

Impact of the isospin symmetry breaking on the neutron-skin thickness and the nuclear equation of state

内藤 智也 (Tomoya Naito)

RIKEN iTHEMS Program, JAPAN

Department of Physics, Graduate School of Science, The University of Tokyo, JAPAN

15 June 2023

7th International Conference on Collective Motion in Nuclei under Extreme Conditions (COMEX7)

Dipartimento di Fisica e Astronomia, Università degli Studi di Catania, Catania, ITALY



- Nuclear interaction: isospin symmetric

$$v_{pp}^{T=1} = v_{pn}^{T=1} = v_{nn}^{T=1}$$

Miller, Opper, and Stephenson. *Annu. Rev. Nucl. Part. Sci.* **56**, 253 (2006)

Nuclear Interaction and Isospin T

- Nuclear interaction: *almost* isospin symmetric

$$v_{pp}^{T=1} \simeq v_{pn}^{T=1} \simeq v_{nn}^{T=1}$$

- Origin of isospin symmetry breaking: $m_u \neq m_d$

Miller, Opper, and Stephenson. *Annu. Rev. Nucl. Part. Sci.* **56**, 253 (2006)

Nuclear Interaction and Isospin T

- Nuclear interaction: *almost* isospin symmetric

$$v_{pp}^{T=1} \simeq v_{pn}^{T=1} \simeq v_{nn}^{T=1}$$

- Origin of isospin symmetry breaking: $m_u \neq m_d$
- Charge symmetry breaking (CSB)**
 - Difference between p - p int. and n - n int.

$$v_{\text{CSB}} \equiv v_{nn}^{T=1} - v_{pp}^{T=1} \sim \tau_{zi} + \tau_{zj}$$

- Originates from mass difference of nucleons ($m_p \neq m_n$) and π^0 - η & ρ^0 - ω mixings in meson-exchange process
- Charge independence breaking (CIB)**
 - Difference between like-particle int. and diff.-particle int.

$$v_{\text{CIB}} \equiv \frac{v_{nn}^{T=1} + v_{pp}^{T=1}}{2} - v_{np}^{T=1} \sim \tau_{zi}\tau_{zj}$$

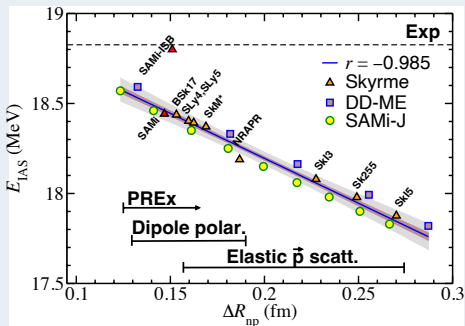
- Originates from mass difference of pions ($m_{\pi^0} \neq m_{\pi^\pm}$)

Miller, Opper, and Stephenson. *Annu. Rev. Nucl. Part. Sci.* **56**, 253 (2006)

Isospin Symmetry Breaking of Atomic Nuclei

- Isospin symmetry of atomic nuclei is *slightly* broken due to
 - Coulomb interaction
 - Isospin symmetry breaking (ISB) terms of nuclear interaction
- Different properties of mirror nuclei
 - Mass
 - Coulomb int. is not enough (Okamoto-Nolen-Schiffer anomaly)
 - Ground-state spin ($^{73}_{38}\text{Sr}$ ($5/2^-$) and $^{73}_{35}\text{Br}$ ($1/2^-$) at NSCL)
 - Shape ($^{70}_{36}\text{Kr}$ and $^{70}_{34}\text{Se}$ at RIBF)
- Finite (negative) neutron-skin thickness $\Delta R_{np} = R_n - R_p$ of $N = Z$ nuclei
 - Okamoto. *Phys. Lett.* **11**, 150 (1964)
 - Nolen and Schiffer. *Annu. Rev. Nucl. Sci.* **19**, 471 (1969)
 - Hoff *et al.* *Nature* **580**, 52 (2020)
 - Wimmer *et al.* *Phys. Rev. Lett.* **126**, 072501 (2021)

