

Temperature evolution of the **nucleon**  
**effective mass** and **symmetry energy**  
**coefficient** in the  $^{68-78}\text{Ni}$  isotopic chain

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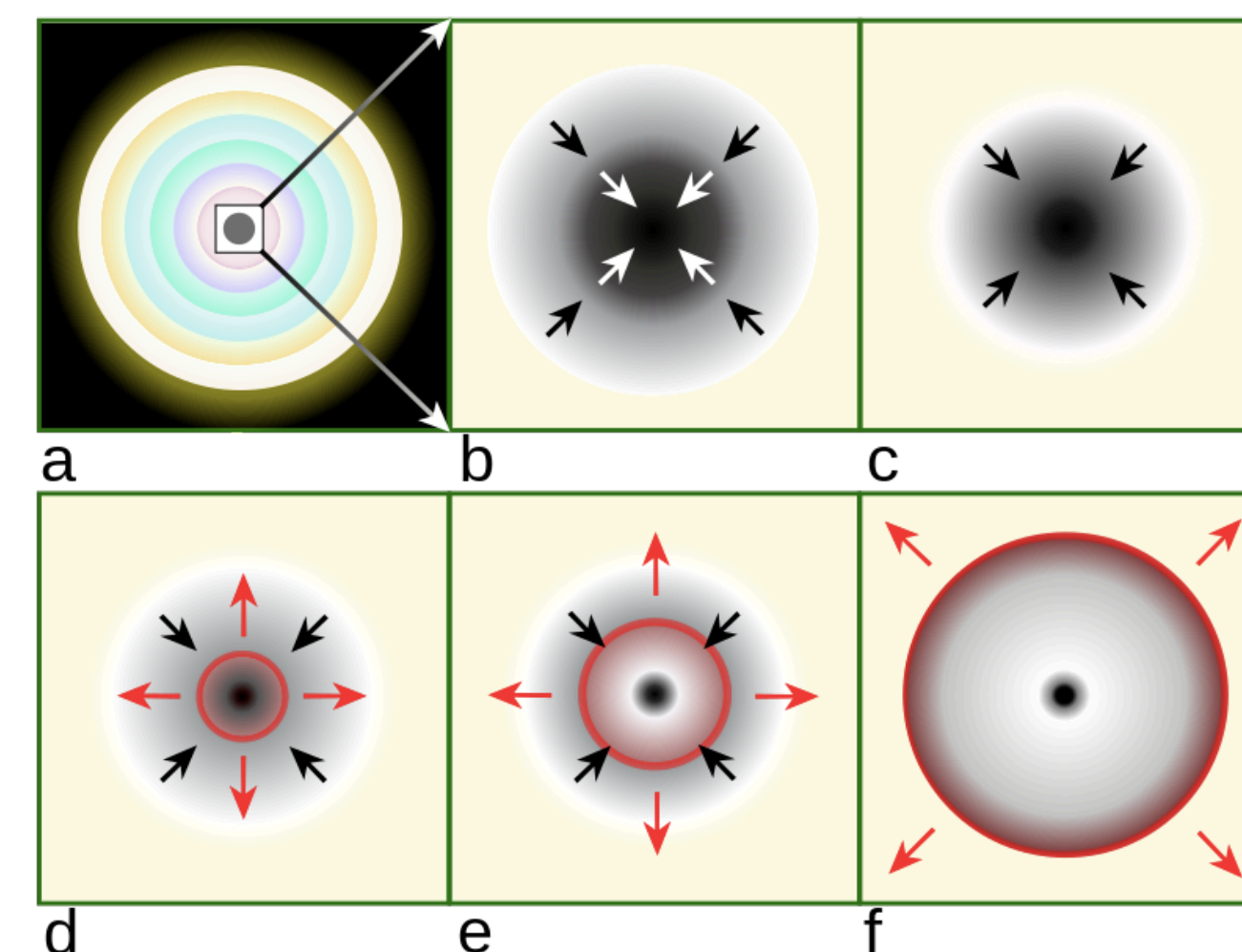
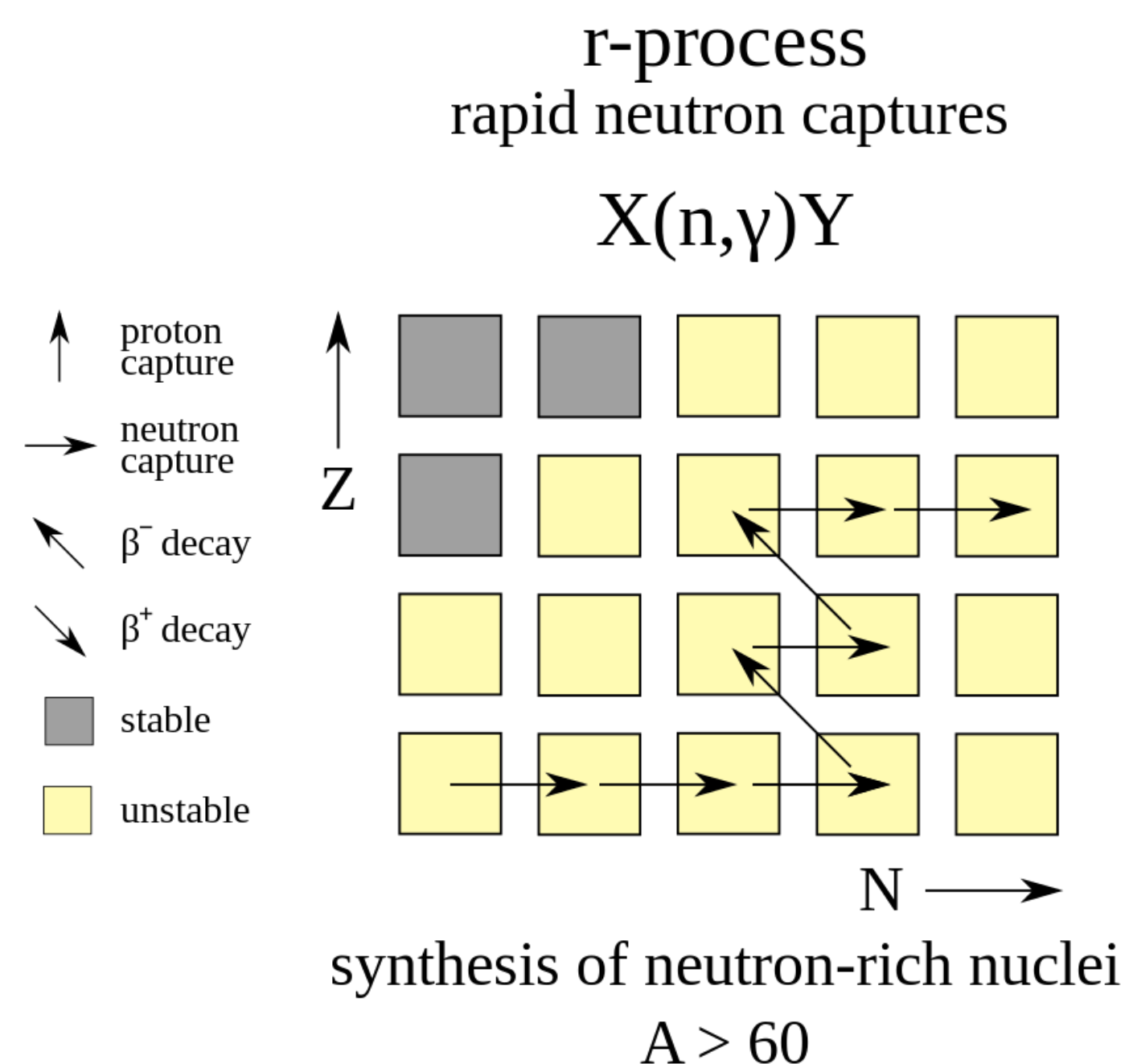


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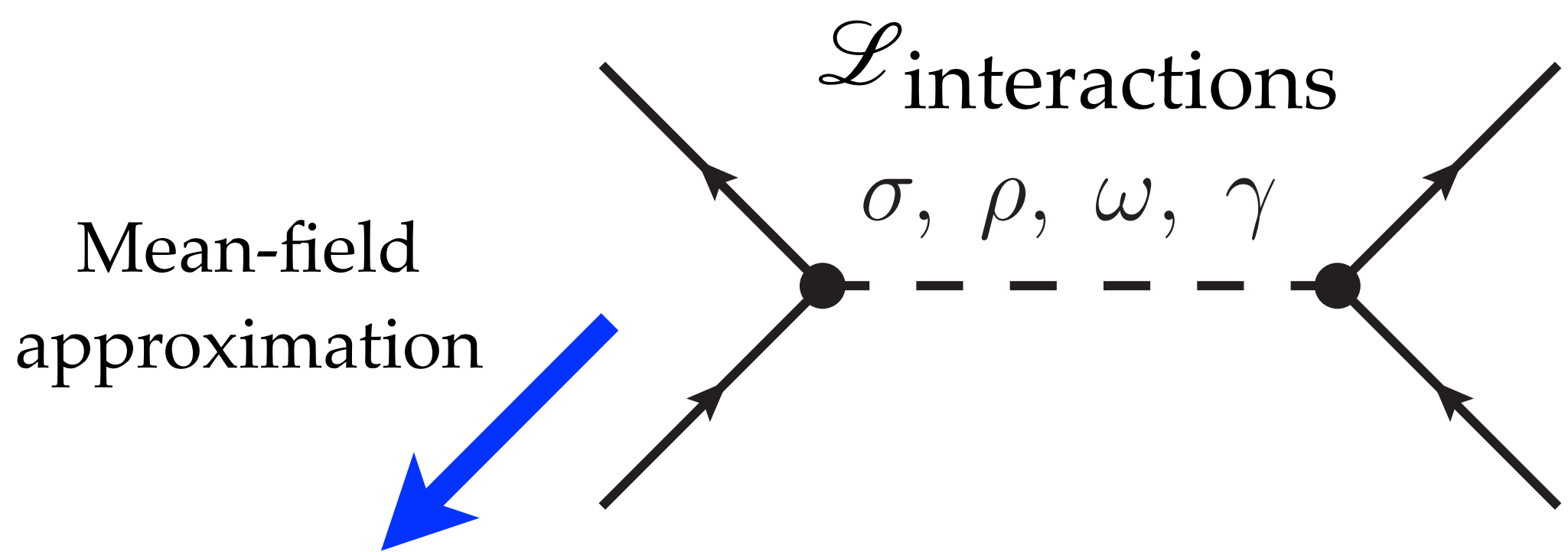
# Astrophysical motivation



