Electric dipole strength functions of Lambda hypernuclei obtained by the time-dependent mean-field calculation

Shuichiro Ebata[1],
Atsushi Umeya[2], Naotaka Yoshinaga[1]

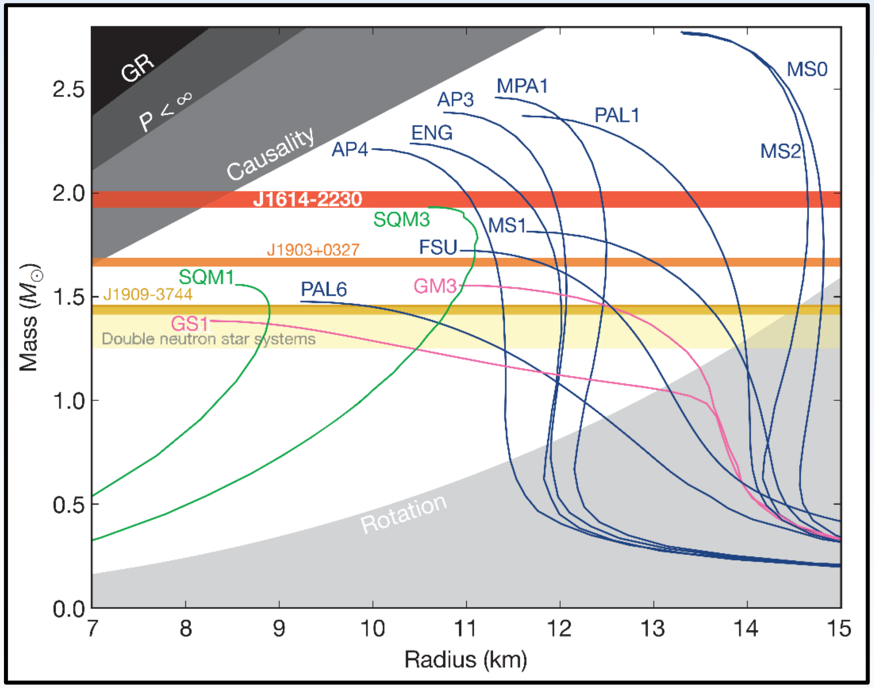
Saitama University[1]

Nippon Inst. Tech.[2]

- ✓ Systematic calculation for E1 strength function of C isotopes
- ✓ LED(PDR) of Lambda hypernuclei

Motivation

P.B. Demorest, et al. Nature **467** 1081 (2010)



E.g.) Heavy neutron stars (2 solar mass) indicate Hyperon existing strongly. → *Hyperon Puzzle* etc.

Electric dipole (E1) mode is used to investigate the nuclear structure and nuclear matter.

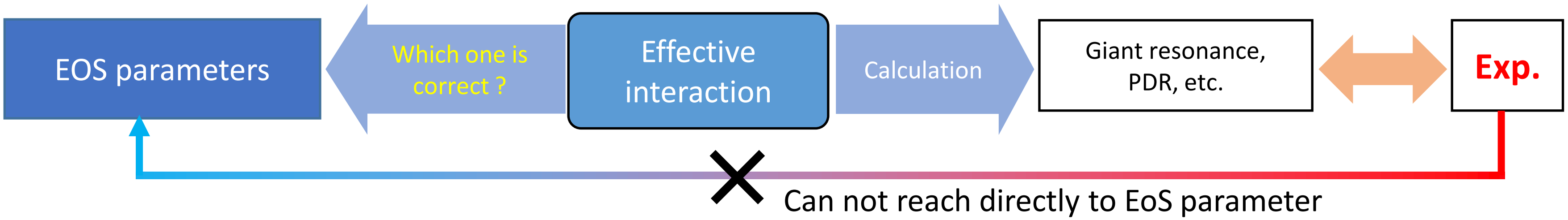
For the ground states of “normal” nuclei, Neutron-skin structure in neutron-rich nuclei Shell-structure evolution (nuclear deformation, new magic numbers)

The excited states of exotic nuclei (neutron-rich, **hypernuclei**) might be important to understand
the high-density nuclear matter and nucleosynthesis.

HIHR (High-Intensity High-Resolution) Hypernuclear Collaboration is planned in Japan to tackle the Hyperon Puzzle in nuclear matter.

Approach to Equation of state (EoS) from the studies of normal nucleus without hyperon

Points : The EoS parameters **cannot be directory deduced** from finite nuclear system (**A → ∞ limit**).
We need to connect between finite nuclear properties (size, GDR, PDR, …) through the effective Int.



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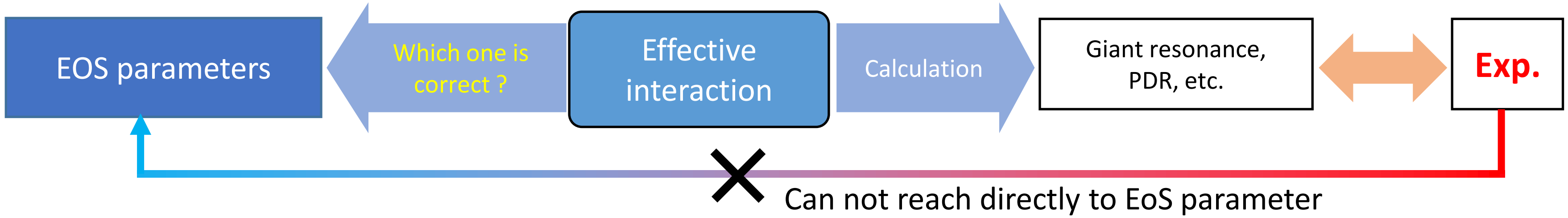


Using a similar approach to a normal nuclear system, we want to develop a new method to extract the information of heigh-density nuclear **matter** from **finite** hypernuclei.