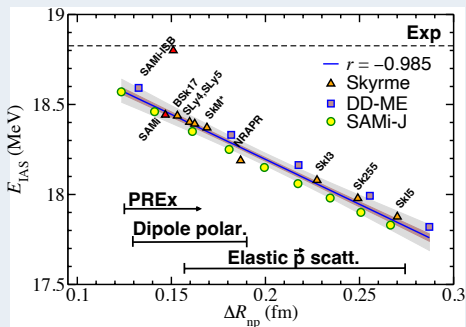
ISB Effects on Isobaric Analog State and Neutron-Skin Thickness



- There is a correlation between E_{IAS} and ΔR_{np} of ^{208}Pb
- Without ISB terms, exp. values of E_{IAS} and ΔR_{np} cannot be described at the same time

Roca-Maza, Colò, and Sagawa. *Phys. Rev. Lett.* **120**, 202501 (2018)

ISB Effects on Isobaric Analog State and Neutron-Skin Thickness



- There is a correlation between E_{IAS} and ΔR_{np} of ^{208}Pb
- Without ISB terms, exp. values of E_{IAS} and ΔR_{np} cannot be described at the same time
- With ISB terms in DFT, exp. values of E_{IAS} and ΔR_{np} can be described at the same time

Roca-Maza, Colò, and Sagawa. *Phys. Rev. Lett.* **120**, 202501 (2018)

Current Status of ISB Interaction

- ISB terms of bare interaction: Constrained (e.g. AV18)
- Systematic study on nuclear chart → Effective int. (EDF) is needed
- Effective int. with ISB terms: Only a few
- Accuracy of effective nuclear int. (EDF)
 E_{tot} within 1 MeV → Final goal: $\lesssim 500$ keV? (c.f., mass model)
 - ISB terms may contribute E_{tot} in several MeV
→ ISB terms may be needed
 - Coulomb int: Not accurately treated
- Systematic study of effects of ISB terms: Not yet
 - Both ISB terms and Coulomb int. break the isospin symmetry
→ We should distinguish two contributions
 - ISB effects are not large
→ Precise calculation is needed
 - Thus, accuracy of the Coulomb EDF should be carefully considered

Series of Works

- Towards systematic studies of ISB effects on nuclear ground state, Coulomb interaction is treated precisely in nuclear DFT
 - Effect of $|\nabla\rho|$ in Coulomb exchange
 - Charge form factors of nucleons in Coulomb EDF (effects of $\rho_{\text{ch}} \neq \rho_p$)
 - Vacuum polarization (e^-e^+ creation)
- Effects of ISB terms on g.s. are compared with those of Coulomb int.
 - Mass difference of mirror nuclei
 - Neutron-skin thickness of $N = Z$ and $N > Z$ nuclei
 - $\rho_{\text{IV}} = \rho_n - \rho_p$

- ISB effects on $L-\Delta R_{np}$ and $L-\Delta R_{\text{ch}}$ correlation are discussed

Naito, Akashi, and Liang. *Phys. Rev. C* **97**, 044319 (2018)

Naito, Roca-Maza, Colò, and Liang. *Phys. Rev. C* **99**, 024309 (2019)

Naito, Roca-Maza, Colò, and Liang. *Phys. Rev. C* **101**, 064311 (2020)

Sagawa, Yoshida, Naito et al. *Phys. Lett. B* **829**, 137072 (2022)

Naito, Roca-Maza, Colò, Liang, and Sagawa. *Phys. Rev. C* **106**, L061306 (2022)

Naito, Colò, Liang, Roca-Maza, and Sagawa. *Phys. Rev. C* **107**, 064302 (2023)

Series of Works

- Towards systematic studies of ISB effects on nuclear ground state, Coulomb interaction is treated precisely in nuclear DFT
 - Effect of $|\nabla\rho|$ in Coulomb exchange
 - Charge form factors of nucleons in Coulomb EDF (effects of $\rho_{\text{ch}} \neq \rho_p$)
 - Vacuum polarization (e^-e^+ creation)
- Effects of ISB terms on g.s. are compared with those of Coulomb int.
 - Mass difference of mirror nuclei
 - Neutron-skin thickness of $N = Z$ and $N > Z$ nuclei
 - $\rho_{\text{IV}} = \rho_n - \rho_p$
- ISB effects on $L-\Delta R_{np}$ and $L-\Delta R_{\text{ch}}$ correlation are discussed

