COEX7

Density dependence of pairing functionals for the rotational excitations in neutron-rich nuclei

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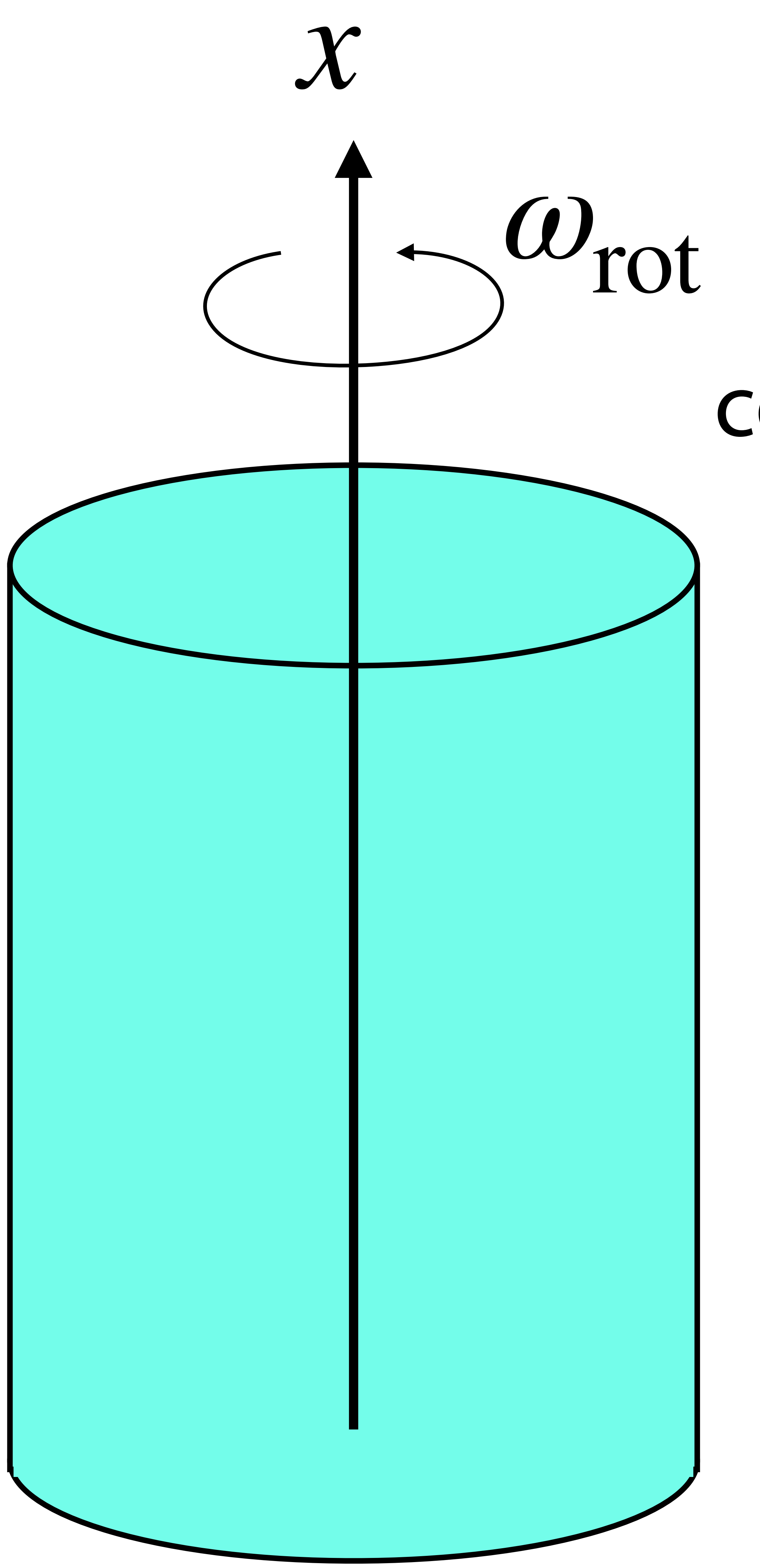
Rotating superfluid



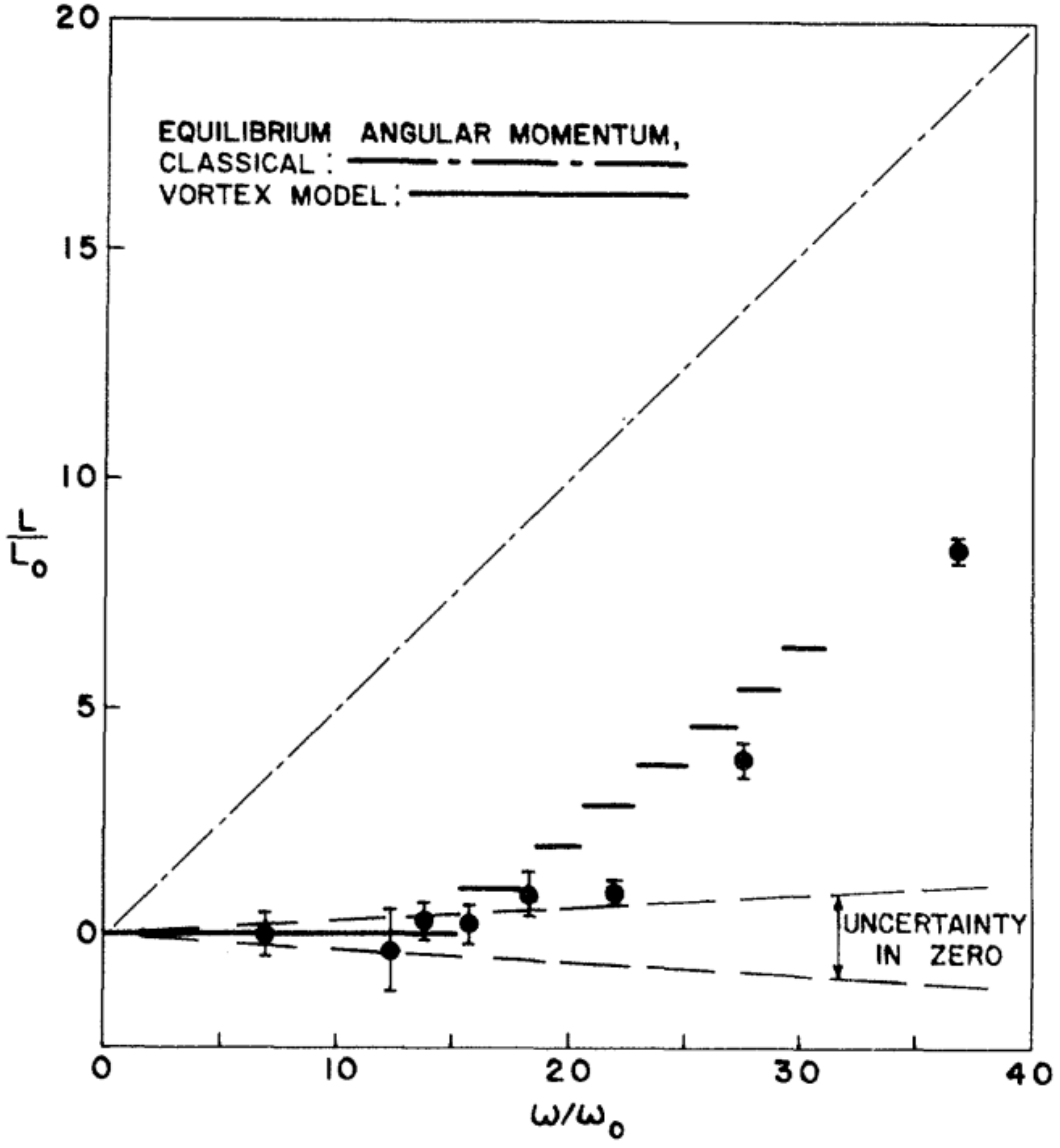
container with helium: $T > T_c$

rotating *slowly* with ω_{rot}

$$L_x = \mathcal{J}_{cl}\omega_{rot}$$

cool the liquid below T_c

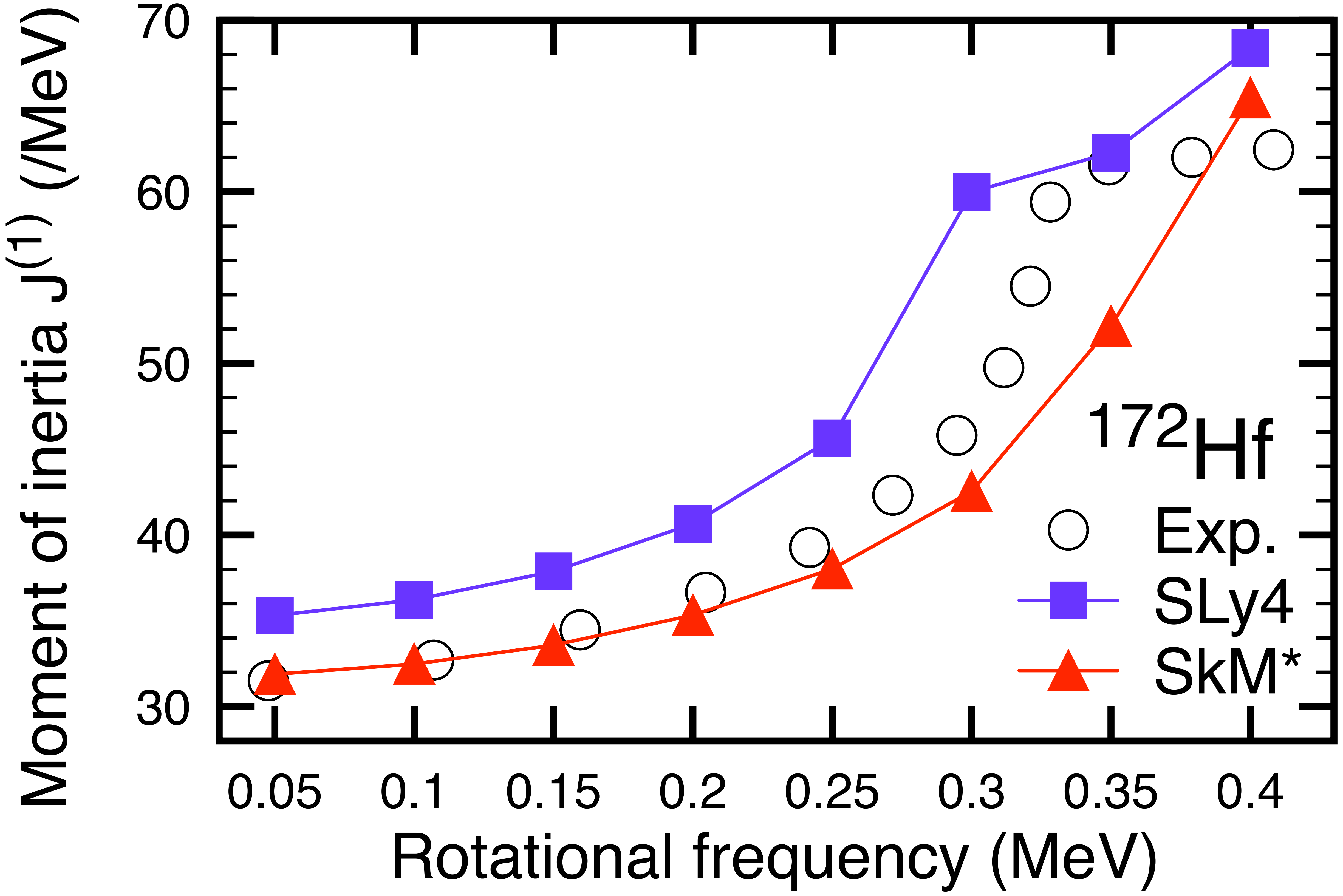
$$L_x = f_{normal}(T)\mathcal{J}_{cl}\omega_{rot}$$



Hess–Fairbank effect (1967)

