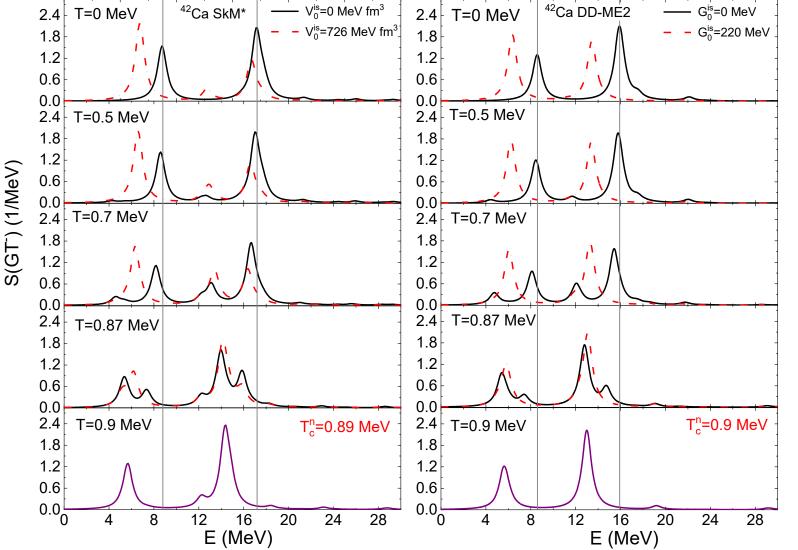
• The isoscalar pairing has an attractive nature...

By increasing the isoscalar pairing strength

- Excitation energies and transition strengths decrease in the GT region
- The excited states start to shift downward, and strength increases in the low-energy region.

w/(wo/) IS pairing (wrt daughter nucleus) SkM* = 0.6 (1.52) MeV; B(GT-)=2.96 (2.41) DD-ME2 = 0.6 (1.36) MeV; B(GT-)=2.36 (2.02) ⁴²Ca Exp: E=0.61 MeV and B(GT-) = 2.7(4) for E < 12 MeV Y. Fujita, et.al, Phys. Rev. C 91, 064316 (2015)

Competition between pairing and temperature effects



E. Yüksel, N. Paar, G. Colò, E. Khan, and Y. F. Niu, Phys. Rev. C 101, 044305, (2020).

Gamow-Teller response for ⁴²*Ca with increasing temperature.*

• Without the isoscalar pairing, the changes in the GT states are caused by the decrease of the isovector pairing effects and the softening of the repulsive ph interaction due to the temperature factors with increasing temperature.

1)
$$T \uparrow \Longrightarrow IV \text{ pairing } \downarrow \Longrightarrow E_{conf} \downarrow$$

2) $T \uparrow \Longrightarrow V_{res} \downarrow \Longrightarrow E_x \downarrow$

- With the inclusion of the isoscalar pairing, the decrease in the excitation energies depends on the competition between the temperature and isoscalar pairing effects.
- The temperature reduces the impact of the both isoscalar and the isovector pairing.

Temperature dependence of weak interaction processes

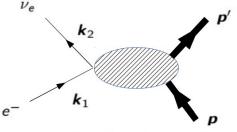
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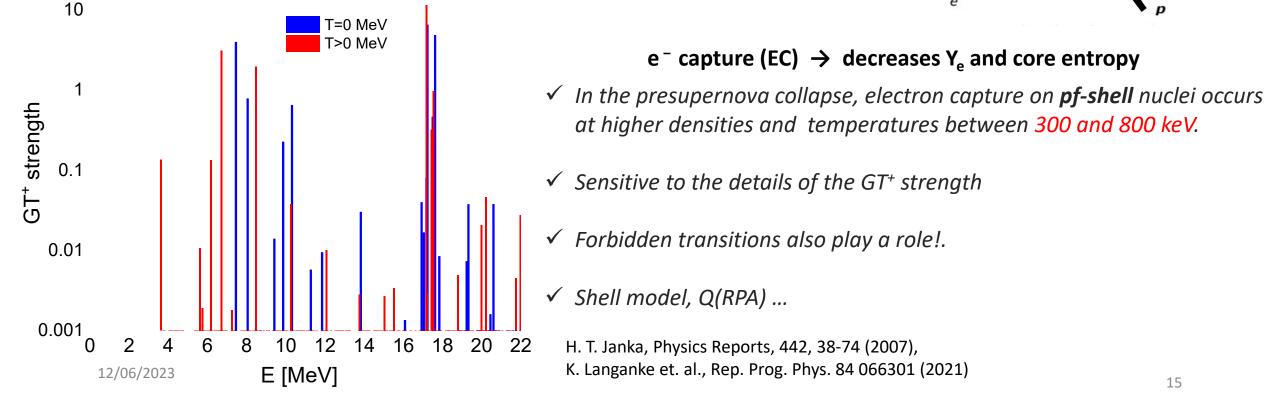
Electron capture in stellar environments