Naito, Akashi, and Liang. *Phys. Rev. C* **97**, 044319 (2018)

Naito, Roca-Maza, Colò, and Liang. *Phys. Rev. C* **99**, 024309 (2019)

Naito, Roca-Maza, Colò, and Liang. *Phys. Rev. C* **101**, 064311 (2020)

Sagawa, Yoshida, Naito et al. *Phys. Lett. B* **829**, 137072 (2022)

Naito, Roca-Maza, Colò, Liang, and Sagawa. *Phys. Rev. C* **106**, L061306 (2022)

Naito, Colò, Liang, Roca-Maza, and Sagawa. *Phys. Rev. C* **107**, 064302 (2023)

Method in This Work

- **Nuclear density functional theory (DFT)** is used because only this can be applied to whole nuclear chart at this moment

$$E[\rho_p, \rho_n] = T_{\text{KS}}[\rho_p, \rho_n] + E_{\text{nucl}}[\rho_p, \rho_n] + E_{\text{CH}}[\rho_{\text{ch}}] + E_{\text{Cx}}[\rho_{\text{ch}}]$$

- **Skyrme-type effective interaction** is used for E_{nucl} (non-rel. calc.)

$$v_{\text{Sky}}^{\text{IS}}(\mathbf{r}) = t_0(1 + x_0 P_\sigma) \delta(\mathbf{r}) + \frac{t_1}{2}(1 + x_1 P_\sigma) [\mathbf{p}^{\dagger 2} \delta(\mathbf{r}) + \delta(\mathbf{r}) \mathbf{p}^2] + t_2(1 + x_2 P_\sigma) \mathbf{p}^\dagger \cdot \delta(\mathbf{r}) \mathbf{p} \\ + \frac{t_3}{6}(1 + x_3 P_\sigma) \delta(\mathbf{r}) [\rho(\mathbf{R})]^\alpha + iW_0 \boldsymbol{\sigma} \cdot \mathbf{p}^\dagger \times \delta(\mathbf{r}) \mathbf{p}$$

SAMi and its family are used for E_{nucl}

- Doubly-magic nuclei are focused on
→ Spherical symmetry is assumed and pairing is neglected

- To perform mean-field (DFT) calculation, the Skyrme-like ISB interaction is introduced

$$v_{\text{Sky}}^{\text{CSB}}(\mathbf{r}) = s_0 (1 + y_0 P_\sigma) \delta(\mathbf{r}) \frac{\tau_{1z} + \tau_{2z}}{4}$$

$$v_{\text{Sky}}^{\text{CIB}}(\mathbf{r}) = u_0 (1 + z_0 P_\sigma) \delta(\mathbf{r}) \frac{\tau_{1z} \tau_{2z}}{2}$$

$$\mathcal{E}_{\text{CSB}} = \frac{s_0 (1 - y_0)}{8} (\rho_n^2 - \rho_p^2)$$

$$\mathcal{E}_{\text{CIB}} = \frac{u_0}{8} \left[(1 - z_0) (\rho_n^2 + \rho_p^2) - 2(2 + z_0) \rho_n \rho_p \right]$$

Note: $\tau_z = -1$ for protons and $\tau_z = +1$ for neutrons (low-energy convention)

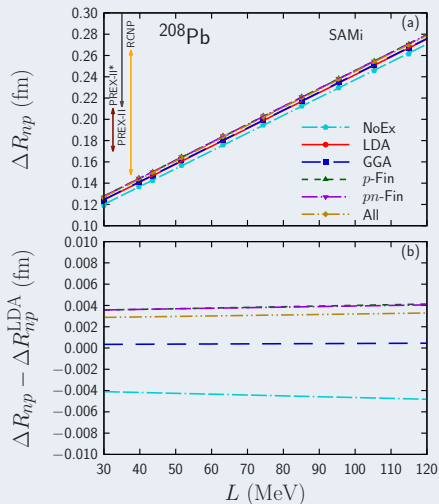
- SAMi-ISB EDF is used in this work
 - $y_0 = z_0 = -1$ to select the spin-singlet ($S = 0$) channel
 - s_0 and u_0 are parameters
 - All the parameters including the main part ($t_j, x_j, W_0, W'_0, \alpha$) are optimized altogether

