



Nuclear equation of state from nuclear collective excited state properties

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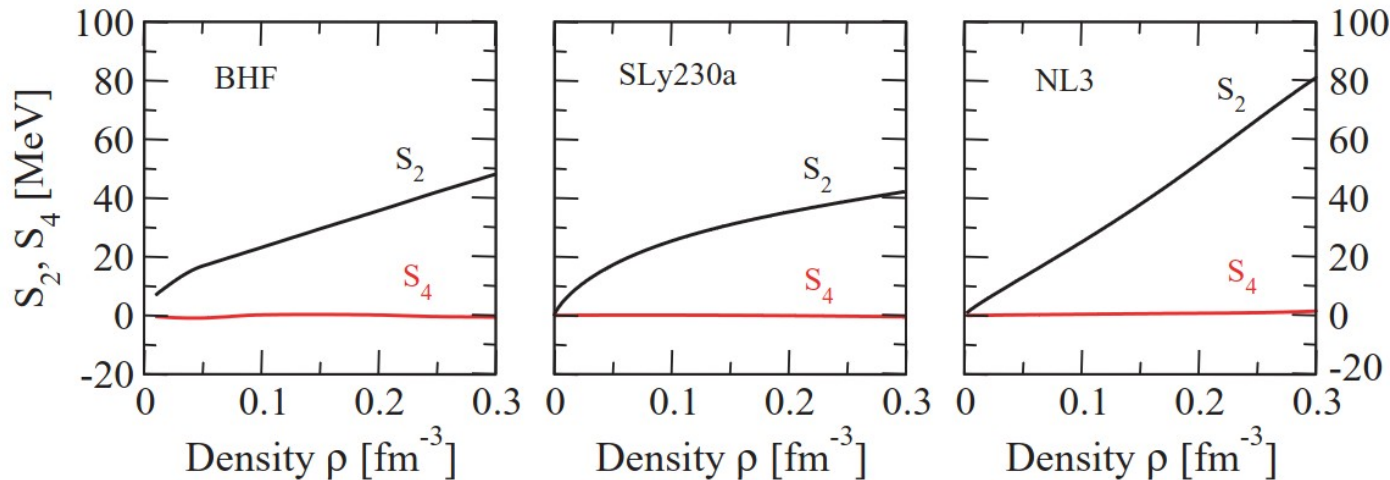
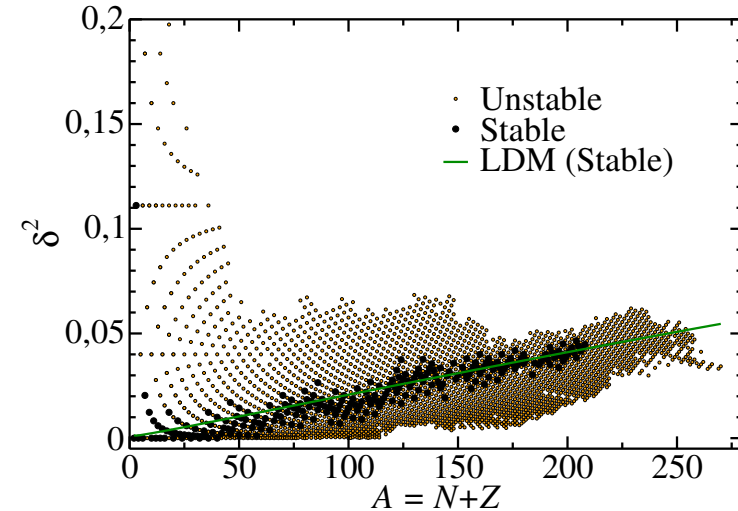
Dipartimento di Fisica e Astronomia "Ettore Majorana"

June 11th - 16th 2023, Catania, Italy.

Nuclear Equation of State (EoS)

Energy per nucleon (e) as a function of the **total density** $\rho = \rho_n + \rho_p$ and the **relative difference** $\delta = (\rho_n - \rho_p) / \rho$ for **unpolarized uniform matter at $T=0$ assuming isospin symmetry** (even powers of δ). For $\delta \rightarrow 0$:

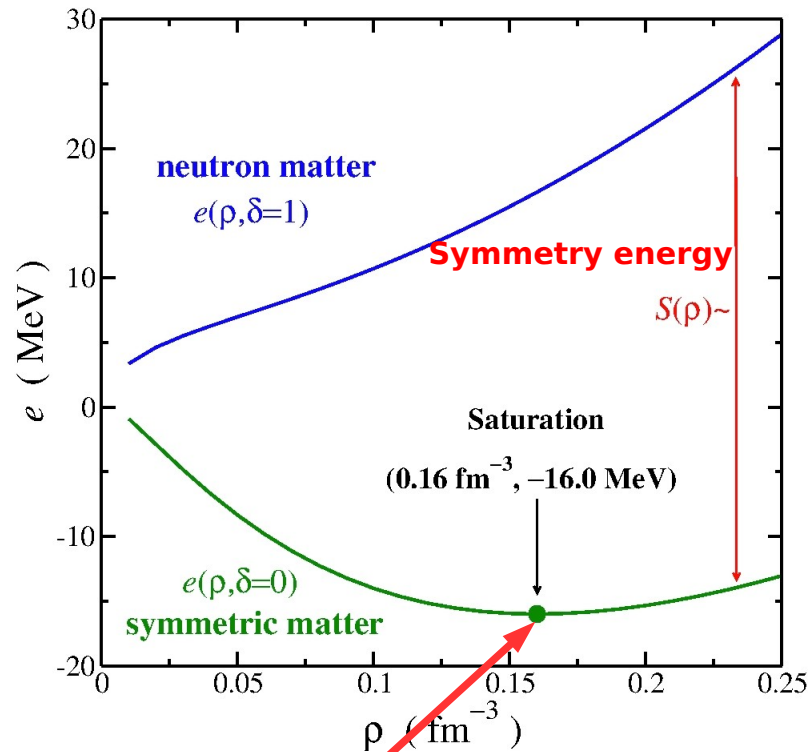
$$e(\rho, \delta) = e(\rho, 0) + S_2(\rho)\delta^2 + S_4(\rho)\delta^4 + \mathcal{O}[\delta^6]$$