- \checkmark Dynamics of core-collapse supernovae are determined by electron-to-baryon ratio Y_e and the core entropy.
- These two parameters are mainly determined by the weak interaction processes in nuclei, in particular electron capture and β-decay.

The electron capture on nuclei is a weak interaction process

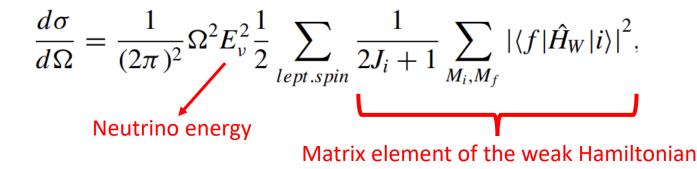
$$(Z, N) + e^- \to (Z - 1, N + 1) + \nu_e$$





Electron capture in stellar environments

✓ In order to derive the electron-capture cross section, we start from the Fermi golden rule



✓ Final expression written in terms of charge \hat{M}_J , longitudinal $\hat{\mathcal{L}}_J$, transverse electric \hat{T}_J^{el} and transverse magnetic \hat{T}_J^{mag} operators for multipole J^{π} .

✓ Electron capture rates are calculated with
$$\lambda_{ec} = \frac{1}{\pi^2 \hbar^3} \int_{E_e^0}^{\infty} p_e E_e \sigma_{ec}(E_e) f(E_e, \mu_e, T) dE_e$$