We study the excited states of hypernucleus using a microscopic mean-field model

Skyrme HF+BCS w/ ΛN effective Int. represented in 3D
and Canonical-basis TDHFB

Method (ΛN effective interaction)

Y. Zhang, H. Sagawa, and E. Hiyama, Phys. Rev. C **103** (2021) 034321

$$E = \int d\boldsymbol{r}\,(\mathcal{H}_N + \mathcal{H}_\Lambda)$$

$$\mathcal{H}_N(\boldsymbol{r}) = \mathcal{H}_{\text{Skrme}} + \mathcal{H}_{\text{Coul.}}$$

$$\begin{aligned}\mathcal{H}_\Lambda(\boldsymbol{r}) = &\frac{\hbar^2}{2m_\Lambda}\tau_\Lambda + t_0^\Lambda\left(1+\frac{1}{2}x_0^\Lambda\right)\rho_N\rho_\Lambda + \frac{1}{4}\left(t_1^\Lambda + t_2^\Lambda\right)\left(\tau_\Lambda\rho_N + \tau_N\rho_\Lambda\right)\\ &+\frac{1}{8}\left(3t_1^\Lambda - t_2^\Lambda\right)\nabla\rho_\Lambda\cdot\nabla\rho_N + \frac{1}{2}W_0^\Lambda\left(\nabla\rho_\Lambda\cdot\boldsymbol{J}_N + \nabla\rho_N\cdot\boldsymbol{J}_\Lambda\right)\\ &+\frac{3}{8}t_3^\Lambda\left(1+\frac{1}{2}x_3^\Lambda\right)\rho_N^{\gamma+1}\rho_\Lambda\end{aligned}$$

$\delta\phi_q^*(\boldsymbol{r},\sigma)\quad q = n, p, \Lambda$ → single-particle Hamiltonian → HF Cal.

Original interaction was deduced in D. E. Lanskoy and Y. Yamamoto, Phys. Rev. C **55**, 2330 (1997), which is Skyrme-type (delta function in space).
The original spin-orbital force W was too strong to reproduce the energy splitting of p-orbits in ${}^{13}_{\Lambda}\mathrm{C}$ (0.152 MeV). New one weaken the force (LY5 : W = 62 MeV fm^5→ LY5r : W = 4.7 MeV fm^5)

Method (Ground state and dynamics of Λ-hypernucleus)

Full self-consistent 3D Skyrme HF+BCS

S.E. et al., Phys. Rev. C**82**(2010) 034306.

Interaction (*ph*) : Skyrme (SLy4),

(*pp*, *hh*) : Constant (monopole)

$$\Delta_k(t) = \sum_{l>0} G_{kl}\kappa_l(t) \quad G_{kl} = g f(\epsilon_k^0) f(\epsilon_l^0)$$

$f(\epsilon)$: cutoff function

ϵ_k^0 : s.p. energy at g.s.

For nucleon phase, we consider the **pairing correlation**. And the **odd-nucleon system** is calculated using the constraints on expectation value of nucleon number <N>="odd".

Any nuclear deformation can be treated in the 3D Cartesian coordinate representation.

Canonical-basis time-dependent Hartree-Fock-Bogoliubov (Cb-TDHFB)

S.E. et al., Phys. Rev. C **82** (2010) 034306, S.E. and T. Nakatsukasa, JPS Conf. Proc. **6**, 020056 (2015)

Cb-TDHFB equations

$$i\hbar\frac{\partial}{\partial t}|\phi_k(t)\rangle = \big(h(t) - \eta_k(t)\big)|\phi_k(t)\rangle$$

$$i\hbar\frac{\partial}{\partial t}\rho_k(t) = \kappa_k(t)\Delta_k^*(t) - \Delta_k(t)\kappa_k^*(t)$$

$$i\hbar\frac{\partial}{\partial t}\kappa_k(t) = \big(\eta_k(t) + \eta_{\bar{k}}(t)\big)\kappa_k(t) + \Delta_k(t)(2\rho_k(t) - 1)$$

Properties of Cb-TDHFB

$$d/dt\langle\phi_k(t)|\phi_{k'}(t)\rangle = 0$$

$$d/dt\langle\hat{N}\rangle = 0,\ d/dt E_{\text{Total}} = 0$$

In the limit of Δ=0 ➡ **TDHF**

In the static limit, ➡ **HF+BCS**

$$\eta_k(t) \equiv \langle\phi_k(t)|h(t)|\phi_k(t)\rangle + i\hbar\left\langle\frac{\partial\phi_k}{\partial t}\middle|\phi_k\right\rangle$$

Nucleon parts : Cb-TDHFB Cal., Lambda-particle states: TDHF Cal.

The lowest occupied states for Lambda-particle are used in TD Cal.

Method (Linear response Cal. for Lambda hypernucleus)

Calculate HF or HF+BCS ground state $|\Psi(0)\rangle$

Adding
a **instantaneous** external field
to ground state

$$\hat{V}_{\text{ext}}(t) \equiv -k\hat{F}\delta(t) \qquad k \ll 1$$

$$|\Psi(0_+)\rangle \equiv e^{i\hbar k\hat{F}}|\Psi(0)\rangle$$ $\hat{F}$: one-body operator

$$\hat{F} = \frac{ZM_N}{AM_N + M_\Lambda}\left(\frac{M_\Lambda}{M_N}\hat{z}_\Lambda + \sum_{n=1}^{N}\hat{z}_n\right) - \frac{NM_N + M_\Lambda}{AM_N + M_\Lambda}\sum_{p=1}^{Z}\hat{z}_p$$