Nuclear Equation of State (EoS)

Unpolarized, uniform nuclear matter at $T=0$ assuming isospin symmetry

$$e(\rho, \delta) = e(\rho, 0) + S_2(\rho)\delta^2$$



It is customary to also **expand** $e(\rho, 0)$ and $S(\rho)$ around nuclear **saturation density** $\rho_0 \sim 0.16 \text{ fm}^{-3}$

$$e(\rho, 0) = e(\rho_0, 0) + \frac{1}{2}K_0x^2 + \mathcal{O}[\rho^3] \text{ where } x = \frac{\rho - \rho_0}{3\rho_0}$$
$$S(\rho) = J + Lx + \frac{1}{2}K_{\text{sym}}x^2 + \mathcal{O}[\rho^3, \delta^4]$$

$K_0 \rightarrow$ how **compressible** is symmetric matter at ρ_0

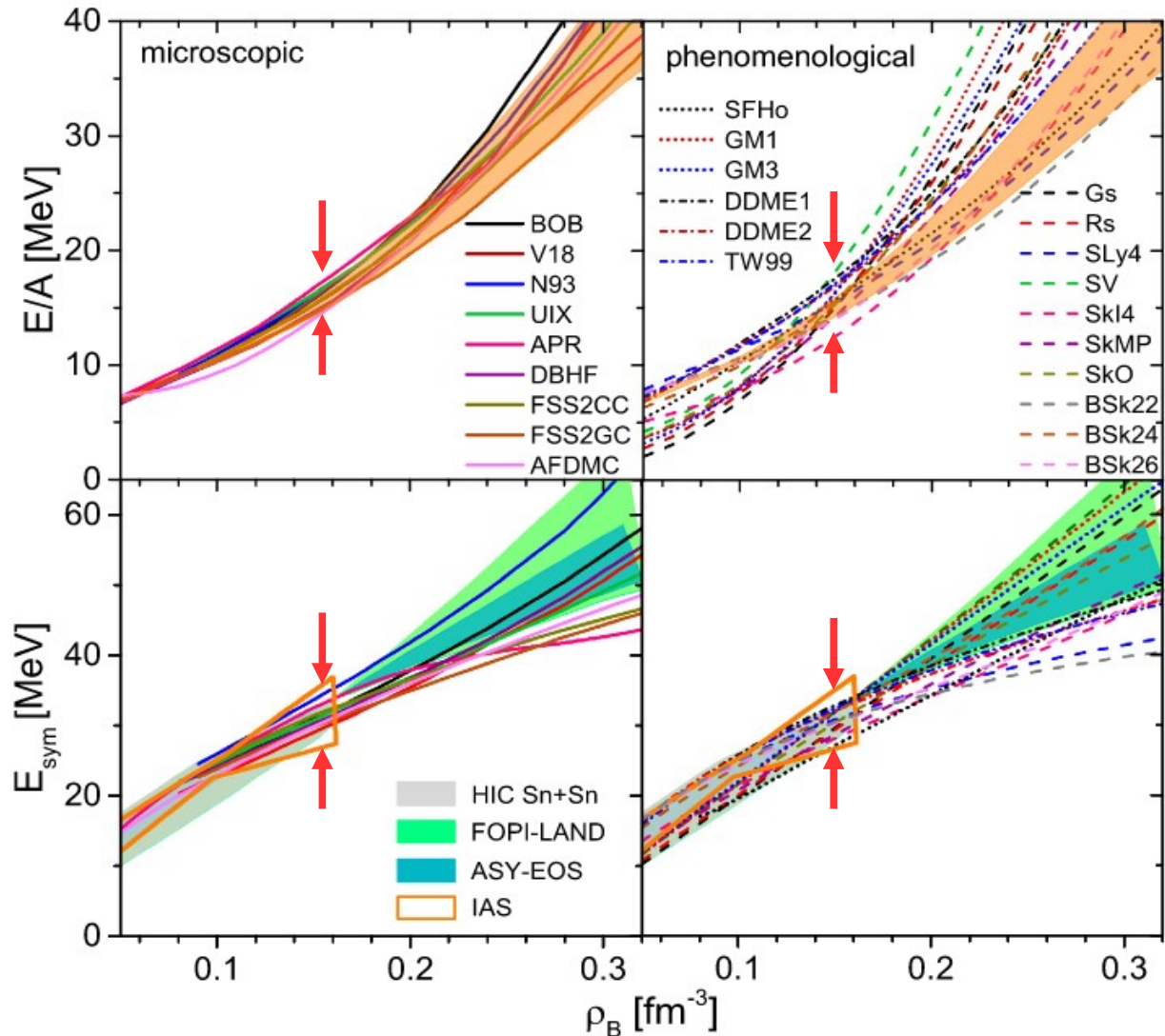
$J \rightarrow$ **penalty energy** for converting all **protons into neutrons** in symmetric matter at ρ_0

$L \rightarrow$ **neutron pressure** in neutron matter at ρ_0

$$P(\rho = \rho_0, \delta = 0) = 0 \text{ MeV fm}^{-3}$$

EoS from current nuclear models

Microroscopic and phenomenological models constrained by different data display similar discrepancies on the EoS



Many-body methods have been shown to agree



Main source of uncertainty in the nuclear Hamiltonian



**Which information can be obtained on the
EoS from nuclear collective excitations?
(personal overview)**

Giant Monopole Resonance do we understand it?

