## Beyond mean field study of Giant resonances (Gamow-Teller), beta-decay and <br> QCD-based Charge Symmetry Breaking Interaction

Hiroyuki Sagawa RIKEN/University of Aizu

COMEX7
Catania, Italy, June 11-16, 2023

1. Introduction
2. Subtracted second RPA with tensor interactions
3. Gamow-Teller states and quenching problem
4. Magnetic dipole (M1) excitations
5. Beta decay
6. QCD-based CSB and Okamoto-Nolen-Schiffer anomaly
7. Summary


## Beyond mean field model

```
Particle - vibration coupling model (RPA+PVC)
Quasi-particle RPA+PVC (QRPA+QPVC)
Relativistic quasiparticle time blocking approximation (RQTBA)
Second RPA model (SQRA)
Subtracted SRPA model. (SSRPA)
Generator coordinate model (GCM)
```

G. Colo

Yifei Niu, Monday
D. Gambacarta, today
E. Letvinova, Friday

Today's topics
SSRPA for Gamow-Teller giant resonance and beta decay

## Targets of Second RPA

- Spreading width of giant resonances
- Quenching of spin-isospin excitations.
- Low-energy pigmy states

SSRPA is based on a proper idea of the EDF theory since it is designed for the mean field model applications (HF, RPA).

## SRPA phonon operator

RPA ground state is defined as

$$
|\Psi\rangle=e^{\hat{S}}|\Phi\rangle
$$

where

$$
\hat{S}=\sum_{p h} C_{p h}(t) a_{p}^{\dagger} a_{h}
$$

SRPA operator is

$$
\hat{S}=\sum_{p h} C_{p h}(t) a_{p}^{\dagger} a_{h}+\frac{1}{2} \sum_{p h p^{\prime} h^{\prime}} \hat{C}_{p p^{\prime} h h^{\prime}}(t) a_{p}^{\dagger} a_{p^{\prime}}^{\dagger} a_{h} a_{h^{\prime}}
$$

$$
\begin{aligned}
Q_{\nu}^{\dagger}= & \sum_{p h}\left(X_{p h}^{\nu} a_{p}^{\dagger} a_{h}-Y_{p h}^{\nu} a_{h}^{\dagger} a_{p}\right) \\
& +\sum_{\substack{p_{1}<p_{2} \\
h_{1}<h_{2}}}\left(X_{p_{1} p_{2} h_{1} h_{2}}^{\nu} a_{p_{1}}^{\dagger} a_{p_{2}}^{\dagger} a_{h_{2}} a_{h_{1}}\right. \\
& \left.-Y_{p_{1} p_{2} h_{1} h_{2}}^{\nu} a_{h_{1}}^{\dagger} a_{h_{2}}^{\dagger} a_{p_{2}} a_{p_{1}}\right)
\end{aligned}
$$

Equation of motion give SRPA matrix equation

$$
\left[H, Q^{\dagger}\right]=\hbar \omega Q^{\dagger}
$$

The basic idea is the same as the coupled cluster model with singlet (s)- and doublet (d)- pairs.

## RPA equation.

$$
\left[\begin{array}{cc}
A & B \\
-B^{*} & -A^{*}
\end{array}\right]\left[\begin{array}{c}
X^{\nu} \\
Y^{\nu}
\end{array}\right]=\hbar \omega_{\nu}\left[\begin{array}{c}
X^{\nu} \\
Y^{\nu}
\end{array}\right]
$$

$$
\begin{aligned}
A & =\left(\begin{array}{ll}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{array}\right), B=\left(\begin{array}{ll}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{array}\right) \\
X & =\binom{X_{1}^{\nu}}{X_{2}^{\nu}}, Y=\binom{Y_{1}^{\nu}}{Y_{2}^{\nu}}
\end{aligned}
$$

$$
\begin{aligned}
A_{11} & =A_{p h ; p^{\prime} h^{\prime}} \\
& =<H F\left|\left[a_{h}^{\dagger} a_{p},\left[H, a_{p^{\prime}}^{\dagger} a_{h^{\prime}}\right]\right]\right| H F> \\
& =\left(E_{p}-E_{h}\right) \delta_{p p^{\prime}} \delta_{h h^{\prime}}+\bar{V}_{p h^{\prime} h p^{\prime}}
\end{aligned}
$$

$$
\begin{aligned}
B_{11} & =B_{p h ; p^{\prime} h^{\prime}} \\
& =-<H F\left|\left[a_{h}^{\dagger} a_{p},\left[H, a_{h^{\prime}}^{\dagger} a_{p^{\prime}}\right]\right]\right| H F> \\
& =\bar{V}_{p p^{\prime} h h^{\prime}}
\end{aligned}
$$

(a)

(b)

$\mathrm{Iplh} \leftrightarrow 2 \mathrm{p} 2 \mathrm{~h}$
(c)


$$
\begin{aligned}
A_{12} & =A_{p h ; p_{1} p_{2} h_{1} h_{2}} \\
& =<H F\left|\left[a_{h}^{\dagger} a_{p},\left[H, a_{p_{1}}^{\dagger} a_{p_{2}}^{\dagger} a_{h_{2}} a_{h_{1}}\right]\right]\right| H F> \\
& =U\left(h_{1} h_{2}\right) \bar{V}_{p_{1} p_{2} p h_{2}} \delta_{h h_{1}}-U\left(p_{1} p_{2}\right) \bar{V}_{h p_{2} h_{1} h_{2}} \delta_{p p_{1}}
\end{aligned}
$$ $U\left(h_{1} h_{2}\right)$ is an anti-symmtrizer.