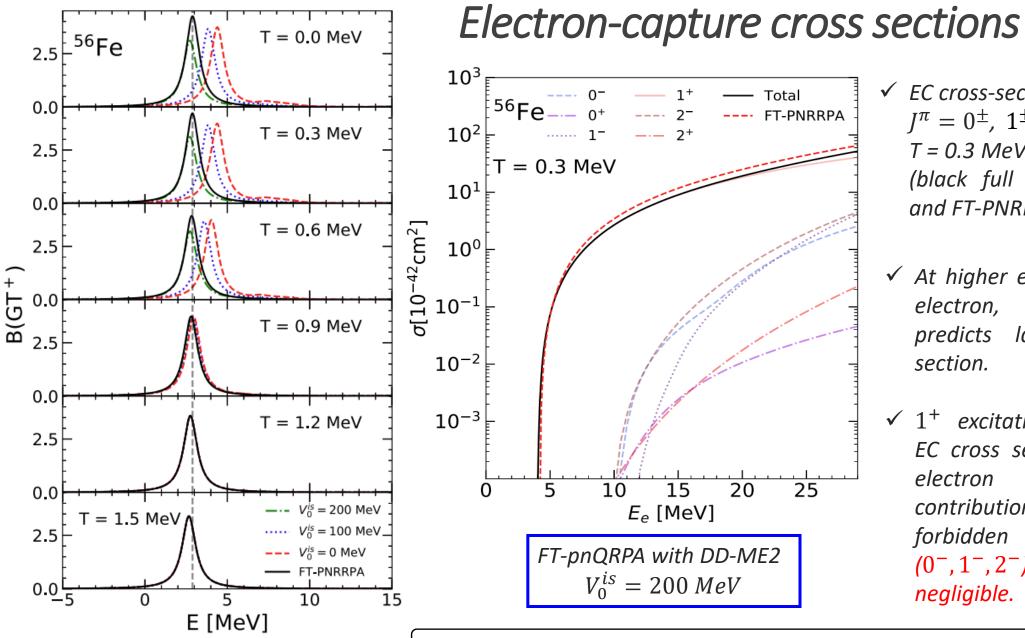
✓ The energy of the outgoing neutrino

$$E_{\nu} = E_e - E_{QRPA} - \Delta_{np} - (\lambda_n - \lambda_p)$$

✓ E_e^0 - threshold energy ✓ μ_e - electron chemical potential ✓ T - temperature

$$p_e = \sqrt{E_e^2 - m_e^2}$$

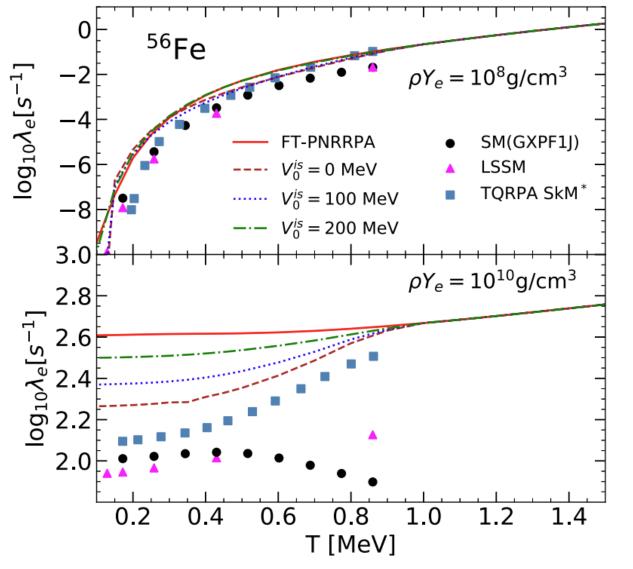
Evolution of electron capture in stellar environments?



- ✓ EC cross-section σ_{EC} on ⁵⁶Fe for $J^{\pi} = 0^{\pm}$, 1^{\pm} , 2^{\pm} multipoles at T = 0.3 MeV using FT-PNRQRPA (black full line + multipoles) and FT-PNRRPA.
- ✓ At higher energies of incident electron, the FT-PNRRPA predicts larger total cross section.
- ✓ 1⁺ excitation dominates the EC cross sections up to high electron energies where contributions from the forbidden multipoles (0⁻, 1⁻, 2⁻) become also nonnegligible.

A. Ravlić, E. Yüksel, Y. F. Niu, G. Colò, E. Khan, and N. Paar, Phys. Rev. C 102, 065804 (2020).

Stellar electron-capture rates

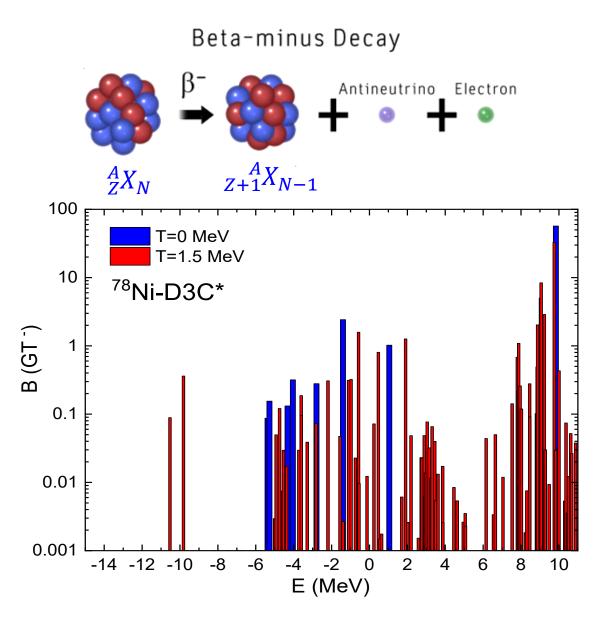


✓ Electron capture rates λ_e for ⁵⁶Fe with respect to temperature **T** for densities $\rho Ye = 10^8$ and 10^{10} g/cm^3 . FT-pnQRPA with DD-ME2