By Illustration by R.J. Hall. Redrawn in Inkscape by Magasjukur2 - File:Core collapse scenario.png, CC BY-SA 3.0, <https://commons.wikimedia.org/w/index.php?curid=12779311>

- Nuclear physics input for the r-process modeling: nuclear masses, beta-decay half-lives, and neutron capture rates.
- In the neutron capture rates calculations, the level density of the excited states and gamma-ray strength functions are the critical statistical properties. [Stephane Goriely's talk, S. Nikas et al., arXiv:2010.01698]
- The evolution of the nuclear shell structure with temperature has an impact on the nuclear level density.
- During the core-collapse supernova, the rate of electron capture is affected by the temperature evolution of the nucleon effective mass and the symmetry energy. [P. Donati et al., Phys. Rev. Lett. **72**, 2835 (1994)]

# Nuclear mean field at finite temperature



Dirac equation  
 Klein-Gordon equations  
 Four baryonic densities

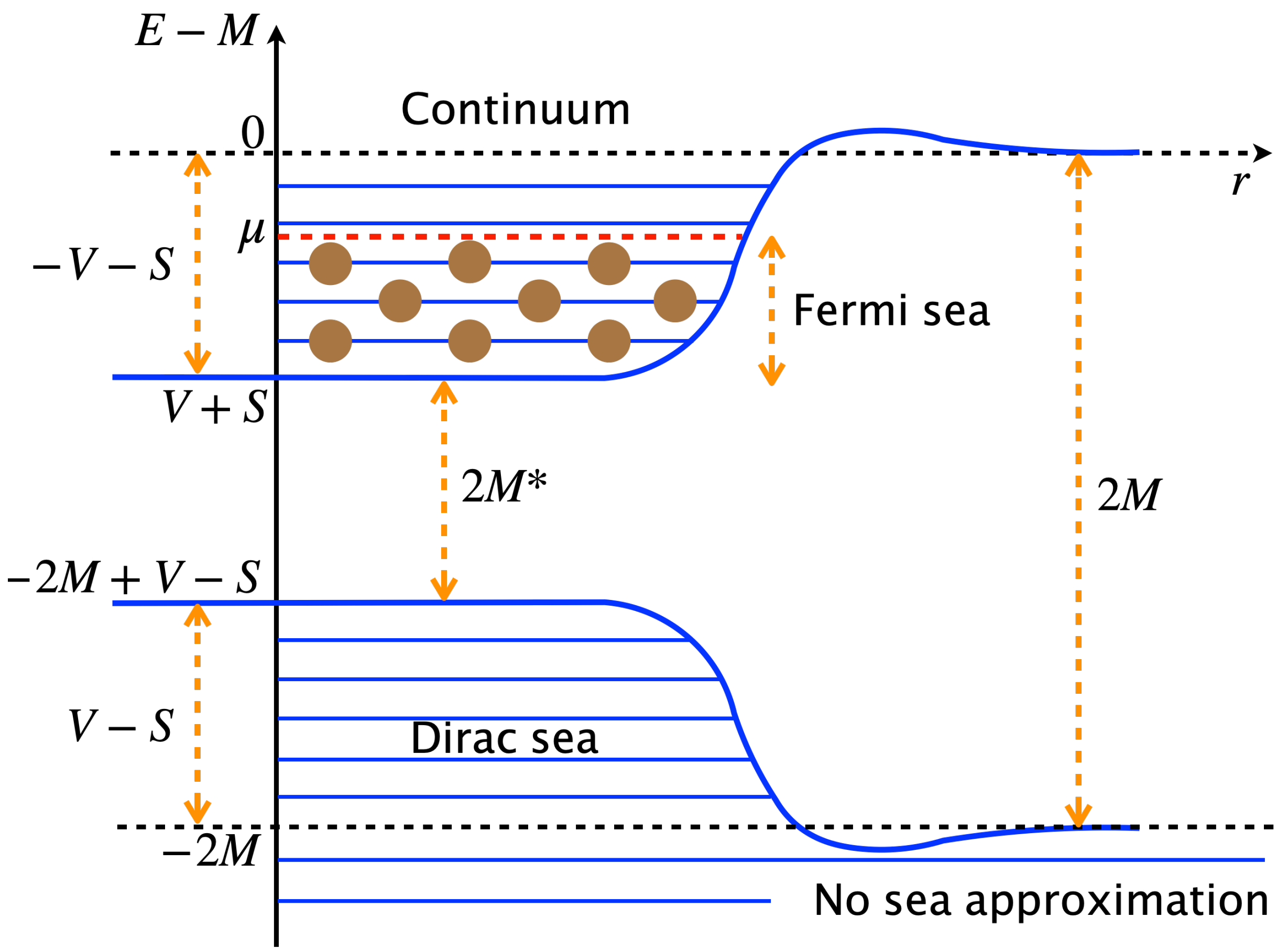
**NL3 parametrization**

**Self-consistent mean field**

$$n(\epsilon, T) = \frac{1}{1 + e^{(\epsilon - \mu)/T}}$$

$$\sum_k n_k = A$$

$\mu(T = 0) = \epsilon_F$



Serot and Walecka, Advance in Nuclear Physics, Vol. 16 (1986)  
 Ring, Prog. Part. Nucl. Phys. **37**, 193 (1996)  
 G. A. Lalazissis, J. König, and P. Ring, Phys. Rev. C **55**, 540 (1997)

# Correlations beyond mean field

Full Dyson equation:

$$\text{---} \xrightarrow{\mathcal{G}(\varepsilon)} \text{---} = \text{---} \xrightarrow{\mathcal{G}^0(\varepsilon)} \text{---} + \text{---} \xrightarrow{\text{---}} \text{---} \xrightarrow{\Sigma} \text{---}$$

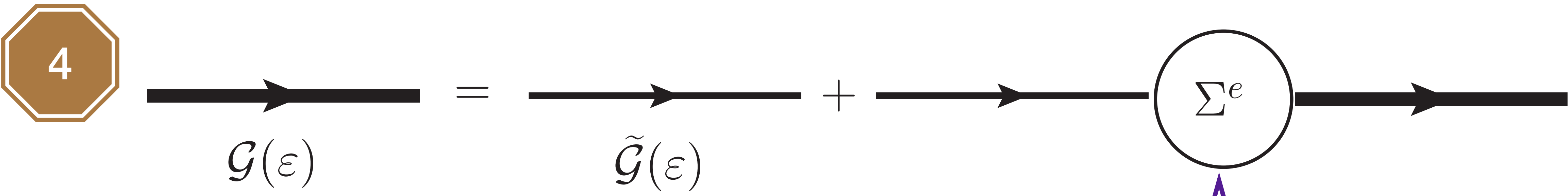
E. Litvinova and P. Schuck, **Nuclear superfluidity at finite temperature**

$$\Sigma(\varepsilon) = \tilde{\Sigma} + \Sigma^e(\varepsilon)$$

is equivalent to two Dyson equations:

$$\text{---} \xrightarrow{\tilde{\mathcal{G}}(\varepsilon)} \text{---} = \text{---} \xrightarrow{\mathcal{G}^0(\varepsilon)} \text{---} + \text{---} \xrightarrow{\text{---}} \text{---} \xrightarrow{\tilde{\Sigma}} \text{---}$$

$$\text{---} \xrightarrow{\mathcal{G}(\varepsilon)} \text{---} = \text{---} \xrightarrow{\tilde{\mathcal{G}}(\varepsilon)} \text{---} + \text{---} \xrightarrow{\text{---}} \text{---} \xrightarrow{\Sigma^e} \text{---}$$

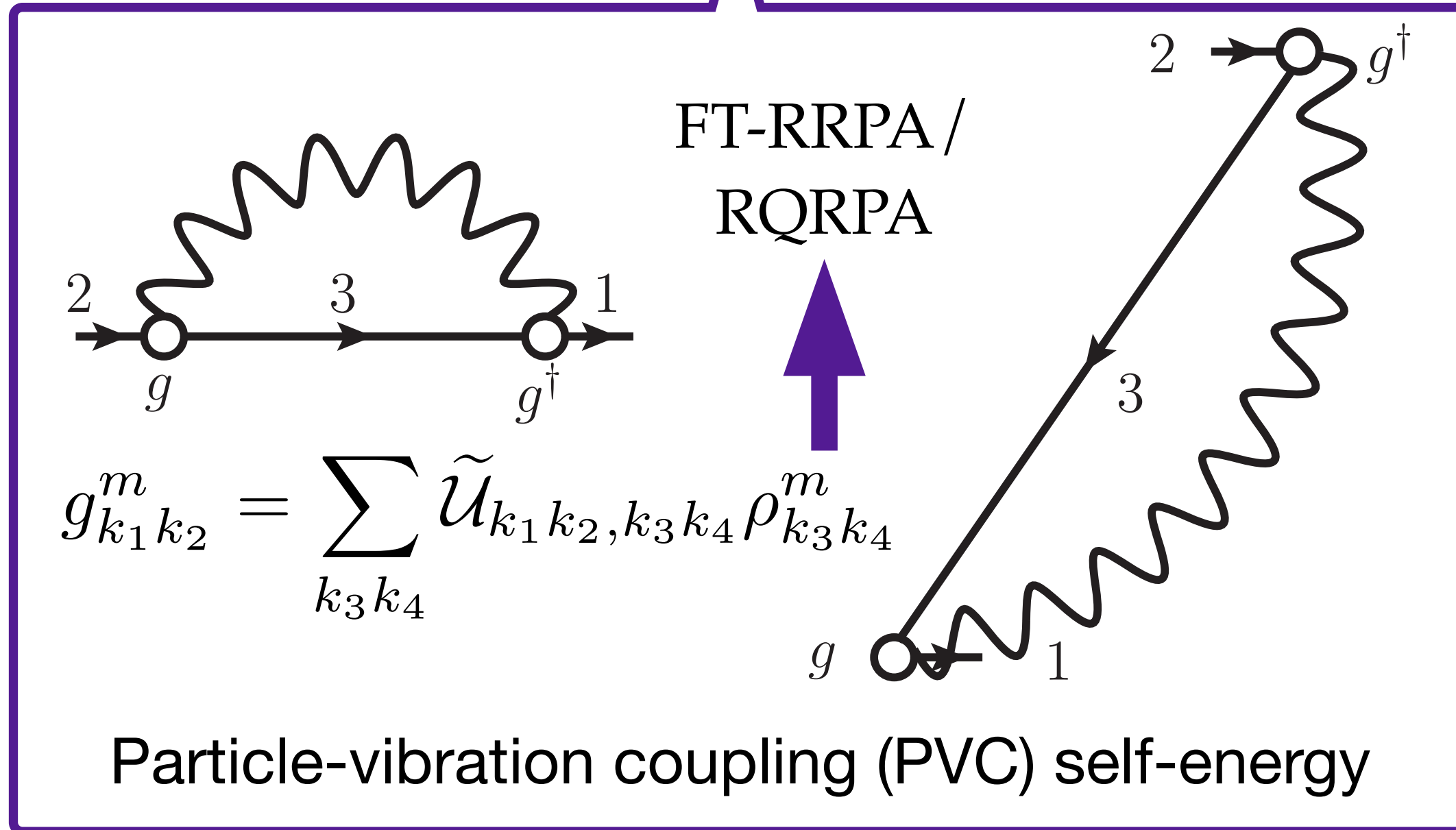


4

$$\tilde{\mathcal{G}}_{k_2 k_1}(\varepsilon_\ell) = \frac{\delta_{k_2 k_1}}{i\varepsilon_\ell - \varepsilon_{k_1} + \mu}$$