The compression-mode giant resonances and nuclear incompressibility

Umesh Garg,^a Gianluca Colò,^{b,c}  

Progress in Particle and Nuclear Physics

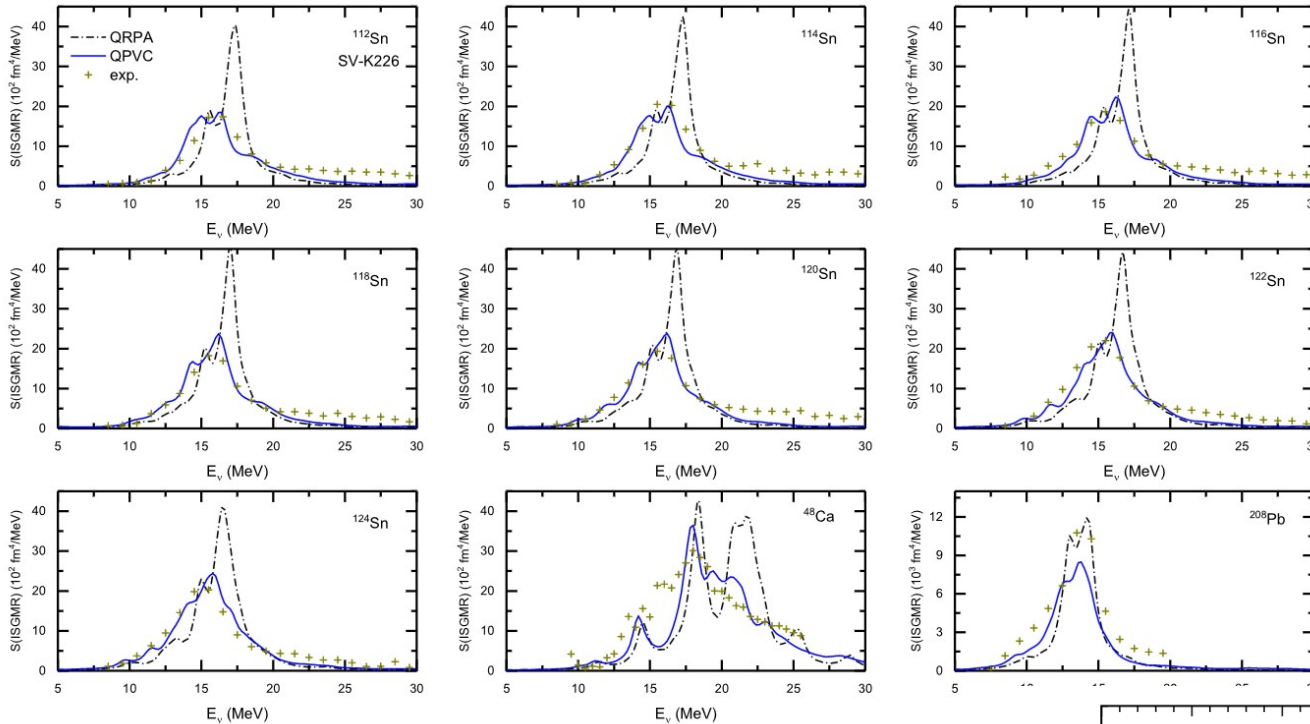
Volume 101, July 2018, Pages 55-95

arXiv:2211.01264 [pdf, ps, other] 

Towards a Unified Description of Isoscalar Giant Monopole Resonances in a Self-Consistent Quasiparticle-Vibration Coupling Approach

Authors: Z. Z. Li, Y. F. Niu, G. Colò

SV-K226



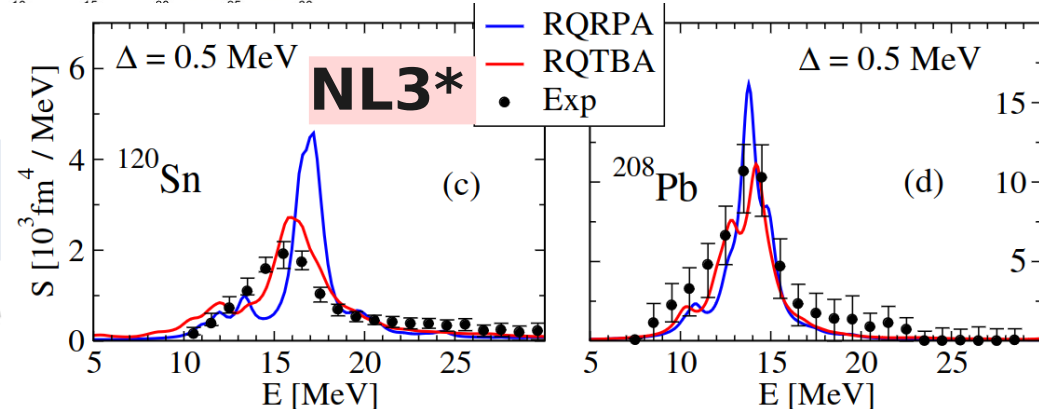
$K_\infty = 226 \text{ MeV}$

These calculations points towards a **plausible estimate** on $K = 220\text{-}260 \text{ MeV}$. **Is that the final word?** Further experiments are planned.

