

Connecting low-lying dipole modes to nuclear structure and equation of state

7th International Conference on Collective Motion
in Nuclei under Extreme Conditions (COMEX7)

Dipartimento di Fisica e Astronomia

Catania, June 11th - 16th, 2023



Authors: S. Burrello, M. Colonna

INFN - Laboratori Nazionali del Sud

Outline of the presentation

1 Theoretical approaches for nuclear many-body problem

- Ab-initio vs phenomenological models based on energy density functionals (EDF)
- Effective interaction and nuclear matter (NM) Equation of State (EoS)

2 EDF-based models: recent results and further developments

⇒ Mean-field dynamical models with phenomenological EDFs

- Nature of low-lying dipole modes and connection with nuclear structure
- Correlation between low-lying dipole excitations and properties of EoS

⇒ Extensions of EDF approaches: bridge with ab-initio and beyond mean-field

- Benchmark ab-initio for low-density pure neutron matter (PNM)
- Embedding many-body correlations and clustering phenomena

3 Summary and perspectives

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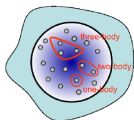
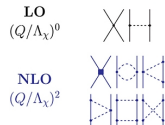
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Theoretical approaches for many-body problem

- **Ab-initio** approaches based on **many-body** expansion
 - Realistic or **effective field theory** (EFT) interactions
 - ⇒ Diagrammatic hierarchy (**power counting**)



- Phenomenological models with **effective** interaction
 - **Self-consistent** mean-field (MF) approximation
 - Fit of parameters to reproduce various observables
- Energy Density Functional (EDF) theory
- Ongoing attempts to **bridge** EDFs with ab-initio

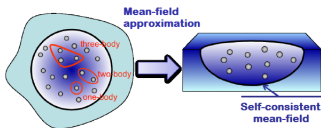
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NLO
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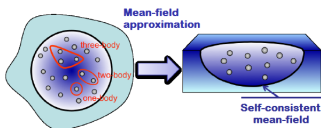
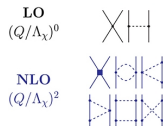


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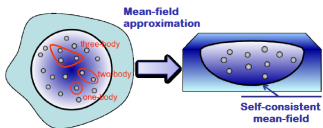
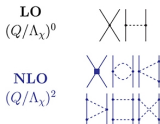
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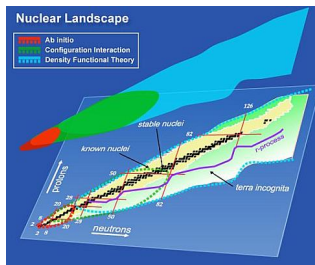
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⇒ Description of **ground state** and **excitations**

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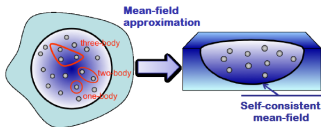
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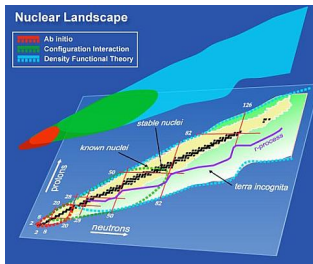
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Recent attempts to bridge EFT with EDF theories

Physics Letters B 811 (2020) 135938

Contents lists available at ScienceDirect

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Towards a power counting in nuclear energy–density–functional theories through a perturbative analysis

Stefano Burrello^{a,*}, Marcella Grasso^a, Chieh-Jen Yang^b

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^b Department of Physics, Chalmers University of Technology, SE-412 96, Göteborg, Sweden


PHYSICAL REVIEW C **106**, L011305 (2022)

Letter

Calculations for nuclear matter and finite nuclei within and beyond energy-density-functional theories through interactions guided by effective field theory

C. J. Yang^{1,2}, W. G. Jiang¹, S. Burrello³ and M. Grasso⁴

¹Department of Physics, Chalmers University of Technology, SE-412 96 Göteborg, Sweden
²Nuclear Physics Institute of the Czech Academy of Sciences, 25069 Řež, Czech Republic
³Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany
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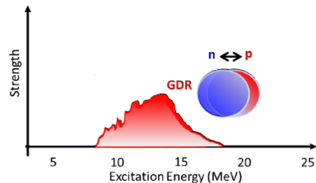
 (Received 4 October 2021; accepted 24 June 2022; published 21 July 2022)

Collective excitations: neutron skin and EoS

- **Collective phenomena** in **many-body** dynamics \Rightarrow properties of **interaction**
- **Dipole excitations** in nuclei:
 - ★ Giant Dipole Resonance (GDR)
 - ★ Pygmy Dipole Resonance (PDR)
- **Isvector terms of effective interaction**
 - \Rightarrow **symmetry energy** in EoS $\left(\delta \equiv \frac{\rho_n - \rho_p}{\rho} \right)$

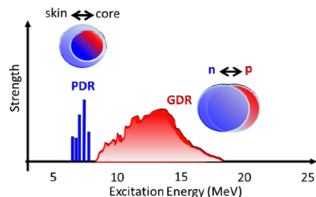
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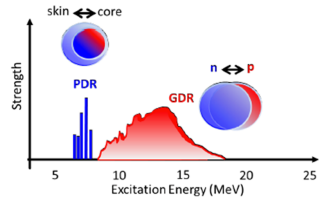
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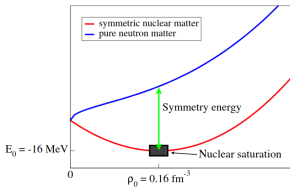
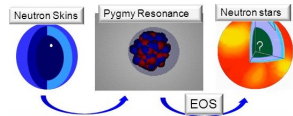


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$$\frac{E}{A}(\rho, \delta) \approx \frac{E}{A}(\rho, \delta = 0) + S(\rho)\delta^2$$