$$\varepsilon_\ell = (2\ell + 1)\pi T$$

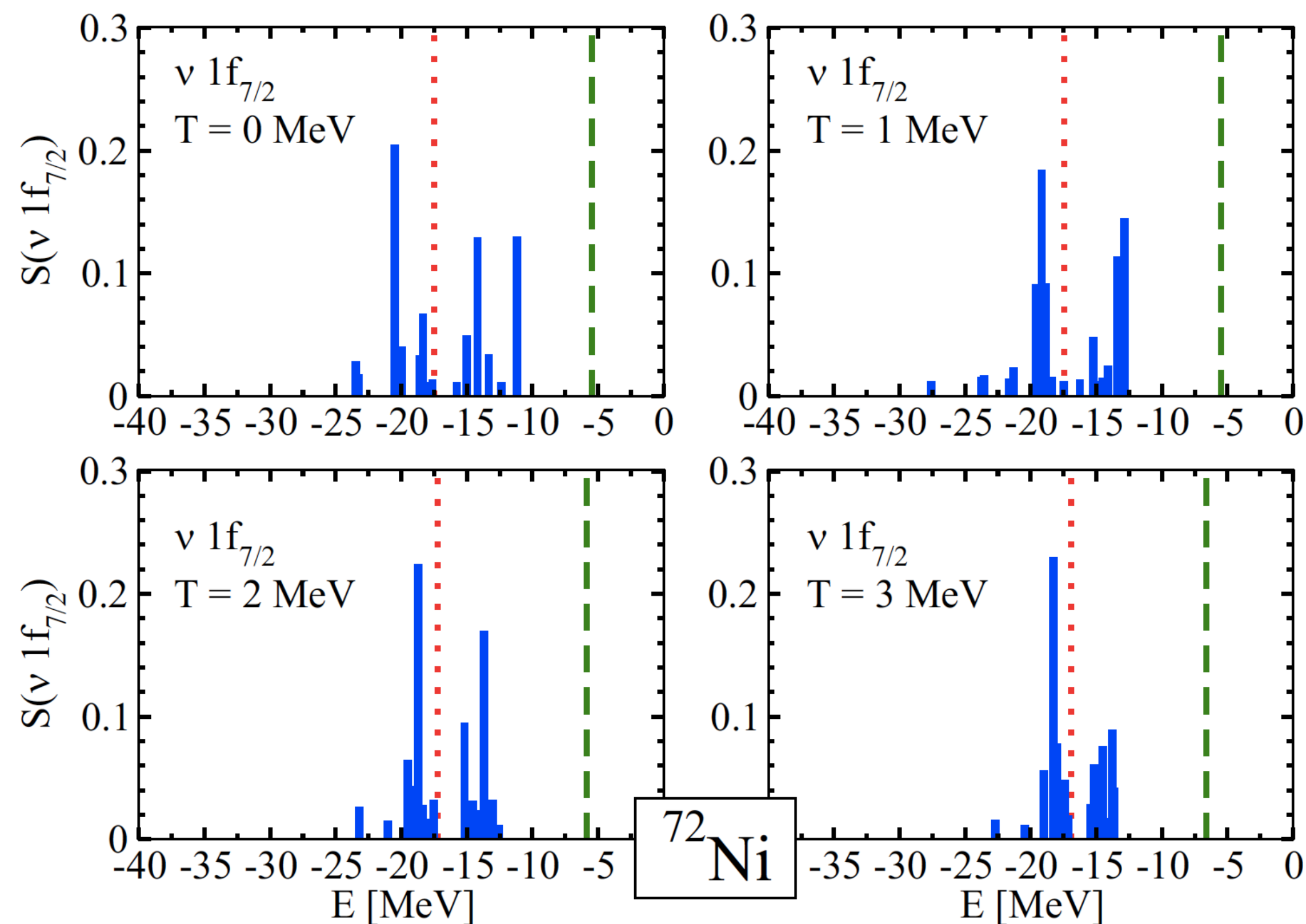
T. Matsubara, Prog. Theor. Phys. **14**, 351 (1995)



FT-Dyson equation:  $[\varepsilon - \varepsilon_k + \mu - \Sigma_k^e(\varepsilon)] \mathcal{G}_k(\varepsilon) = 1 \longrightarrow \left\{ \varepsilon_k^{(\lambda)} \right\}$

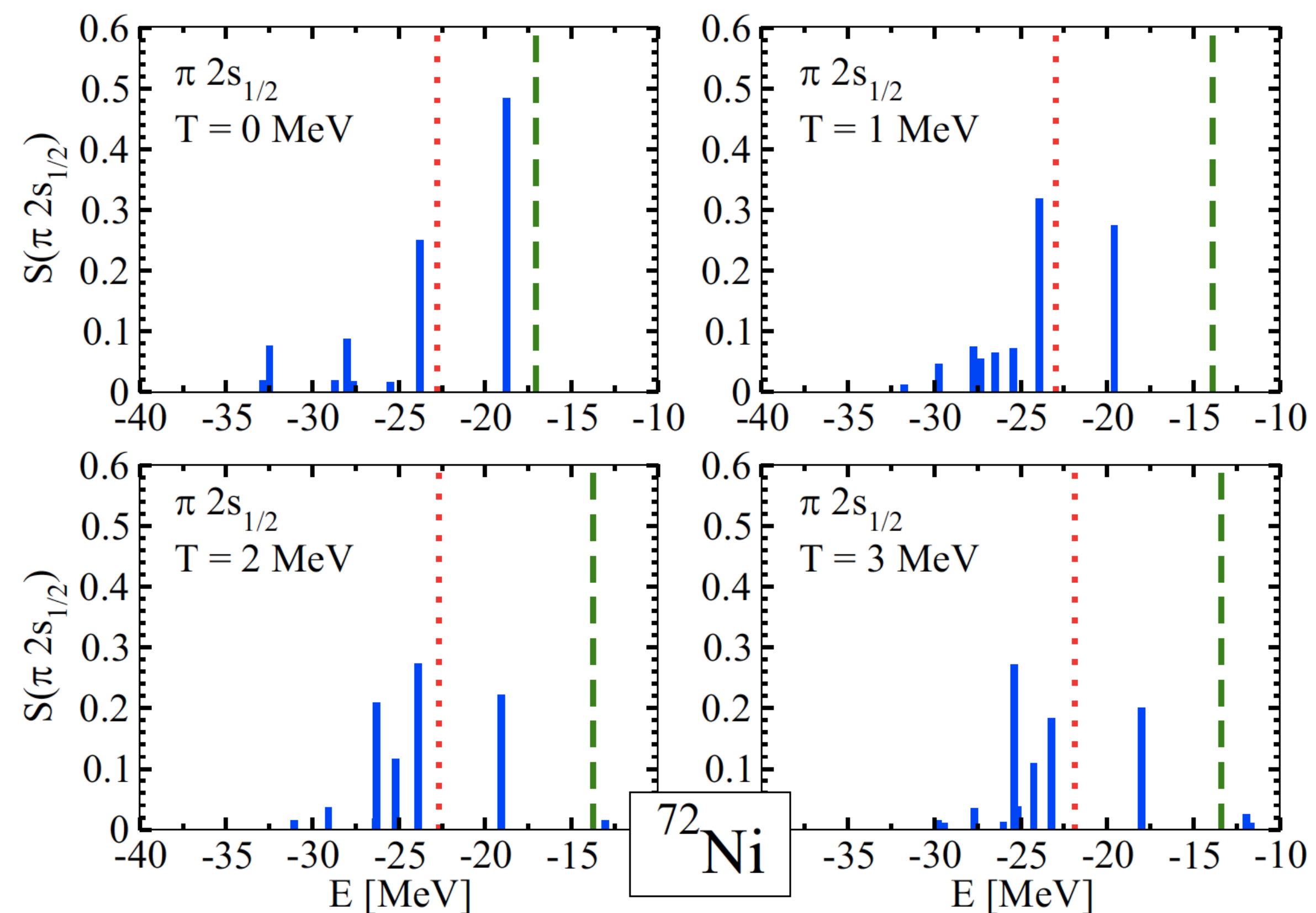
Spectroscopic factor:  $S_k^{(\lambda)} = \left( 1 - \left. \frac{d\Sigma_k^e(\varepsilon)}{d\varepsilon} \right|_{\varepsilon = \varepsilon_k^{(\lambda)}} \right)^{-1}$  ;  $\sum_{\lambda} S_k^{(\lambda)} = 1$

# Fragmentation of single-particle states



H. Wibowo and E. Litvinova, Phys. Rev. C **106**, 044304 (2022)

- : spectroscopic factors of the fragmented states
- - - : spectroscopic factors of the pure mean-field states
- - - : position of chemical potential