 $\rho Ye = \mathbf{10^8} \rightarrow Low \ \lambda_e \ (\approx 2 \ MeV) \rightarrow low \ EC \ rates$ $\rho Ye = \mathbf{10^{10}} \rightarrow high \ \lambda_e \ (\approx 10 \ MeV) \rightarrow high \ EC \ rates$

- ✓ Very good agreement with the LSSM calculations and the shell-model calculations with GXPF1J interaction in the whole temperature range.
- ✓ The EC rates calculated with the FT-PNRQRPA display weak dependence on temperature for high stellar densities.
 - A. Ravlić, E. Yüksel, Y. F. Niu, G. Colò, E. Khan, and N. Paar, Phys. Rev. C 102, 065804 (2020).

Beta-decay half-lives at finite temperatures



- \checkmark Allowed Gamow-Teller transitions mainly determine the β decay half-lives.
- ✓ Transitions below the *Q*-value play a major role.
- \checkmark The general form of β -decay rate in stellar conditions is

$$\lambda = \frac{\ln 2}{K} \int_0^{p_0} p_e^2 (W_0 - W)^2 F(Z, W) C(W) [1 - f(W)] dp_e$$

 \checkmark For the allowed GT transitions, C(W) is equal to the reduced matrix element of the GT- transition

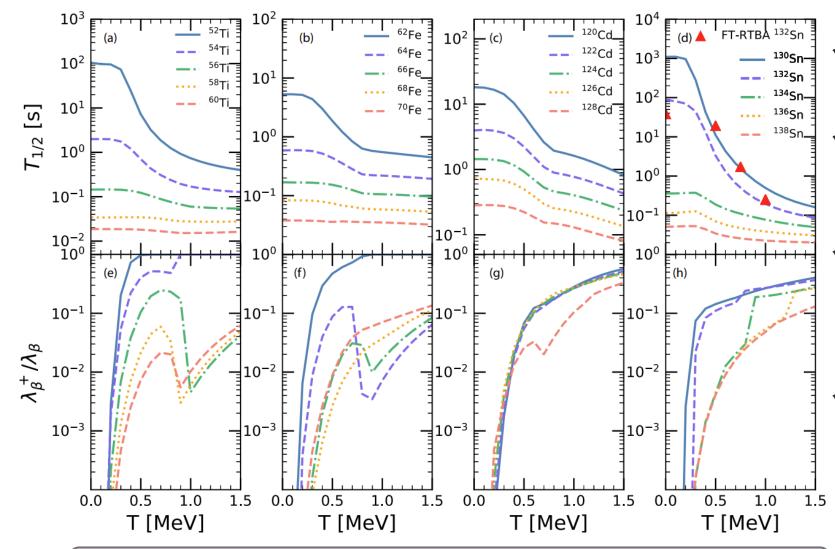
 $\rightarrow -1.0$

$$B(\mathrm{GT}^{-}) = g_A^2 \frac{|\langle f \| \boldsymbol{\sigma} \boldsymbol{\tau}_{-} \| i \rangle|^2}{(2J_i + 1)} \qquad \begin{array}{c} \text{axial coupling} \\ g_A = -1.26 \rightarrow -1 \\ \text{"quenching"} \end{array}$$

- ✓ The excited states shifts downwards
- \checkmark New excited states are obtained due to the unblocking mechanism of temperature.
- \checkmark β-decay phase space increases → half-lives decrease.