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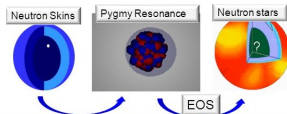
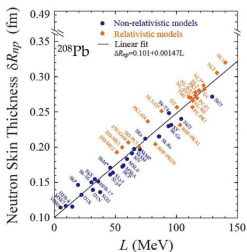
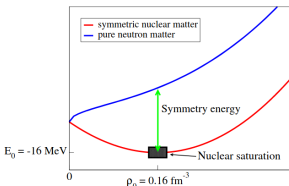
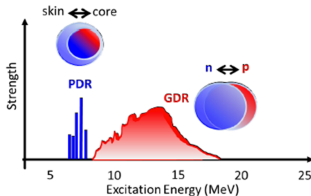
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$$S(\rho) = J + L \left(\frac{\rho - \rho_0}{3\rho_0} \right) + \dots$$

[X. Roca-Maza et al., PRL 106 (2011)]

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Mean-field dynamics and dipole excitations

- **Mean-field** dynamical models with **non-relativistic Skyrme-like** EDFs

PHYSICAL REVIEW C **99**, 054314 (2019)

Interplay between low-lying isoscalar and isovector dipole modes: A comparative analysis between semiclassical and quantum approaches

S. Burrello,¹ M. Colonna,¹ G. Colò,^{2,3} D. Lacroix,⁴ X. Roca-Maza,^{2,3} G. Scamps,^{5,6} and H. Zheng^{1,7}

- **Quantal** Time-Dependent-Hartree-Fock (**TDHF**) (or **RPA** for zero-amplitude)

- **Comparison** also with **semi-classical Vlasov** calculations

[S. Burrello, M. Colonna, and H. Zheng, Front. Phys. 7, 53 (2019)]

[H. Zheng, S. Burrello, M. Colonna, and V. Baran, PRC 94, 014313 (2016)]

$$i\hbar\dot{\hat{\rho}}(t) + [\hat{\rho}, \hat{H}_{\text{eff}}[\rho]] = 0$$

- **Isoscalar (IS)** or **isovector (IV)** dipole operators:

$$\hat{D}_S = \sum_i \left(r_i^2 - \frac{5}{3} \langle r^2 \rangle \right) z_i, \quad \hat{D}_V = \sum_i \tau_i \frac{N}{A} z_i - (1 - \tau_i) \frac{Z}{A} z_i, \quad \tau_i = 0 (1) \quad \text{for } n (p)$$

- **Strength function**: $S_K(E) = \sum_n |\langle n | \hat{D}_K | 0 \rangle|^2 \delta(E - (E_n - E_0))$ $K = S, V$

- **Transition densities**: $\delta\rho_q(r, E) \Rightarrow$ Information on **spatial structure** of excitations

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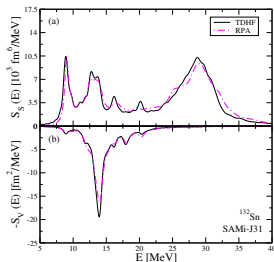
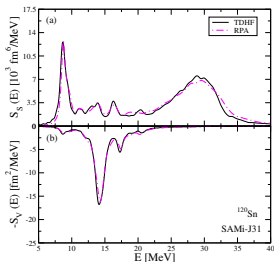
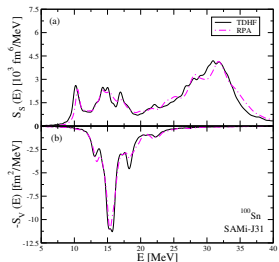
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Nature and structure of low-lying dipole modes

- Evolution with the neutron/proton content \Rightarrow Sn isotopic chain [see Markova's talk]



- PDR \Rightarrow isoscalar-like mode

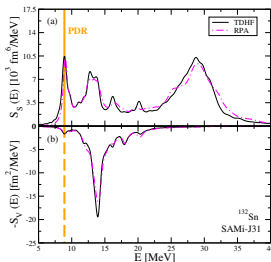
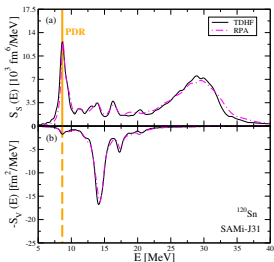
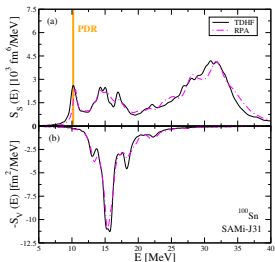
isoscalar-like mode for neutron-rich nuclei
 n and p move in phase for ^{100}Sn
 involves the neutron skin (skin)

- ^{120}Sn surface is more diffuse than ^{132}Sn
 (open vs closed-shell nucleus)

[S. Burrello et al., Phys. Rev. C 99, 054314 (2019)]

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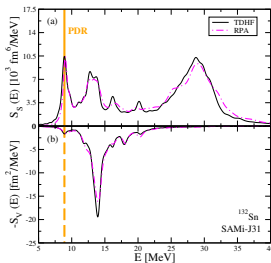
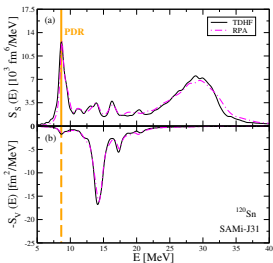
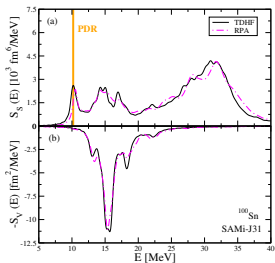


- PDR \Rightarrow **isoscalar-like** mode
 - **IV component** for **neutron-rich** nuclei
 - n and p move in phase for ^{100}Sn
 - involves the **outer surface (skin)**
- ^{120}Sn surface is more diffuse than ^{132}Sn (open vs closed-shell nucleus)