# Role of (q)PVC in nuclear multipole responses

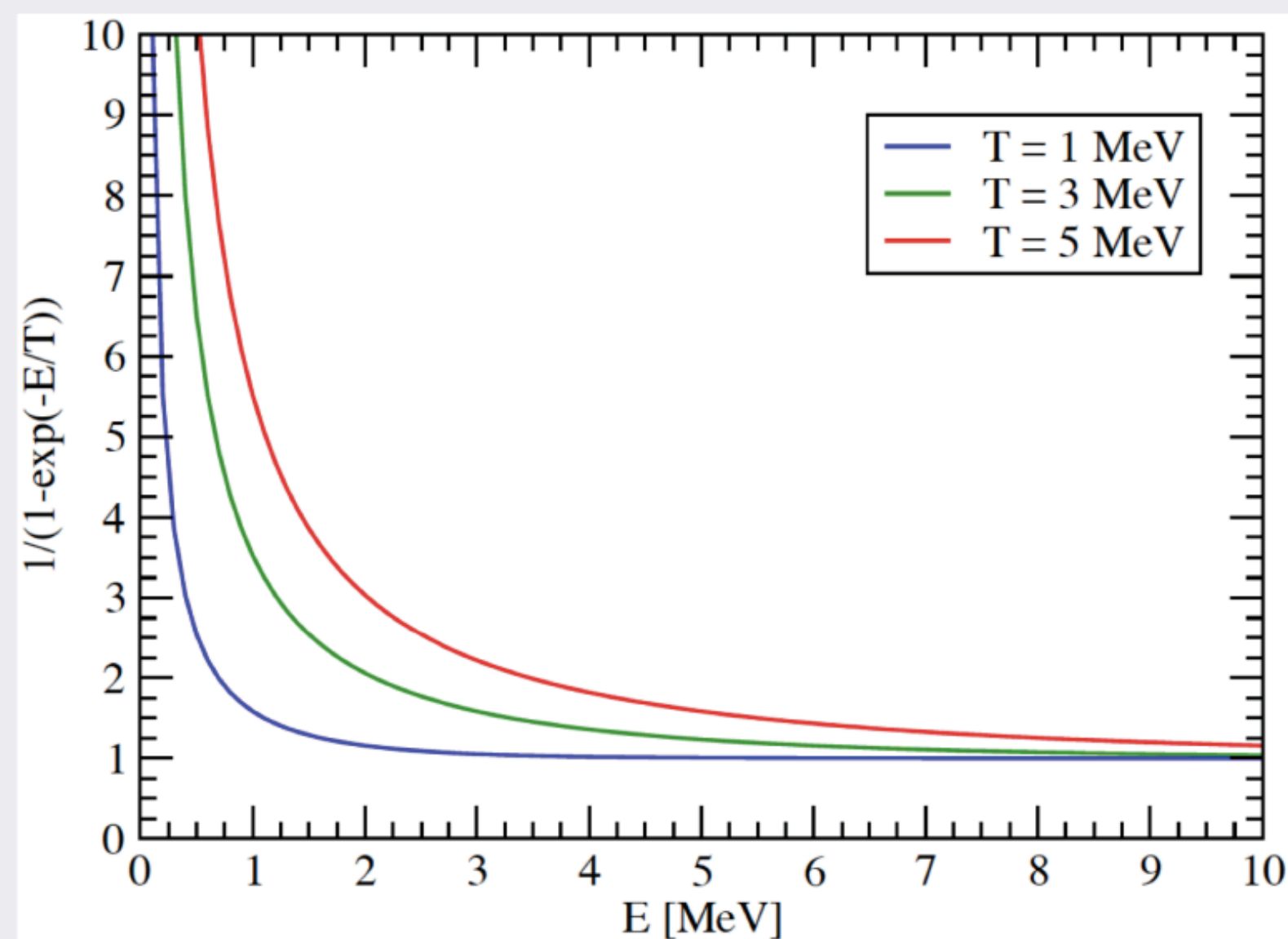
Strength Function at  $T > 0$

$$\tilde{S}(E) = \frac{1}{1 - e^{-E/T}} \lim_{\Delta \rightarrow +0} \frac{1}{\pi} \text{Im} \sum_{1234} V_{21}^{0*} \mathcal{R}_{12,34}(E + i\Delta) V_{43}^0$$

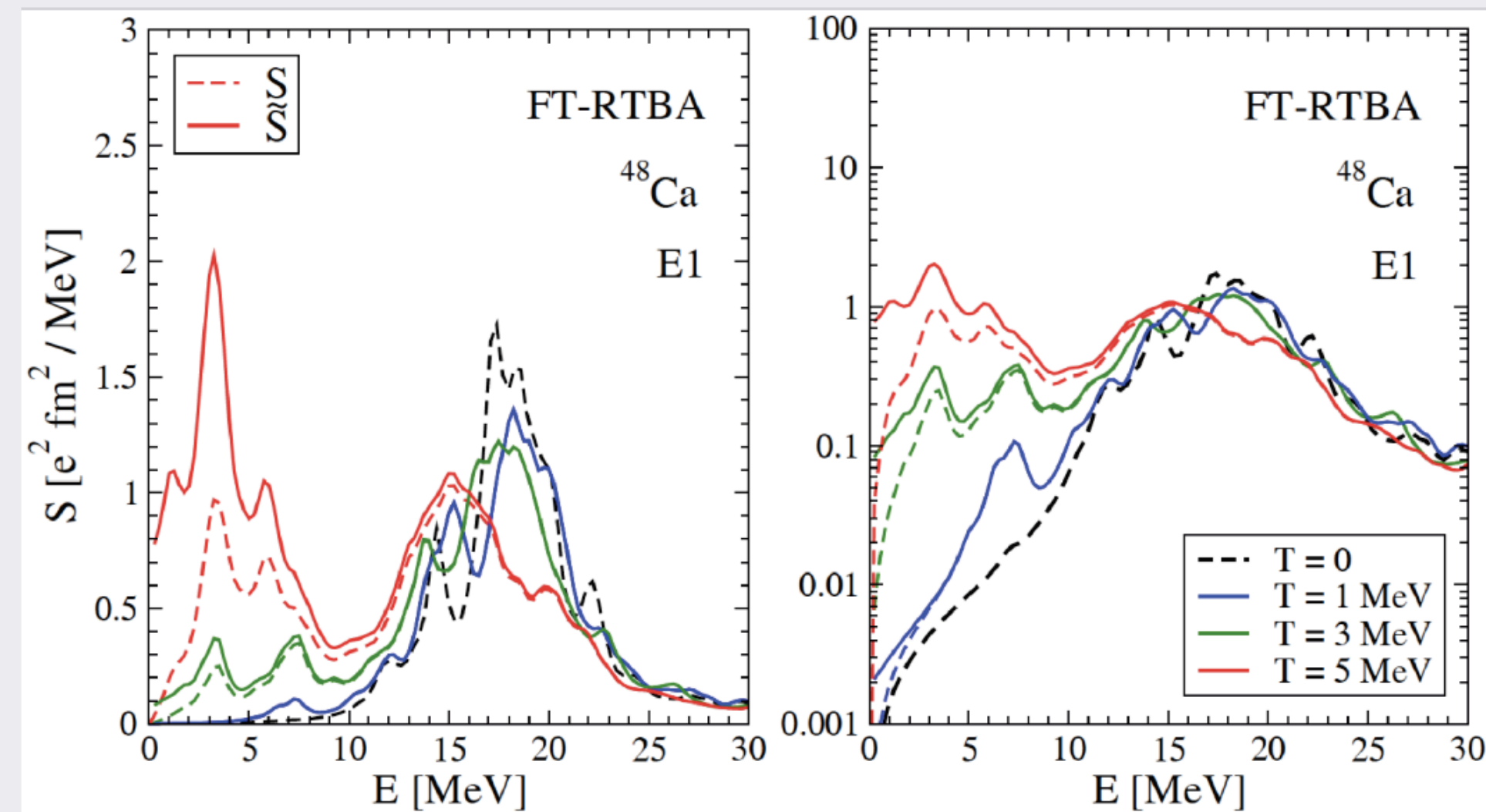
Spectral density  $S(E)$

Exponential factor

Exponential Factor



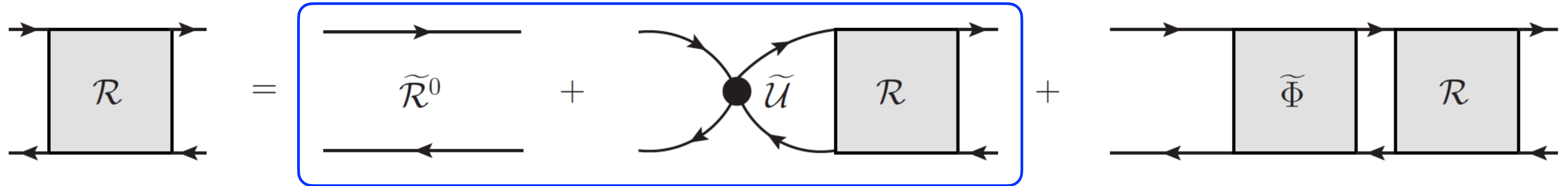
The Role of Exponential Factor



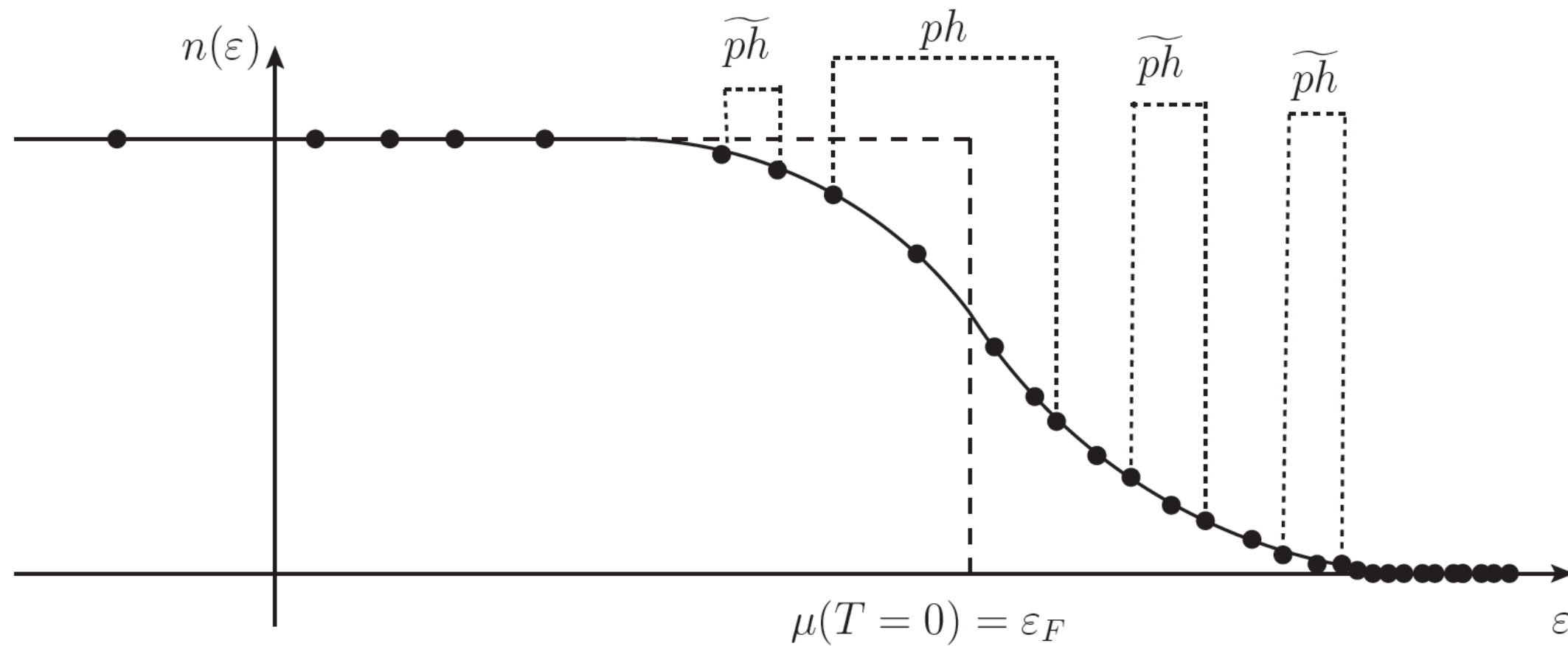
E. Litvinova and H. Wibowo, EPJA 55, 223 (2019)