$$
\begin{aligned}
A_{22}= & A_{p_{1} p_{2} h_{1} h_{2} ; p_{1}^{\prime} p_{2}^{\prime} h_{1}^{\prime} h_{2}^{\prime}} \\
= & <H F\left|\left[a_{h_{1}}^{\dagger} a_{h_{2}}^{\dagger} a_{p_{2}} a_{p_{1}},\left[H, a_{p_{1}^{\prime}}^{\dagger} a_{p_{2}^{\prime}}^{\dagger} a_{h_{2}^{\prime}} a_{h_{1}^{\prime}}\right]\right]\right| H F> \\
= & \left(E_{p_{1}}+E_{p_{2}}-E_{h_{1}}-E_{h_{2}}\right) U\left(p_{1} p_{2}\right) U\left(h_{1} h_{2}\right) \\
& \times \delta_{p_{1} p_{1}^{\prime}} \delta_{p_{2} p_{2}^{\prime}} \delta_{h_{1} h_{1}^{\prime}} \delta_{h_{2} h_{2}^{\prime}} \\
& +U\left(h_{1} h_{2}\right) \bar{p}_{p_{1} p_{2} p_{1}^{\prime} p_{2}^{\prime}} \delta_{h_{1} h_{1}^{\prime}} \delta_{h_{2} h_{2}^{\prime}} \\
& +U\left(p_{1} p_{2}\right) \bar{V}_{h_{1} h_{2} h_{1}^{\prime} h_{2}^{\prime}} \delta_{p_{1} p_{1}^{\prime}} \delta_{p_{2} p_{2}^{\prime}} \\
& -U\left(p_{1} p_{2}\right) U\left(h_{1} h_{2}\right) U\left(p_{1}^{\prime} p_{2}^{\prime}\right) U\left(h_{1}^{\prime} h_{2}^{\prime}\right) \\
& \times \bar{V}_{p_{1} h_{1}^{\prime} p_{1}^{\prime} h_{1}} \delta_{p_{2} p_{2}^{\prime}} \delta_{h_{2} h_{2}^{\prime}}
\end{aligned}
$$

In SRPA with subtraction procedure (SSRPA), $\mathrm{A}_{11}$ and $\mathrm{B}_{11}$ are modified.

$$
\begin{gathered}
A_{11^{\prime}}^{S}=A_{11^{\prime}}+\sum_{2} A_{12}\left(A_{22}\right)^{-1} A_{21^{\prime}}+\sum_{2} B_{12}\left(A_{22}\right)^{-1} B_{21^{\prime}} \\
B_{11^{\prime}}^{S}=B_{11^{\prime}}+\sum_{2} A_{12}\left(A_{22}\right)^{-1} B_{21^{\prime}}+\sum_{2} B_{12}\left(A_{22}\right)^{-1} A_{21^{\prime}} \\
\mathcal{A}_{F}^{S}=\left(\begin{array}{cc}
A_{11^{\prime}}+\sum_{2,2^{\prime}} A_{12}\left(A_{22^{\prime}}\right)^{-1} A_{2^{\prime} 1^{\prime}}+\sum_{2,2^{\prime}} B_{12}\left(A_{22^{\prime}}\right)^{-1} B_{2^{\prime} \prime^{\prime}} & A_{12} \\
A_{21} & A_{22^{\prime}}
\end{array}\right), \\
\mathcal{B}_{F}^{S}=\left(\begin{array}{cc}
B_{11^{\prime}}+\sum_{2,2^{\prime}} A_{12}\left(A_{22^{\prime}}\right)^{-1} B_{2^{\prime} 1^{\prime}}+\sum_{2,2^{\prime}} B_{12}\left(A_{22^{\prime}}\right)^{-1} A_{2^{\prime} 1^{\prime}} & B_{12} \\
B_{21} & 0
\end{array}\right) .
\end{gathered}
$$

Gamow-Teller transitions in magic nuclei calculated by the charge-exchange subtracted second random-phase approximation

M. J. Yang $\odot,{ }^{1}$ C. L. Bai $\odot,{ }^{1}$ H. Sagawa, ${ }^{2,3}$ and H. Q. Zhang ${ }^{4}$

## Skyrme tensor interactions

$$
\begin{aligned}
v_{T}= & \frac{T}{2}\left\{\left[\left(\vec{\sigma}_{1} \cdot k^{\prime}\right)\left(\vec{\sigma}_{2} \cdot k^{\prime}\right)-\frac{1}{3}\left(\vec{\sigma}_{1} \cdot \vec{\sigma}_{2}\right) k^{2}\right] \delta\left(\vec{r}_{1}-\vec{r}_{2}\right)+\delta\left(\vec{r}_{1}-\vec{r}_{2}\right)\left[\left(\vec{\sigma}_{1} \cdot k\right)\left(\vec{\sigma}_{2} \cdot k\right)-\frac{1}{3}\left(\vec{\sigma}_{1} \cdot \vec{\sigma}_{2}\right) k^{2}\right]\right\} \\
& +U\left\{\left(\vec{\sigma}_{1} \cdot k^{\prime}\right) \delta\left(\vec{r}_{1}-\vec{r}_{2}\right)\left(\vec{\sigma}_{2} \cdot k\right)-\frac{1}{3}\left(\vec{\sigma}_{1} \cdot \vec{\sigma}_{2}\right)\left[k^{\prime} \cdot \delta\left(\vec{r}_{1}-\vec{r}_{2}\right) k\right]\right\},
\end{aligned}
$$