Very **recently** two works explain **ISGMR** in different nuclei within the **PVC** approach

$$(E_x^{\text{ISGMR}})^2 \equiv K_A \frac{\hbar^2}{m \langle r^2 \rangle}$$

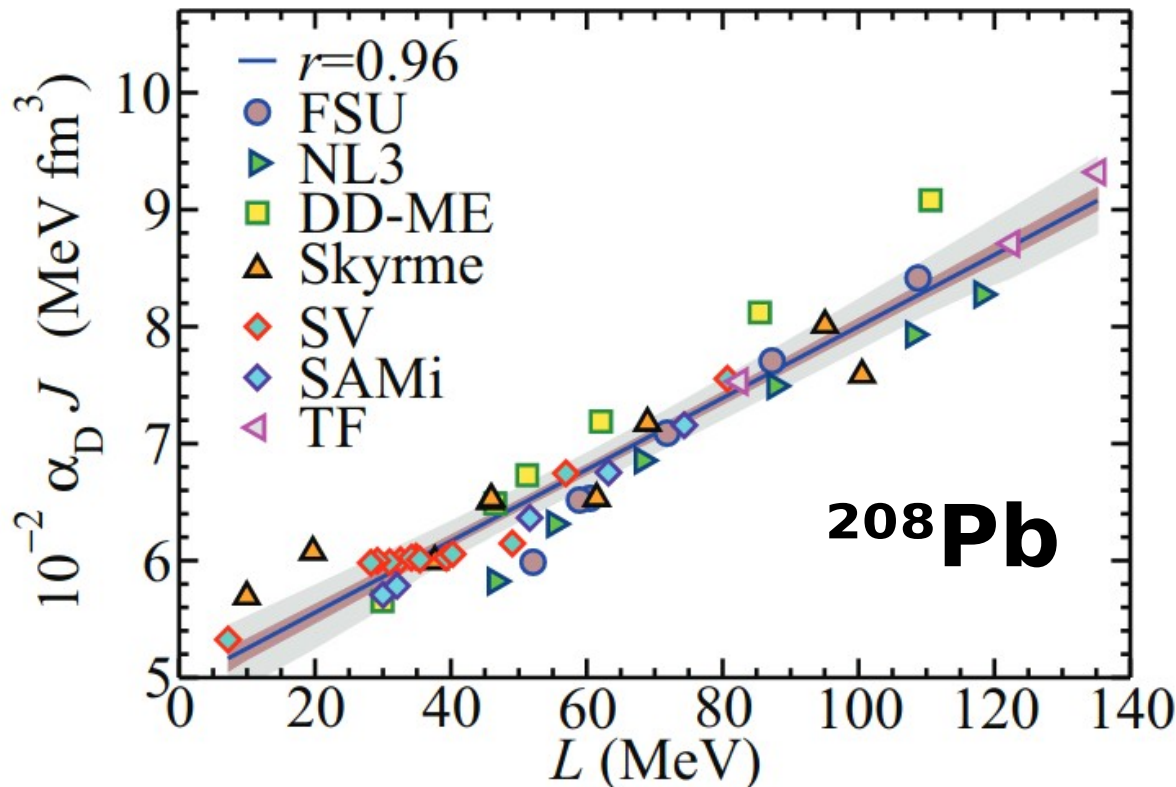
$$K_\infty = 258 \text{ MeV}$$



Dipole polarizability, J and L

$$\alpha_D = \frac{8\pi e^2}{9} m_{-1}(E1)$$

Dipole polarizability (α_D) probes the neutron skin thickness



$$\alpha_D \approx \frac{\pi e^2 \langle r^2 \rangle}{54 J} A \left(1 + \frac{5 \Delta r_{np} - \Delta r_{np}^{\text{surf}} - \Delta r_{np}^{\text{Coul}}}{2 \langle r^2 \rangle^{1/2} (I - I_{\text{Coul}})} \right)$$