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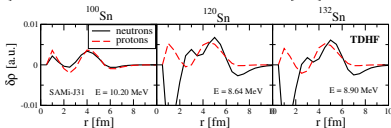
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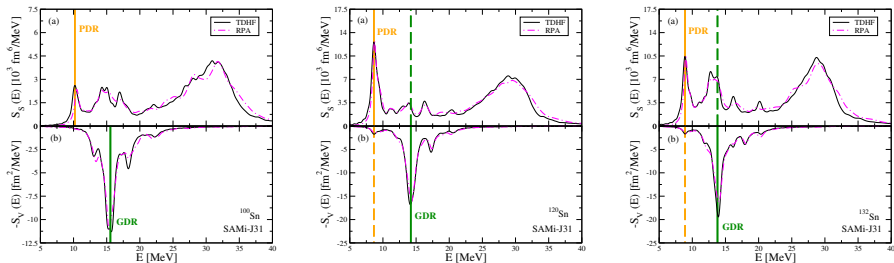
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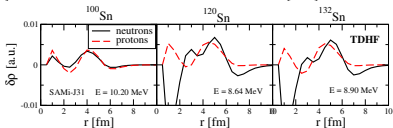
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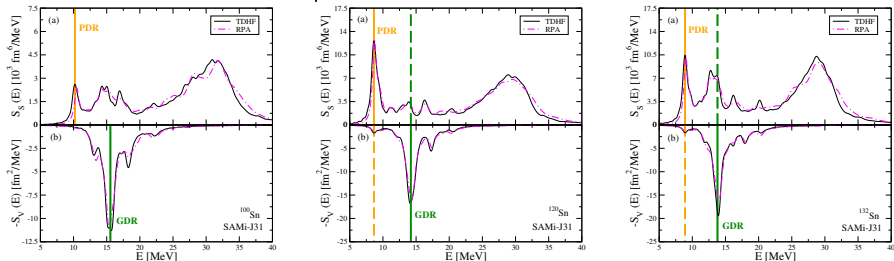


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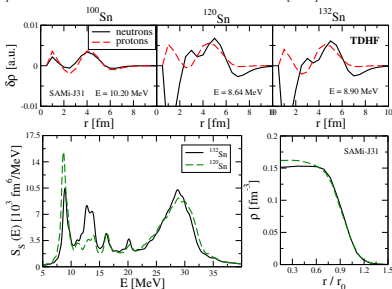


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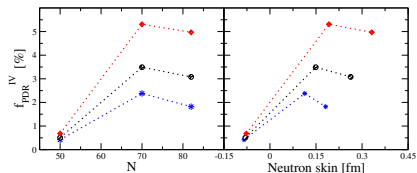
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Evolution of IS/IV PDR along Sn isotopic chain

- **Question:** IV PDR fraction of **Energy Weighted Sum Rule** does not grow from N
- **Explanation:** it reflects the **decrease** in the **IS fraction** and IS dipole strength

[S. Burrello et al., Phys. Rev. C 99, 054314 (2019)]

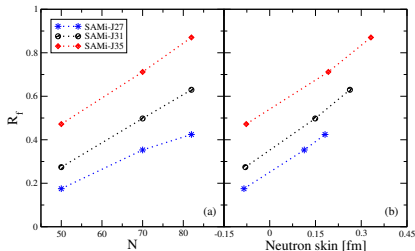
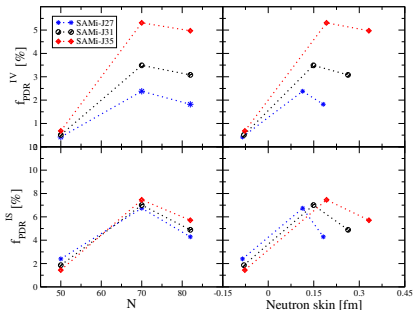


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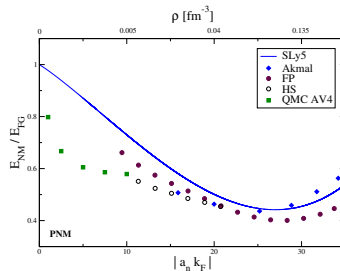
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3 Summary and perspectives

EFT-inspired EDFs: YGLO and ELYO

- Dilute PNM ($a_s = -18.9$ fm) \Rightarrow close to **unitary** limit of interacting **Fermi** gas
- Lee-Yang (LY) expansion in $(a_s k_F)$ from EFT ($\nu_i = 2, 4$ for PNM, SNM)

$$\frac{E}{N} = \frac{\hbar^2 k_F^2}{2m} \left[\frac{3}{5} + (\nu_i - 1) \frac{2}{3\pi} (k_F a_s) + (\nu_i - 1) \frac{4}{35\pi^2} (11 - 2 \ln 2) (k_F a_s)^2 + \dots \right]$$



- Combining EDF with LY expansion:

● Yang Grasso Lacroix Orsay (YGLO)

[C.J. Yang, M. Grasso, D. Lacroix, PRC 94, 031301 (2016)]

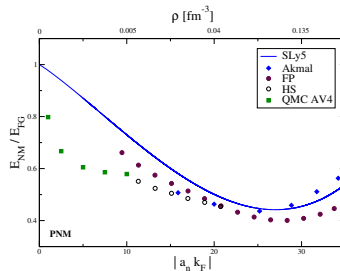
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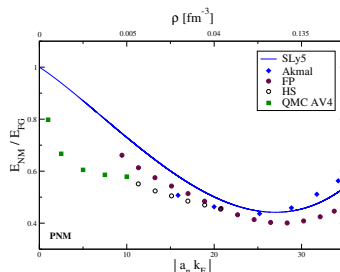
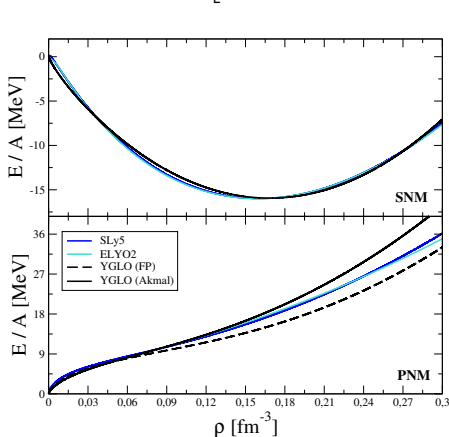
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YGLO: EFT resummed formula and benchmark