## Mean field potential

$$
h_{s o}=U_{s o} l \cdot \sigma
$$

$$
\alpha_{T}=\frac{5}{12} U
$$

$$
\beta_{T}=\frac{5}{24}(T+U)
$$

$$
U_{S O}=\frac{W_{0}}{r}\left(\frac{d \rho_{q}}{d r}+\frac{d \rho_{1-q}}{d r}\right)+\alpha \frac{J_{q}}{r}+\beta \frac{J_{1-q}}{r}
$$

$$
J_{q}(r)=\frac{1}{4 \pi r^{3}} \sum_{i \in q} v_{i}^{2}\left(2 j_{i}+1\left[j_{i}\left(j_{i}+1\right)-l_{i}\left(l_{i}+1\right)-\frac{3}{4}\right] R_{i}^{2}(r)\right.
$$



FIG. 3: GT_ strength distributions of ${ }^{48} \mathrm{Ca}\left[\right.$ panel (a)], ${ }^{90} \mathrm{Zr}$ [panel (b)], ${ }^{132} \mathrm{Sn}\left[\right.$ panel (c)] and ${ }^{208} \mathrm{~Pb}$ [panel (d)] calculated with the SGII and SAMi EDFs by RPA (dash lines) and SSRPA (solid lines). The results obtained by SGII and SAMi are shown by the red and blue lines, respectively. The experimental data of ${ }^{48} \mathrm{Ca}[41],{ }^{90} \mathrm{Zr}$ [42], ${ }^{132} \mathrm{Sn}[43]$, and ${ }^{208} \mathrm{~Pb}$ [44] are shown by the black filled circles. The calculated discrete strength distributions are smoothed by a Lorentzian weighting function of 1 MeV


The quenching factor of Gamow-Teller strength
The cumulative sums are taken up to $\mathrm{E}_{\text {max }}=25 \mathrm{MeV}$ for ${ }^{48} \mathrm{Ca}$ and ${ }^{90} \mathrm{Zr}$, 23 MeV for ${ }^{132} \mathrm{Sn}$, and 25 MeV for ${ }^{208} \mathrm{~Pb}$,

| Force | $(\mathrm{T}, \mathrm{U})$ | ${ }^{48} \mathrm{Ca}$ | ${ }^{90} \mathrm{Zr}$ | ${ }^{132} \mathrm{Sn}$ | ${ }^{208} \mathrm{~Pb}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| SAMi | $(0,0)$ | $14.4 \%$ | $15.2 \%$ | $12.5 \%$ | $10.0 \%$ |
| SAMi-T | $(415.5 .-95.5)$ | $18.6 \%$ | $16.3 \%$ | $14.2 \%$ | $12.7 \%$ |
| SGII | $(0,0)$ | $34.4 \%$ | $29.4 \%$ | $31.4 \%$ | $33.2 \%$ |
| SGII $^{\mathrm{O}}+\mathrm{Te} 1$ | $(500,-350)$ | $39.8 \%$ | $35.0 \%$ | $42.8 \%$ | $43.2 \%$ |
| SGII $^{\mathrm{O}}+\mathrm{Te} 2$ | $(600,0)$ | $37.9 \%$ | $31.0 \%$ | $35.8 \%$ | $31.7 \%$ |
| SGII $^{\mathrm{O}}+\mathrm{Te} 3$ | $(650,200)$ | $34.8 \%$ | $32.6 \%$ | $40.6 \%$ | $35.0 \%$ |
| Exp. |  | $36.7 \%$ | $34.9 \%$ | $44.5 \%$ | $38.6 \%$ |

```
SAMi-T: tensor 1-4% more quenching
SGII+Te1: tensor 5-10 % more quenching
Triplet-odd tensor plays an effective role to Increase the quenching.
```

```
Mingjun Yang, Chunlin Bai and HS
```

Triplet-odd tensor for the spin-orbit splitting of $p$-h excitations of the same particles

> Preliminary

$T e 1=(500,-350)$

$$
U_{s o}=\frac{W_{0}}{r}\left(\frac{d \rho_{q}}{d r}+\frac{d \rho_{1-q}}{d r}\right)+\alpha \frac{J_{q}}{r}+\beta \frac{J_{1-q}}{r}
$$

$$
\alpha_{T}=\frac{5}{12} U
$$

TABLE III: The excitation energy of $1^{+}$state in ${ }^{90} \mathrm{Zr}$. The energy is calculated with SGII $+(\mathrm{T}, \mathrm{U})=(500,-280)$ and SAMi-T, $(\mathrm{T}, \mathrm{U})=(415.5,-91.4) . \Delta \mathrm{E}^{T}$ is the difference of energy between with and without the tensor interaction. The experimental energy is $\sim 9.0 \mathrm{MeV}$. The unit is in MeV .