Bethe-Salpeter equation:

FT-RRPA



Free response: 
$$\tilde{\mathcal{R}}_{12,34}^0(\omega) = \delta_{13}\delta_{24} \frac{n_2(T) - n_1(T)}{\omega - \varepsilon_1 + \varepsilon_2}$$

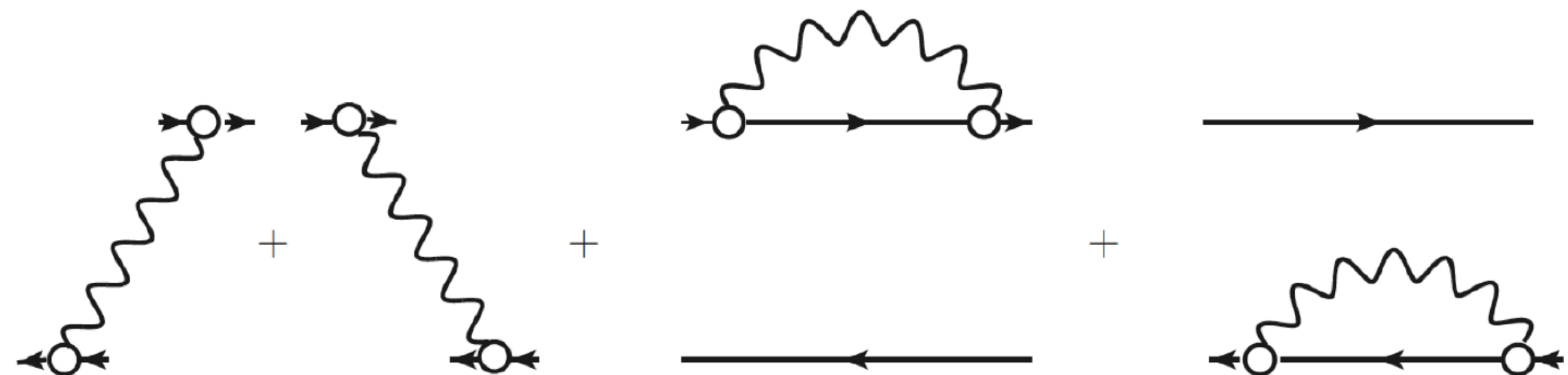


Effective meson-exchange interaction:

$$\tilde{\mathcal{U}}_{kl,k'l'} = \frac{\delta \tilde{\Sigma}_{k'l'}}{\delta \rho_{kl}}$$

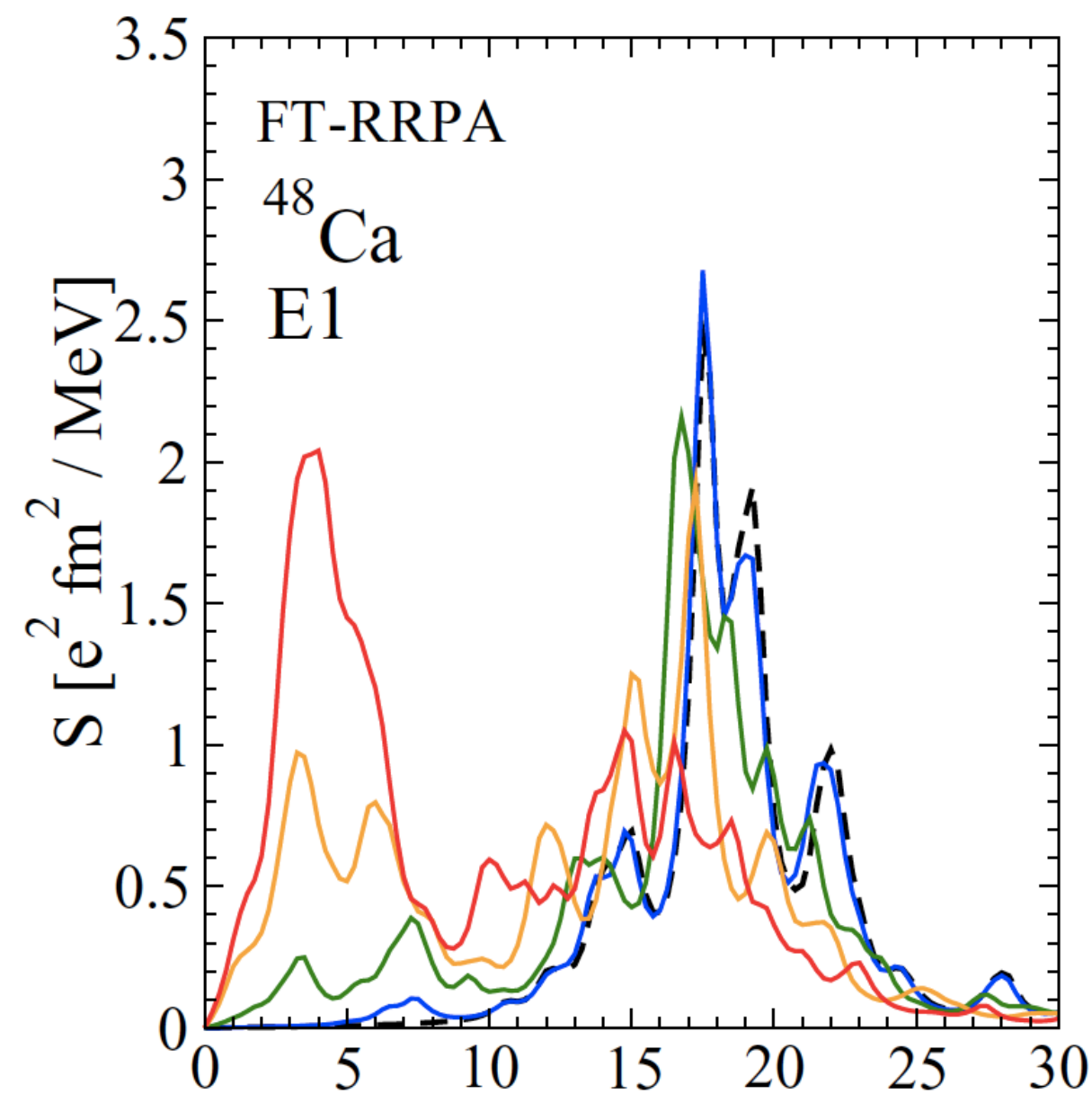
PVC amplitude:  
(FT-RTBA)

$$\tilde{\Phi}(\omega) =$$

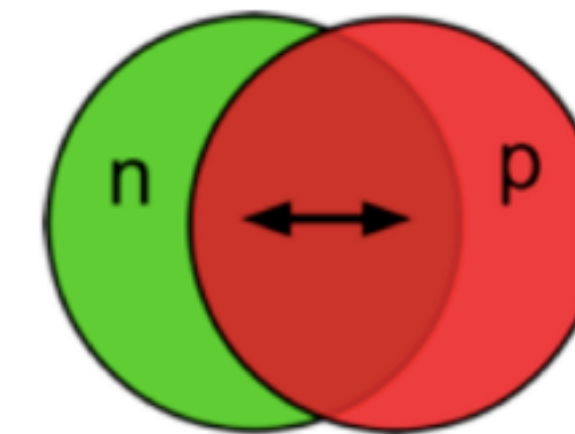
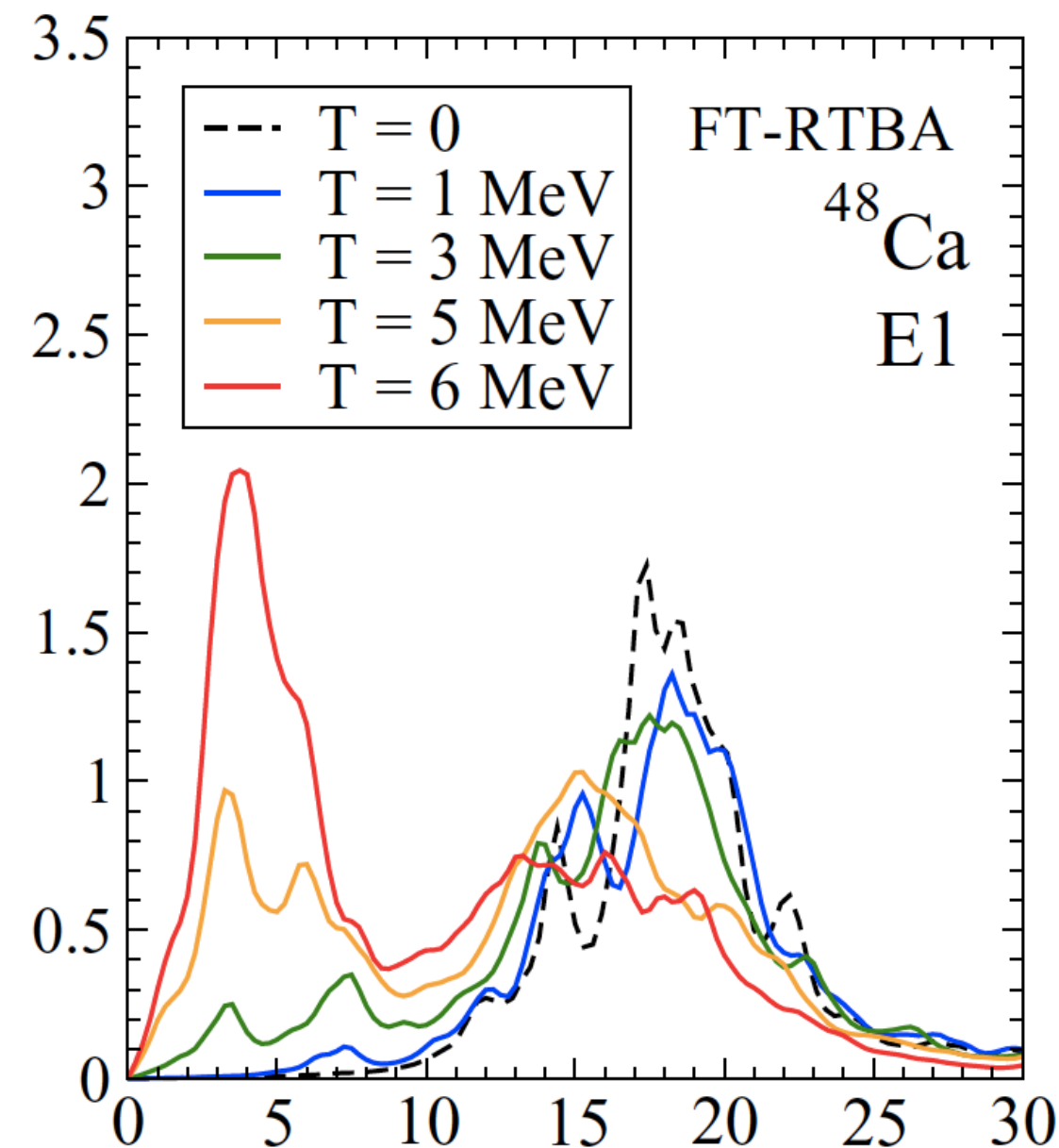