- Potential part of **YGLO functional** ($a_i = -18.9(-20.0)$ fm, $i = S, N$)

$$\mathcal{E}_Y = Y_i[\rho]\rho^2 + D_i\rho^{8/3} + F_i\rho^{(\alpha+2)}, \quad Y_i[\rho] = \frac{B_i}{1 - R_i\rho^{1/3} + C_i\rho^{2/3}}$$
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- Benchmark on **ab-initio** \Rightarrow fit of PNM Quantum Monte-Carlo calculations
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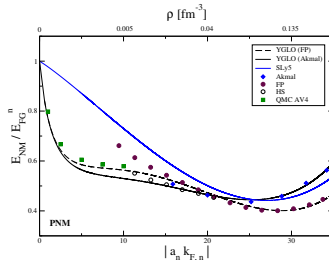
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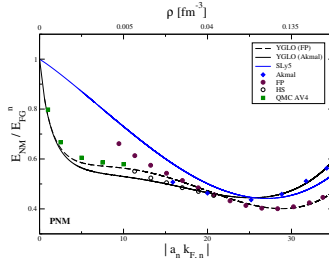
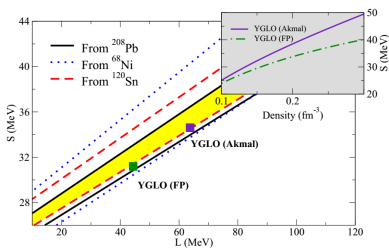
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\hookrightarrow extra term $\mathcal{E}'_3 \propto \nabla^2 D$ (with $\nu_i = 2/3$)

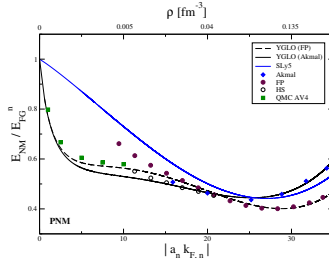
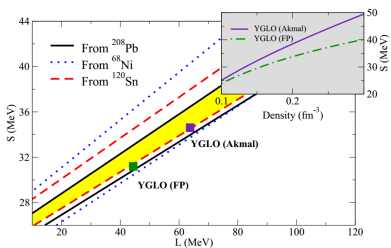
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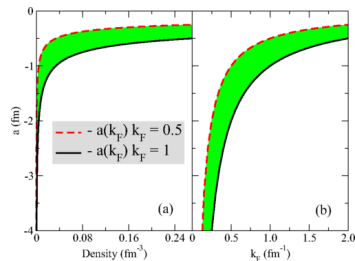
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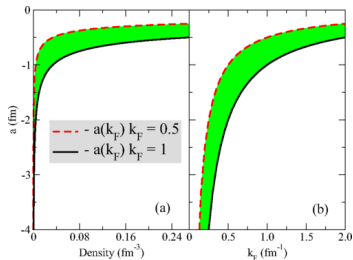
- Mapping with **s-wave** Skyrme-like EDF

$$t_0(1 - x_0) = \frac{4\pi\hbar^2}{m} a_s(\rho)$$

$$t_3(1 - x_3) = \frac{144\hbar^2}{3} 5m(3\pi^2)^{1/3}(11 - 2 \ln 2)a_s^2(\rho)$$

$$t_1(1 - x_1) = W_1 \frac{2\pi\hbar^2}{m} (a_s^2(\rho)r_s + 0.19\pi a_s^3(\rho))$$

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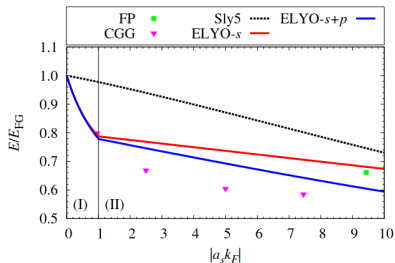
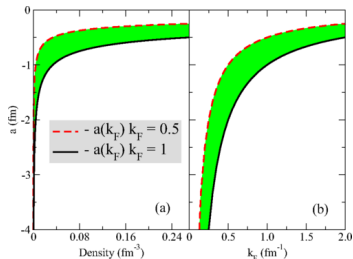
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$$t_2(1 - x_2) = W_2 \frac{4\pi\hbar^2}{m} a_p^3(\rho)$$

[J. Bonnard, M. Grasso, D. Lacroix, PRC 101, 064319 (2020)]



Energy of neutron drops and effective mass

- **Application** of YGLO and ELYO on **finite systems** \Rightarrow **neutron drops**

[J. Bonnard, M. Grasso, D. Lacroix, PRC 98, 034319 (2018); PRC 103, 039901(E) (2021)]

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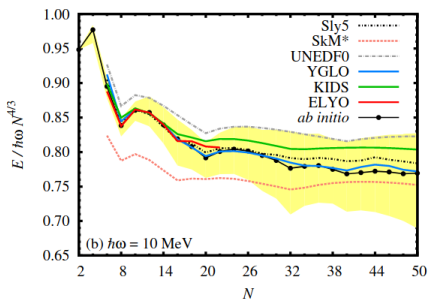
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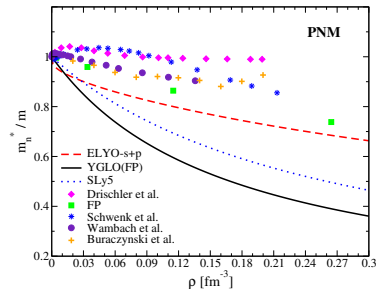
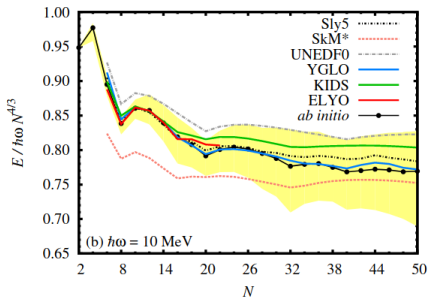
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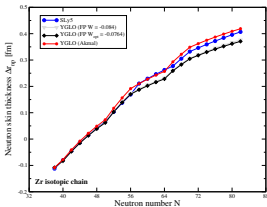
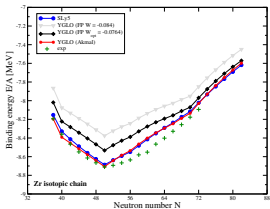
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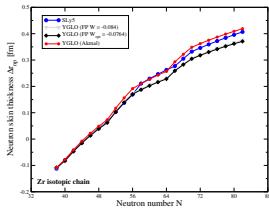
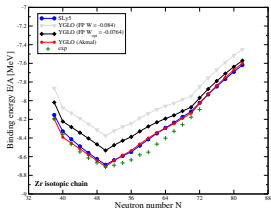