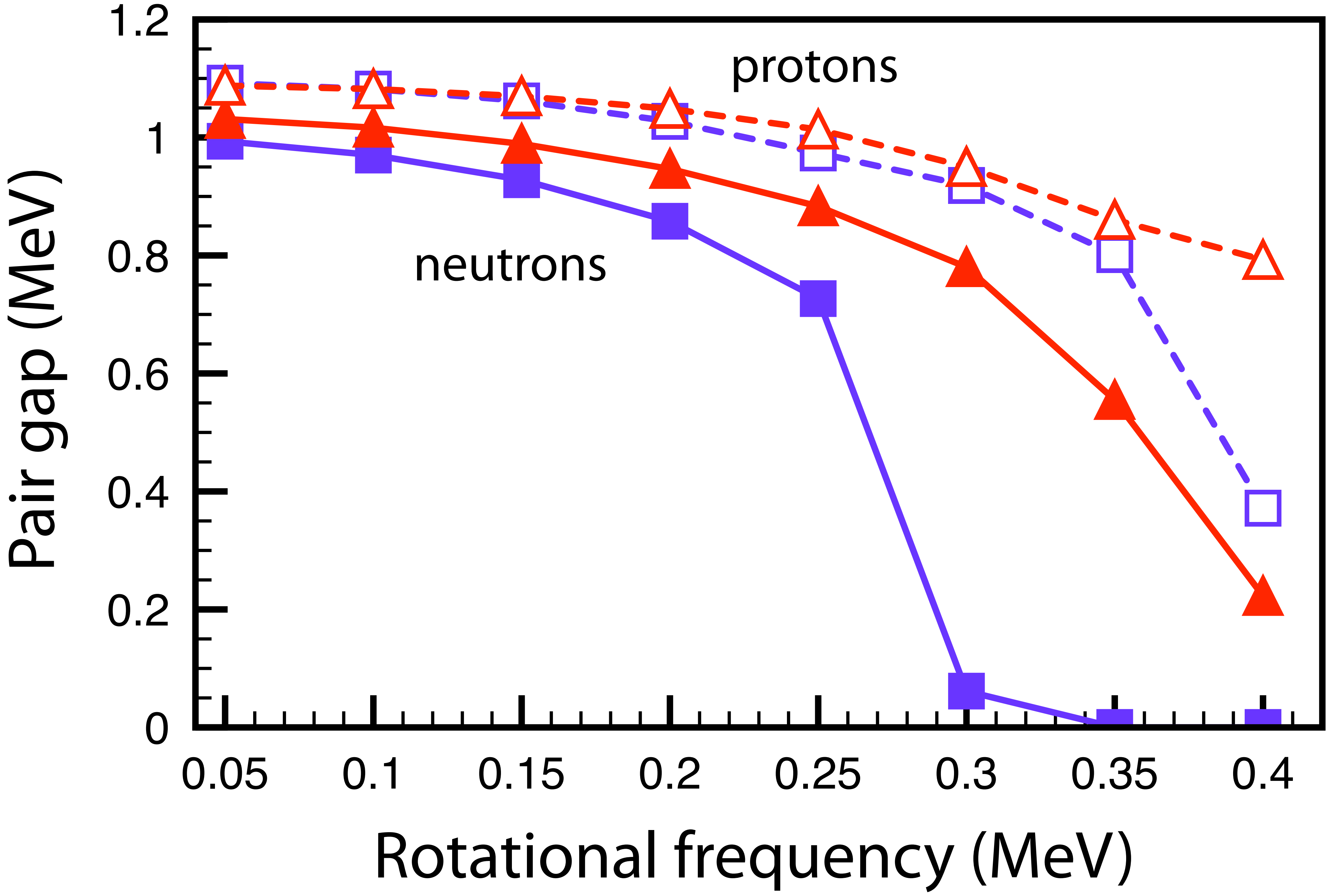
Pairing in rotating nuclei

KY, PRC105(2022)024313



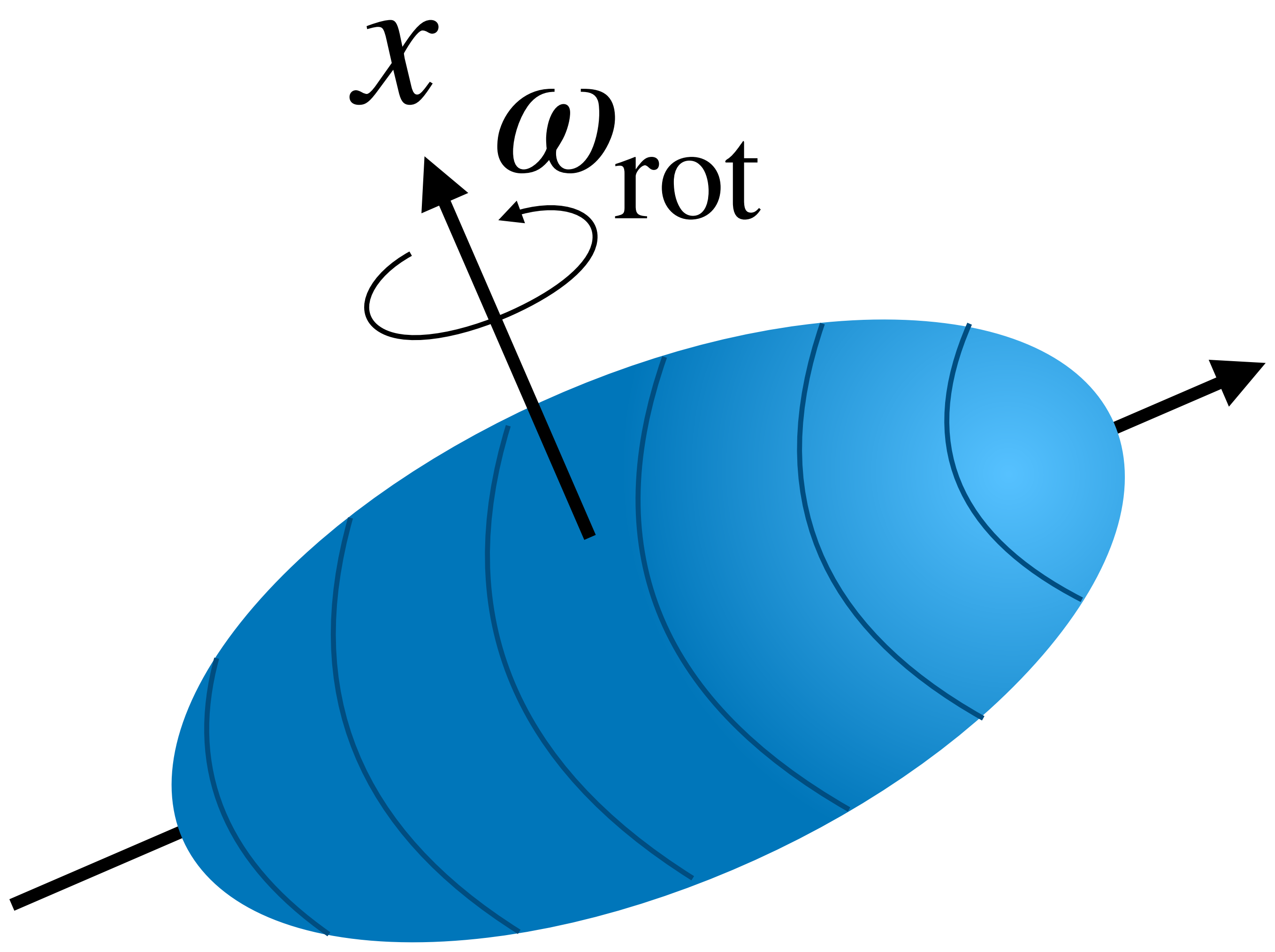
"alignment" at $\omega_{\rm rot} \sim 0.3$ MeV

breaking of the Cooper pairs



pairing plays a key role

Role of pairing in the moment of inertia



$$E = E_0 + \frac{1}{2}\mathcal{J}\omega_{\rm rot}^2$$

Inglis formula: perturbation $\omega_{\rm rot} \ll 1$

$$\mathcal{J}_{\rm I} = 2\sum_{ph} \frac{|\langle m|J_x|i\rangle|^2}{e_m - e_i}$$

$$E_\alpha = \sqrt{(e_\alpha - \lambda)^2 + \Delta^2} > |e_\alpha - \lambda|$$

$$0 \le u_\alpha, v_\alpha \le 1$$

Belyaev formula including pairing

$$\mathcal{J}_{\rm B} = 2\sum_{\alpha\beta} \frac{|\langle \alpha|J_x|\beta\rangle|^2}{E_\alpha + E_\beta}(u_\alpha v_\beta - v_\alpha u_\beta)^2 \le \mathcal{J}_I$$

Nuclear density functional theory (DFT)

Nuclear EDF $E[\rho, \tilde{\rho}, \tilde{\rho}^*]$: Skyrme, Gogny, covariant,…

Kohn–Sham–Bogoliubov–de Gennes (KSB or HFB)

for the equilibrium configuration

Bulgac(1980), Dobaczewski+(1984)
Oliveira+(1988)

$$\delta(E[\rho, \tilde{\rho}, \tilde{\rho}^*] - \sum_q \lambda^q \langle \hat{N}_q \rangle) = 0$$

$$(\mathscr{H}^q_{\text{HFB}} - \lambda^q \mathscr{N})\Phi^q_\alpha(x) = E^q_\alpha \Phi^q_\alpha(x)$$

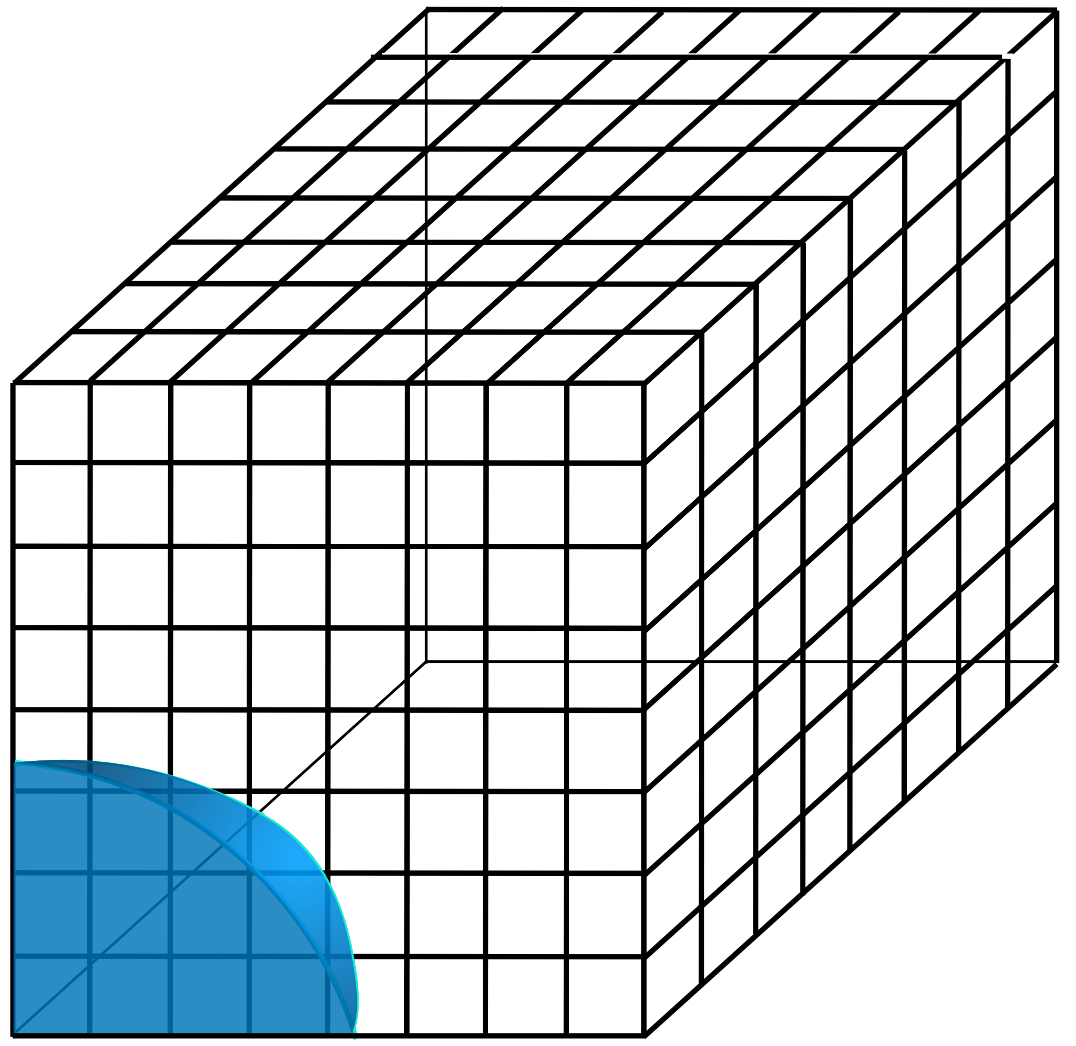
$$\mathscr{H}^q_{\text{HFB}}[\rho, \tilde{\rho}, \tilde{\rho}^*] = \sum_{\sigma'} \begin{bmatrix} h^q_{\sigma\sigma'}(\boldsymbol{r}) & \tilde{h}^q_{\sigma\sigma'}(\boldsymbol{r}) \\ 4\sigma\sigma' \tilde{h}^{q*}_{-\sigma-\sigma'}(\boldsymbol{r}) & -4\sigma\sigma' h^{q*}_{-\sigma-\sigma'}(\boldsymbol{r}) \end{bmatrix}, \quad \mathscr{N} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad h = \frac{\delta E}{\delta \rho}, \quad \tilde{h} = \frac{\delta E}{\delta \tilde{\rho}^*}$$

appropriate framework for describing neutron-rich nuclei

asymptotic behavior of densities at large distances

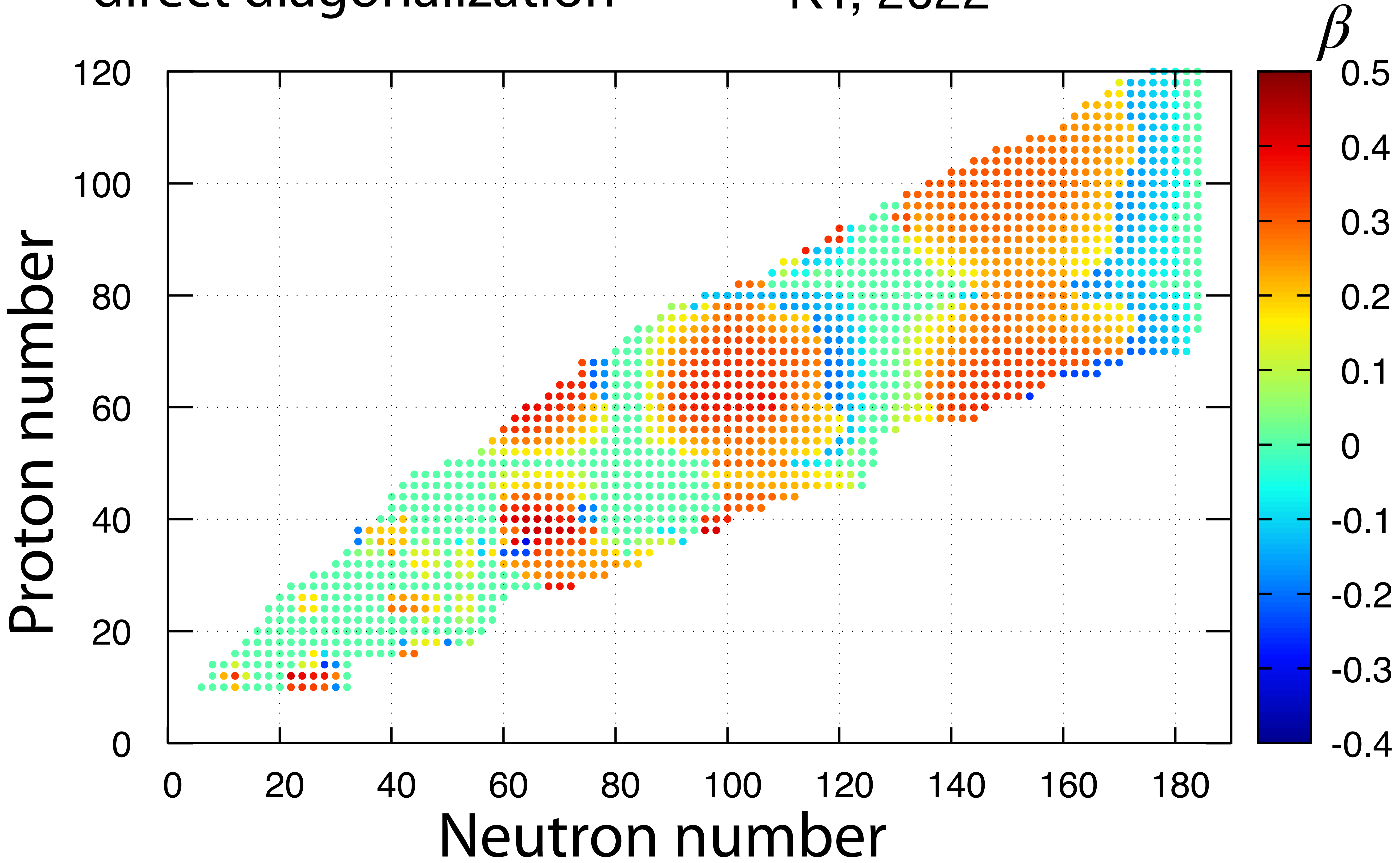
pairing involving the continuum states

Nuclear DFT for equilibrium deformed shapes



3D-mesh calculation is now available

- Krylov subspace method for the Greens func.
  - Seattle–Warsaw (Jin–Bulgac–Roche–Wlazłowski, 2017)
  - Tsukuba (Kashiwaba–Nakatsukasa, 2020)
- canonical basis and FFT — East Lansing–Erlangen (Chen+, 2022)
- direct diagonalization — KY, 2022



parity, $R(=e^{-i\pi j_x})$ symmetries

M^3: number of mesh points

$\dim(\mathscr{H}_{\rm HFB}) = 8M^3$

$\sim 14{,}000 \; (M=12)$

$\times$ ~100 times iteration

Nuclear DFT for rotational motions

Time-dependent DFT for dynamics: TD-KSB approach

$$i\partial_t R^q(t) = [\mathscr{H}^q_{\rm HFB}(t) - \lambda^q \mathscr{N}, R^q(t)] \qquad R = \Phi\Phi^\dagger$$

for collective rotations

$\Phi'(t) = U\Phi(t) = \exp[i\omega \hat{J}_x \mathscr{N} t]\Phi(t)$ in a uniformly rotating system about x-axis

$$i\partial_t R'^q(t) = [\mathscr{H}^q_{\rm HFB} - (\lambda^q + \omega \hat{J}_x)\mathscr{N}, R'^q(t)]$$