Calculate the time-evolution with TDHF or Cb-TDHFB

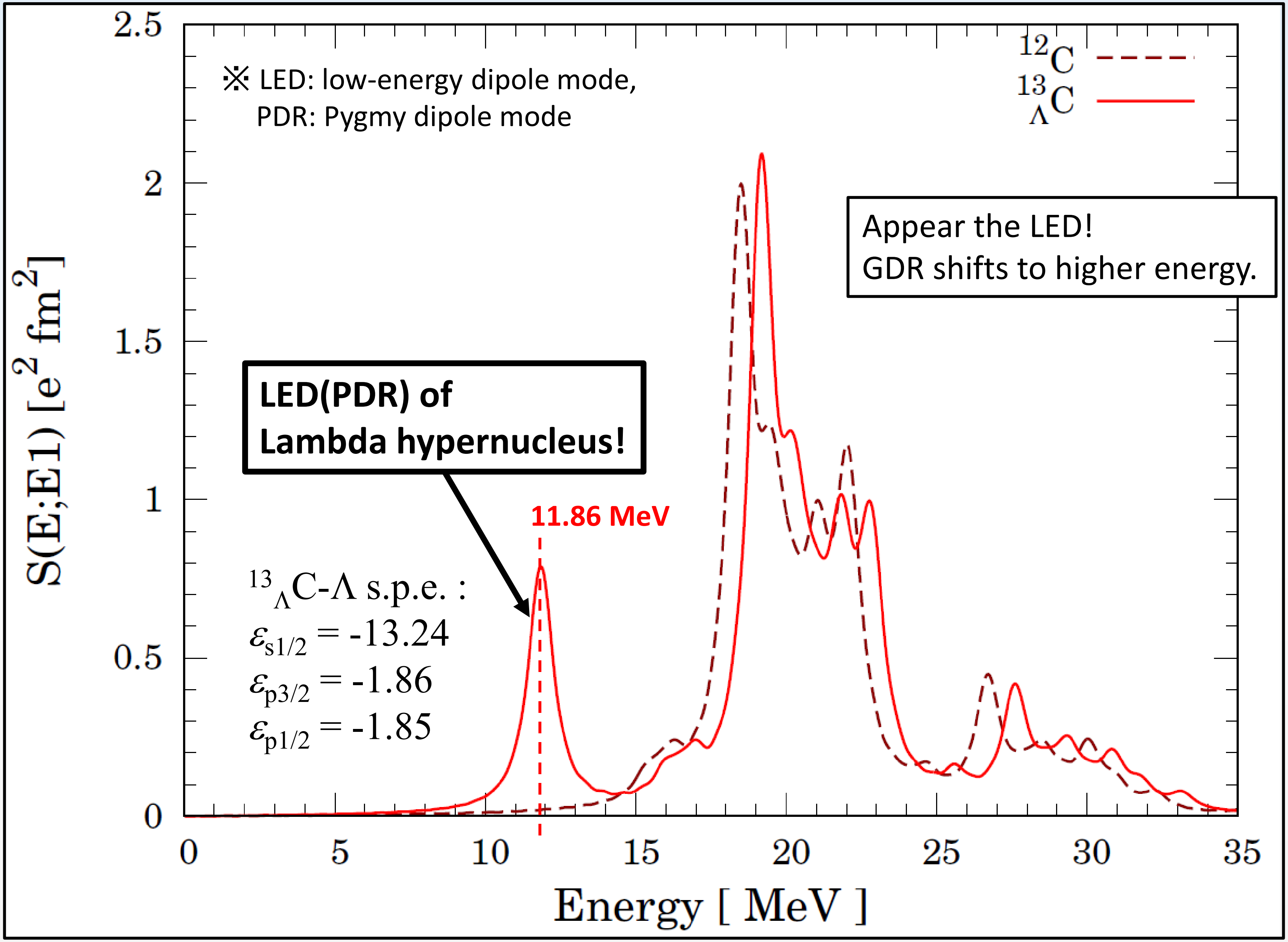
Strength function *S(E;F)* is gotten as Fourier transformed TD- $\langle\hat{F}\rangle$.

$$S(E;\hat{F}) = \sum_n \left|\langle n|\hat{F}|0\rangle\right|^2 \delta(E - \tilde{E}_n) \qquad \tilde{E}_n \equiv E_n - E_0, \quad E_n > E_0$$