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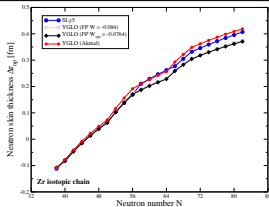
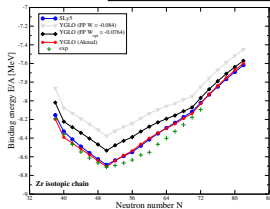
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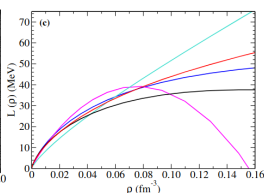
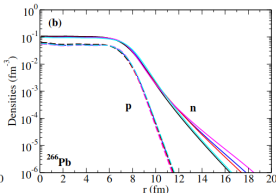
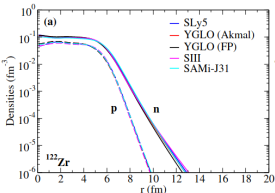
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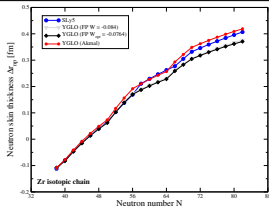
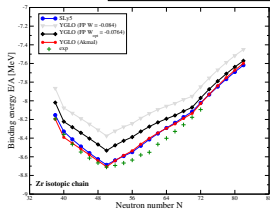
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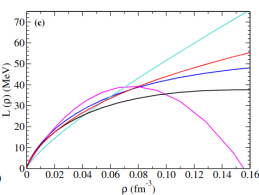
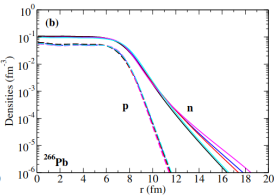
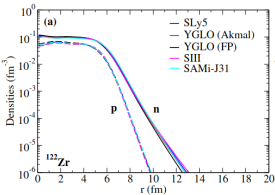
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What next?

- Improving **effective mass** predictions
- **Implementation** in **dynamical** models

● Correlation between tail of density profiles and slope of $S(\rho)$ at low-density



Outline of the presentation

1 Theoretical approaches for nuclear many-body problem

- ⌘ Ab-initio vs phenomenological models based on energy density functionals (EDF)
- ⌘ Effective interaction and nuclear matter (NM) Equation of State (EoS)

2 EDF-based models: recent results and further developments

⇒ Mean-field dynamical models with phenomenological EDFs

- ⌘ Nature of low-lying dipole modes and connection with nuclear structure
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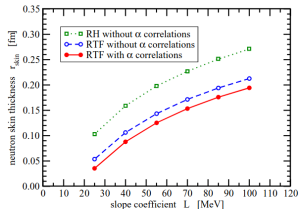
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 - Formation of nucleon **bound** states (**clustering**)
- **Phenomenological** EDF-based models with **clusters**
 - Generalized relativistic density functional (**GRDF**)
[S. Typel et al., PRC 81, 015803 (2010)]
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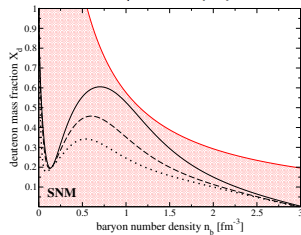
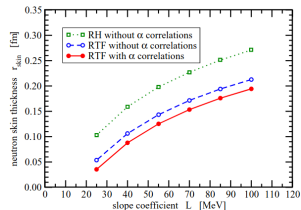
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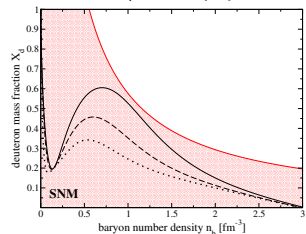
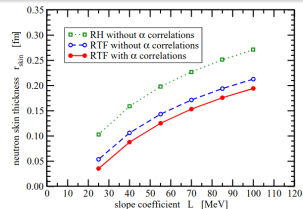
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Err. Phys. J. A (2022) 58:120
 THE EUROPEAN PHYSICAL JOURNAL A
Embedding short-range correlations in relativistic density functionals through quasi-deuterons
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 THE EUROPEAN PHYSICAL JOURNAL A
Embedding short-range correlations in relativistic density functionals through quasi-deuterons
 S. Burrello^{1,a}, S. Typel^{1,2,b}

What next?

- Inclusion of **light clusters** (and quasi-deuterons) within a **kinetic** approach
- Study of collective **excitation modes** [in coll. with R. Wang + INFN CT]

Outline of the presentation

1 Theoretical approaches for nuclear many-body problem

- Ab-initio vs phenomenological models based on energy density functionals (EDF)
- Effective interaction and nuclear matter (NM) Equation of State (EoS)

2 EDF-based models: recent results and further developments

- Many-body perturbation theory with χ -mixing interactions (MBPT)

- χ -mixing interactions: a new paradigm for the construction of energy density functionals

3 Summary and perspectives

Final remarks and conclusions

Main topic

- **MF** calculations for nuclear **structure** and small amplitude **dynamics**
- Extension of EDFs to **bridge** with **ab-initio** approaches and include **clusters**

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- Characterization of the **nature** of **low-lying** response, in view of **IS/IV mixing**
- **Evolution** of low-lying modes with **density profiles** and **neutron skin**
- Application to **finite systems** of EDFs grounded on **ab-initio** at **low-density**
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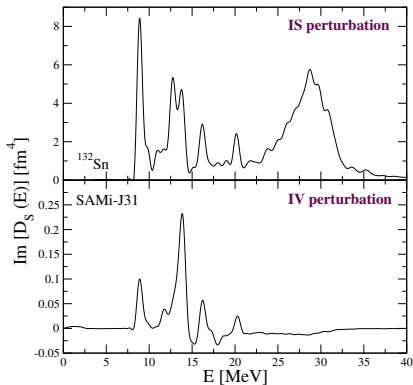
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THANK YOU FOR YOUR KIND ATTENTION!