| in MeV. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| SGII+(T,U) | HF | RPA | $\Delta \mathrm{E}($ RPA-HF $)$ | SSRPA | $\Delta E$ (SSRPA-HF) |
| w/o | 6.21 | 8.23 | 2.02 | 6.81 | 0.60 |
| with | 8.77 | 10.46 | 1.69 | 8.27 | -0.50 |

## Excitation <br> Energy

| $\Delta \mathrm{E}^{T}$ | 2.56 | 2.23 |  | 1.46 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| SAMi-T+(T,U) | HF | RPA | $\Delta \mathrm{E}(\mathrm{RPA}-\mathrm{HF})$ | SSRPA | $\Delta E(\mathrm{SSRPA}-\mathrm{HF})$ |
| w/o | 6.25 | 8.42 | 2.17 | 7.52 | 1.27 |
| with | 7.05 | 9.19 | 2.14 | 8.20 | 1.15 |
| $\Delta \mathrm{E}^{T}$ | 0.80 | 0.77 |  | 1.68 |  |


| SGII+(T,U) | HF | RPA | SSRPA |
| :---: | :---: | :---: | :---: |
| w/o | 15.53 | $15.24(98.1 \%)$ | $12.56(80.9 \%)$ |
| with |  | $14.73(94.8 \%)$ | $10.93(70.4 \%)$ |
| $\Delta B^{T}$ |  | $-0.51(3.2 \%)$ | $-1.63(10.5 \%)$ |
| SAMi-T+(T,U) | HF | RPA | SSRPA |
| w/o | 15.53 | $15.37(99.1 \%)$ | $13.27(85.4 \%)$ |
| with |  | $15.22(98.0 \%)$ | $13.05(84 . .0 \%)$ |
| $\Delta B^{T}$ | $-0.15(1.1 \%)$ | $-0.22(1.4 \%)$ |  |





Iwamoto, Tamii (p,p')

## Beyond mean field study of Giant resonances (Gamow-Teller), beta-decay

 andQCD-based Charge Symmetry Breaking Interaction
Hiroyuki Sagawa RIKEN/University of Aizu

COMEX7
Catania, Italy, June 11-16, 2023

1. Introduction
2. Subtracted second RPA with tensor interactions
3. Gamow-Teller states and quenching problem
4. Magnetic dipole (M1) excitations
5. Beta decay
6. QCD-based CSB and Okamoto-Nolen-Schiffer anomaly
7. Summary


Effects of two particle-two hole configurations and tensor force on $\beta$ decay of magic nuclei

> M. J. Yang, ${ }^{1}$ H. Sagawa, ${ }^{2,3}$ C. L. Bai, ${ }^{1}$ and H. Q. Zhang ${ }^{4}$
> ${ }^{1}$ College of Physics, Sichuan University, Chendu 610065, China
> ${ }^{2}$ Center for Mathematics and Physics, University of Aizu, Aizu-Wakamatsu, Fukushima 965-8560, Japan
> ${ }^{3}$ RIKEN, Nishina Center, Wako 351-0198, Japan
> ${ }^{4}$ China Institute of Atomic Energy, Beijing 102413, China

Phys. Rev. C 107, 014325 (2023) - Published 30 January 2023

$$
\begin{gathered}
B_{1_{n}^{+}}^{G T^{ \pm}}=\left|<1_{n}^{+}\left\|\hat{O}_{G T}^{ \pm}\right\| 0>\right|^{2}, \\
\hat{O}_{G T}^{ \pm}=\sum_{i=1}^{A} \sigma(i) t_{ \pm}(i), \\
T_{1 / 2}=\frac{D}{g_{A}^{2} \sum_{n}^{\Delta_{n H}} B_{1_{n}^{\prime}}^{G T-} f_{0}\left(Z, A, \omega_{n}\right)},
\end{gathered}
$$




FIG. 5: The GT strength distributions with respect to the daughter nucleus in ${ }^{132} \mathrm{Sn},{ }^{68} \mathrm{Ni},{ }^{34} \mathrm{Si}$, and ${ }^{78} \mathrm{Ni}$ calculated by RPA with SGII and SSRPA with SGII and SGII+T. The observed excitation energies of GT states are marked by the arrows [13].



Niu, Colo et al., RPA+PVC


SSRPA

PRL 114, 142501 (2015)

## Beyond mean field study of Giant resonances (Gamow-Teller), beta-decay and <br> QCD-based Charge Symmetry Breaking Interaction

Hiroyuki Sagawa RIKEN/University of Aizu

COMEX7
Catania, Italy, June 11-16, 2023


1. Introduction
2. Subtracted second RPA with tensor interactions
T. Naito, this morning
3. Gamow-Teller states and quenching problem
4. Magnetic dipole (M1) excitations
5. Beta decay
6. QCD-based CSB and Okamoto-Nolen-Schiffer anomaly
7. Summary

```
Isospin Breaking Interactions (ISB)
```