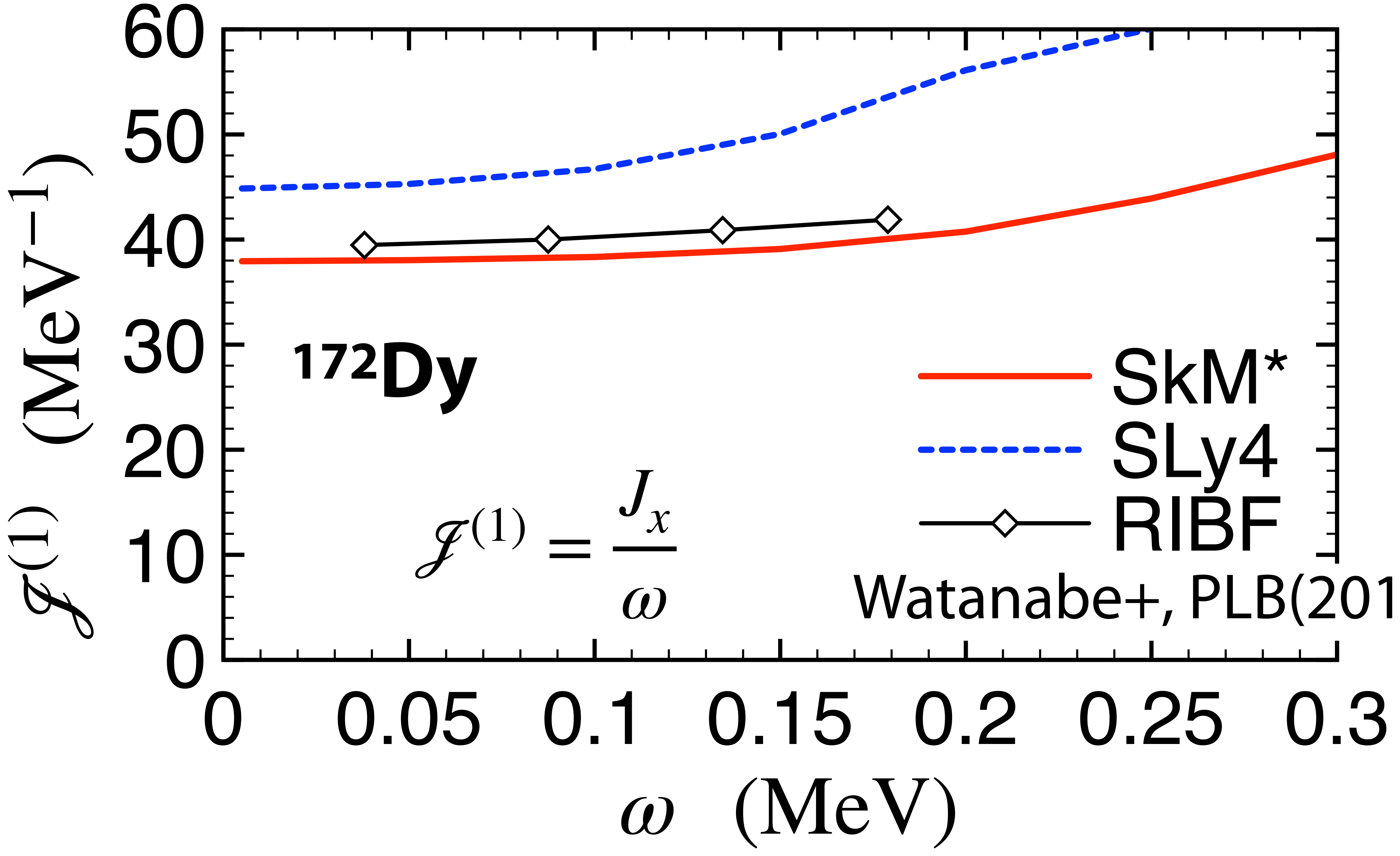
stationary

cranked KSB equation

$$[\mathscr{H}^q_{\rm HFB} - (\lambda^q + \omega \hat{J}_x)\mathscr{N}]\Phi'^q_\alpha(x) = E'^q_\alpha \Phi'^q_\alpha(x)$$

equivalent to

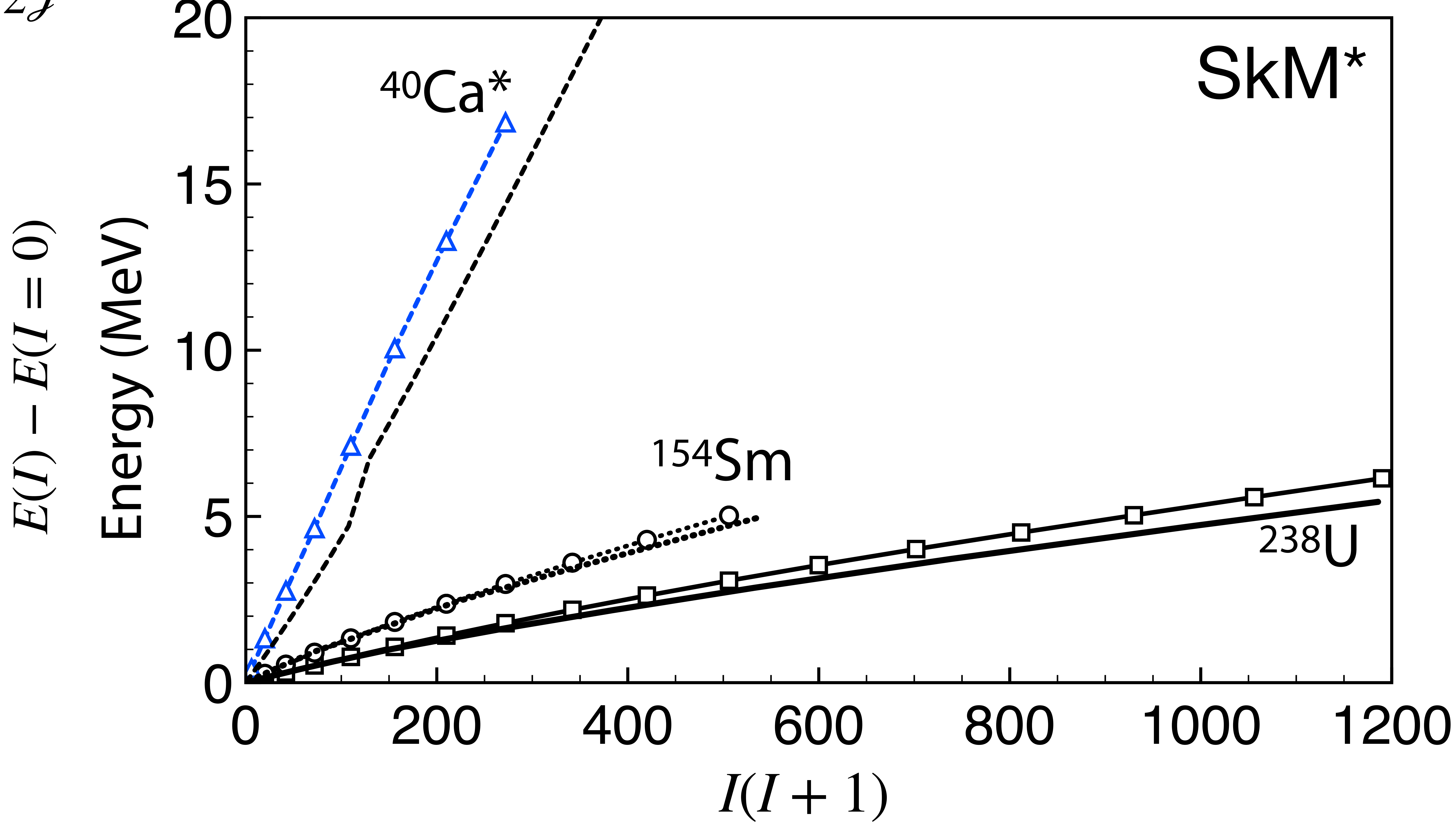
$$\delta(E[\rho, \tilde{\rho}, \tilde{\rho}^*] - \sum_q \lambda^q \langle \hat{N}_q \rangle - \omega \langle \hat{J}_x \rangle) = 0$$



Watanabe+, PLB(2016)