Back-up slides

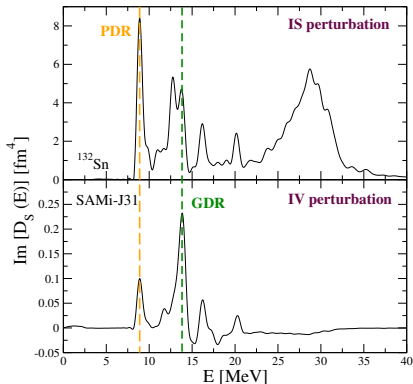
Coupling between IS and IV modes

- **Symmetric** nuclear matter: **IS** and **IV** modes are **decoupled**
- **Neutron-rich** systems: n and p oscillate with **different amplitudes** \Rightarrow **coupling**



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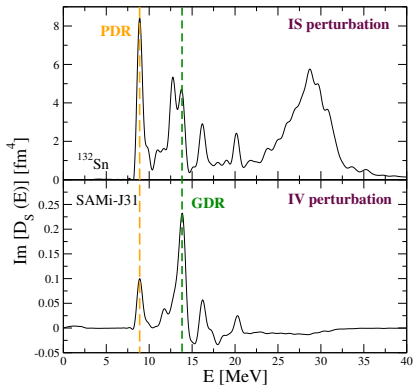
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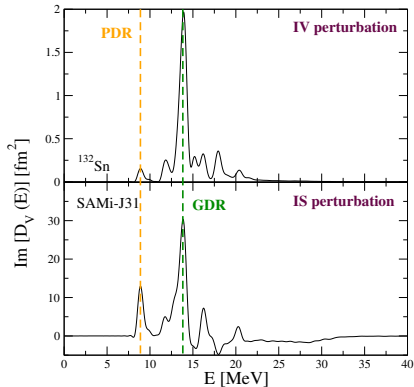
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IS response



IV response

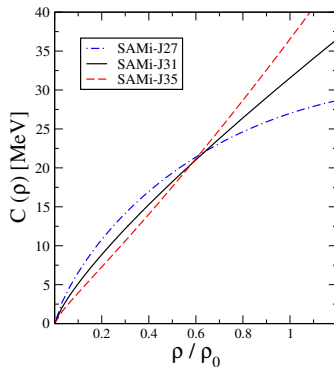
Influence of the effective interaction

- SAMi-J interactions

[X. Roca-Maza et al., PRC87, (2013)]

⇒ isolate influence of **IV channel**

$$E_{\text{sym}}(\rho) = C(\rho)I^2$$



- Sensitivity of $E_{\text{IV-GDR}}$ to E_{sym} at crossing
- Role of symmetry energy slope:
 - IV PDR
- Agreement with Vlasov results

[Zheng, H. et al., PRC 94 (2016)]

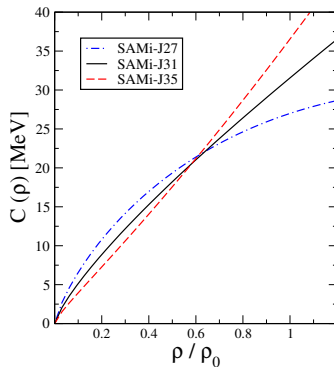
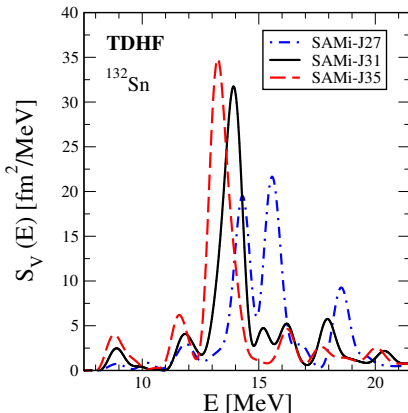
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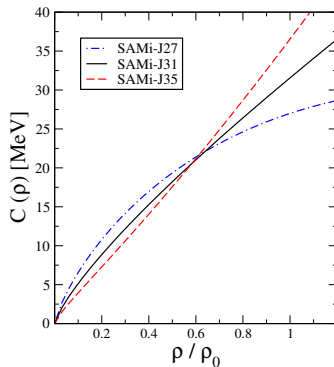
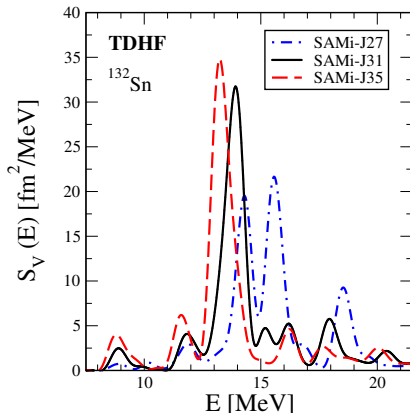
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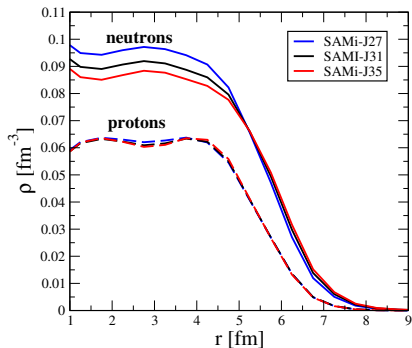
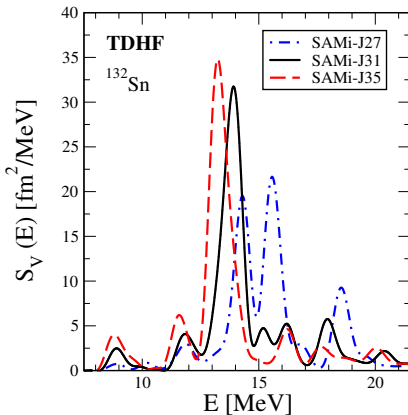
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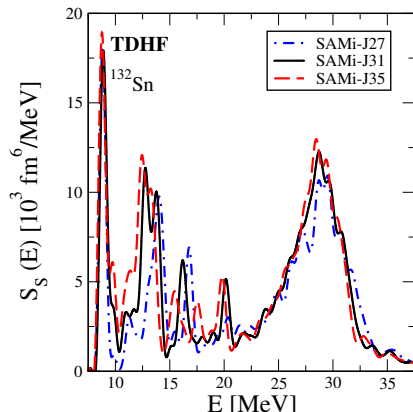
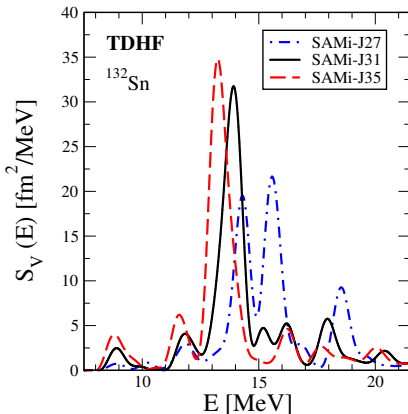
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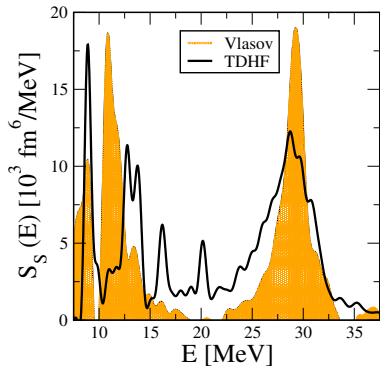
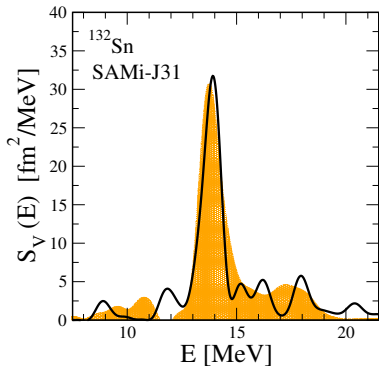
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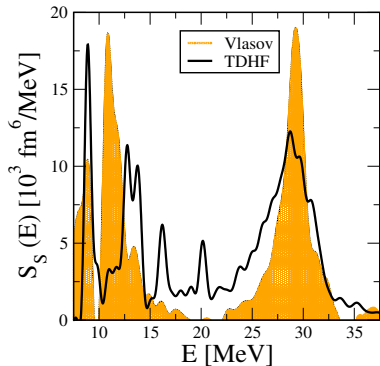
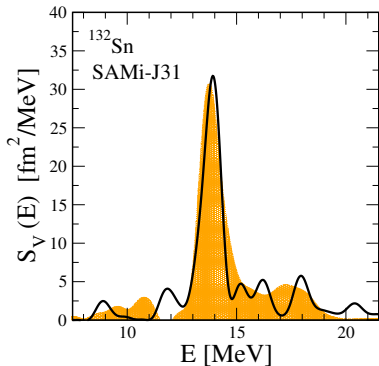