Concept of Isospin proposed by J. Heisenberg, 1932 and E. P. Wigner, 1937

Isospin conservation $\quad[H, T]=0$

$$
[H, T]=\left[V_{C}+V_{C S B}+V_{C I B}, T\right] \neq 0
$$

## Scattering Length

$$
\begin{aligned}
a_{(S=0)}^{p p} & =-17.3 \pm 0.4 \mathrm{fm} \\
a_{(S=0)}^{n n} & =-18.7 \pm 0.6 \mathrm{fm} \\
a_{(S=0)}^{p n} & =-23.70 \pm 0.03 \mathrm{fm}
\end{aligned}
$$

charge symmetry breaking

$$
V_{\mathrm{CSB}}=V_{\mathrm{nn}}-V_{\mathrm{pp}}
$$

Charge independence breaking

$$
V_{\mathrm{CIS}}=\left(\mathrm{V}_{\mathrm{nn}}+\mathrm{V}_{\mathrm{pp}}\right) / 2-\mathrm{V}_{\mathrm{np}}
$$

The difference between $a_{0}^{p p}$ and $a_{0}^{n n}$ is an evidence of CSB (charge symmetry breaking) nuclear force, while the difference between $a_{0}^{p n}$ and the average $\left(a_{0}^{p p}+a_{0}^{n n}\right) / 2$ is due to CIB (charge invariance breaking) force. These negative

```
[Okamoto-Nolen-Schiffer anomaly]
Coulomb energy differences between mirror nuclei and Isobaric analogue
states from A=3~220 are always 3-9 % larger than theoretical
calculations of Independent particle model.
```

This anomaly suggests

1) $10-20 \%$ smaller proton radii of valence particles than those of core
2) The effect of Charge symmetry interaction (CSB)

Standard model on CSB interactions

1) Meson exchange model sigma-rho and pi-eta meson exchange potentials.
2) Phenomenological Skyrme EDF to reproduce ground state properties of mirror nuclei.

QCD -based
approach

Proton $=$ (uud) $\quad m_{u} c^{2}{ }_{\sim} 2.3 \mathrm{MeV}$
Neutron=(udd) $\quad m_{d} C^{2} \sim 4.8 \mathrm{MeV}$
Explicit Chiral symmetry breaking
QCD dynamics of strong interaction
(Spontaneous chiral symmetry breaking)

## QCD-based CSB interaction

1. QCD sum rule approach to evaluate mass difference of proton and neutron in nuclear medium
2. Partial restoration of Spontaneous symmetry breaking (SSB) in nuclear medium

The binding energy (mass) of neutron and proton is formulated in nuclear matter by the QCD sum rule approach in leading order of the quark mass difference and QED effect

$$
\begin{aligned}
\Delta_{n p}(\rho) & \simeq C_{1} G(\rho)-C_{2}, \\
G(\rho) & =\left(\frac{\langle\bar{q} q\rangle}{\langle\bar{q} q\rangle_{0}}\right)^{1 / 3} .
\end{aligned}
$$

IN VACUUM,

$$
\Delta_{n p}(0)=m_{n}-m_{p} \simeq 1.29 \mathrm{MeV} .
$$

Here, $\langle\bar{q} q\rangle$ and $\langle\bar{q} q\rangle_{0}$ are, respectively, the isospin averaged in-medium and in-vacuum chiral condensate. The coefficient $C_{1}$ is proportional to the $u$ - $d$ quark mass difference $\delta m^{1}$, through the isospin-breaking constant $\gamma \equiv\langle\bar{d} d\rangle_{0} /\langle\bar{u} u\rangle_{0}-1$ as $C_{1}=-a \gamma$ with a positive numerical constant $a$ determined by the Borel QSR


The in-medium chiral condensate has a general form in the leading order of Fermi motion corrections;
T. Nishi et al., Nature Physics, March 23, 2023 Pionic atom experiments

## Goda and Jido

$$
\begin{align*}
\frac{\langle\bar{q} q\rangle}{\langle\bar{q} q\rangle_{0}} & \simeq 1+k_{1} \frac{\rho}{\rho_{0}}+k_{2}\left(\frac{\rho}{\rho_{0}}\right)^{5 / 3}  \tag{2a}\\
k_{1} & =-\frac{\sigma_{\pi N} \rho_{0}}{f_{\pi}^{2} m_{\pi}^{2}}<0, \quad k_{2}=-k_{1} \frac{3 k_{\mathrm{F} 0}^{2}}{10 m_{N}^{2}}>0 \tag{2b}
\end{align*}
$$

where $\sigma_{\pi N}$ is the $\pi-N$ sigma term, $m_{\pi}\left(m_{N}\right)$ is the pion (nucleon) mass, and $f_{\pi}$ is the pion decay constant. The