DFT for the rotational band

$$E = \frac{I(I+1)}{2\mathcal{J}}$$



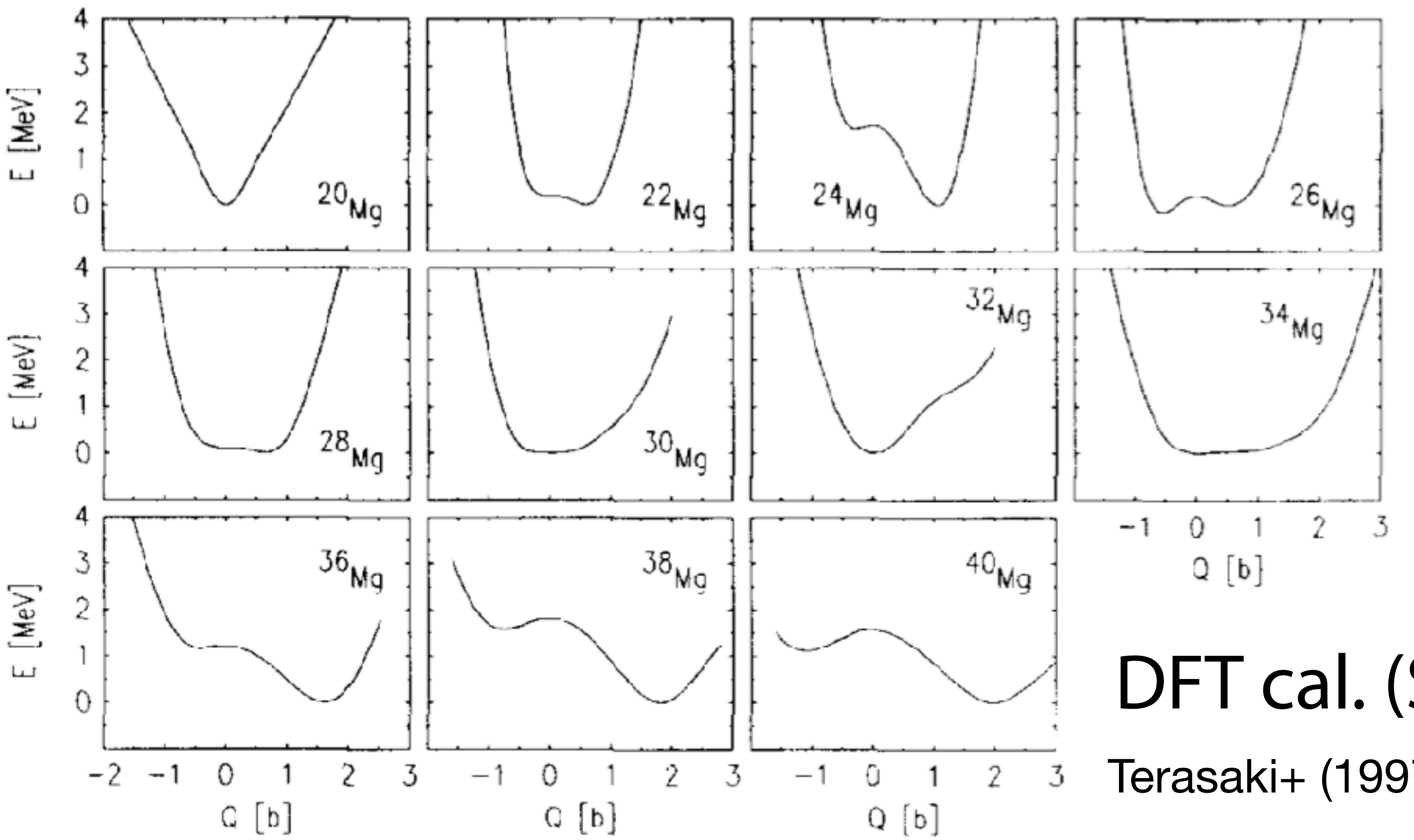
Neutron-rich Mg isotopes

strong quadrupole correlation

✓ breaking of the spherical N=20 magic number at ^{32}Mg

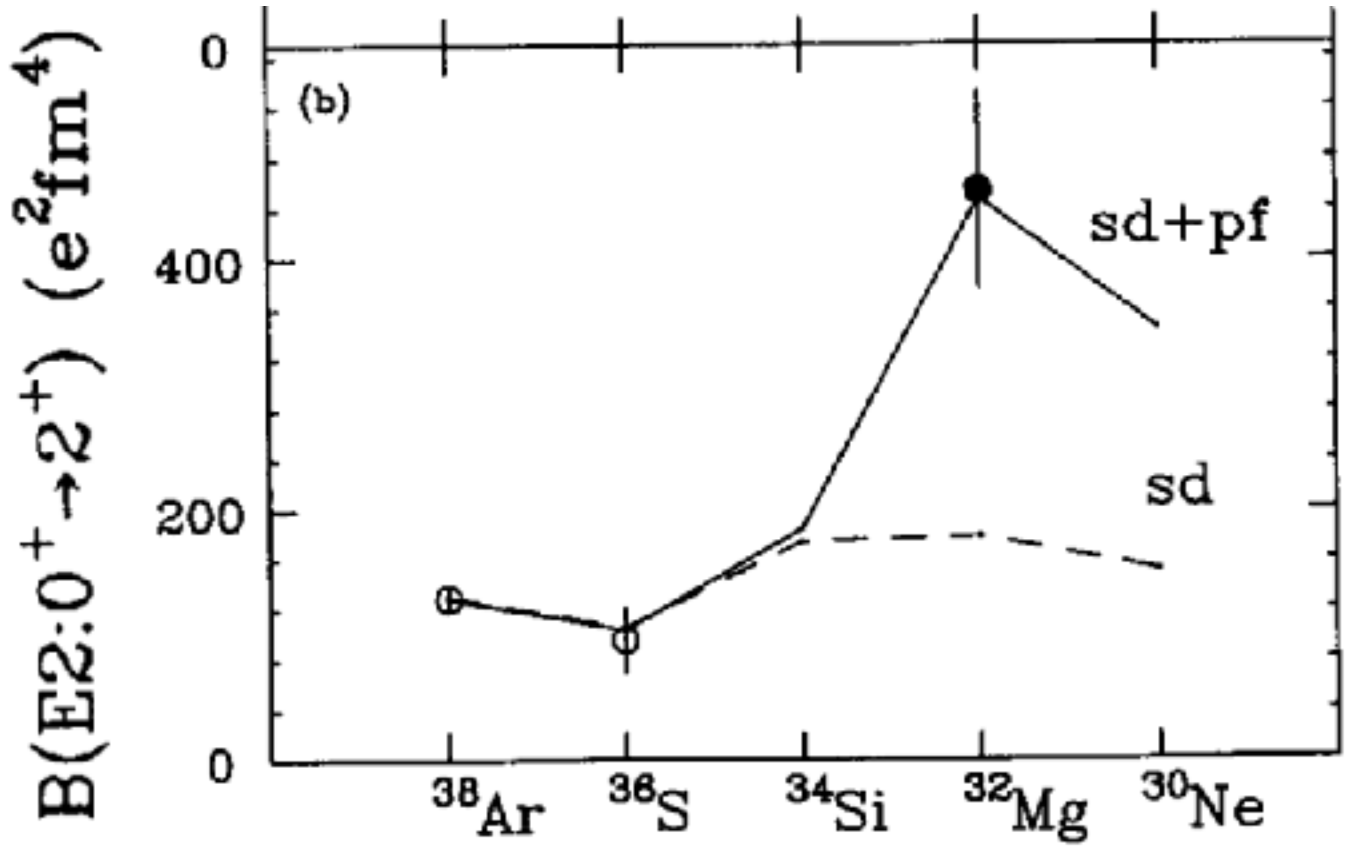
low $E(2_1^+)$ and high $B(\mathrm{E2})$

✓ quadrupole deformation near the drip line

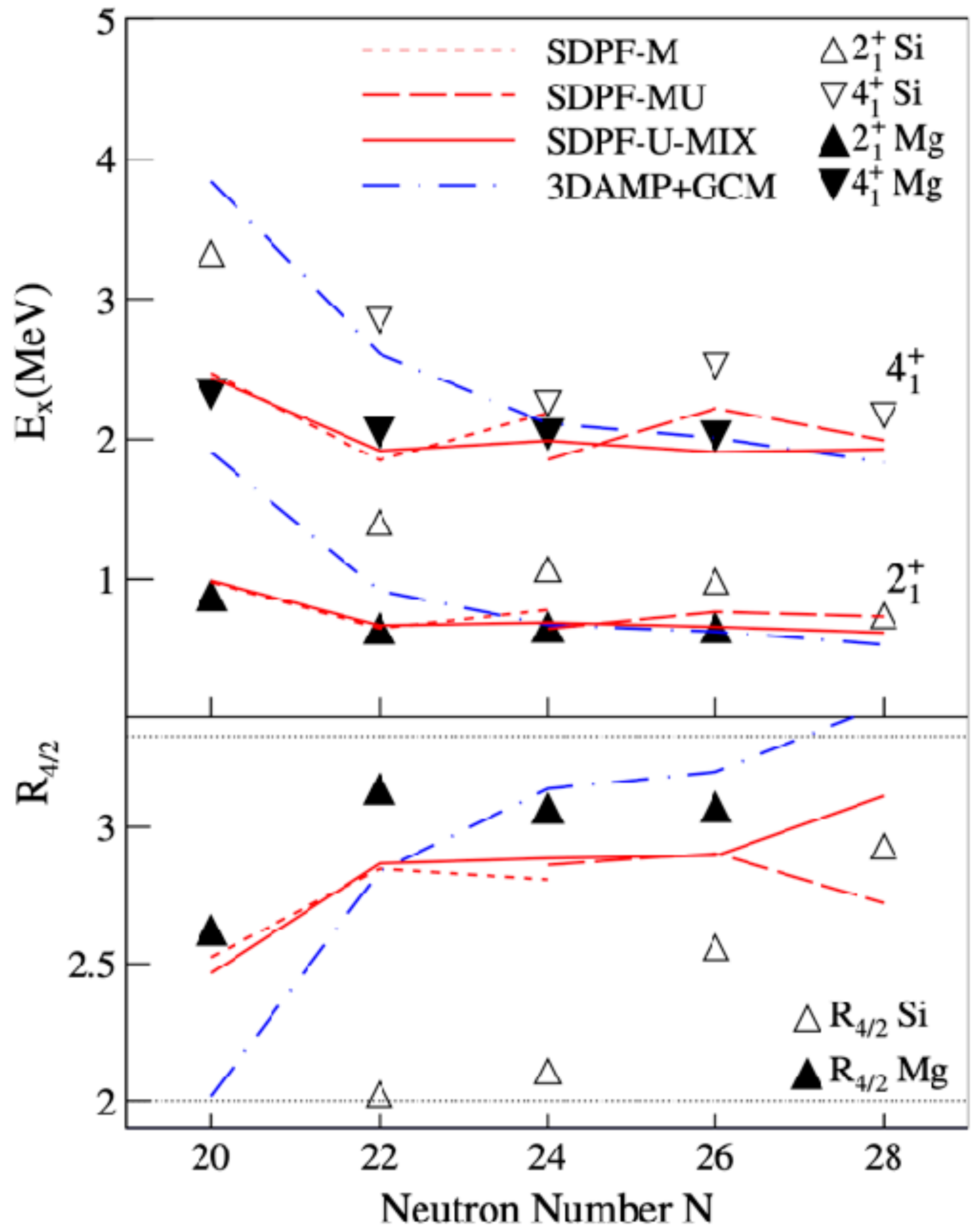


DFT cal. (SIII)

Terasaki+ (1997)



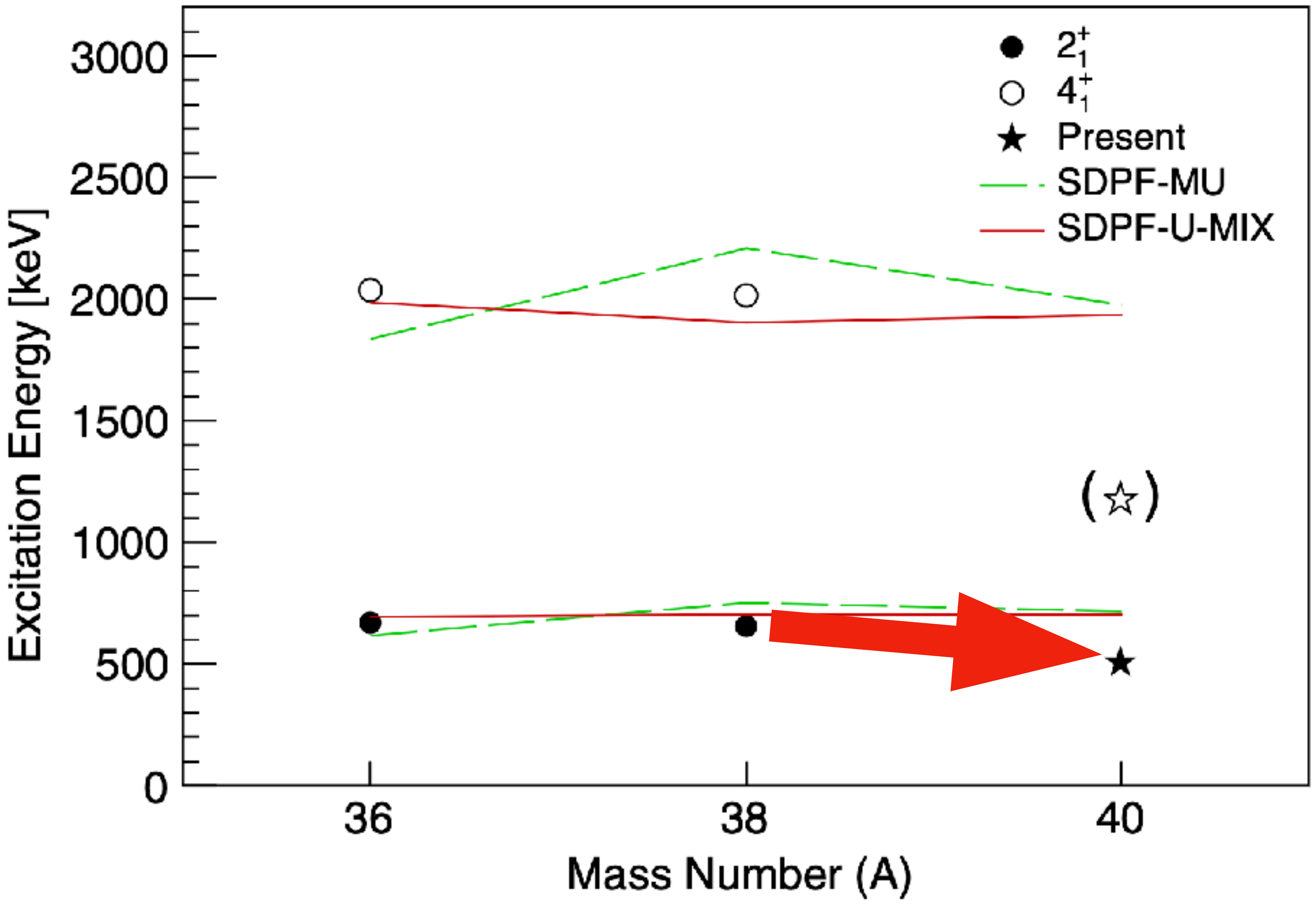
Motobayashi+ (1995)



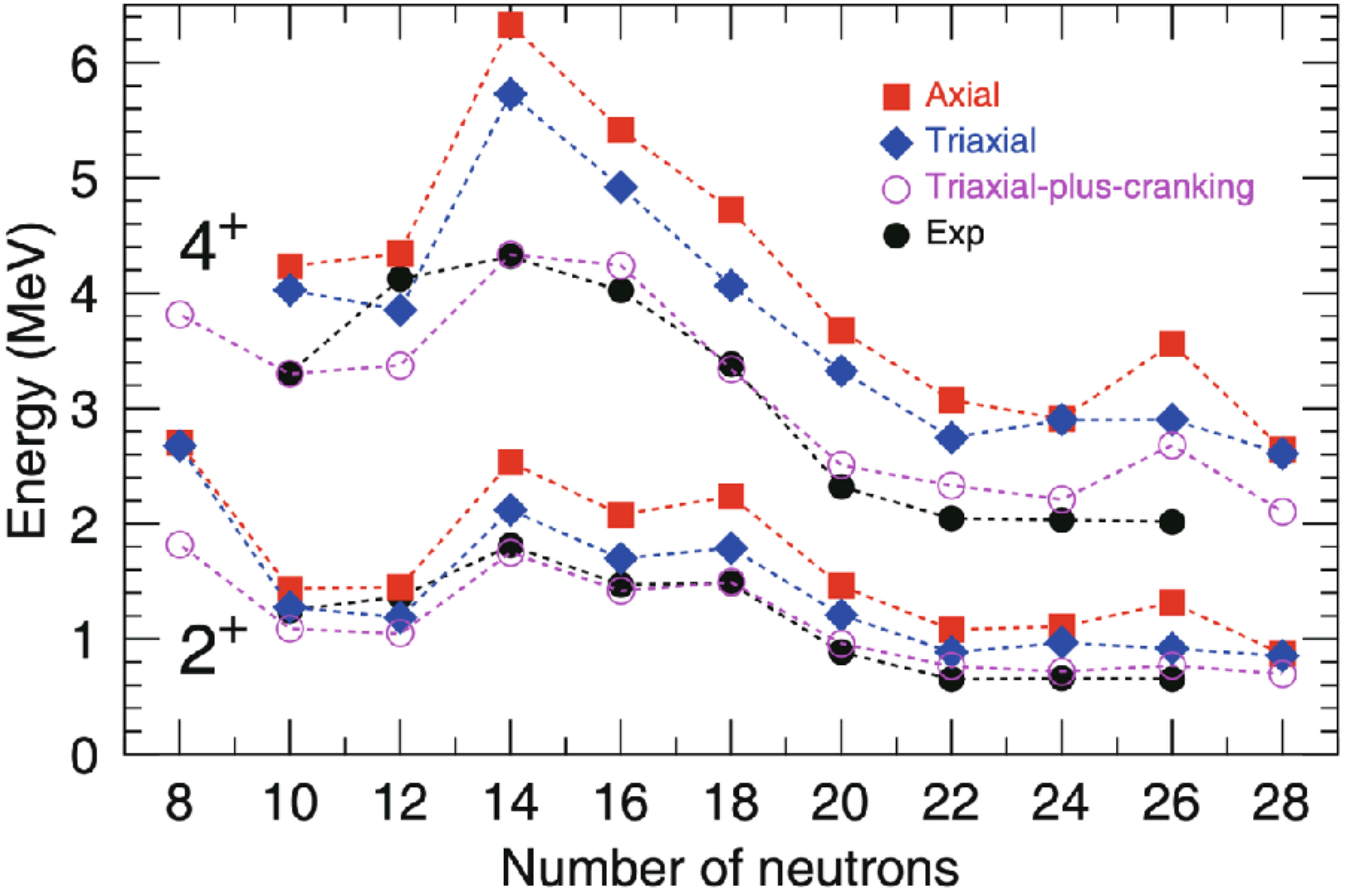
Doornenbal+ (2013)

Is the low-lying 2^+ state in ^{40}Mg unique?