Comparison between Vlasov and TDHF model



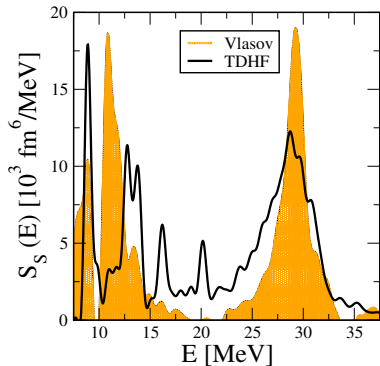
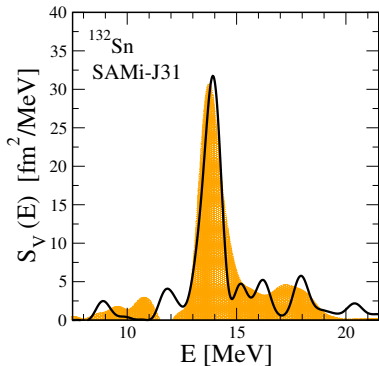
- Good reproduction of **IV GDR** and **IS GDR**
- Two contributions in low-energy region: [see M. Urban, PRC85, (2012)]
 - PDR mode (outer surface)
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- Displacement of PDR peaks \Rightarrow numerical treatment of surface

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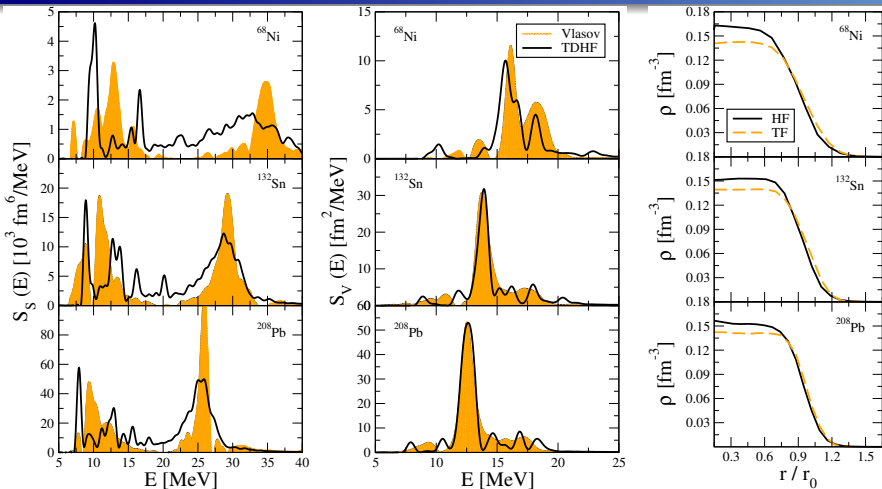
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Link between nuclear response and density profiles



- **Sharper evolution** from **bulk** to **surface** region favor **toroidal** mode

Smoother density profile leads to **robust PDR** oscillations

[S. Burrello et al., Phys. Rev. C 99, 054314 (2019)]