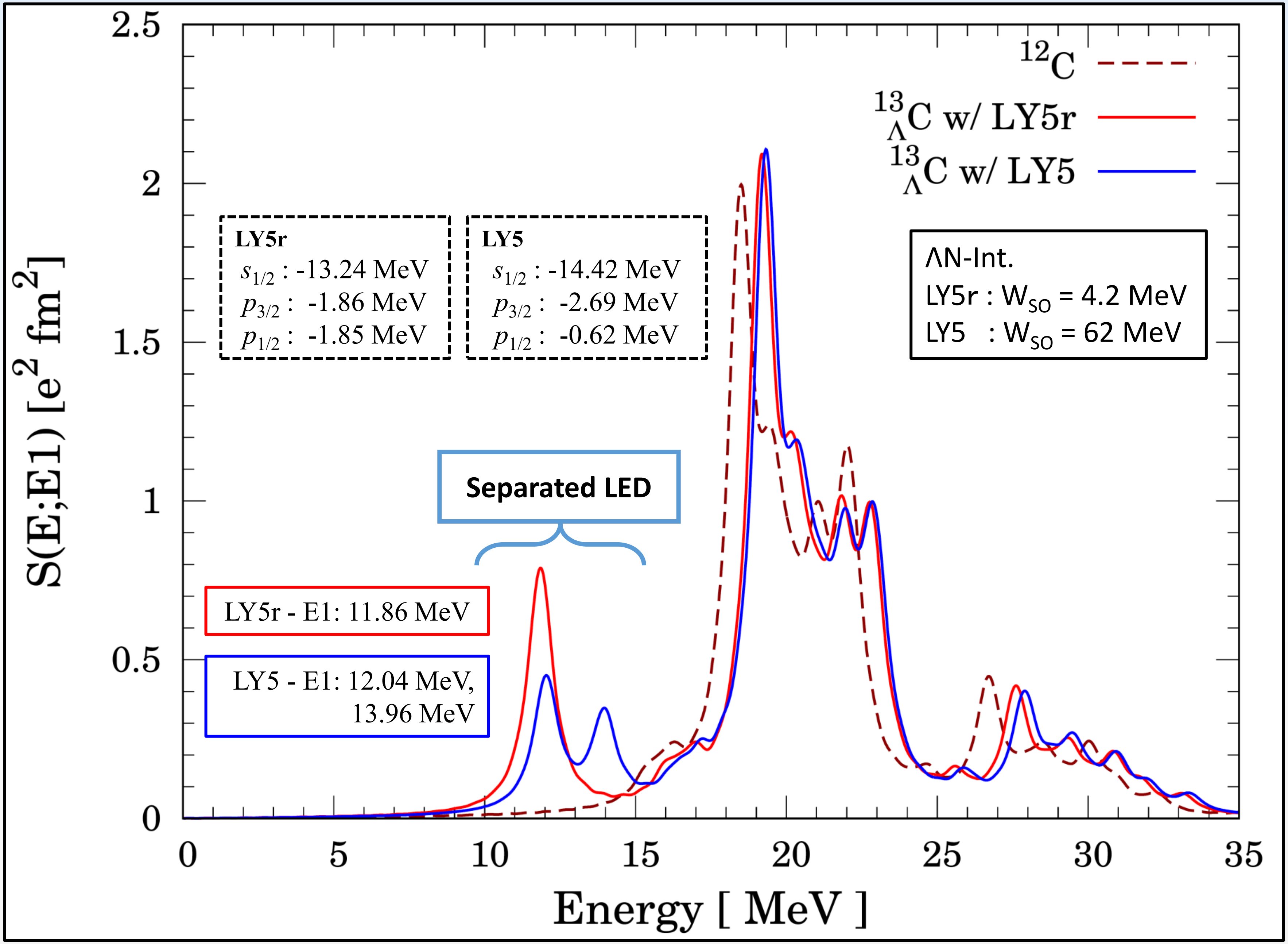
$$= -\frac{1}{k\pi}\lim_{\Gamma\to 0}\text{Im}\int_0^\infty dt\, e^{(iE-\Gamma/2)t/\hbar}\left(f(t) - f(0)\right) \qquad f(t) \equiv \langle\Psi(t)|\,\hat{F}\,|\Psi(t)\rangle$$

Γ: Smoothing parameter

Results (E1 strength function: ^{12}C, $^{13}_{\Lambda}$C)



Results (E1 strength function: ^{12}C, $^{13}_{\Lambda}C$)



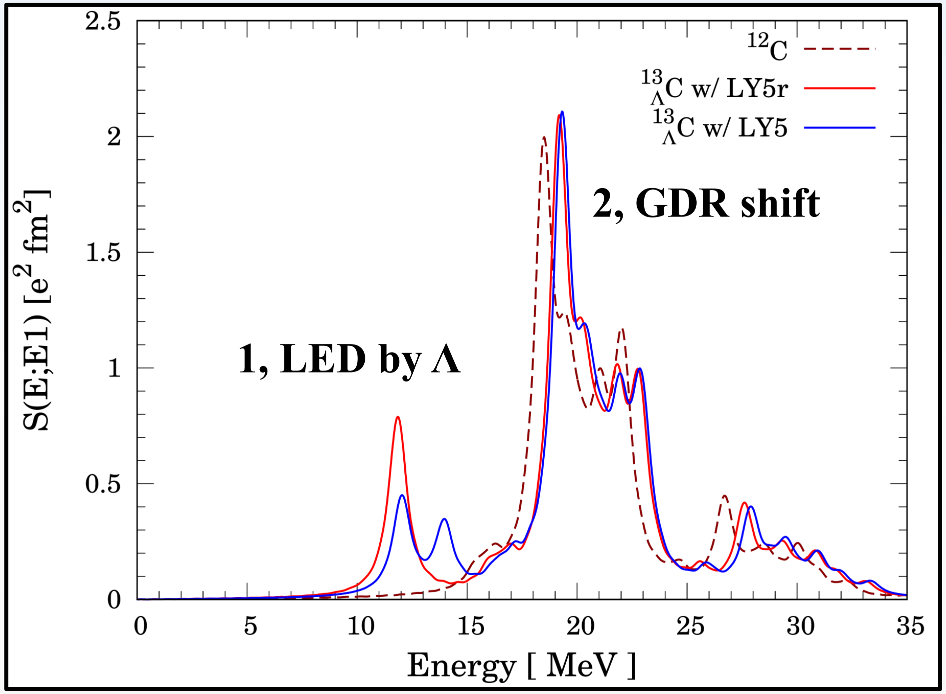
Results (E1 strength function: ^{12}C, $^{13}_{\Lambda}$C)

What are the remarkable points?