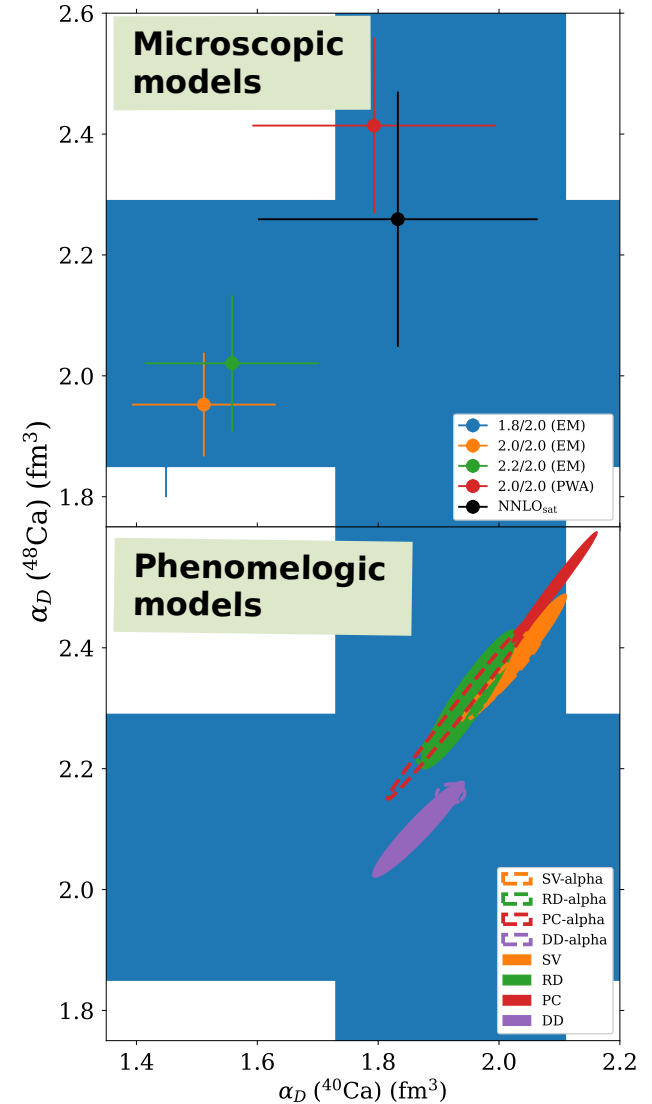
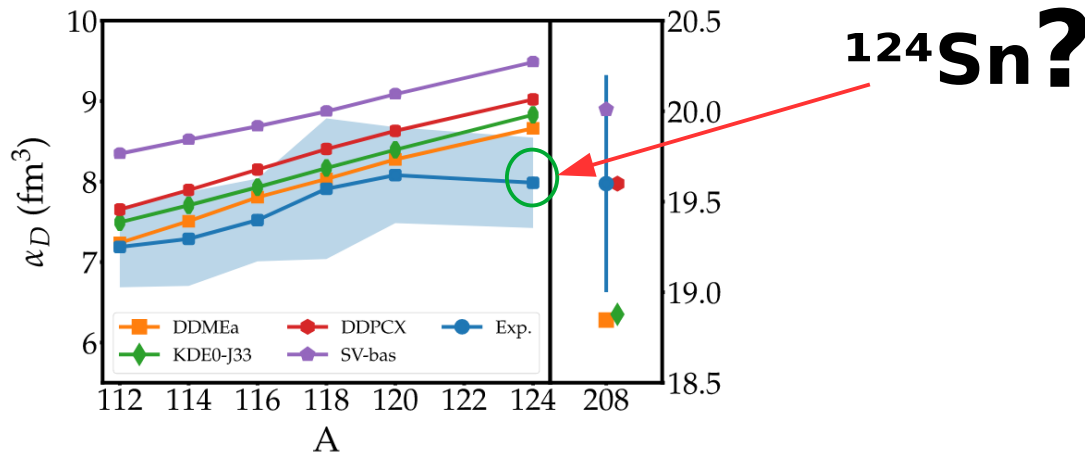
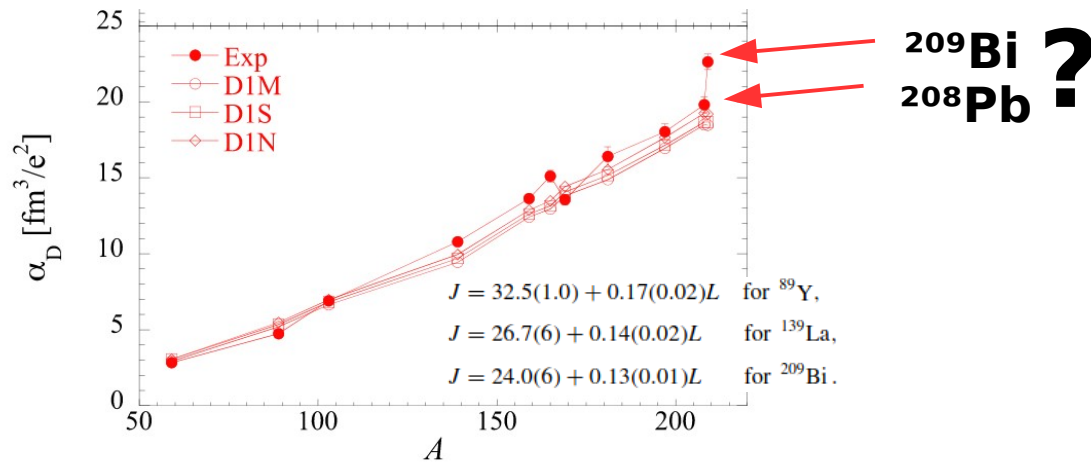
$$\begin{aligned} J &= 25.0(2) + 0.19(2)L && \text{for } ^{68}\text{Ni}, \\ J &= 25.4(1.1) + 0.17(1)L && \text{for } ^{120}\text{Sn}, \\ J &= 24.5(8) + 0.17(1)L && \text{for } ^{208}\text{Pb}. \end{aligned}$$