Crawford+, PRL122(2019)052501



Rodríguez, EPJA52(2016)190



635(6) keV →500(14) keV

~20% decrease in energy

$E(2_1^+)$ looks constant

Is it a qualitatively unique feature?

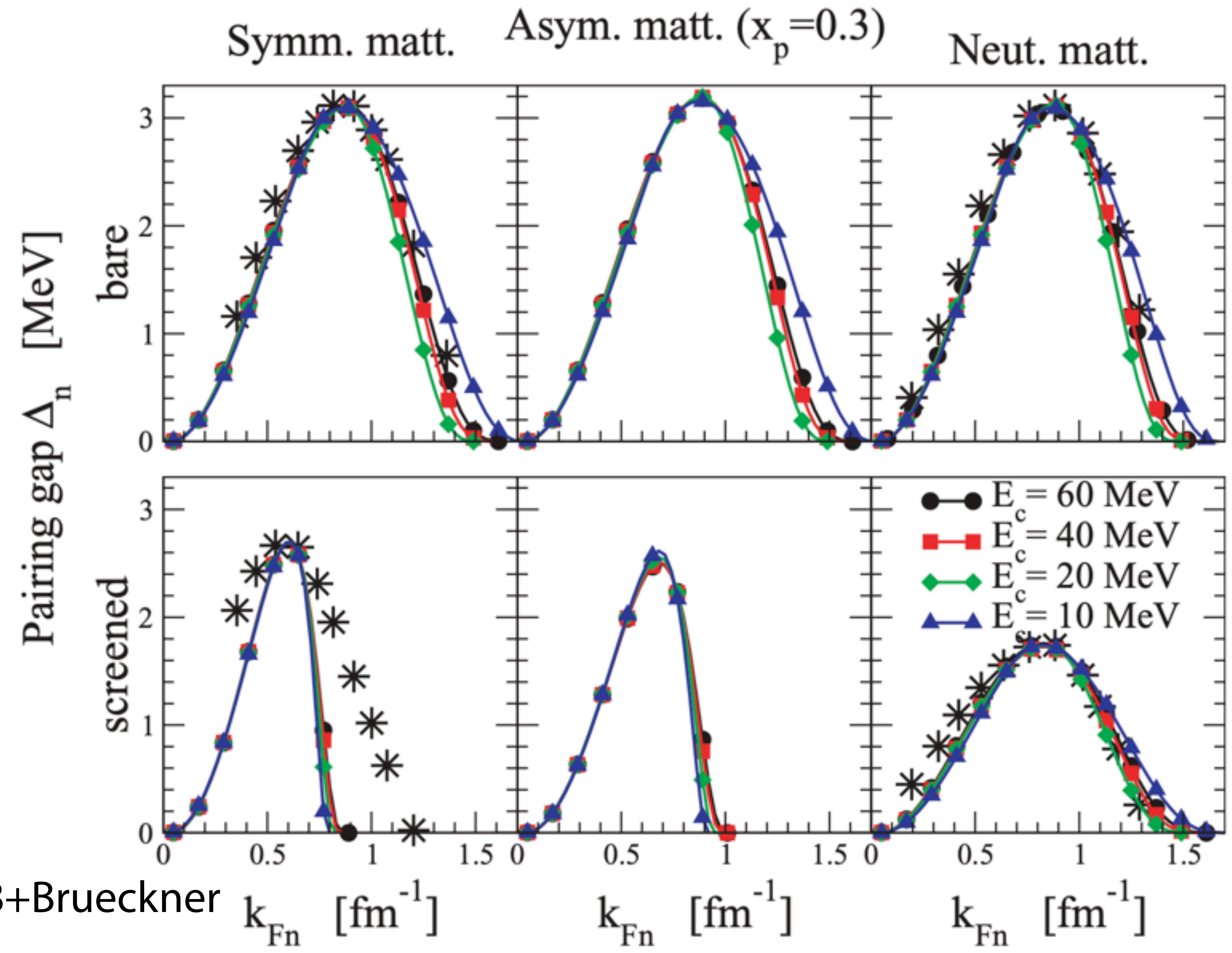
Pairing functional for neutron-rich systems

Margueron–Sagawa–Hagino, PRC76('07),PRC77('08)

isospin-asymmetry dependence in pairing EDF

$$\Delta_n = -\frac{v_0 g[\rho_n, \rho_p]}{2(2\pi)^3} \int d^3k \frac{\Delta_n}{E_n(k)} \theta(k,k), \qquad I = \frac{N-Z}{A}$$

$$g_1[\rho_n, \rho_p] = 1 - f_s(I)\eta_s \left(\frac{\rho}{\rho_0}\right)^{\alpha_s} - f_n(I)\eta_n \left(\frac{\rho}{\rho_0}\right)^{\alpha_n}$$

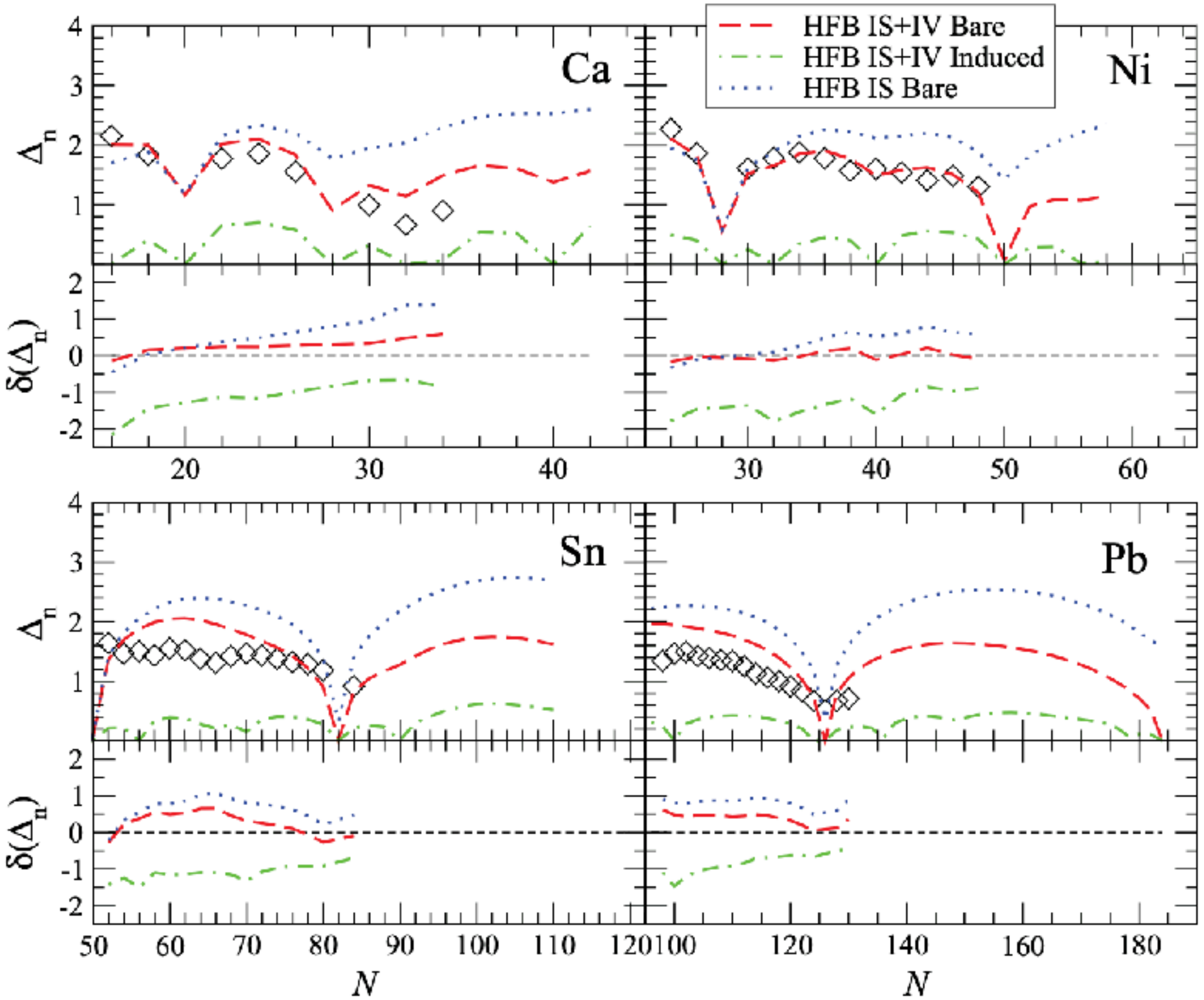


✳ AV18+Brueckner

pairing gaps of

nuclear and neutron matter

stable and n-rich nuclei



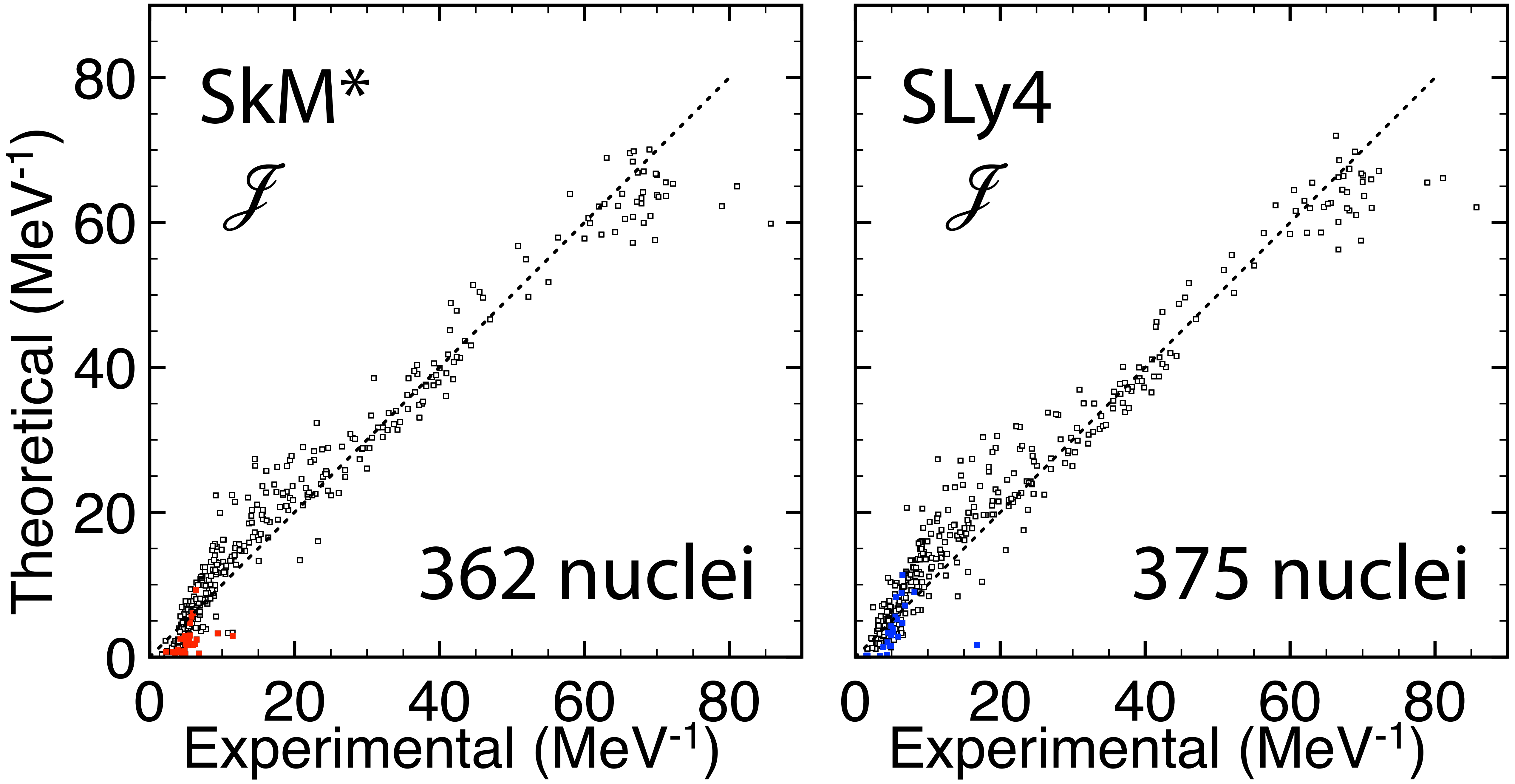
Microscopic MoI with a global pairing functional

$$H(\boldsymbol{r}) = \frac{V_0}{4} \sum_{\tau=n,p} g_\tau[\rho, \rho_1]\, |\, \tilde{\rho}_\tau(\boldsymbol{r})\, |^2$$