The mass difference between ( $Z+/-1, N$ ) and $(Z, N+/-1)$ with $N=Z$


$$
\begin{align*}
\frac{\langle\bar{q} q\rangle}{\langle\bar{q} q\rangle_{0}} & \simeq 1+k_{1} \frac{\rho}{\rho_{0}}+k_{2}\left(\frac{\rho}{\rho_{0}}\right)^{5 / 3}  \tag{2a}\\
k_{1} & =-\frac{\sigma_{\pi N} \rho_{0}}{f_{\pi}^{2} m_{\pi}^{2}}<0, \quad k_{2}=-k_{1} \frac{3 k_{\mathrm{F} 0}^{2}}{10 m_{N}^{2}}>0 \tag{2b}
\end{align*}
$$

where $\sigma_{\pi N}$ is the $\pi-N$ sigma term, $m_{\pi}\left(m_{N}\right)$ is the pion (nucleon) mass, and $f_{\pi}$ is the pion decay constant. The

$$
\tilde{s}_{0}=-\frac{4}{3} \frac{C_{1} \sigma_{\pi N}}{f_{\pi}^{2} m_{\pi}^{2}}, \quad \tilde{s}_{1}+3 \tilde{s}_{2}=\frac{1}{m_{N}^{2}} \frac{C_{1} \sigma_{\pi N}}{f_{\pi}^{2} m_{\pi}^{2}}
$$

$$
\begin{aligned}
& V_{\mathrm{CSB}}(\boldsymbol{r})= \mid s_{0}\left(1+y_{0} P_{\sigma}\right) \delta(\boldsymbol{r}) \\
&+\frac{s_{1}}{2}\left(1+y_{1} P_{\sigma}\right)\left(\boldsymbol{k}^{\dagger 2} \delta(\boldsymbol{r})+\delta(\boldsymbol{r}) \boldsymbol{k}^{2}\right) \\
&\left.+s_{2}\left(1+y_{2} P_{\sigma}\right) \boldsymbol{k}^{\dagger} \cdot \delta(\boldsymbol{r}) \boldsymbol{k}\right] \frac{\tau_{1 z}+\tau_{2 z}}{4} \\
& \frac{E}{A} \simeq \varepsilon_{0}(\rho)+\varepsilon_{1}(\rho) \beta+\varepsilon_{2}(\rho) \beta^{2}
\end{aligned}
$$

$$
\beta=(N-Z) / A
$$

$$
\begin{gather*}
\Delta E=-2 \varepsilon_{1}(\rho) \\
\delta_{\text {Skyrme }}=-\frac{\tilde{s}_{0}}{4} \rho-\frac{1}{10}\left(\frac{3 \pi^{2}}{2}\right)^{2 / 3}\left(\tilde{s}_{1}+3 \tilde{s}_{2}\right) \rho^{5 / 3} \tag{7}
\end{gather*}
$$

where we have defined the effective coupling strengths,

$$
\begin{equation*}
\tilde{s}_{0} \equiv s_{0}\left(1-y_{0}\right), \quad \tilde{s}_{1} \equiv s_{1}\left(1-y_{1}\right), \quad \tilde{s}_{2} \equiv s_{2}\left(1+y_{2}\right) \tag{8}
\end{equation*}
$$



Figure 47: Lattice results and FLAG averages for the nucleon sigma term, $\sigma_{\pi N}$, for the $N_{f}=2,2+1$, and $2+1+1$ flavour calculations. Determinations via the direct approach are indicated by squares and the Feynman-Hellmann method by triangles. Results from calculations which analyze more than one lattice data set within the Feynman-Hellmann approach [204, 211-219] are shown for comparison (pentagons) along with those from recent analyses of $\pi-N$ scattering [186-188, 220] (circles).

TABLE II. Parameters of the Skyrme-type CSB interactions constrained from the low-energy constants in QCD. To evaluate the CSB effect in finite nuclei where $\tilde{s}_{1}$ and $\tilde{s}_{2}$ contribute independently, two characteristic parameter sets (Case I and Case II) are introduced.

| $\tilde{s}_{0}\left(\mathrm{MeV} \mathrm{fm}^{3}\right)$ | $-15.5_{-12.5}^{+8.8}$ |  |
| :--- | :---: | :---: |
| $\tilde{s}_{1}+3 \tilde{s}_{2}\left(\mathrm{MeV} \mathrm{fm}^{5}\right)$ | $0.52_{-0.29}^{+0.42}$ |  |
| $\tilde{s}_{0}\left(\mathrm{MeV} \mathrm{fm}^{3}\right)$ | Case I | Case II |
| $\tilde{s}_{1}\left(\mathrm{MeV} \mathrm{fm}^{5}\right)$ | $-15.5_{-12.5}^{+8.8}$ | $-15.5_{-12.5}^{+8.8}$ |
| $\tilde{s}_{2}\left(\mathrm{MeV} \mathrm{fm}^{5}\right)$ | $0.52_{-0.29}^{+0.42}$ | 0.00 |

## a conservative estimate.

$$
\sigma_{\pi N}=45 \pm 15 \mathrm{MeV}
$$



FIG. 2. Comparisons of the experimental ONS anomaly $\Delta E_{\text {Expt. }}-\Delta E_{\mathrm{C}}$ (grey hatched bars) and the corresponding theoretical estimates in two EDFs (SGII and SAMi). The contribution from the QCD-based CSB interaction (CSBI) in Case I and the extra contributions are indicated by the red bars with error bars and the blue bars, respectively.