Sagawa, Van Giai, and Suzuki. *Phys. Lett. B* **353**, 7 (1995)

Roca-Maza, Colò, and Sagawa. *Phys. Rev. Lett.* **120**, 202501 (2018)

Neutron-Skin Thickness and Charge Radii Difference

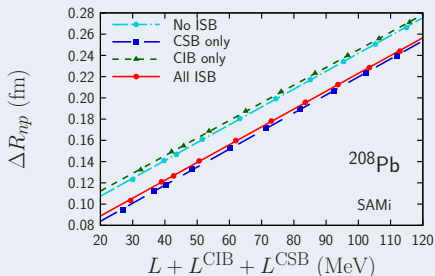
Coulomb Effect on Neutron-Skin Thickness



- ISB terms are not considered
- Treatment of Coulomb int. does not change the slope of L - ΔR_{np} dependence
- Absolute value of ΔR_{np} changes, but its value is tiny \rightarrow negligible

Neutron-Skin Thickness and Charge Radii Difference

Neutron-Skin Thickness of ^{208}Pb



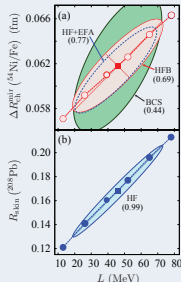
- L vs ΔR_{np} correlation is estimated using SAMi-J family
- SAMi-J family
Same as SAMi but different J
→ Different L
- On top of SAMi-J family, ISB terms are considered

- If we assume the same ΔR_{np} , difference between estimated L_{full} without & that with ISB is 11.1 MeV
CSB contribution 13.9 MeV
CIB contribution -2.7 MeV
- $L_{\text{CIB}} = 2.3$ MeV and $L_{\text{CSB}} = -3.2$ MeV → Change of L is 12 MeV
(predicted value of L : $30 \lesssim L \lesssim 100$ MeV)

Neutron-Skin Thickness and Charge Radii Difference

Charge-Radii Difference of Mirror Nuclei

- Without Coulomb nor ISB, $R_p^{(Z,N)} = R_n^{(N,Z)}$ and $R_n^{(Z,N)} = R_p^{(N,Z)}$ hold
 $\rightarrow \Delta R_{\text{ch}}^{(Z,N)} = R_{\text{ch}}^{(Z,N)} - R_{\text{ch}}^{(N,Z)} \simeq R_p^{(Z,N)} - R_p^{(N,Z)} = \Delta R_{np}^{(Z,N)}$
- R_{ch} can be measured precisely
- Therefore, it was proposed that ΔR_{ch} is correlated to L
- It was also pointed out that the correlation is weak due to pairing and deformation, in contrast to ΔR_{np}



Brown. *Phys. Rev. Lett.* **119**, 112502 (2017)

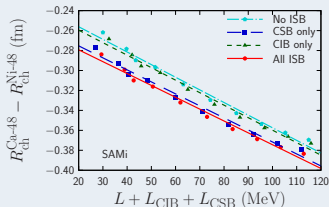
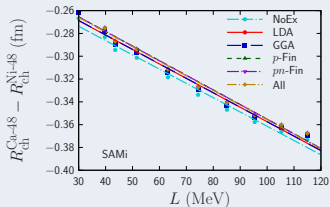
Reinhard and Nazarewicz. *Phys. Rev. C* **105**, L021301 (2022)

Charge-Radii Difference of Mirror Nuclei ^{48}Ca - ^{48}Ni

- Both ^{48}Ca and ^{48}Ni are doubly-magic nuclei in our calculation
→ No deformation nor pairing correlation
- Therefore, the weakly correlation problem may be avoidable