For the general form of reaction rate: T. Marketin et al., Phys. Rev. C, 93, 025805, (2016)

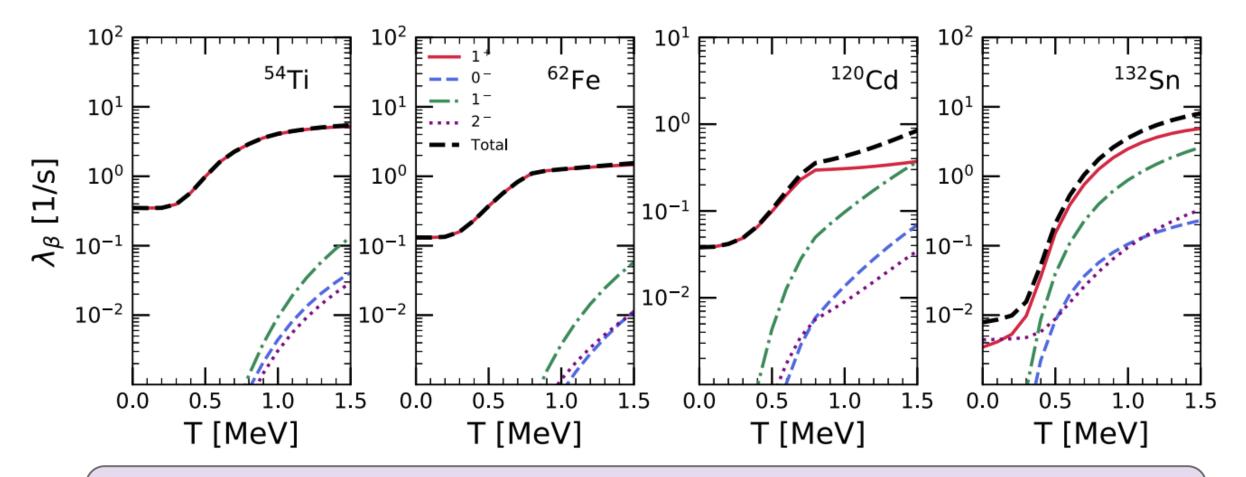
β-decay half-lives in stellar environments



- ✓ T = 0-1.5 MeV and stellar density is fixed to ρ Ye = 10^7 g/cm³.
- Nuclei with long β-decay half-lives at zero temperature are impacted more
- ✓ Transitions from highly excited initial states with negative transition energy (de-excitations) start to play a role already at T ≈ 0.3 MeV
- ✓ Its contribution increases for all considered nuclei with increasing temperature.

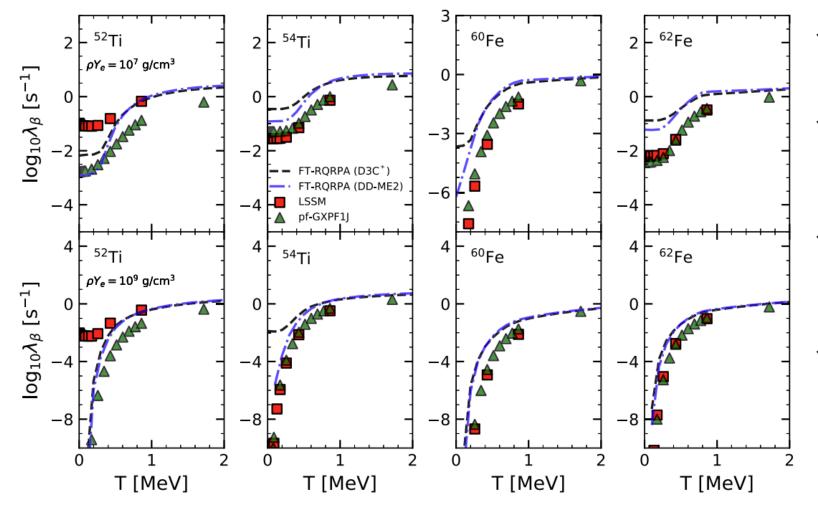
Total β -decay half-lives $T_{1/2}$ for Ti, Fe, Cd, and Sn isotopes as a function of temperature, including negative-energy transitions (de-excitations).