Comparison between TDHF and RPA

- **TDHF** and **RPA equivalent** in zero-amplitude limit, despite **technical procedures**
- Question: which **numerical parameters** ensure the best agreement?
- Dependence on **box size** (i.e. **discretization** of **continuum** single-particle states)
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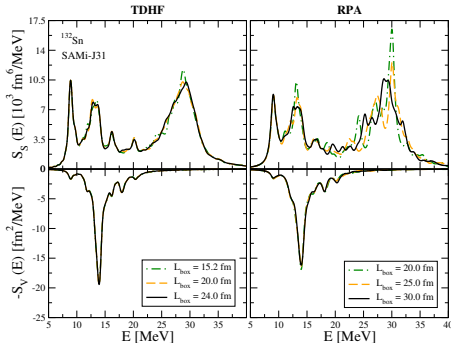
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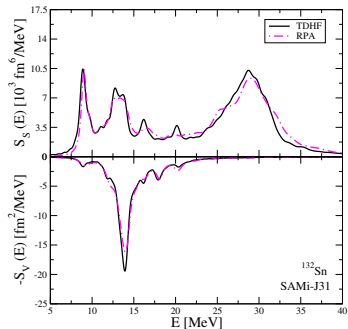
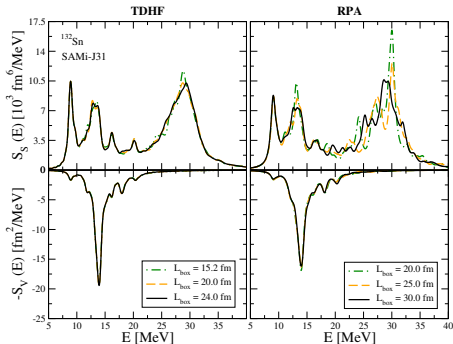


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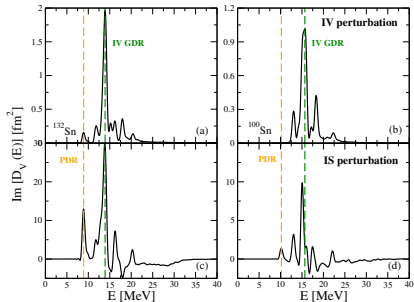
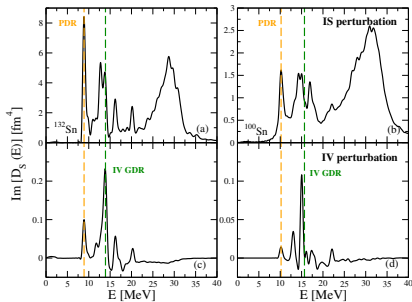
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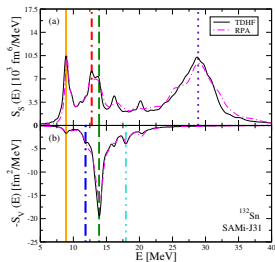
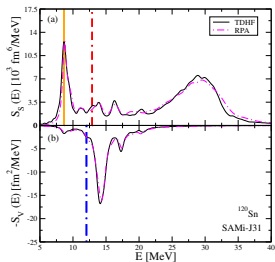
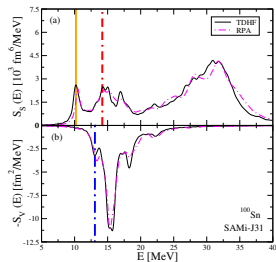


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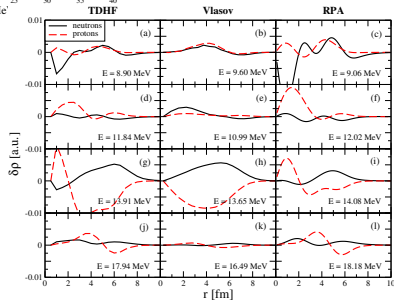
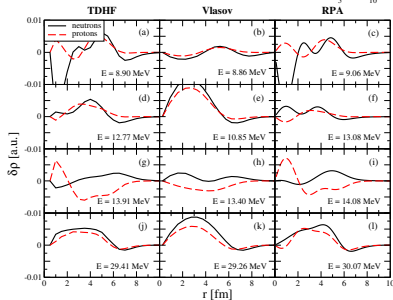
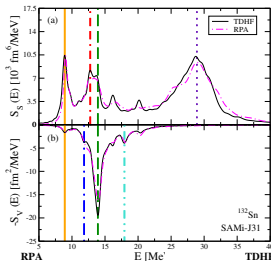
Back-up slides: focus on IS/IV mixing



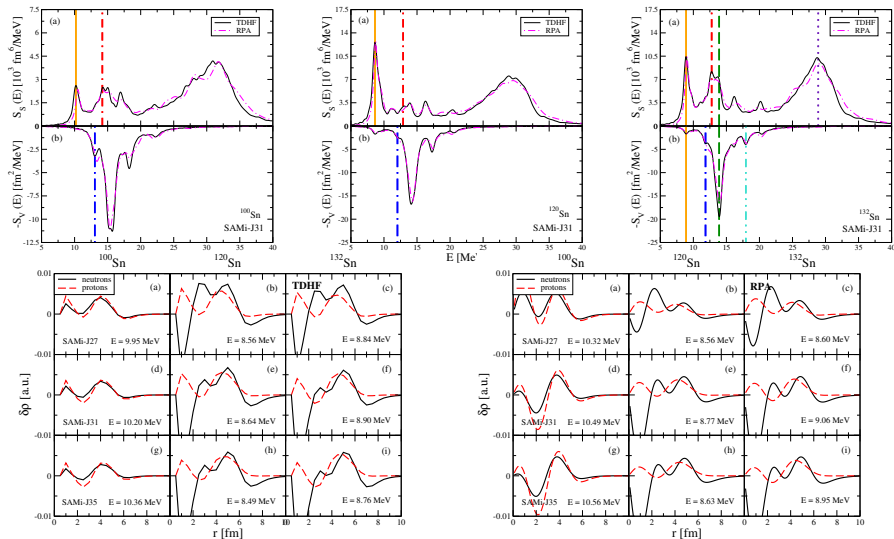
Back-up slides: dipole strength in Sn isotopes



Back-up slides: transition densities comparison



Back-up slides: transition densities of PDR



Back-up slides: torodail mode and 2nd IV peak