Charge-Radius Difference of Mirror Nuclei ^{48}Ca - ^{48}Ni

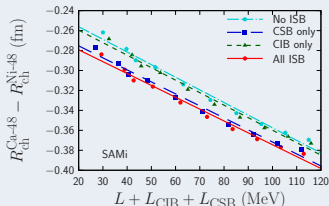
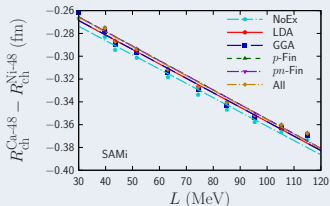
- Both ^{48}Ca and ^{48}Ni are doubly-magic nuclei in our calculation
 \rightarrow No deformation nor pairing correlation
- Therefore, the weakly correlation problem may be avoidable



- Coulomb does not affect such correlation
- ISB terms change R_{ch} , and thus ΔR_{ch}
- If we assume the same ΔR_{ch} ,
 difference between estimated L_{full} without & that with ISB is 14.7 MeV
 (CSB: 12.7 MeV, CIB: 2.0 MeV) **Non-negligible**

Charge-Radius Difference of Mirror Nuclei ^{48}Ca - ^{48}Ni

- Both ^{48}Ca and ^{48}Ni are doubly-magic nuclei in our calculation
 \rightarrow No deformation nor pairing correlation
- Therefore, the weakly correlation problem may be avoidable



- Coulomb does not affect such correlation
- ISB terms change R_{ch} , and thus ΔR_{ch}
- If we assume the same ΔR_{ch} ,
 difference between estimated L_{full} without & that with ISB is 14.7 MeV
 (CSB: 12.7 MeV, CIB: 2.0 MeV) **Non-negligible**
- Future perspectives: more realistic pair of mirror nuclei (e.g., ^{36}S - ^{36}Ca)

- **Phenomenological determination**—Referring experimental data
- ***Ab initio* determination**
—CSB strength s_0 extracted from *ab initio* calculation

Naito, Colò, Liang, Roca-Maza, and Sagawa. *Phys. Rev. C* **105**, L021304 (2022)

s_0 -value: Roca-Maza, Colò, and Sagawa. *Phys. Rev. Lett.* **120**, 202501 (2018)

s_0 -value: Bączyk, Dobaczewski *et al.* *Phys. Lett. B* **778**, 178 (2018)

CC & χ EFT: Novario, Lonardonì, Gandolfi, and Hagen. *Phys. Rev. Lett.* **130**, 032501 (2023)

VMC & AV18: Wiringa. Private communication

Mysterious of CSB Strength

- **Phenomenological determination**—Referring experimental data
 - $s_0 = -26.3 \text{ MeV fm}^3$ (IAE of ^{208}Pb)
 - $s_0 \simeq -10 \text{ MeV fm}^3$ (MDE and TDE)
- **Ab initio determination**
—CSB strength s_0 extracted from *ab initio* calculation

$O(10) \text{ MeV fm}^3$

Naito, Colò, Liang, Roca-Maza, and Sagawa. *Phys. Rev. C* **105**, L021304 (2022)

s_0 -value: Roca-Maza, Colò, and Sagawa. *Phys. Rev. Lett.* **120**, 202501 (2018)

s_0 -value: Bączyk, Dobaczewski *et al.* *Phys. Lett. B* **778**, 178 (2018)

CC & χ EFT: Novario, Lonardonì, Gandolfi, and Hagen. *Phys. Rev. Lett.* **130**, 032501 (2023)

VMC & AV18: Wiringa. Private communication

Mysterious of CSB Strength

- **Phenomenological determination**—Referring experimental data

- $s_0 = -26.3 \text{ MeV fm}^3$ (IAE of ^{208}Pb)

$$O(10) \text{ MeV fm}^3$$

- $s_0 \simeq -10 \text{ MeV fm}^3$ (MDE and TDE)

- **Ab initio determination**

—CSB strength s_0 extracted from *ab initio* calculation

- $s_0 \simeq -2 \text{ MeV fm}^3$ (ΔE_{tot} of ^{48}Ca - ^{48}Ni , CC & χ EFT)

- $s_0 \simeq -3 \text{ MeV fm}^3$ (ΔE_{tot} of ^{10}Be - ^{10}C , VMC & AV18)