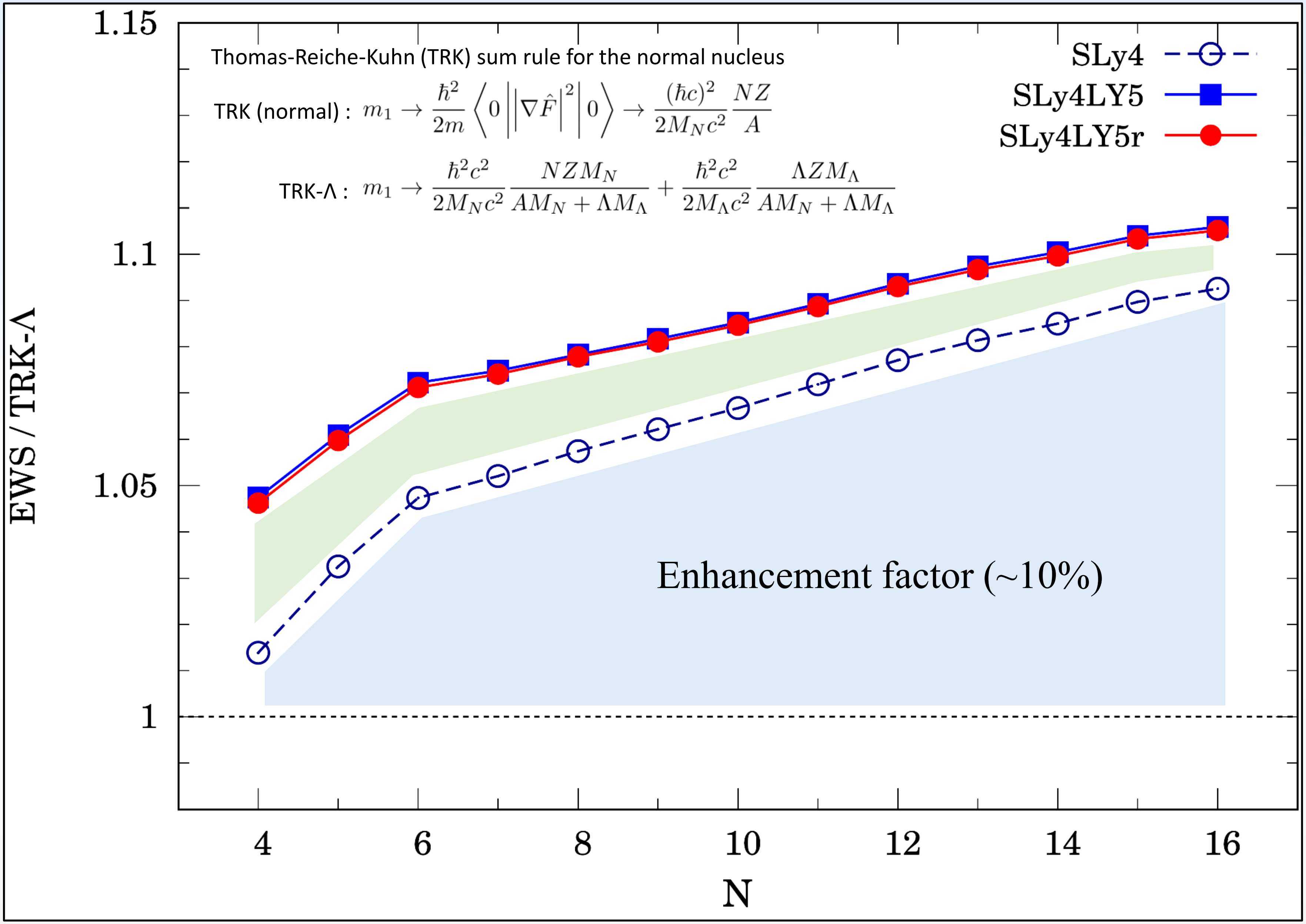
- What is the mechanism of LED induced by the Lambda particle?
  ← **Single-particle excitation?**
  ← **Collective motion?** (single-Λ particle …)

- What does the LED strength ratio in the isotopes?
  ← **Check their strength**.

- Why is the GDR shifted to higher energy?
  ← **Nuclear radius?** (G.S. property?)
  ← **Residual interaction?** (Excited states?)



Results (EWS vs TRK-Λ sum rule)



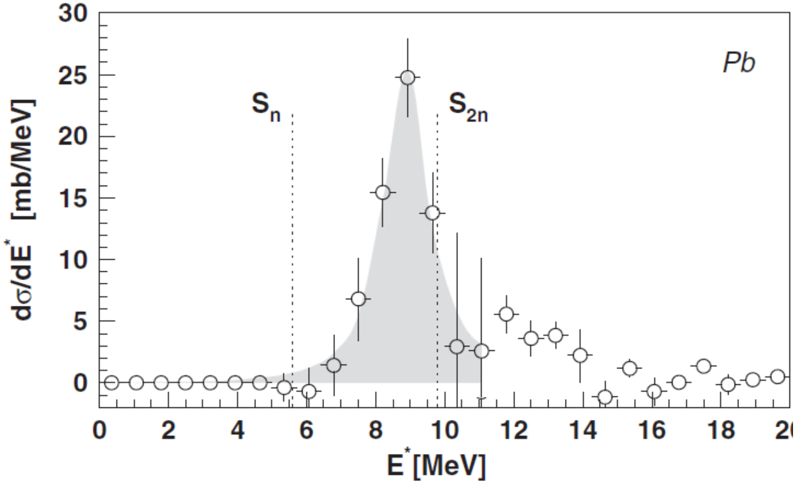
Results (Low-energy dipole fraction)

S. Ebata, T. Nakatsukasa and T. Inakura, Phys. Rev. C**90** (2014) 024303

Fraction is introduced to evaluate the LED.