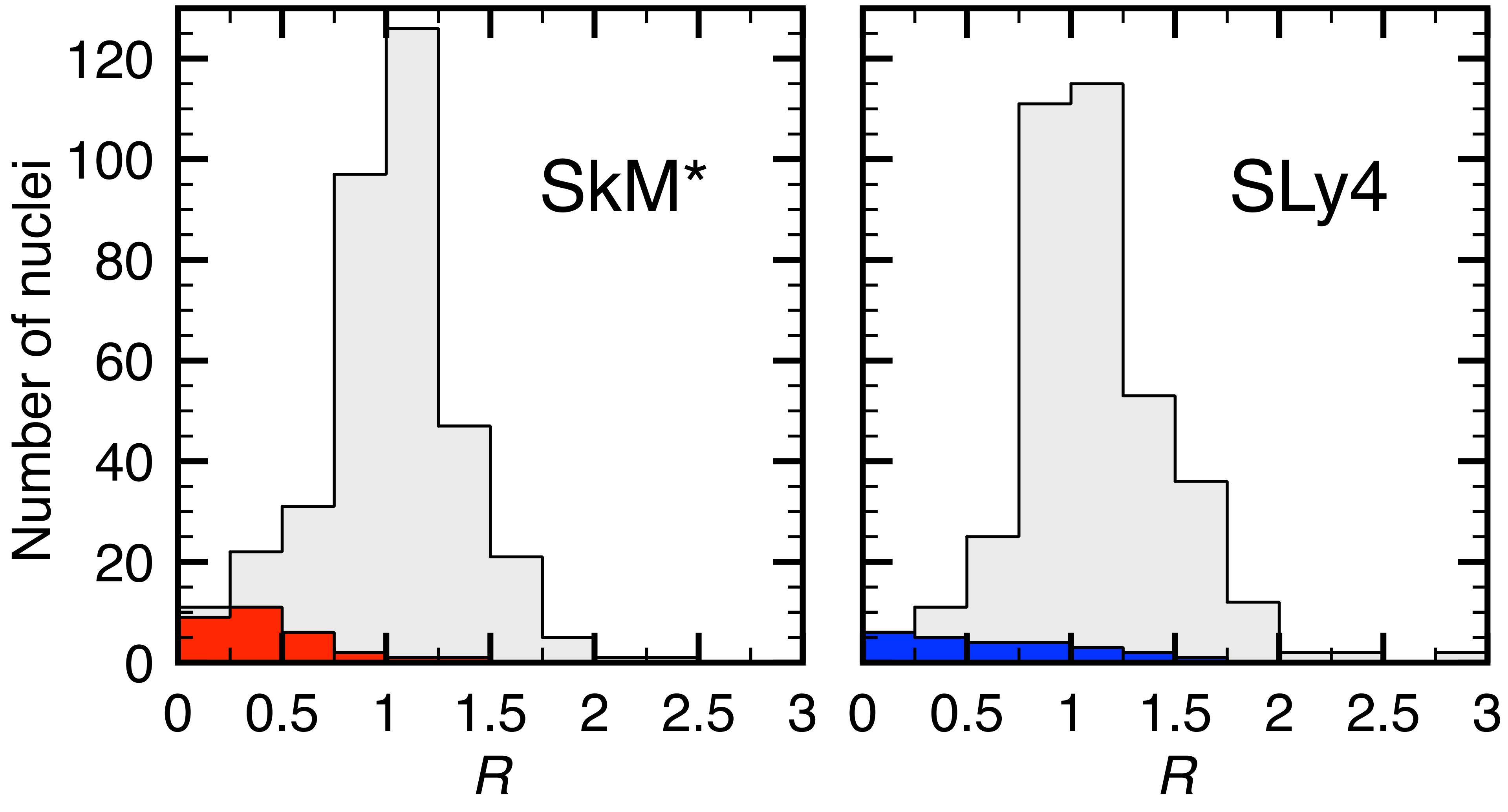
Yamagami–Shimizu–Nakatsukasa, PRC80('09)

$$g_\tau[\rho, \rho_1] = 1 - \eta_0 \frac{\rho_0(\boldsymbol{r})}{\rho_{sat}} - \eta_1 \frac{\tau_3 \rho_1(\boldsymbol{r})}{\rho_{sat}} - \eta_2 \left(\frac{\rho_1(\boldsymbol{r})}{\rho_{sat}} \right)^2$$

Cranking model: $E(2^+) = \dfrac{3}{\mathscr{J}}, \quad \mathscr{J} = \lim_{\omega \to 0} \dfrac{J_x}{\omega}$ numerically evaluated at $\omega = 0.01$ MeV



$R = \mathscr{J}_{th} / \mathscr{J}_{exp}$



Validity of the cranking model for the 2_1^+ state energy

model	# of nuclei	$\bar{R}_E$	σ_E
CHFB(SkM*)	332	-0.021	0.33
CHFB(SLy4)	335	-0.095	0.30
MAP(SL4)	359	0.28	0.49
MAP(SLy4,def)	135	0.20	0.30
GCM(SLy4)	359	0.51	0.38
GCM(SLy4,def)	135	0.27	0.33
5DCH(D1S)	519	0.12	0.33
5DCH(D1S,def)	146	-0.05	0.19

MAP (minimization after projection), Sabby+(2007)
GCM (Hill–Wheeler), Sabby+(2007)
5DCH (GCM+GOA), Bertsch+(2007)

$$R_E = \ln(E_{\rm th}(2^+)/E_{\rm exp}(2^+))$$

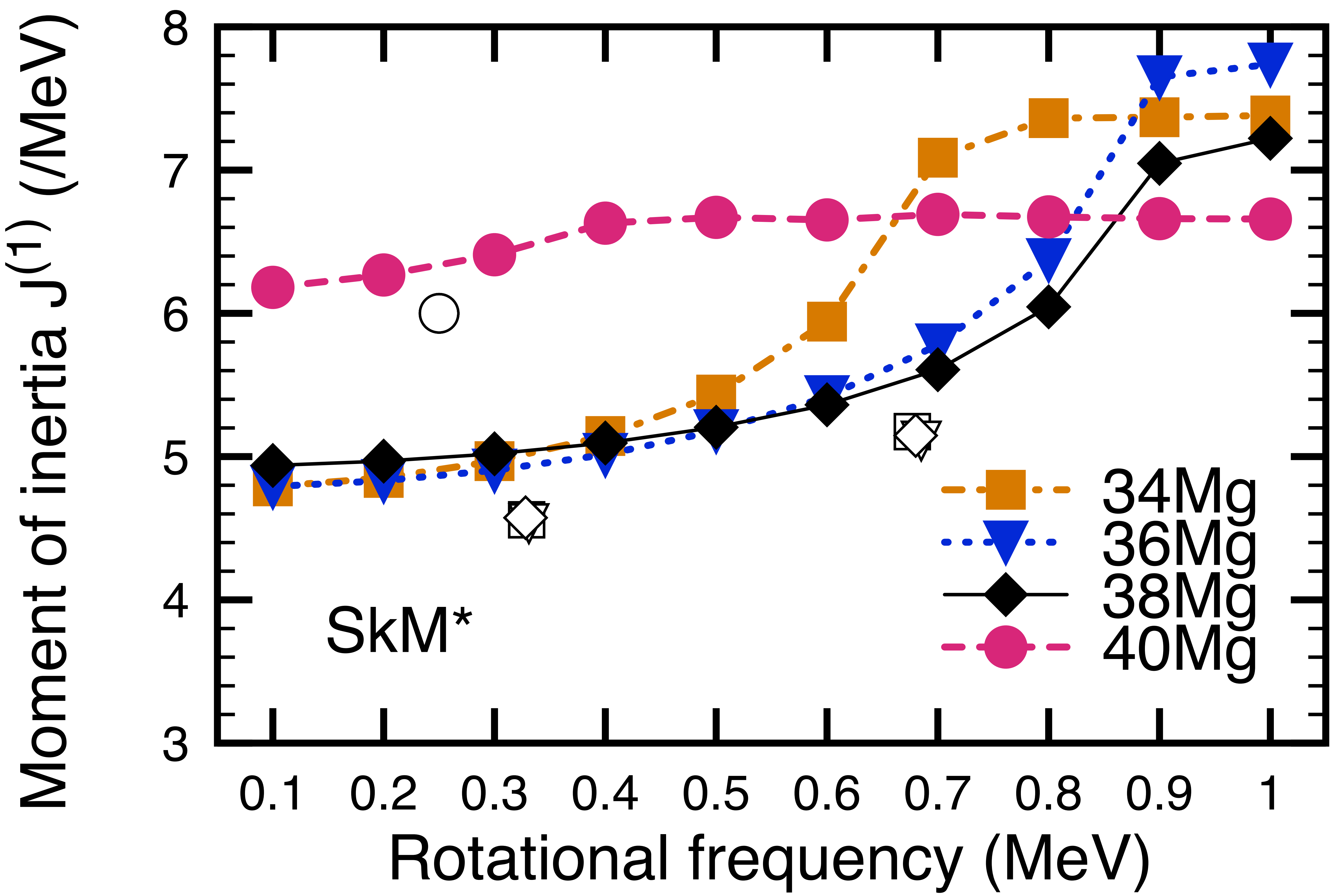
$$\sigma_E = \sqrt{\langle (R_E - \bar{R}_E)^2 \rangle}$$