Naito, Colò, Liang, Roca-Maza, and Sagawa. *Phys. Rev. C* **105**, L021304 (2022)

s_0 -value: Roca-Maza, Colò, and Sagawa. *Phys. Rev. Lett.* **120**, 202501 (2018)

s_0 -value: Bączyk, Dobaczewski *et al.* *Phys. Lett. B* **778**, 178 (2018)

CC & χ EFT: Novario, Lonardonì, Gandolfi, and Hagen. *Phys. Rev. Lett.* **130**, 032501 (2023)

VMC & AV18: Wiringa. Private communication

Mysterious of CSB Strength

- **Phenomenological determination**—Referring experimental data

- $s_0 = -26.3 \text{ MeV fm}^3$ (IAE of ^{208}Pb)

$$O(10) \text{ MeV fm}^3$$

- $s_0 \simeq -10 \text{ MeV fm}^3$ (MDE and TDE)

- **Ab initio determination**

—CSB strength s_0 extracted from *ab initio* calculation

- $s_0 \simeq -2 \text{ MeV fm}^3$ (ΔE_{tot} of ^{48}Ca - ^{48}Ni , CC & χ EFT)

$$O(1) \text{ MeV fm}^3$$

- $s_0 \simeq -3 \text{ MeV fm}^3$ (ΔE_{tot} of ^{10}Be - ^{10}C , VMC & AV18)

Naito, Colò, Liang, Roca-Maza, and Sagawa. *Phys. Rev. C* **105**, L021304 (2022)

s_0 -value: Roca-Maza, Colò, and Sagawa. *Phys. Rev. Lett.* **120**, 202501 (2018)

s_0 -value: Bączyk, Dobaczewski *et al.* *Phys. Lett. B* **778**, 178 (2018)

CC & χ EFT: Novario, Lonardonì, Gandolfi, and Hagen. *Phys. Rev. Lett.* **130**, 032501 (2023)

VMC & AV18: Wiringa. Private communication

Mysterious of CSB Strength

- **Phenomenological determination**—Referring experimental data

- $s_0 = -26.3 \text{ MeV fm}^3$ (IAE of ^{208}Pb)

$$O(10) \text{ MeV fm}^3$$

- $s_0 \approx -10 \text{ MeV fm}^3$ (MDE and TDE)

- **Ab initio determination**

—CSB strength s_0 extracted from *ab initio* calculation

- $s_0 \approx -2 \text{ MeV fm}^3$ (ΔE_{tot} of ^{48}Ca - ^{48}Ni , CC & χ EFT)

$$O(1) \text{ MeV fm}^3$$

- $s_0 \approx -3 \text{ MeV fm}^3$ (ΔE_{tot} of ^{10}Be - ^{10}C , VMC & AV18)

- CSB effect in *ab initio* is $\times 0.1$ of that in DFT?!?!

^{48}Ca - ^{48}Ni Convergence of χ EFT?

^{10}Be - ^{10}C Deformation? Too light?

DFT Side s_1 & s_2 terms?

Naito, Colò, Liang, Roca-Maza, and Sagawa. *Phys. Rev. C* **105**, L021304 (2022)

s_0 -value: Roca-Maza, Colò, and Sagawa. *Phys. Rev. Lett.* **120**, 202501 (2018)

s_0 -value: Bączyk, Dobaczewski *et al.* *Phys. Lett. B* **778**, 178 (2018)

CC & χ EFT: Novario, Lonardonì, Gandolfi, and Hagen. *Phys. Rev. Lett.* **130**, 032501 (2023)

VMC & AV18: Wiringa. Private communication

Mysterious of CSB Strength

- **Phenomenological determination**—Referring experimental data

- $s_0 = -26.3 \text{ MeV fm}^3$ (IAE of ^{208}Pb)

$O(10) \text{ MeV fm}^3$

- $s_0 \simeq -10 \text{ MeV fm}^3$ (MDE and TDE)

- **Ab initio determination**

—CSB strength s_0 extracted from *ab initio* calculation

- $s_0 \simeq -2 \text{ MeV fm}^3$ (ΔE_{tot} of ^{48}Ca - ^{48}Ni , CC & χ EFT)

$O(1) \text{ MeV fm}^3$

- $s_0 \simeq -3 \text{ MeV fm}^3$ (ΔE_{tot} of ^{10}Be - ^{10}C , VMC & AV18)

- CSB effect in *ab initio* is $\times 0.1$ of that in DFT?!?!?