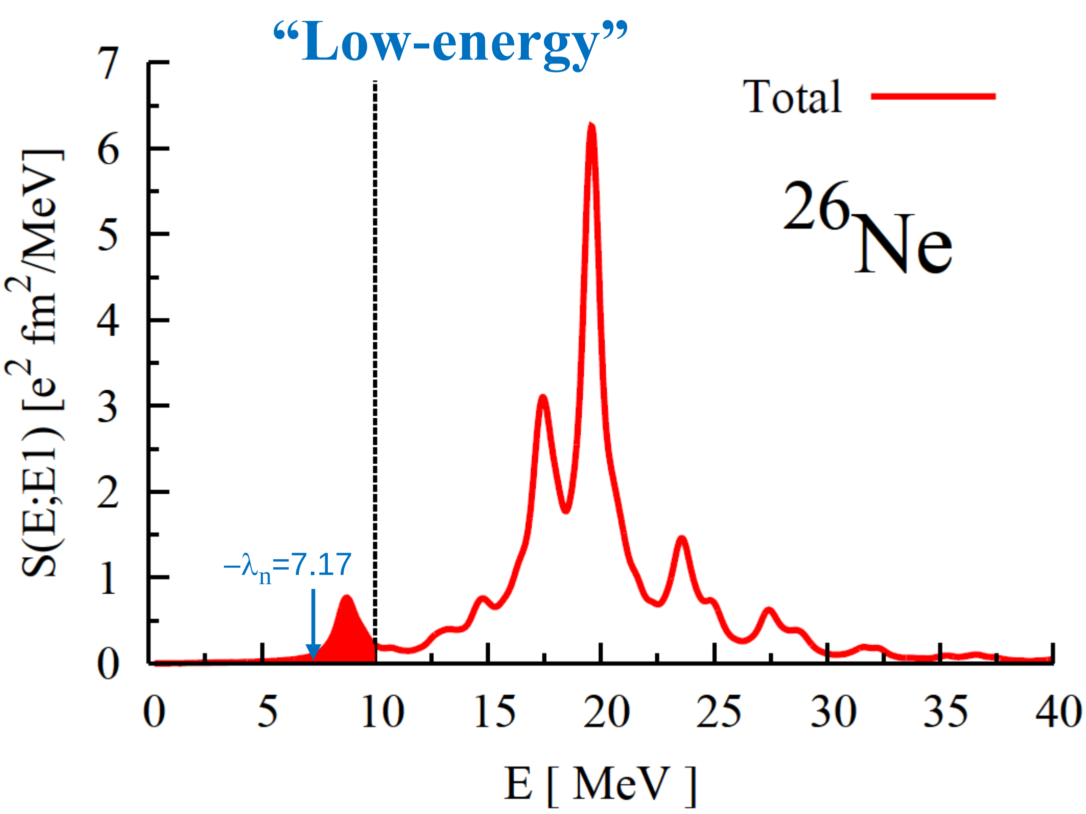
$$\frac{m_1(E_c=10)}{m_1} \equiv \frac{\int_0^{10[\mathrm{MeV}]} E\,S(E;E1)dE}{\int E\,S(E;E1)dE}$$

^{26}Ne on a ^{208}Pb target



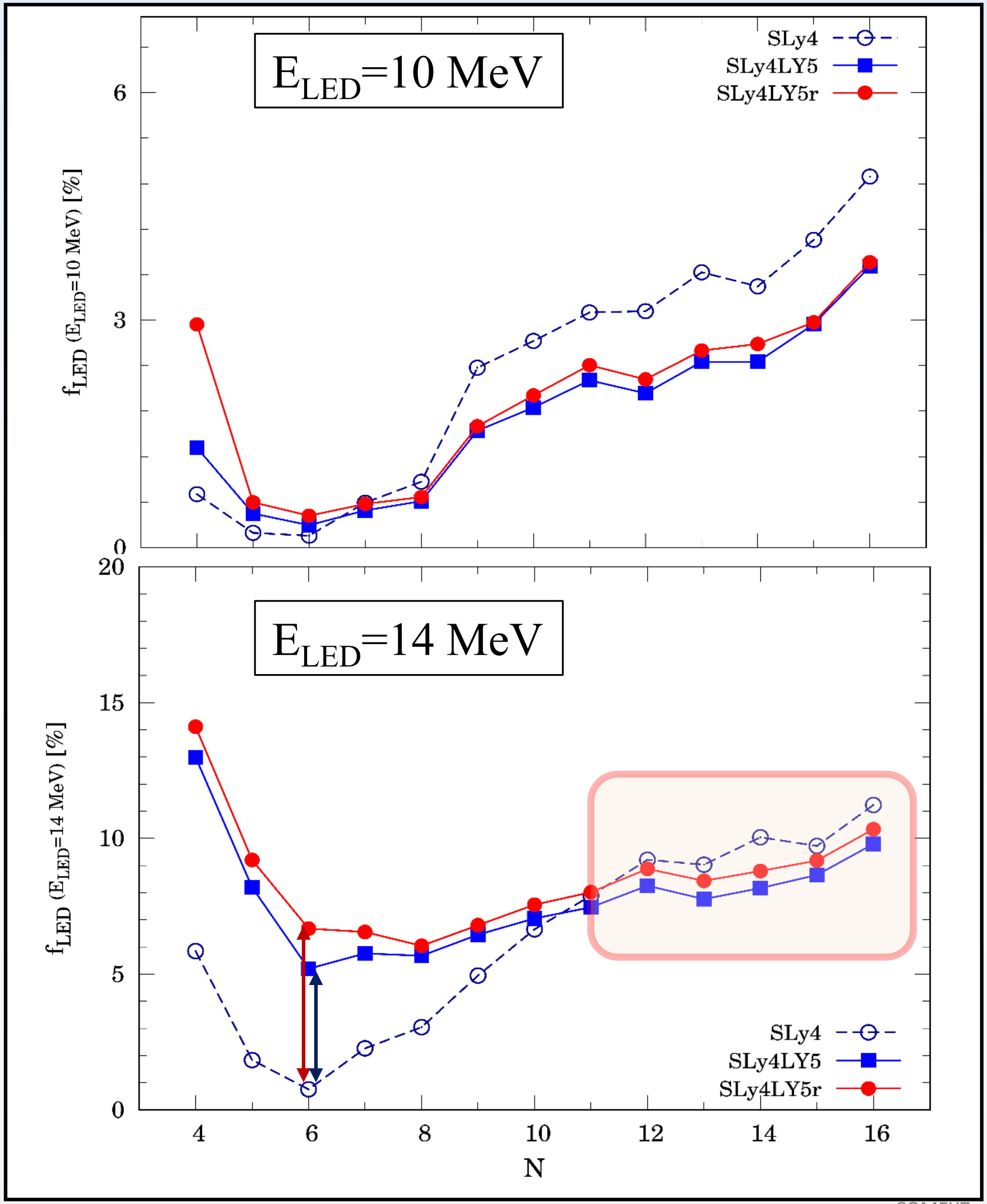
J. Gibelin, et al., Phys. Rev Lett. **101**, 212503 (2008)

ex.)