β-decay half-lives in stellar environments



Temperature dependence of β -decay rates λ_{β} for both allowed (1⁺) and first-forbidden transitions (0⁻, 1⁻, 2⁻) together with their total sum (Total) for ⁵⁴Ti, ⁶²Fe, ¹²⁰Cd, and ¹³²Sn. Calculations are performed at stellar density $\rho Ye = 10^7 \ g/cm^3$

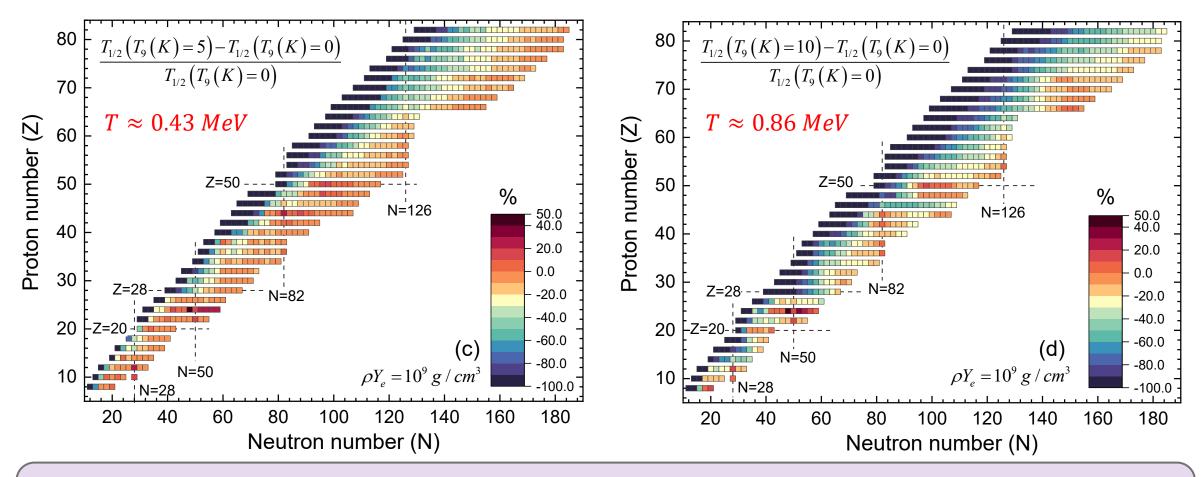
Comparison of β -decay half-lives with different models



- β-decay rates significantly decrease with increasing density.
- ✓ Using both D3C* and DD-ME2 interactions are in good agreement with both shell-model results
- ✓ Trends of increasing rates with increasing temperature are well reproduced.
- The agreement is better at higher temperatures

- LSSM (red squares): K. Langanke and G. Martínez-Pinedo, At. Data Nucl. Data Tables 79, 1 (2001).
- Shell-model calculations based on the pf-GXPF1J interaction (green triangles): K. Mori, M. A.et.al., Astrophys. J. 833, 179 (2016), T. Suzuki (private communication).

Large scale calculations for even-even nuclei



In the majority of the nuclide map, a decrease in the β-decay half-lives is obtained with increasing temperature.
 Nuclei showing the most change are those with initially long half-lives (magic, semi-magic, and close to the valley of stability).

Conclusions

We developed self-consistent Finite temperature (pn)QRPA: includes both pairing and temperature

Using both relativistic (DD-ME2, D3C^{*}) and nonrelativistic Skyrme-type EDFs.

Spin-isospin excitations at finite temperature

As T increases \rightarrow Unblog

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s in the low-energy region A proper description of nuclei and weak nuclear Impact of the attracti nd pairing correlations is weak interaction processes in stellar environments decisive on the location requires self-consistent calculations that include β-decay half-lives in st In the majority of the **both pairing and finite temperature effects....** ing temperature. Nuclei showing the house and close to the valley of

stability). *Forbidden transitions starts to play a role with increasing temperature.* ۲

Negative energy transitions (de-excitations) are essential for proper description, and they already start to play a role at $T \approx 0.3$ MeV.

Perspectives on the future of work

- M1 excitations at FT?
- *Proper treatment of nuclei at finite temperatures: exact canonical treatment of pairing problem.*
- Inclusion of beyond-mean field effects?
- Odd-nuclei?
- Large-scale calculations at Finite Temperature (A. Ravlic, E. Yüksel, T. Niksic, N. Paar)

In Collaboration with

INFŃ



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