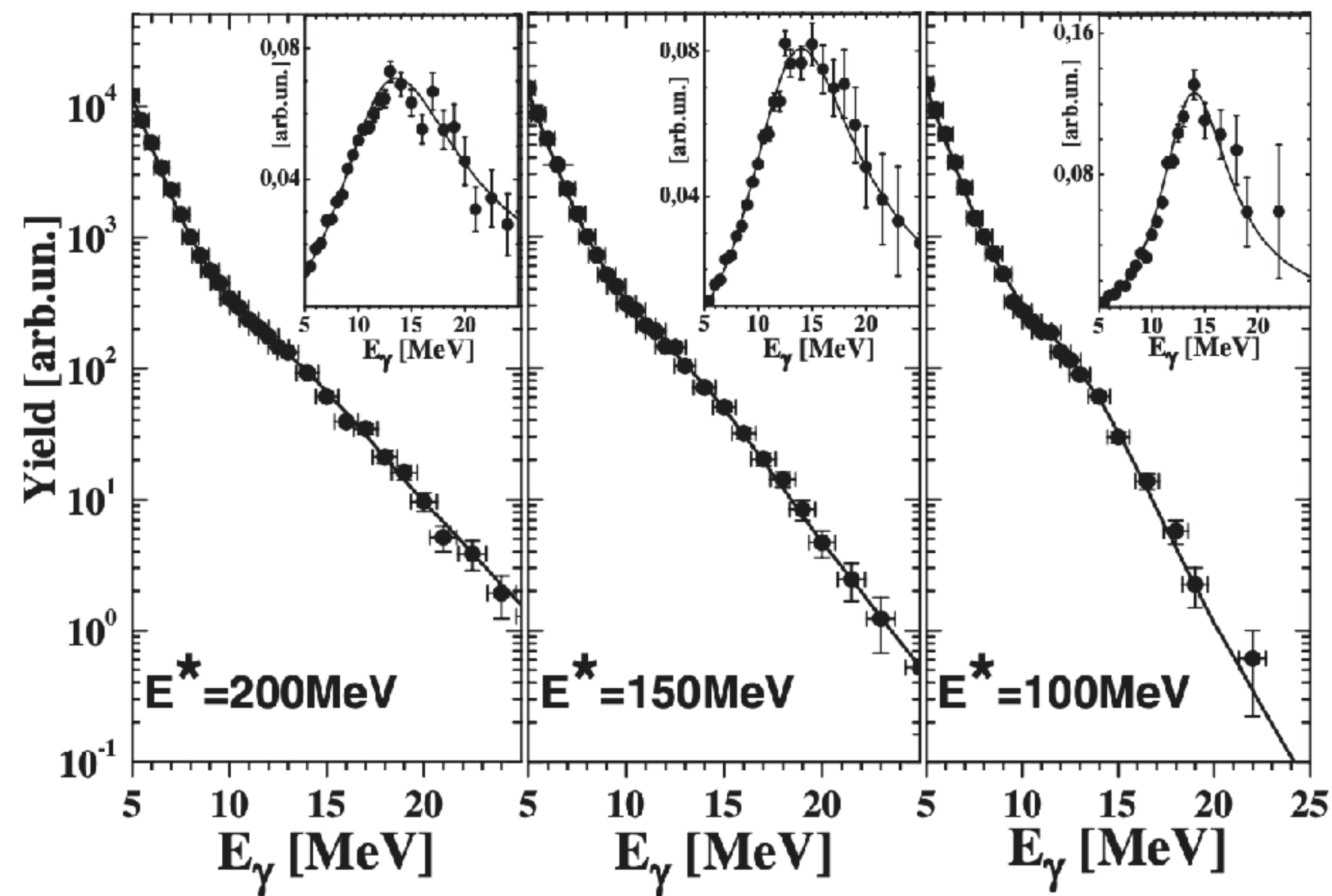
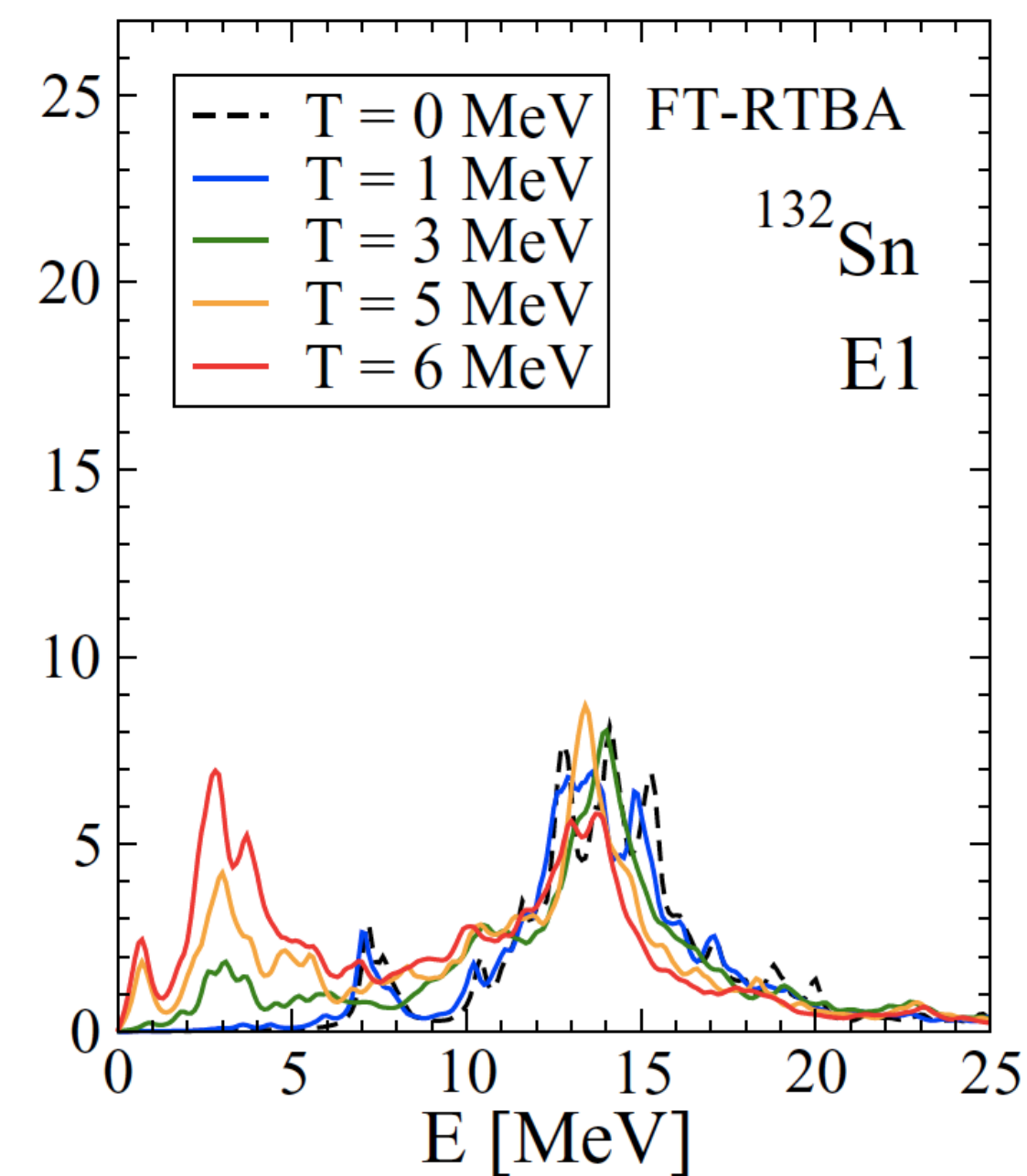
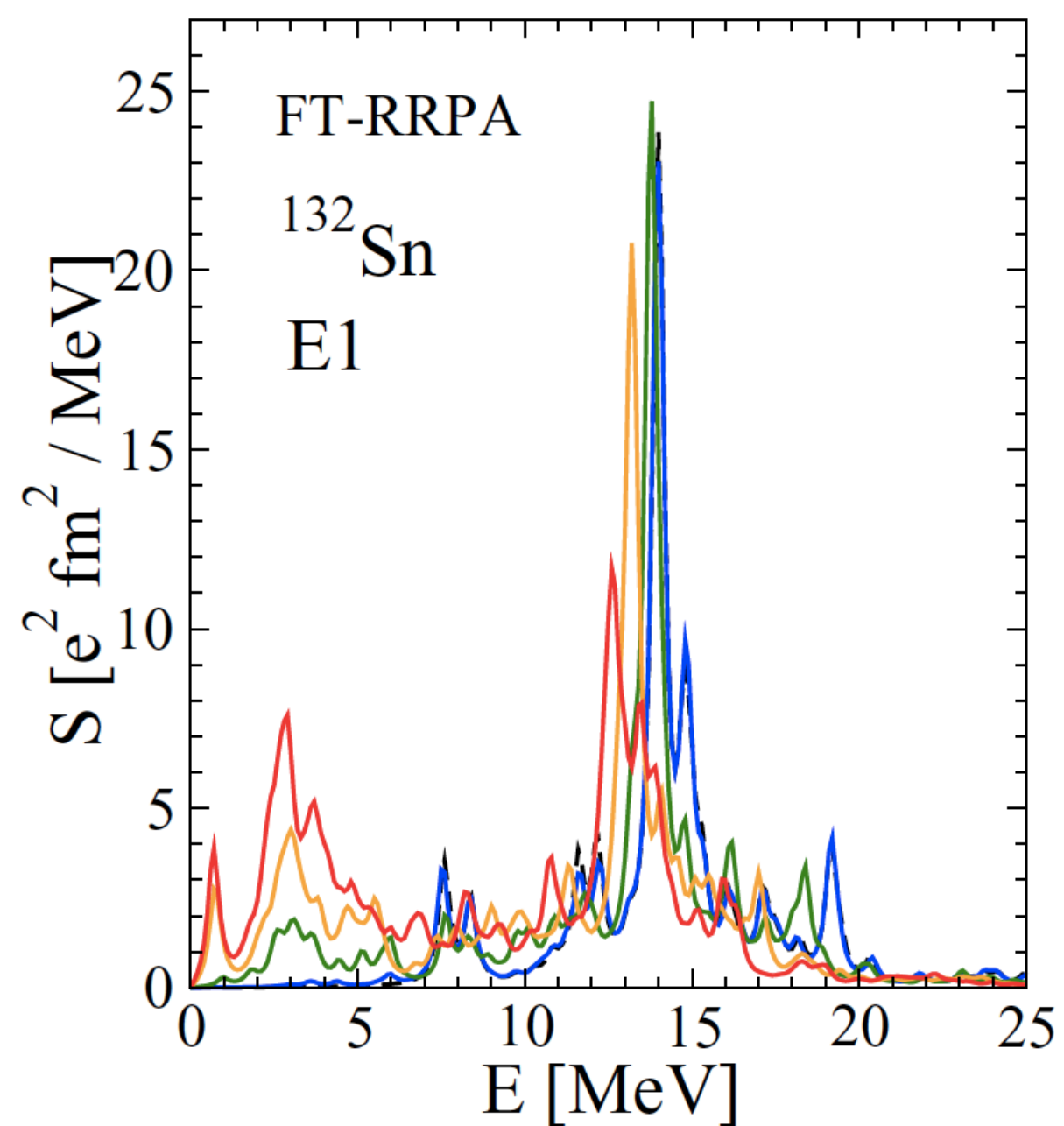
(FT-RRPA)