10 and 14 MeV are used as the "Low energy" and
we check the LED fraction in the energy weighted sum-rule.

Results (LED fraction for C isotopes)



Because the LED of Λ hypernuclei might depend on the Λ S.P. energy, E_{LED}=10 MeV is not suitable to estimate the LED in the isotopes.

$$f_{\rm LED} = \frac{\int^{E_{\rm LED}} dE ES(E;E1)}{\int dE ES(E;E1)}$$

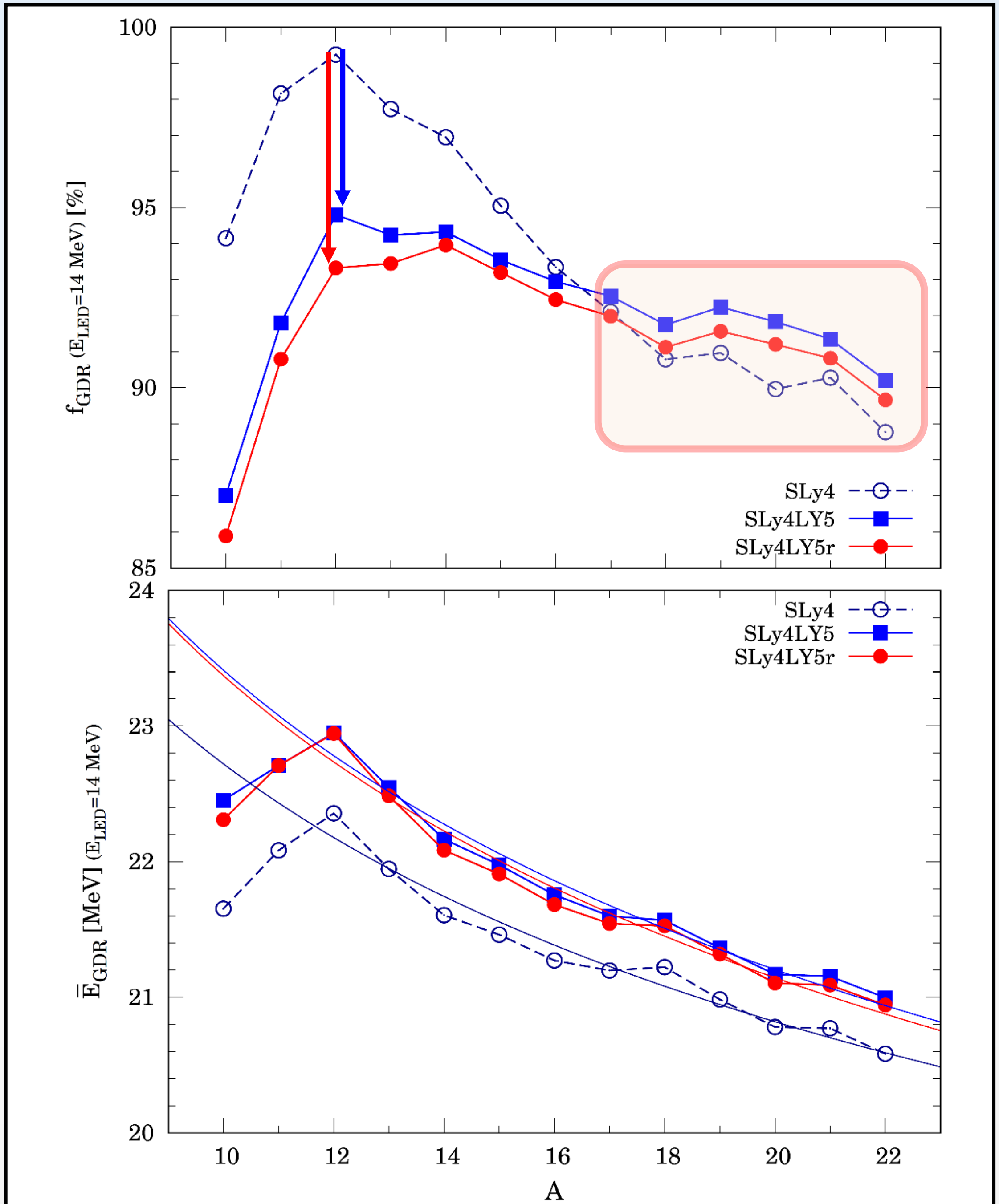
Here, we evaluate LED and GDR using E_{LED}=14MeV to separate them.

Stable nucleus + Λ particle has almost 7%.
When the LS Int. is strong,
the LED peak is separated, and its strength ratio is underestimated.

The LED contribution to the EWS might get relatively small since the Λ particle in the neutron-rich nucleus is deeply bound.

→ Next, go to "GDR strength"

Results (GDR fraction & mean E for C isotopes)



$$f_{\mathrm{GDR}} = \frac{\int_{E_{\mathrm{LED}}} dEES(E;E1)}{\int dEES(E;E1)}$$

For Λ-hyper nuclei, the LEDs take about 5% of GDR strength.
But for neutron-rich Λ-hyper nuclei, the LEDs seem to keep GDR strength at a glance.
← The LEDs locate and remain in high energy.