self-consistent cranking model surprisingly well describes $E(2^+)$

30–35% error implies a variety of characters of individual nuclides

Microscopic MoI of neutron-rich Mg isotopes close to the drip line

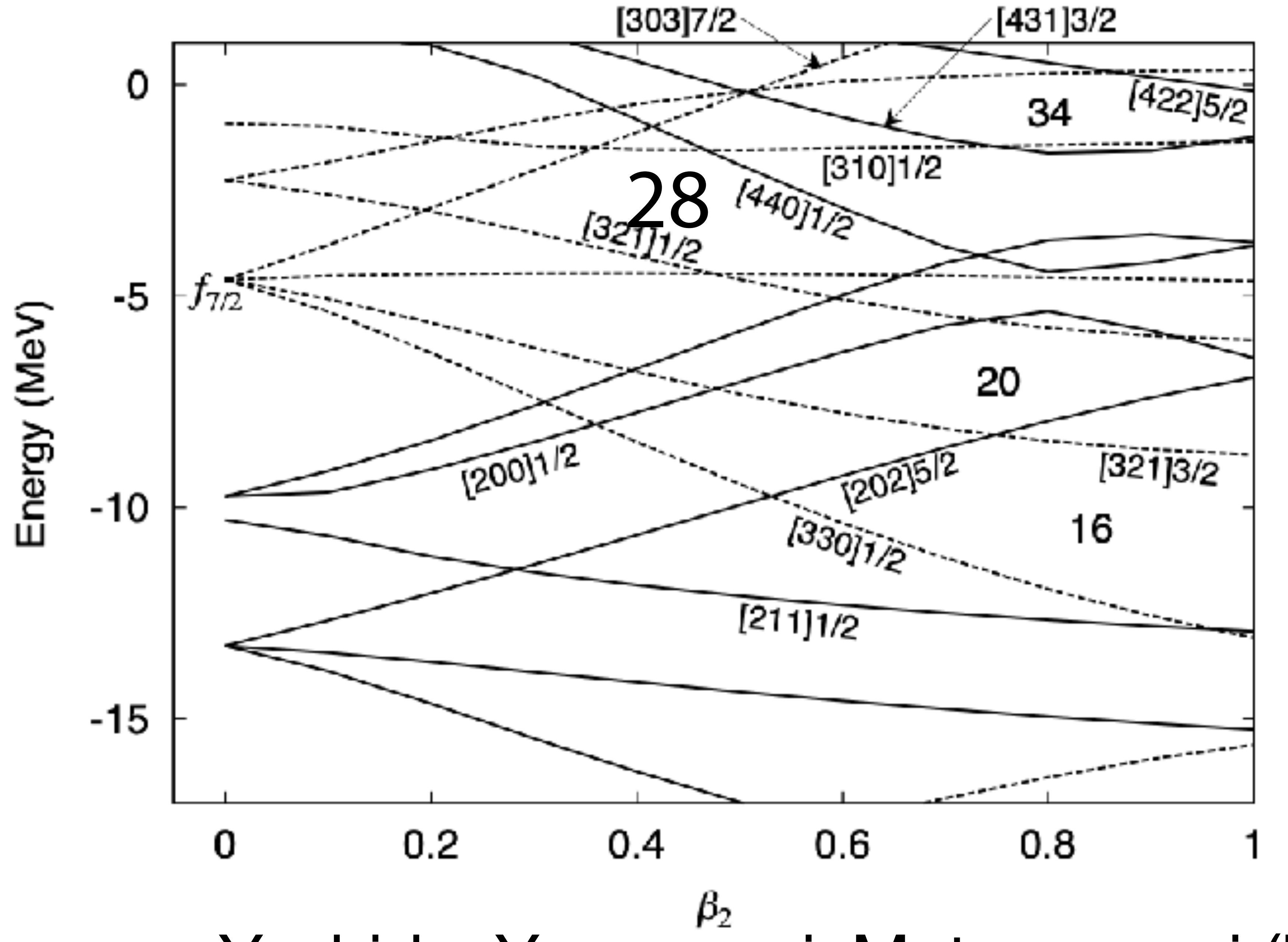
KY, PRC105(2022)024313



weakening of pairing at ^{40}Mg

isospin-dependence of the paring

deformed gap at $N = 28$

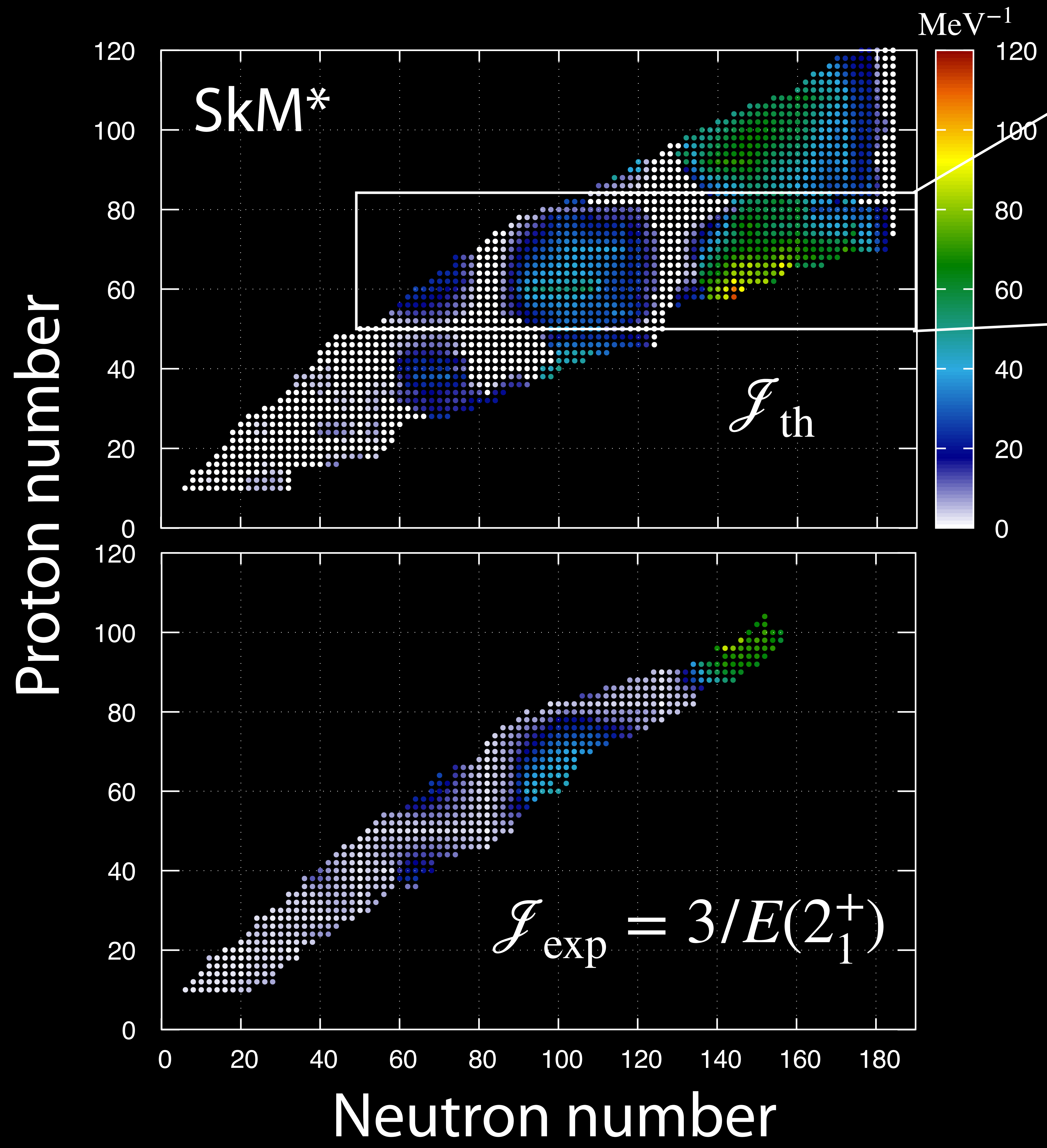


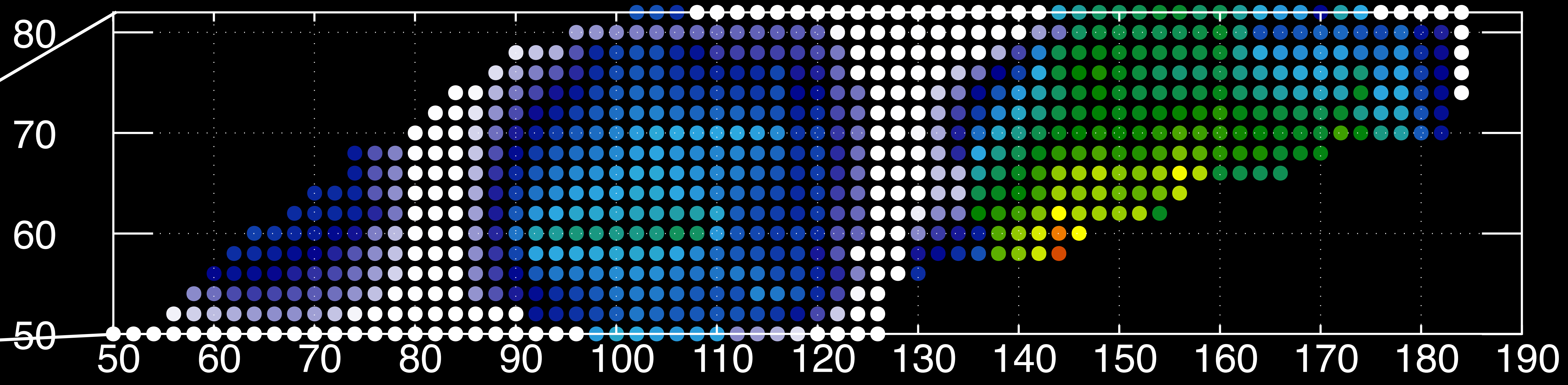
Yoshida–Yamagami–Matsuyanagi ('05)

MoI of neutron-rich nuclei

KY, PLB834(2022)137458

$E(2^+)$: indicator for the evolution of shell structure and deformation   cf. SEASTAR





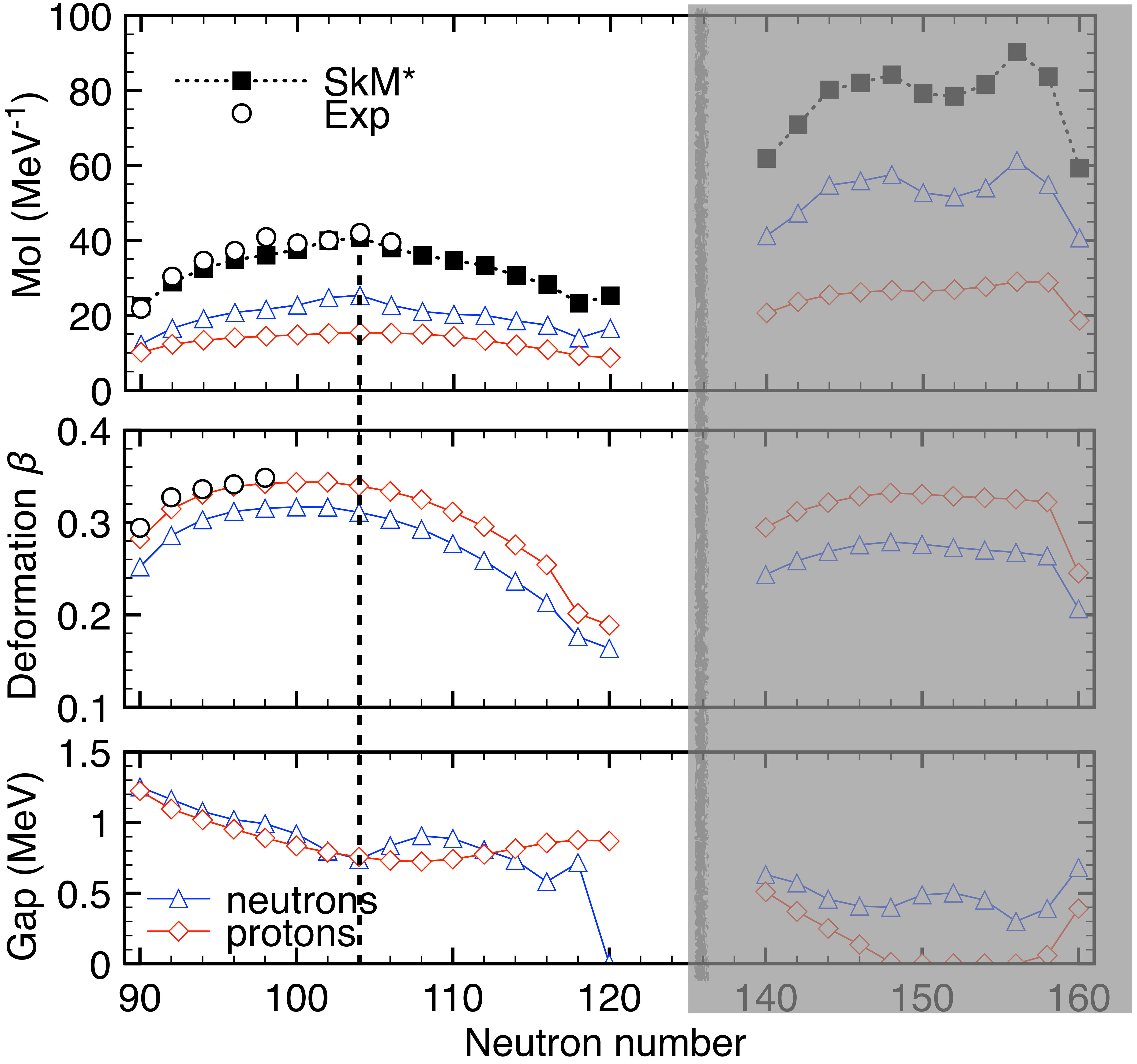
$Z = 60$ deformed shell

$N \simeq 100$ deformed shell

island of high $\mathscr{J}$ near the drip line

comparable to heavy actinides

MoI of neutron-rich Dy isotopes



$\mathscr{J}$ is highest at $N = 104$
both in exp. and cal.