$$S(\langle \rho \rangle \approx 0.08 \text{ fm}^{-3}) \approx 25 \text{ MeV}$$

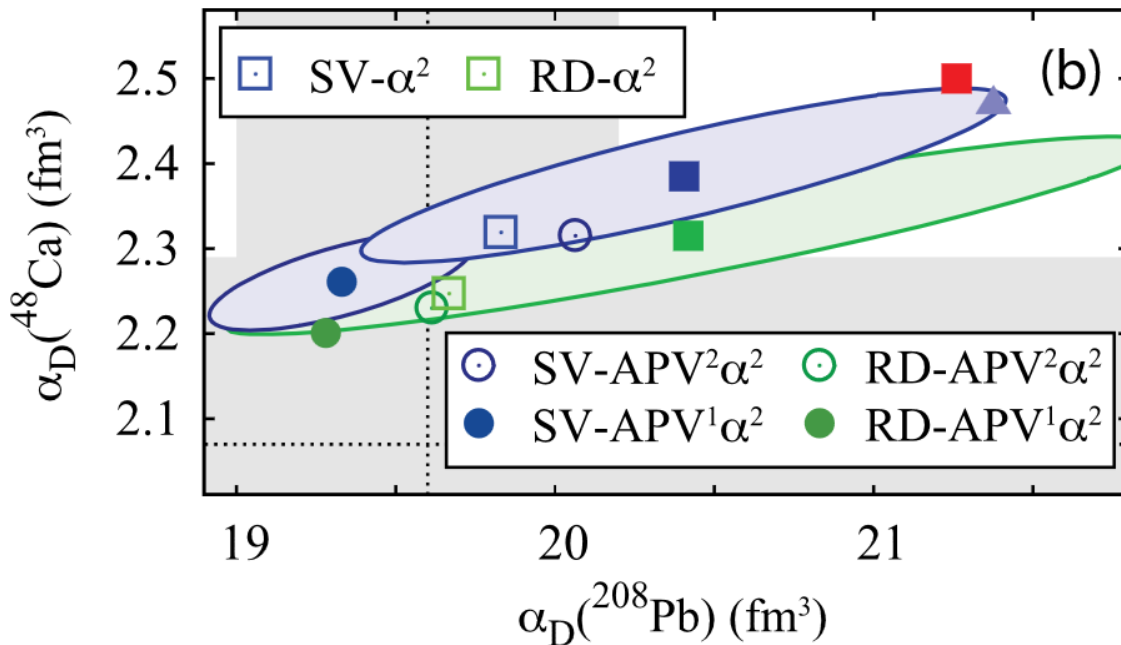
Determination of the J vs L relation from experimental data according to Energy Density Functionals (EDFs)

Dipole polarizability: do we understand it?

S. Goriely, S. Péru, G. Colò, X. Roca-Maza, I. Gheorghe, D. Filipescu, and H. Utsunomiya
 Phys. Rev. C **102**, 064309 – Published 9 December 2020



A_{PV} (sensitive to Δr_{np}) VS α_D (sensitive to J and Δr_{np}) in ^{48}Ca and ^{208}Pb



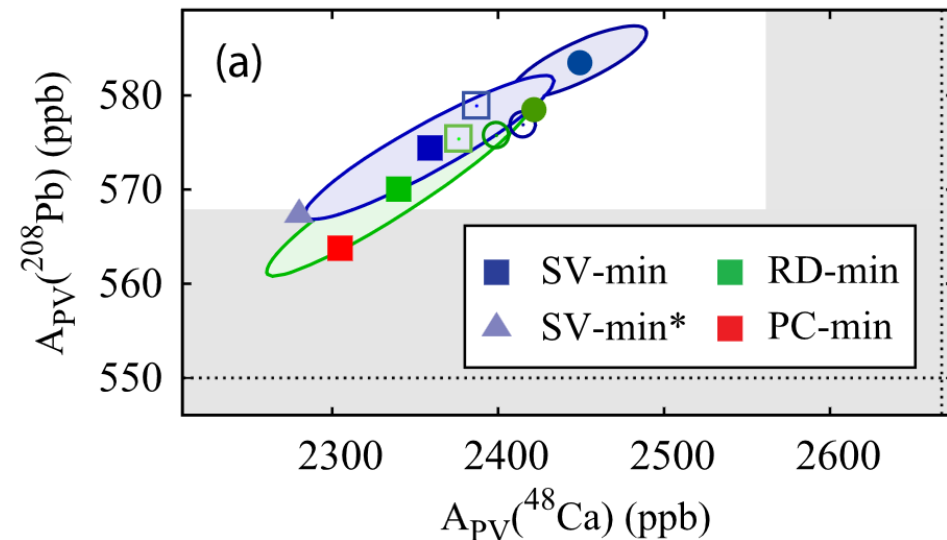
No simultaneous description of **parity violating asymmetries** (ground state observable) \rightarrow point to a **deficient understanding** of **neutron skins**

Simultaneous description of **dipole polarizabilities** \rightarrow point to a **good understanding** of **symmetry energy** and **neutron skins**

Ab-initio (B. Hu) Nature Physics (2022)