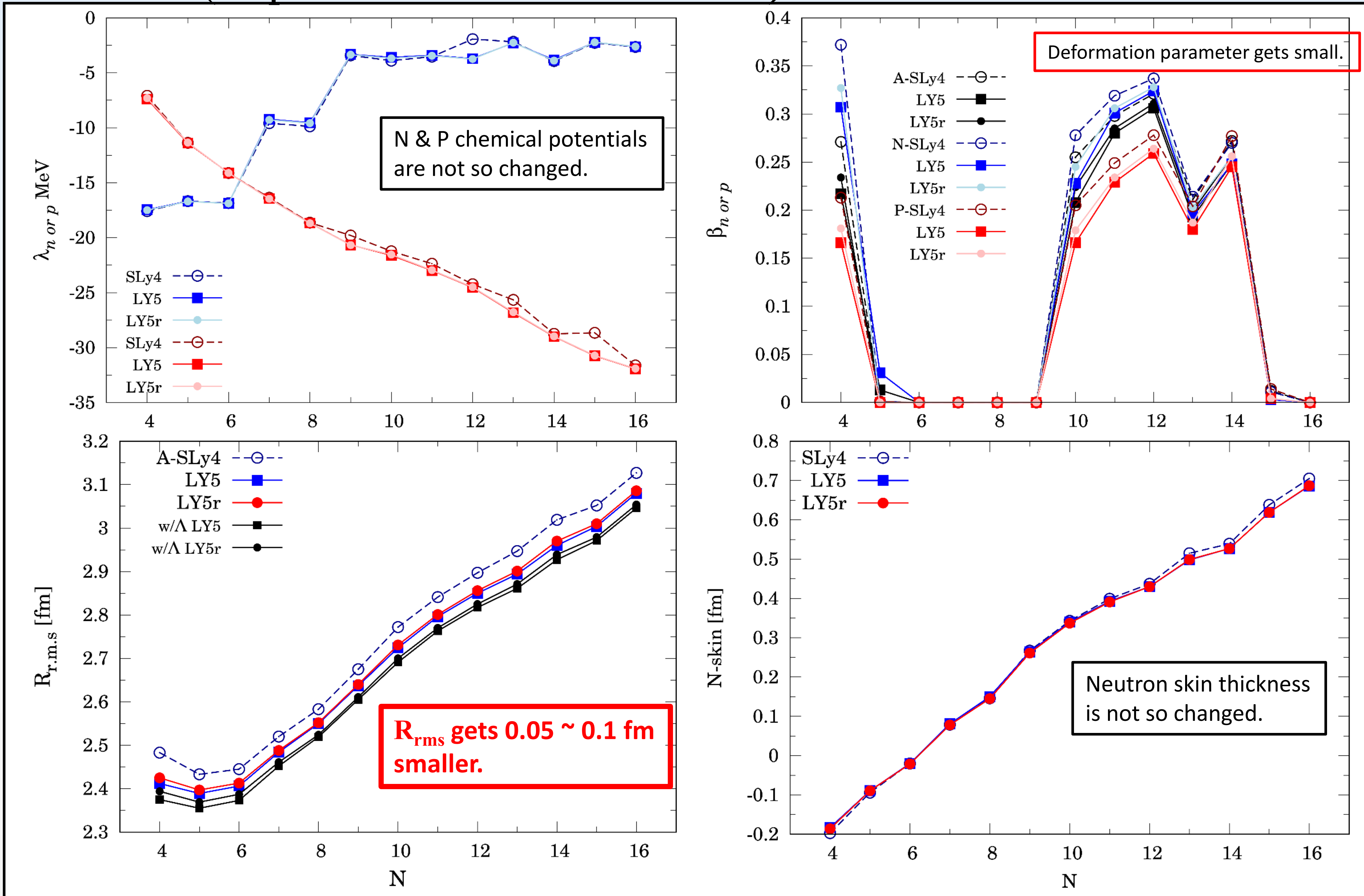
$$\bar{E}_{\mathrm{GDR}} = \frac{\int_{E_{\mathrm{LED}}} dEES(E;E1)}{\int_{E_{\mathrm{LED}}} dES(E;E1)}$$

The GDR mean energy of Λ-hyper nuclei is 0.4∼0.8 MeV higher than normal nuclei.

$$\bar{E}_{\mathrm{GDR}} \sim 20A^{-1/3} + 13.5\ \mathrm{MeV}$$

$$\bar{E}^{\Lambda}_{\mathrm{GDR}} \sim 23A^{-1/3} + 12.5\ \mathrm{MeV}$$

Results (Λ-particle influence on G.S.)



Summary

- ✓ We calculate the E1 strength functions of C and $_{\Lambda}$C isotopes using TD mean-field model (Cb-TDHFB) with Skyrme-type ΛN effective interaction.
- ✓ We found some strength of $^{13}_{\Lambda}$C in lower energy than GDR.
  The energy position of LED depends on the s.p.e. of Lambda particle in the mean-field.
  - ✓ We obtained the E1 strength of deformed Lambda hypernuclei.
  - ✓ For proton-rich nuclei, the LED of single-Lambda hypernuclei clearly appear.
  - ✓ For neutron-rich nuclei, the effects of single-Lambda becomes small.
  - ✓ The Lambda particle shifts the GDR to a higher energy region.
    ← Nuclear shrinking by a Lambda particle might be the major cause.

Perspective

- Compare calculations and measurements
- Extend the target hypernucleus to more massive one
- Check the dynamical effects of ΛN effective interaction on the excited states.
  3-body force of Lambda particles, LS force, etc.
- Fission, Fusion, *Lambda-particle transfer*

Thank you!