Open problem

^{48}Ca - ^{48}Ni Convergence of χ EFT?

^{10}Be - ^{10}C Deformation? Too light?

DFT Side s_1 & s_2 terms?

Naito, Colò, Liang, Roca-Maza, and Sagawa. *Phys. Rev. C* **105**, L021304 (2022)

s_0 -value: Roca-Maza, Colò, and Sagawa. *Phys. Rev. Lett.* **120**, 202501 (2018)

s_0 -value: Bączyk, Dobaczewski *et al.* *Phys. Lett. B* **778**, 178 (2018)

CC & χ EFT: Novario, Lonardonì, Gandolfi, and Hagen. *Phys. Rev. Lett.* **130**, 032501 (2023)

VMC & AV18: Wiringa. Private communication

Determination from QCD Sum Rule

- Chiral condensation $\langle \bar{q}q \rangle / \langle \bar{q}q \rangle_0$ is related to p - n mass difference
- $\langle \bar{q}q \rangle / \langle \bar{q}q \rangle_0$ can be calculated by using QCD sum rule
- Comparing mirror nuclei mass difference obtained by
 - Skyrme HF calculation with s_0 , s_1 , and s_2
 - QCD sum rule and the local density approximation

we obtain

$$s_0 = -7.8_{-6.3}^{+4.4} \text{ MeV fm}^3$$
$$s_1 (1 - y_1) + 3s_2 (1 + y_2) = 0.52_{-0.29}^{+0.42} \text{ MeV fm}^5$$

Sagawa, Naito, Roca-Maza, Hatsuda. arXiv:2305.17481 [nucl-th]

Determination from QCD Sum Rule

- Chiral condensation $\langle \bar{q}q \rangle / \langle \bar{q}q \rangle_0$ is related to p - n mass difference
- $\langle \bar{q}q \rangle / \langle \bar{q}q \rangle_0$ can be calculated by using QCD sum rule
- Comparing mirror nuclei mass difference obtained by
 - Skyrme HF calculation with s_0 , s_1 , and s_2
 - QCD sum rule and the local density approximation

we obtain

$$s_0 = -7.8_{-6.3}^{+4.4} \text{ MeV fm}^3$$

$$s_1 (1 - y_1) + 3s_2 (1 + y_2) = 0.52_{-0.29}^{+0.42} \text{ MeV fm}^5$$

s_1 & s_2 terms may be small

Sagawa, Naito, Roca-Maza, Hatsuda. arXiv:2305.17481 [nucl-th]

Conclusion

- CSB and CIB terms contribute to ΔR_{np} of ^{208}Pb in -0.02 fm
this corresponds to 12 MeV in L value
- CSB and CIB terms contribute to ΔR_{ch} of ^{48}Ca - ^{48}Ni in -0.02 fm
this corresponds to 14 MeV in L value
- CSB effects on ΔE_{tot} in *ab initio* calculation is
about $\times 0.1$ of that in DFT Open problem
- QCD sum rule approach gives $\times 1/2$ – $1/3$ of DFT value
- Perspectives:
pairing, deformation, reveal the open problem, (Q)RPA calc., ...

Conclusion

- CSB and CIB terms contribute to ΔR_{np} of ^{208}Pb in -0.02 fm
this corresponds to 12 MeV in L value
- CSB and CIB terms contribute to ΔR_{ch} of ^{48}Ca - ^{48}Ni in -0.02 fm
this corresponds to 14 MeV in L value
- CSB effects on ΔE_{tot} in *ab initio* calculation is
about $\times 0.1$ of that in DFT Open problem
- QCD sum rule approach gives $\times 1/2$ – $1/3$ of DFT value
- Perspectives:
pairing, deformation, reveal the open problem, (Q)RPA calc., ...

Grazie Mille!!