$$\alpha_D(^{48}\text{Ca}) \quad 2.30^{+0.31}_{-0.26}$$

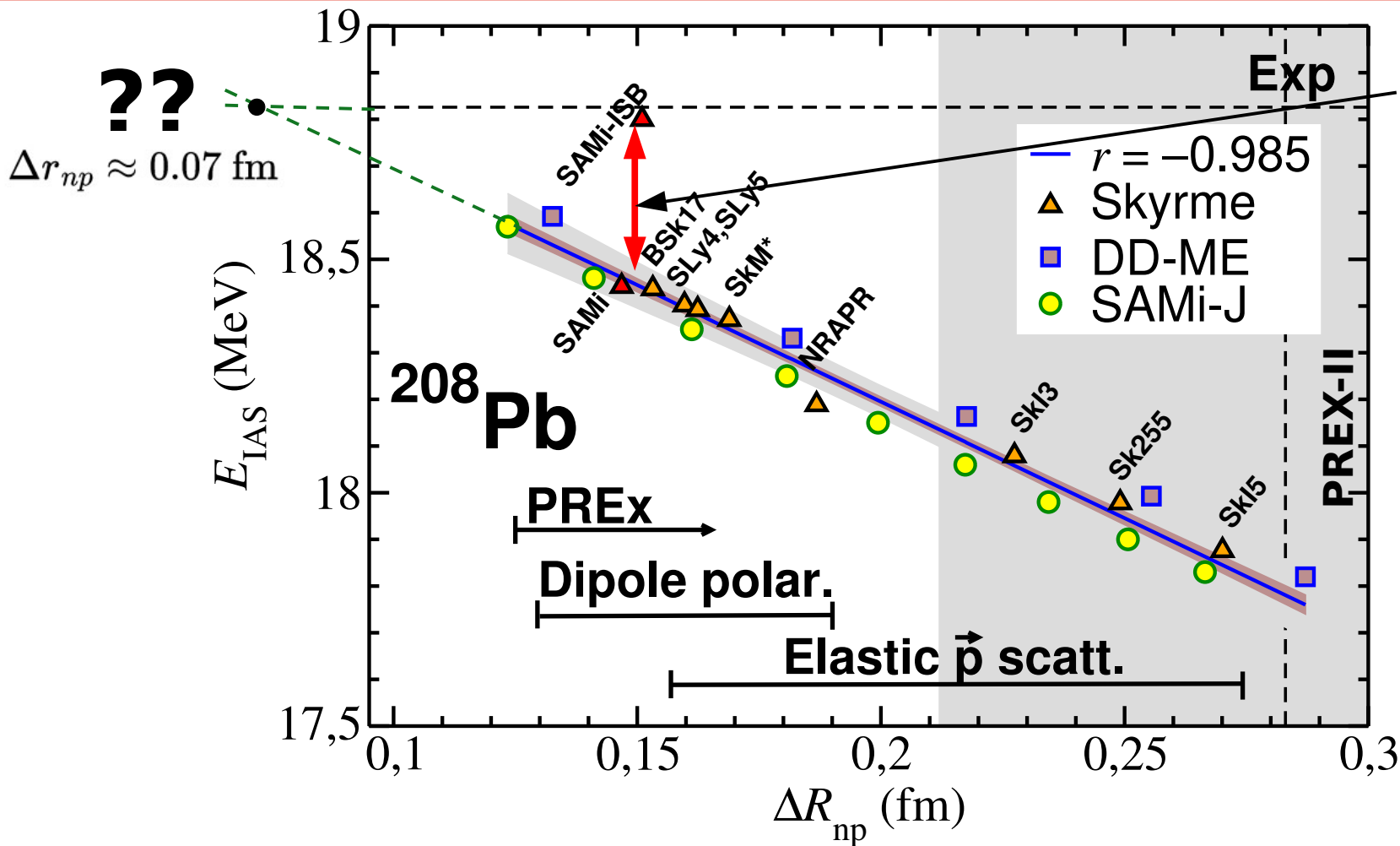
$$\alpha_D(^{208}\text{Pb}) \quad 22.6^{+2.1}_{-1.8}$$



Isobaric Analog State, ISB and Δr_{np}

$$E_{IAS} = \frac{\langle 0|T_+[\mathcal{H}, T_-]|0\rangle}{\langle 0|T_+T_-|0\rangle} \approx \frac{6}{5} \frac{Ze^2}{r_0 A^{1/3}} \left(1 - \sqrt{\frac{5}{12}} \frac{N}{N-Z} \frac{\Delta R_{np}}{r_0 A^{1/3}} \right)$$

$$F = T_{\pm} = \sum_i^A t_{\pm}(i)$$



Isospin symmetry breaking (ISB) missing effects:

- 1) **Nuclear strong int.**
- 2) **Coulomb corrections**

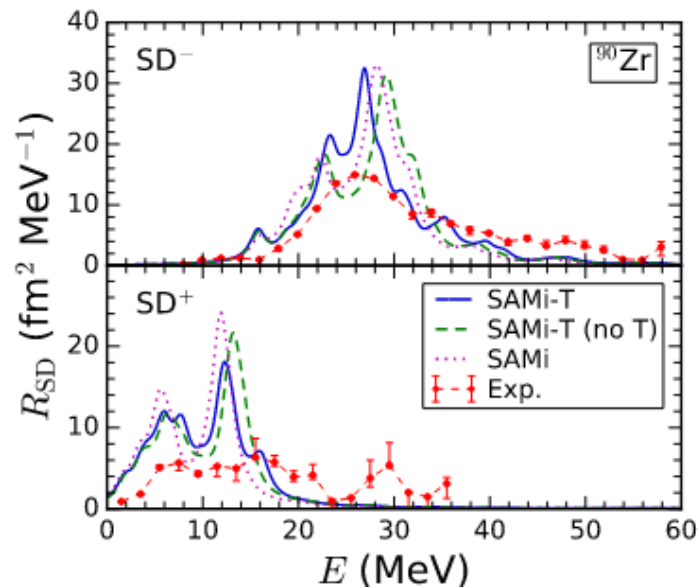
The larger the Δr_{np} , the larger the ISB contributions to IAS in ^{208}Pb