(FT-RTBA = FT-RRPA + PVC)



*O. Wieland et al., PRL 97, 012501 (2006):  
GDR in  $^{132}\text{Ce}$*



H. Wibowo and E. Litvinova, Phys. Rev. C **100**, 024307 (2019)

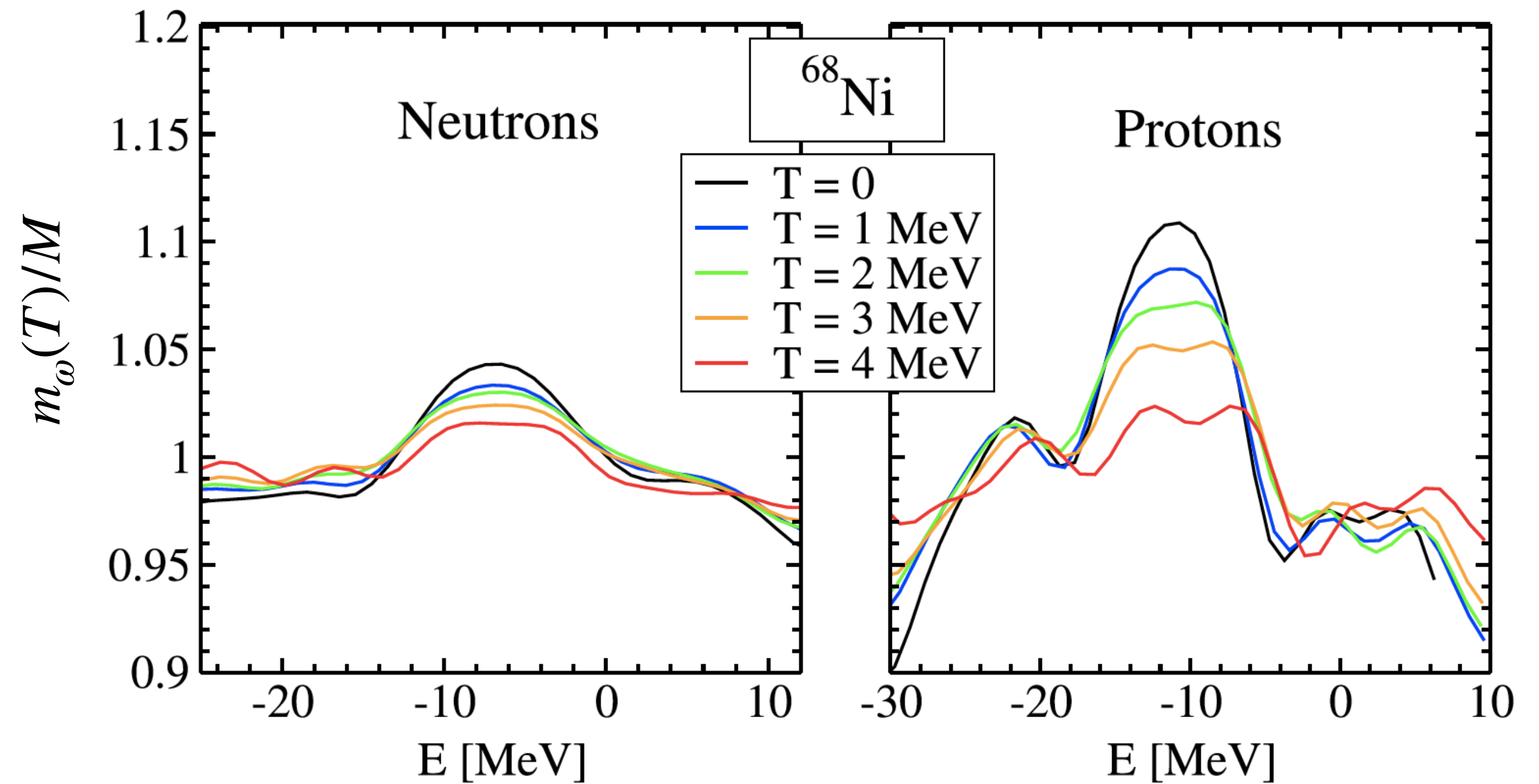
# Temperature evolution of nucleon effective mass

- ▶ The  $T$ -dependent nucleon effective mass:

$$\frac{m^*(T)}{M} = \frac{\tilde{m}}{M} \frac{m_\omega(T)}{M}$$

P. F. Bortignon, A. Bracco, and R. A. Broglia, *Giant Resonances: Nuclear Structure at Finite Temperature*

- ▶ For NL3 parametrization, the value of  $k$  mass,  $\tilde{m}$ , is  $0.6M$ .
- ▶ The omega mass,  $m_\omega(T)$ , accounts for (q)PVC and finite temperature effects.



H. Wibowo, E. Litvinova, Y. Zhang, and P. Finelli, *Phys. Rev. C* **102**, 054321 (2020)

# Procedure to determine the omega mass

1. For each  $T$ , we determine  $\bar{m}_{(k)}(E, T)/M$  as function of  $E$ :

$$\frac{\bar{m}_{(k)}(E, T)}{M} = 1 - \frac{\partial}{\partial \varepsilon} \text{Re} \Sigma_{(k)}^e(\varepsilon)$$

$$\varepsilon = E + i\Delta$$

- ▶ Interval of real part  $E$ :  $\mu - 5 \leq E \leq \mu + 5$  [MeV].
- ▶ The imaginary part  $\Delta$  is the average distance between the energy fragments with the spectroscopic factors larger than 0.5 within the given interval of  $E$  values.

2. For each  $T$ , we determine  $m_\omega(T)$  by the average over s.p. states:

$$\frac{m_\omega(T)}{M} = \max_E \left[ \frac{\sum_{(k)} (2j_{(k)} + 1) \left( \bar{m}_{(k)}(E, T)/M \right) \left( 1/v_{(k)}^2 \right)}{\sum_{(k)} (2j_{(k)} + 1)} \right]$$