TABLE V. The breakdown of the mass difference of mirror nuclei $\Delta E$ into each contribution (Coulomb, Extra and CSB interaction (CSBI) for Case I with Skyrme EDF, SGII. Numbers are given in the unit of MeV .

| Nuclei | ${ }^{17} \mathrm{~F}-{ }^{17} \mathrm{O}$ | ${ }^{15} \mathrm{O}^{15} \mathrm{~N}$ | ${ }^{41} \mathrm{Sc}^{41} \mathrm{Ca}$ | ${ }^{39} \mathrm{Ca}-{ }^{39} \mathrm{~K}$ |
| :--- | ---: | ---: | ---: | ---: |
| Orbital | $d_{5 / 2}$ | $\left(1 p_{1 / 2}\right){ }^{-1}$ | $1 f_{7 / 2}$ | $\left(1 d_{3 / 2}\right)^{-1}$ |
| $\Delta E_{\mathrm{D}}$ (Coulomb) | 3.596 | 3.272 | 7.133 | 6.717 |
| $\Delta E_{\mathrm{E}}$ (Coulomb) | -0.203 | 0.026 | -0.267 | 0.260 |
| Extra | 0.040 | 0.028 | 0.102 | 0.011 |
| CSBI (Case I) | 0.224 | 0.264 | 0.287 | 0.315 |
| Sum (w/o CSBI) | 3.432 | 3.326 | 6.965 | 6.985 |
| Sum (w/ CSBI) | 3.656 | 3.590 | 7.252 | 7.300 |
| Expt. [29] | 3.543 | 3.537 | 7.278 | 7.307 |

TABLE VI. The same as Table V, but with Skyrme EDF, SAMi.

| Nuclei | ${ }^{17} \mathrm{~F}-{ }^{17} \mathrm{O}$ | ${ }^{15} \mathrm{O}^{-}{ }^{15} \mathrm{~N}$ | ${ }^{41} \mathrm{Sc}^{41} \mathrm{Ca}$ | ${ }^{39} \mathrm{Ca}^{-{ }^{39} \mathrm{~K}}$ |
| :--- | ---: | ---: | ---: | ---: |
| Orbital | $d_{5 / 2}$ | $\left(1 p_{1 / 2}\right){ }^{-1}$ | $1 f_{7 / 2}$ | $\left(1 d_{3 / 2}\right)^{-1}$ |
| $\Delta E_{\mathrm{D}}$ (Coulomb) | 3.506 | 3.242 | 7.025 | 6.697 |
| $\Delta E_{\mathrm{E}}$ (Coulomb) | -0.193 | 0.022 | -0.259 | 0.281 |
| Extra | 0.043 | 0.075 | 0.104 | 0.092 |
| CSBI (Case I) | 0.206 | 0.269 | 0.271 | 0.321 |
| Sum (w/o CSBI) | 3.356 | 3.339 | 6.870 | 7.070 |
| Sum (w/ CSBI) | 3.562 | 3.608 | 7.141 | 7.391 |
| Expt. [29] | 3.543 | 3.537 | 7.278 | 7.307 |

## Summary

Gamow-Teller states of ${ }^{48} \mathrm{Ca},{ }^{90} \mathrm{Zr},{ }^{132} \mathrm{Sn}$ and ${ }^{208} \mathrm{~Pb}$ are studied by SSRPA and $2 p-2$ states make a larger spreading width with the proper excitation energies compared with experimental ones.

```
Quenching (SGII+Te1): }\mp@subsup{}{}{48}\textrm{Ca}~35%\textrm{Ex}<20\textrm{MeV
    208Pb 30-40% Ex<25 MeV
Triplet-odd tensor plays an important role to increase the quenching
```

Excitation energy of magnetic dipole state is very sensitive to the triplet-odd tensor in RPA level, but the tensor effect is smacked in the SSRPA.

Beta decay life time has a strong effect by the $2 \mathrm{p}-2 \mathrm{~h}$ correlations and also the tensor correlations.

Ab initio QCD-base CSB interaction is proposed for the first time and cures largely
Okamoto-Nolen-Shiffer of the energydifference between mirror nuclei.

## Collaborators

T. Naito, iTHEMS, RIKEN
T. Hatsuda, iTHEMS, RIKEN

Chunlin Bai, Sichun University, China
Mingjun Yang, Sichun Univerisity, China
H. Q. Zhang, CAEC, China

Xavi Roca-Maza, University of Milano, Italy

TABLE IV. Contributions from the Skyrme CSB interactions to $\delta_{\text {ONS }}$ in Case I and Case II with theoretical uncertainties. The values are given in unit of keV . The core density and the wave function of valence orbit are calculated by HF model with Skyrme EDFs, SGII and SAMi. All the values are obtained self-consistently.