Spin Dipole Resonance and Δr_{np}

Difficult to measure
and analyze?

$$\sum_{i=1}^A \sum_M \tau_{\pm}(i) r_i^L [Y_L(\hat{r}_i) \otimes \sigma(i)]_{JM}$$

$$m_0(t_-) - m_0(t_+) = \frac{9}{4\pi} (N \langle r_n^2 \rangle - Z \langle r_p^2 \rangle)$$

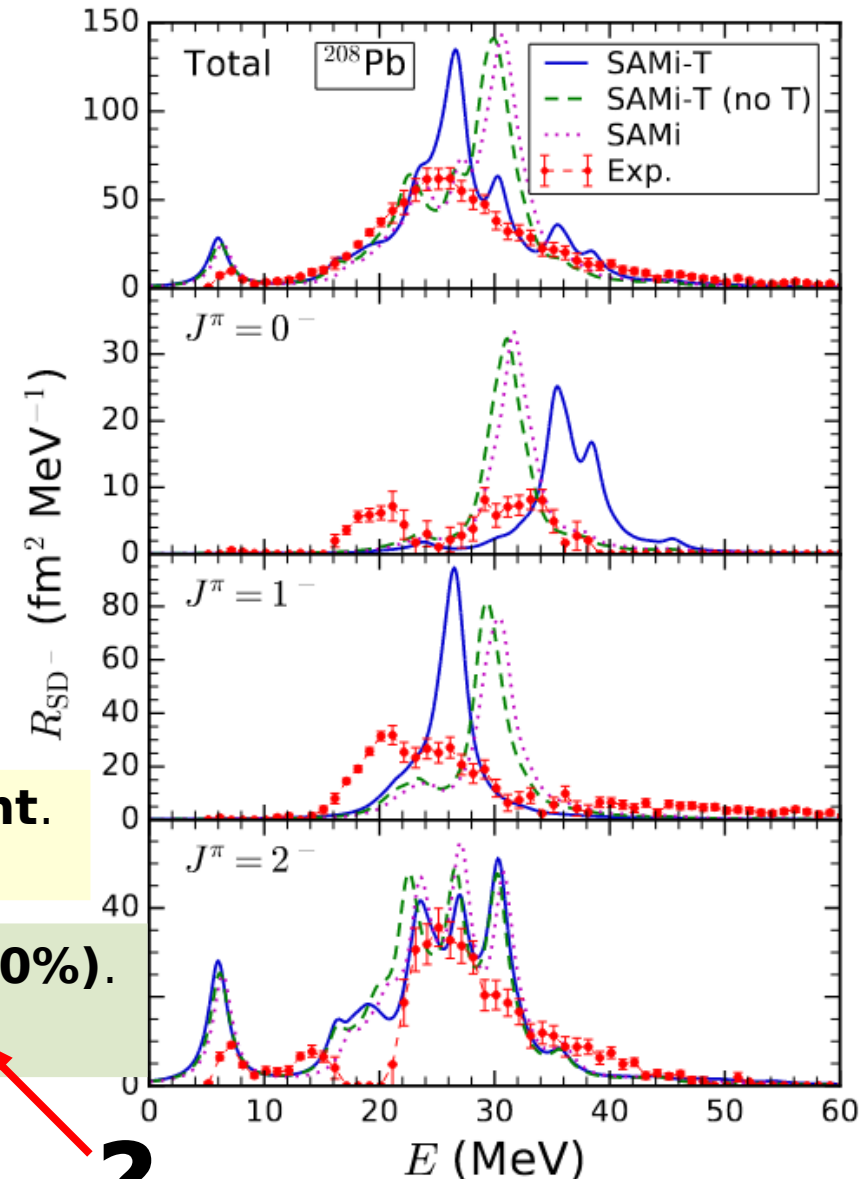


→ ^{90}Zr exp and theo sum rules in agreement.

From exp. sum rule: $\Delta r_{np} = 0.07 \pm 0.04 \text{ fm}$

→ ^{208}Pb exp sum rule < theo sum rule (~20%).

From exp. sum rule: $\Delta r_{np} = 0.07 \pm 0.03 \text{ fm}$



Conclusions

Different ways to investigate properties of the symmetry energy and/or the Δr_{np} provide different answers:

→ current nuclear models likely miss relevant physics

→ interest in repeating and/or improving the analysis of some key experiments (or propose new ones)

Summary

with qualitative indication of accuracy needed to describe experiment
(note that absolute values might be subject to systematics)

- $\rho_0 \in [0.154, 0.159] \text{ fm}^{-3} \rightarrow$ variation range **2%**
 - needed to describe experiment (Rch) $\leq 0.1\%$
- $e_0 \in [15.6, 16.2] \text{ MeV} \rightarrow$ variation range **4%**
 - needed to describe experiment (B) $\leq 0.0001\%$
- $K_0 \in [220, 260] \text{ MeV} \rightarrow$ variation range **15%**
 - needed to describe experiment (E_x^{GMR}) $\leq 7\%$
- $J \in [30, 35] \text{ MeV} \rightarrow$ variation range **15%**
 - needed to describe experiment (α) $\leq 15\%$
- $L \in [20, 120] \text{ MeV} \rightarrow$ variation range **150%**
 - needed to describe experiment (α) $\leq 50\%$
- ...

Nuclear equation of state from ground and collective excited state properties of nuclei

X. Roca-Maza^a, N. Paar^b

Progress in Particle and Nuclear Physics

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