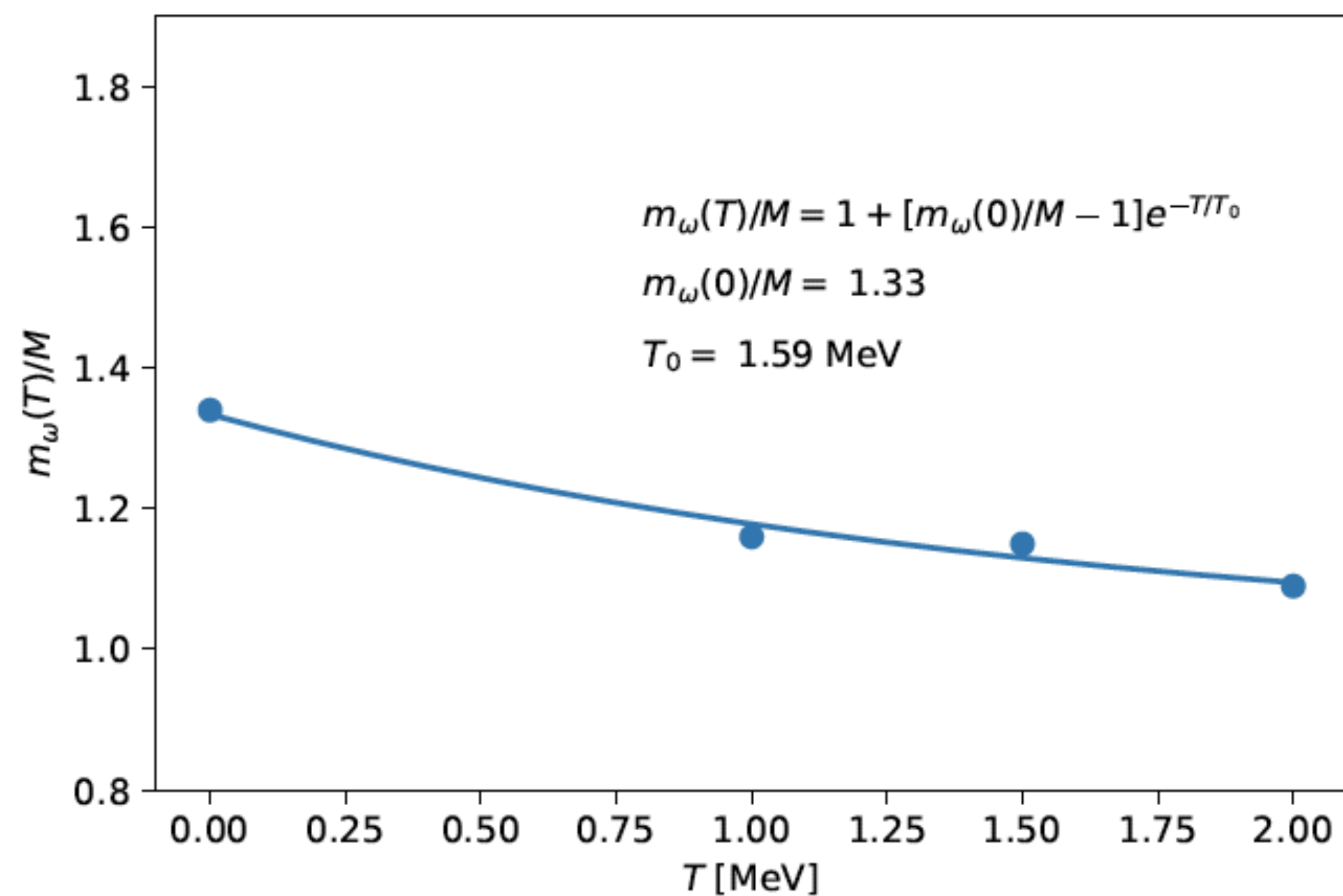
3. Exponential fit:

$$\frac{m_\omega(T)}{M} = 1 + \left[ \frac{m_\omega(T=0)}{M} - 1 \right] e^{-T/T_0}$$

P. Donati, P. M. Pizzochero, P. F. Bortignon, and R. A. Broglia, Phys. Rev. Lett. **72**, 2835 (1994)

# Parameters for the exponential fit

Ni-68



	$^{68}\text{Ni}$	$^{70}\text{Ni}$	$^{72}\text{Ni}$	$^{74}\text{Ni}$	$^{76}\text{Ni}$	$^{78}\text{Ni}$
$m_\omega(T = 0)/M$	1.33	1.34	1.39	1.54	1.34	1.14
$T_0$ (MeV)	1.59	1.99	1.61	0.96	1.26	4.80

H. Wibowo and E. Litvinova, Phys. Rev. C **106**, 044304 (2022)

Average values:  $m_\omega(T = 0)/M = 1.39$ ;  $T_0 = 1.48 \text{ MeV}$

# Temperature evolution of symmetry coefficient

- ▶ The symmetry energy term in the nuclear EOS:

$$E_S = S(T = 0) \left( 1 - 2 \frac{Z}{A} \right)^2$$

P. Donati, P. M. Pizzochero, P. F. Bortignon, and R. A. Broglia, Phys. Rev. Lett. **72**, 2835 (1994)

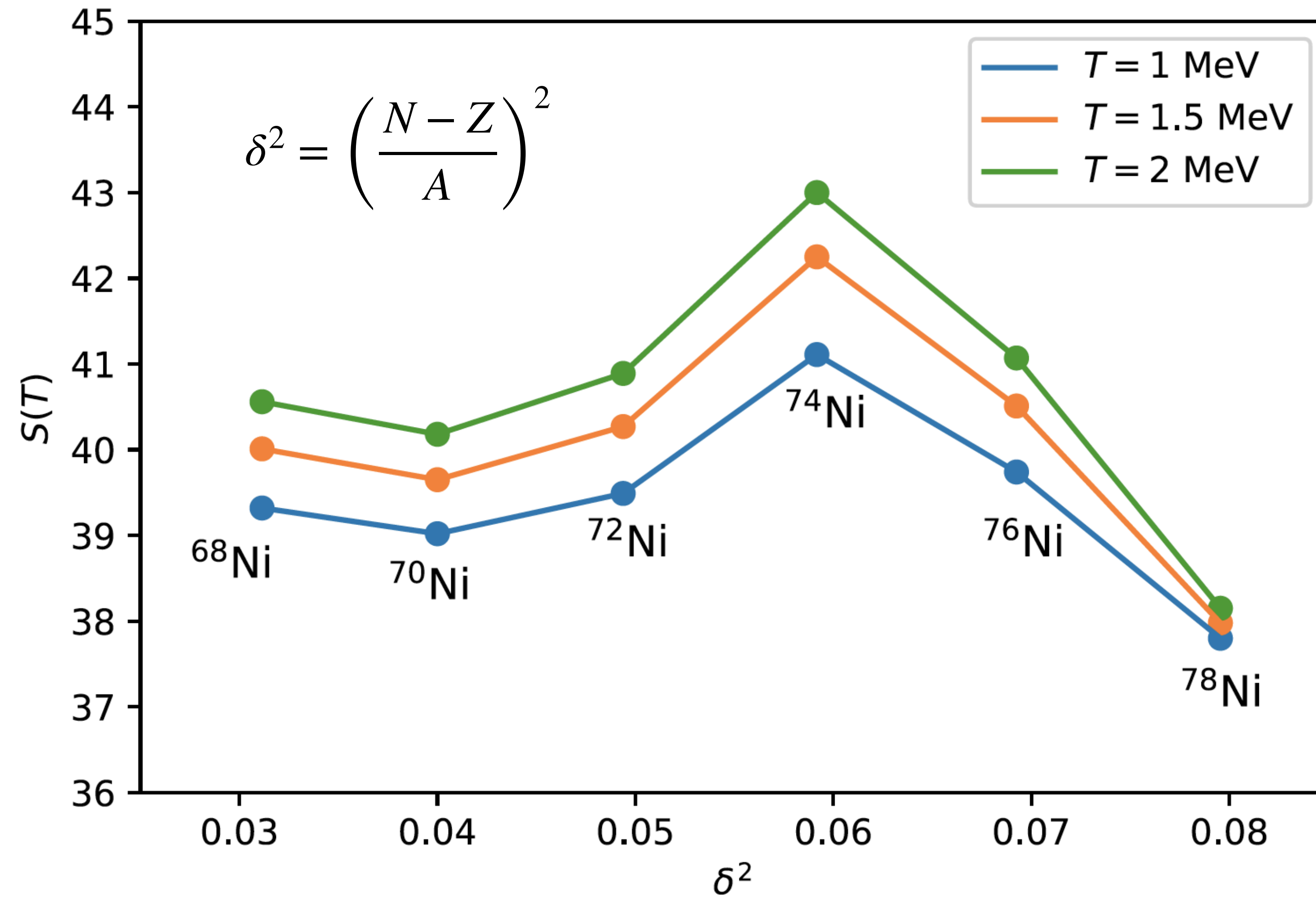
- ▶ The symmetry coefficient  $S(T)$  at finite temperature:

$$S(T) = S(T = 0) + \frac{\hbar^2 c^2 k_F^2}{6M} \left[ \frac{M}{m^*(T)} - \frac{M}{m^*(T = 0)} \right] \quad k_F = \left( \frac{3}{2} \pi^2 \rho_0 \right)^{1/3}$$

- ▶ For the NL3 parametrization:

$$S(T = 0) = 37.4 \text{ MeV}; \quad M = 939 \text{ MeV}; \quad \rho_0 = 0.148 \text{ fm}^{-3}$$

	$T = 0$	$T = 1 \text{ MeV}$	$T = 1.5 \text{ MeV}$	$T = 2.0 \text{ MeV}$
$m^*/M$	0.83	0.72	0.68	0.66
$S \text{ (MeV)}$	37.4	39.6	40.4	41.1

Dependence of  $S(T)$  on  $\delta^2$ 

$$S(T) = S(T = 0) + \frac{\hbar^2 c^2 k_F^2}{6M} \left[ \frac{M}{m^*(T)} - \frac{M}{m^*(T = 0)} \right]$$

$$\frac{m^*(T)}{M} = \frac{\tilde{m}}{M} \frac{m_\omega(T)}{M}$$

$$\frac{m_\omega(T)}{M} = \max_E \left[ \frac{\sum_{(k)} (2j_{(k)} + 1) \left( \bar{m}_{(k)}(E, T)/M \right) \left( 1/v_{(k)}^2 \right)}{\sum_{(k)} (2j_{(k)} + 1)} \right]$$

Nucleus	$2d_{5/2}$	$2d_{3/2}$	$3s_{1/2}$	$1g_{9/2}$	$2p_{1/2}$	$2p_{3/2}$	$1f_{5/2}$
$^{68}\text{Ni}$				0.812	0.839	0.765	0.790
$^{70}\text{Ni}$	0.540			0.808	0.824	0.676	0.756
$^{72}\text{Ni}$				0.730	0.733	0.522	0.617
$^{74}\text{Ni}$			0.653	0.752	0.708	0.537	0.580
$^{76}\text{Ni}$		0.597	0.755	0.655	0.752	0.608	0.605
$^{78}\text{Ni}$	0.884		0.892	0.800	0.852	0.709	0.708

Dominant spectroscopic factors

Nucleus	$2d_{5/2}$	$2d_{3/2}$	$3s_{1/2}$	$1g_{9/2}$	$2p_{1/2}$	$2p_{3/2}$	$1f_{5/2}$
$^{68}\text{Ni}$				0.931	0.790	0.918	0.933
$^{70}\text{Ni}$	0.982			0.817	0.930	0.929	0.970
$^{72}\text{Ni}$				0.640	0.962	0.978	0.982
$^{74}\text{Ni}$			0.974	0.537	0.968	0.990	0.984
$^{76}\text{Ni}$		0.982	0.976	0.736	0.980	0.988	0.990

BCS occupation numbers  $v_{(k)}^2$

# Summary

- ▶ We model a hot nucleus in the framework of the thermal relativistic mean-field theory.
- ▶ Particle-vibration coupling (PVC) significantly modifies the mean-field single-particle states.
- ▶ The nucleon effective mass decreases with temperature, whereas the symmetry energy coefficient grows with temperature.
- ▶ The pairing correlations below the critical temperature and the PVC effects determine the isotopic dependence of  $S(T)$ .



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**THANK YOU FOR YOUR ATTENTION!!!**