|  | Nuclei | ${ }^{17} \mathrm{~F}_{-}^{17} \mathrm{O}$ | ${ }^{15} \mathrm{O}_{-}{ }^{15} \mathrm{~N}$ | ${ }^{41} \mathrm{Sc}^{41} \mathrm{Ca}$ | ${ }^{39} \mathrm{Ca}^{39} \mathrm{~K}$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | Orbital | $1 d_{5 / 2}$ | $\left(1 p_{1 / 2}\right)^{-1}$ | $1 f_{7 / 2}$ | $\left(1 d_{3 / 2}\right)^{-1}$ |
| SGII | $\tilde{s}_{0}$ | $229_{-125}^{+192}$ | $269_{-148}^{+221}$ | $292_{-160}^{+245}$ | $322_{-176}^{+264}$ |
|  | $\tilde{s}_{1}\left(\tilde{s}_{2}=0\right)$ | $-5.0_{-4.0}^{+2.8}$ | $-5.6_{-4.5}^{+3.1}$ | $-6.6_{-5.3}^{+3.7}$ | $-6.0_{-4.9}^{+3.4}$ |
|  | $\tilde{s}_{2}\left(\tilde{s}_{1}=0\right)$ | $-6.4_{-5.2}^{+3.5}$ | $-3.3_{-2.7}^{+1.8}$ | $-5.3_{-4.3}^{+2.9}$ | $-5.0_{-4.1}^{+2.8}$ |
|  | Case I | $224_{-125}^{+192}$ | $264_{-148}^{+221}$ | $287_{-160}^{+245}$ | $315_{-176}^{+264}$ |
|  | Case II | $225_{-125}^{+192}$ | $266_{-148}^{+221}$ | $289_{-160}^{+245}$ | $316_{-176}^{+264}$ |
|  | $\tilde{s}_{0}$ | $211_{-115}^{+174}$ | $274_{-152}^{+225}$ | $278_{-151 .}^{+230}$ | $324_{-180}^{+269}$ |
|  | $\tilde{s}_{1}\left(\tilde{s}_{2}=0\right)$ | $-5.2_{-4.2}^{+2.9}$ | $-5.4_{-4.4}^{+3.0}$ | $-7.3_{-5.9}^{+4.0}$ | $-8.4_{-6.6}^{+4.6}$ |
|  | $\tilde{s}_{2}\left(\tilde{s}_{1}=0\right)$ | $-4.1_{-3.3}^{+2.3}$ | $-3.2_{-2.6}^{+1.8}$ | $-5.7_{-4.6}^{+3.1}$ | $-5.2_{-4.2}^{+2.9}$ |
|  | Case I | $206_{-115}^{+174}$ | $269_{-152}^{+225}$ | $271_{-151}^{+230}$ | $321_{-180}^{+269}$ |
|  | Case II | $207_{-115}^{+174}$ | $271_{-152}^{+225}$ | $272_{-151}^{+230}$ | $322_{-180}^{+269}$ |


| Nuclei | ${ }^{17} \mathrm{~F}-{ }^{17} \mathrm{O}$ | ${ }^{15} \mathrm{O}-{ }^{15} \mathrm{~N}$ | ${ }^{41} \mathrm{Sc}^{41} \mathrm{Ca}$ | ${ }^{39} \mathrm{Ca}-{ }^{39} \mathrm{~K}$ |
| :--- | :---: | ---: | ---: | ---: |
| Particle (hole) | $1 d_{5 / 2}$ | $\left(1 p_{1 / 2}\right)^{-1}$ | $1 f_{7 / 2}$ | $\left(1 d_{3 / 2}\right)^{-1}$ |
| Finite size | -0.053 | -0.070 | -0.066 | -0.082 |
| Center-of-mass | 0.023 | 0.030 | 0.014 | 0.018 |
| $\delta_{\text {NN }}^{1}$ | 0.014 | 0.006 | 0.034 | 0.021 |
| $\delta_{\text {NN }}^{2}$ | 0.050 | -0.136 | 0.134 | -0.176 |
| Spin-orbit | -0.065 | 0.080 | -0.126 | 0.142 |
| $p n$ mass difference | 0.034 | 0.024 | 0.040 | 0.031 |
| $\delta_{\text {pol }}$ | 0.018 | 0.073 | 0.036 | 0.020 |
| Vacuum polarization | 0.019 | 0.021 | 0.036 | 0.037 |
| Sum | 0.040 | 0.028 | 0.102 | 0.011 |

TABLE S.II. The same as Table S.I, but for SAMi EDF.

| Nuclei | ${ }^{17} \mathrm{~F}-{ }^{17} \mathrm{O}$ | ${ }^{15} \mathrm{O}-{ }^{15} \mathrm{~N}$ | ${ }^{41} \mathrm{Sc}^{41} \mathrm{Ca}$ | ${ }^{39} \mathrm{Ca}-{ }^{39} \mathrm{~K}$ |
| :--- | :---: | ---: | ---: | ---: |
| Particle (hole) | $1 d_{5 / 2}$ | $\left(1 p_{1 / 2}\right)^{-1}$ | $1 f_{7 / 2}$ | $\left(1 d_{3 / 2}\right)^{-1}$ |
| Finite size | -0.050 | -0.068 | -0.063 | -0.080 |
| Center-of-mass | 0.021 | 0.029 | 0.014 | 0.018 |
| $\delta_{\text {NN }}^{1}$ | 0.014 | 0.007 | 0.031 | 0.021 |
| $\delta_{\text {NN }}^{2}$ | 0.047 | -0.090 | 0.131 | -0.098 |
| Spin-orbit | -0.061 | 0.078 | -0.121 | 0.140 |
| $p n$ mass difference | 0.035 | 0.026 | 0.041 | 0.034 |
| $\delta_{\text {pol }}$ | 0.018 | 0.073 | 0.036 | 0.020 |
| Vacuum polarization | 0.019 | 0.020 | 0.035 | 0.037 |
| Sum | 0.043 | 0.075 | 0.104 | 0.092 |