Δ_n is lowest at $N = 104$

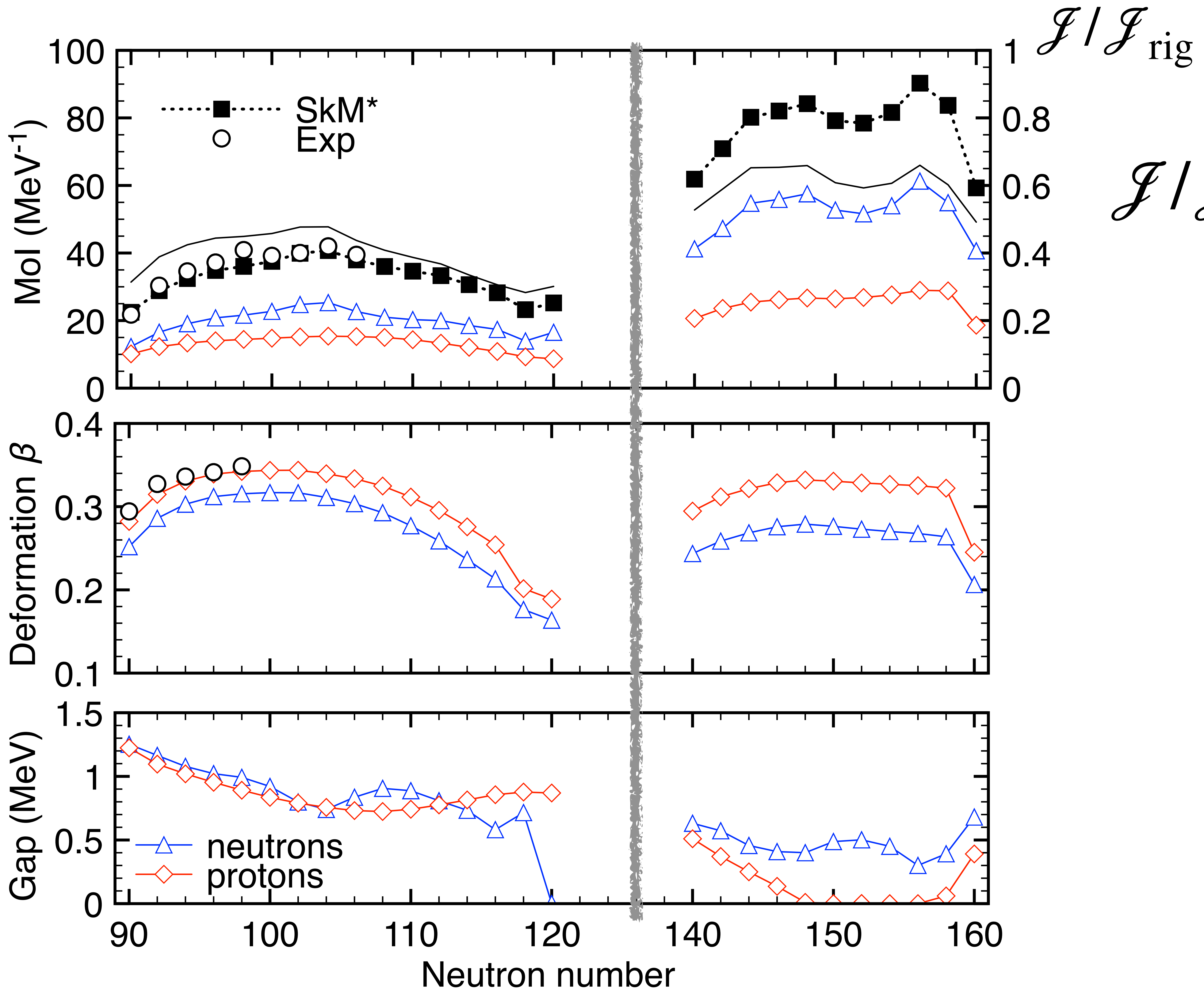
deformation develops toward $N = 100$

$\mathscr{J} \leftrightarrow \beta$ is not a one-to-one correspondence

$E(2^+) \leftrightarrow B(E2)$

$\mathscr{J}$ is much more sensitive to the shell structure and the pairing

MoI of neutron-rich Dy isotopes: A role of the pairing

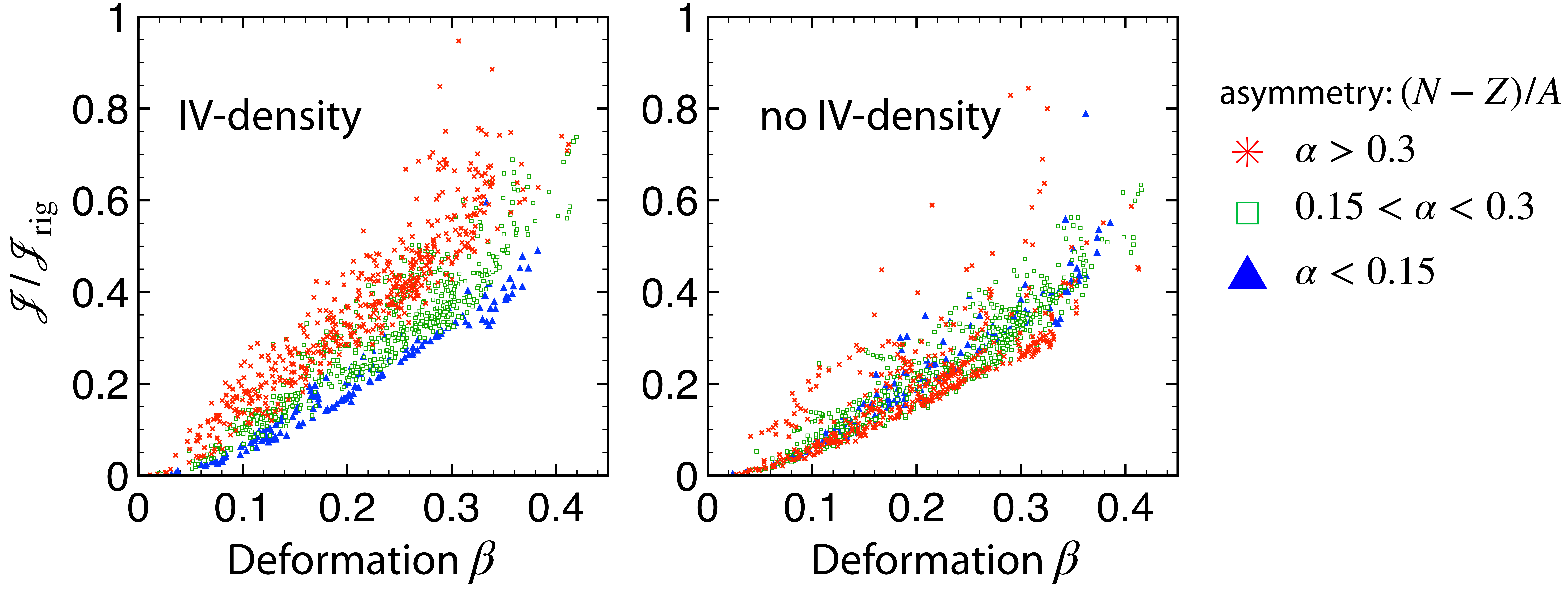


$\mathscr{J}/\mathscr{J}_{\mathrm{rig}}$ is higher near the drip line

weakening of the pairing

Role of the IV-density dependence

~1700 even-even nuclei

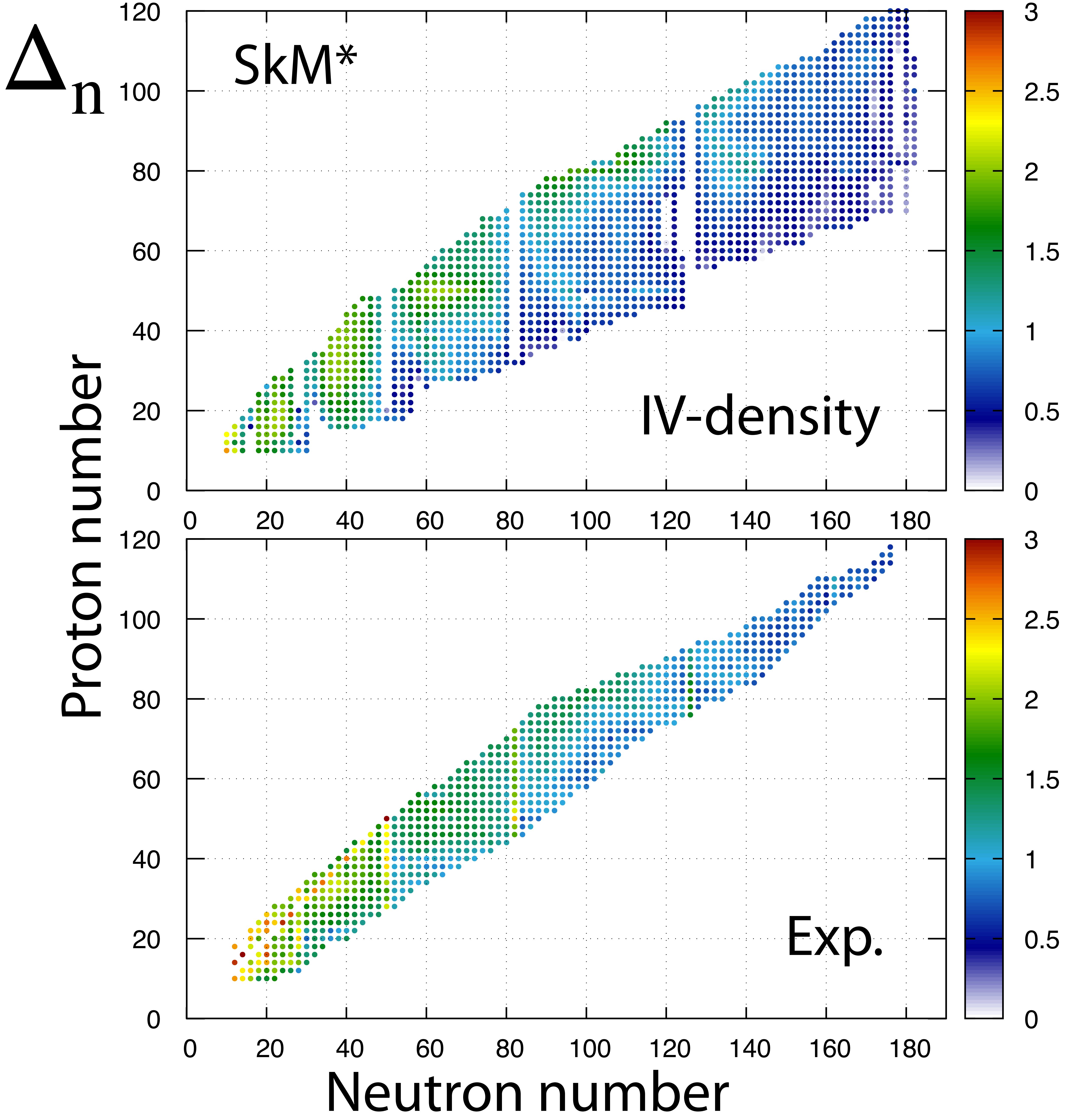


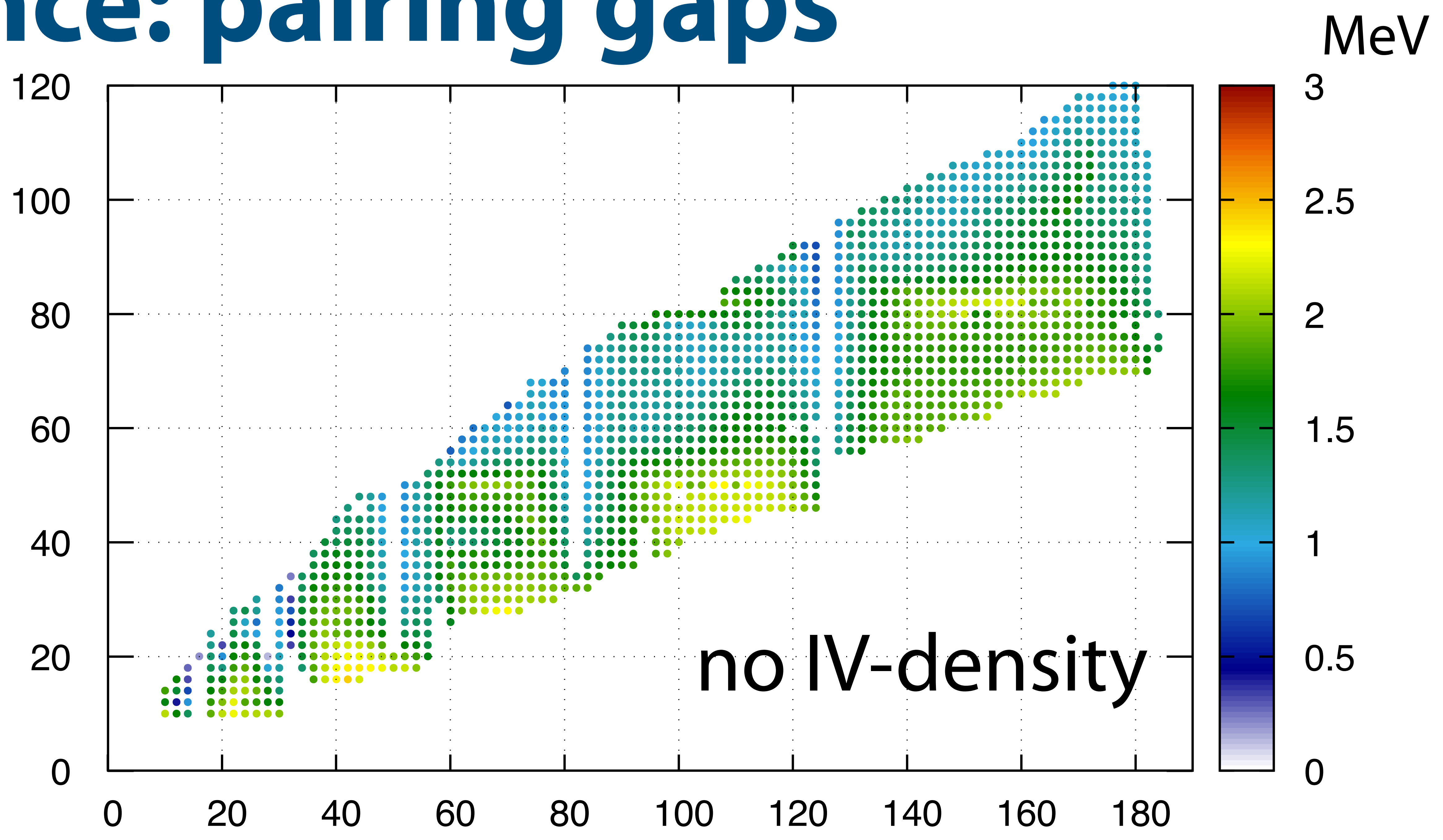
enhancement of $\mathscr{J}$

weakening of pairing in n-rich nuclei

increase ~~in~~ deformation

Role of the IV-density dependence: pairing gaps





pairing-energy functional

w/ IV density — weaker pairing

↕

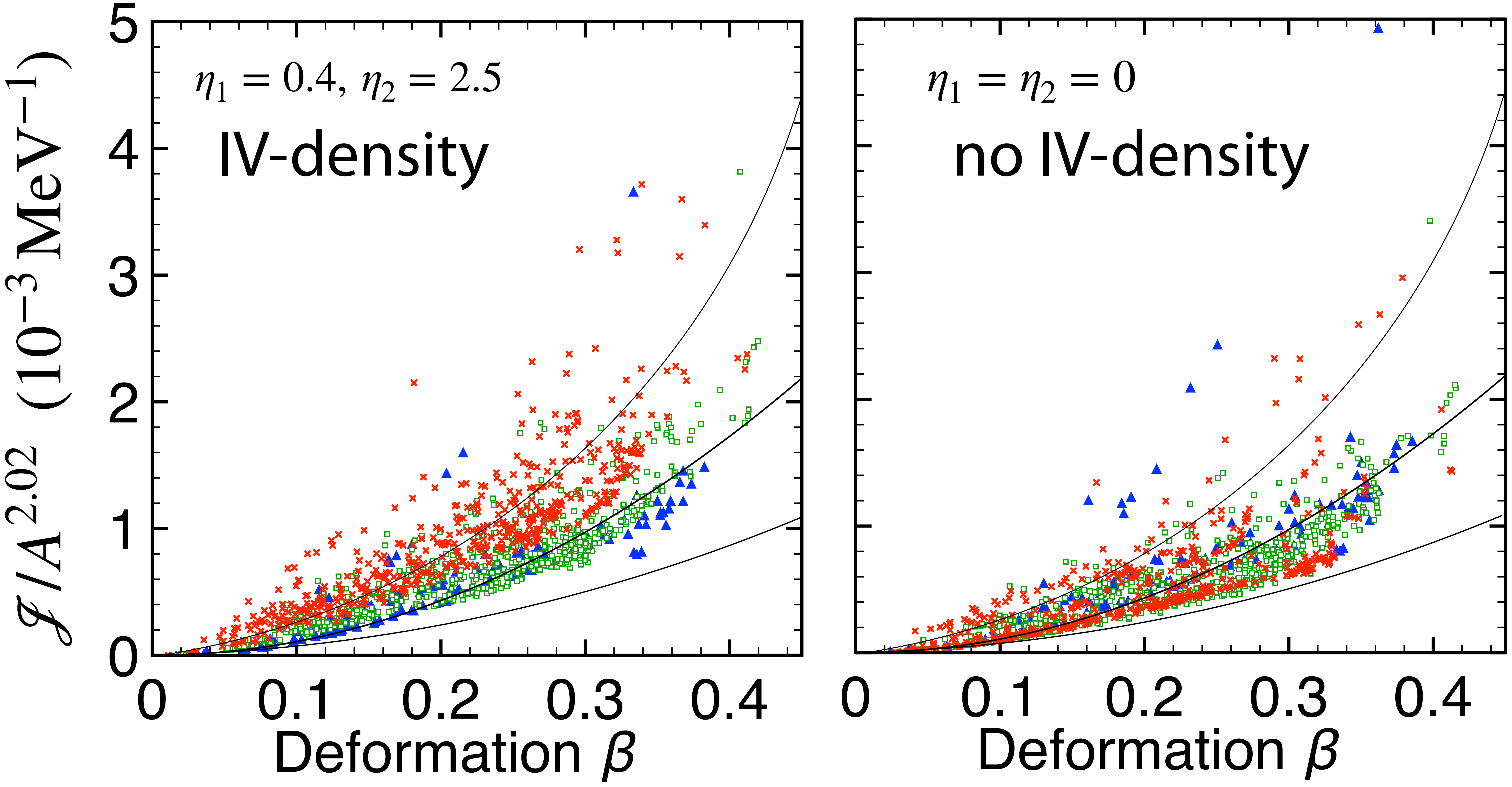
w/o IV density — stronger pairing towards the drip line

Role of the IV-density dependence: MoI

high $\mathscr{J}$ near the drip line

low $E(2^+)$

Role of the IV-density dependence



$E(2^+) \leftrightarrow B(E2)$ Raman+('01)

$$\left[\frac{B(E2;0_1^+ \to 2_1^+)}{1\,e^2\mathrm{fm}^4}\right] \times \left[\frac{E(2_1^+)}{1\,\mathrm{MeV}}\right] = 32.6\frac{Z^2}{A^{0.69}}$$

91% data within a factor of 2

$$\beta = \frac{4\pi}{3ZR_0^2}\sqrt{B(E2)/e^2}$$

$$\mathscr{J} = \frac{3}{32.6}\left(\frac{3}{4\pi}\right)^2 A^{0.69}R_0^4\beta^2 \ \ (\mathrm{MeV}^{-1})$$

R_0 in fm

$$= \frac{3}{32.6}\left(\frac{3}{4\pi}\right)^2 1.2^4 A^{2.02}\beta^2 \ \ (\mathrm{MeV}^{-1})$$

$\mathscr{J}$ is out of line with the empirical formula in very neutron-rich nuclei.

A systematic measurement of $E(2^+)$, $B(E2)$ deepens the understanding of the pairing in neutron-rich nuclei.

Summary

Skyrme-DFT for a systematic investigation (~1700 nuclei) of the ground-state MoI

thanks to high-performance computers

The self-consistent cranking model, though it is simple, describes well the rotational excitation in deformed nuclei

Exotic behavior in nuclear rotation near the neutron dripline

high $\mathscr{J}$—low $E(2^+)$—but not necessarily means a strong deformation

model dependence is strong, and it should be examined more

$\mathscr{J}/\mathscr{J}_{\rm rig}$: pairing indicator

like the Hess–Fairbank effect