Quartet structure of $N = Z$ nuclei

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Outline

- An exact treatment of the isovector pairing Hamiltonian:
  the role of quartets
- The phenomenon of quartet condensation in the ground and excited states of a proton-neutron pairing Hamiltonian
- Spectra of $N = Z$ nuclei in a formalism of quartets

in collaboration with N. Sandulescu (NIPNE, Bucharest)

The isovector pairing Hamiltonian

$$H^{(iv)} = \sum_{i=1}^{\Omega} \epsilon_i \mathcal{N}_i - g \sum_{i,i'=1}^{\Omega} \sum_{M_T=-1}^{1} P^\dagger_{iM_T} P_{i'M_T}$$

$$\mathcal{N}_i = \sum_{\sigma,\tau} a^\dagger_{i\sigma\tau} a_{i\sigma\tau}, \qquad P^\dagger_{iM_T} = [a^\dagger_{i+} a^\dagger_{i-}]^{T=1}_{M_T}, \qquad (P^\dagger_{iM_T})^\dagger = P_{iM_T}$$

$$\left(P^\dagger_{iM_T} : \quad M_T = -1 \ \ (pp), \quad M_T = 0 \ \ (pn), \quad M_T = +1 \ \ (nn) \right)$$

We confine our analysis to $T = 0$ seniority-zero eigenstates.

The Hilbert space of the model is spanned by the states

$$P^\dagger_{i_1 M_{T_1}} P^\dagger_{i_2 M_{T_2}} \cdots P^\dagger_{i_N M_{T_N}} |0\rangle$$

subject to the condition

$$M_{T_1} + M_{T_2} + \cdots + M_{T_N} = 0$$

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First case: 2 protons and 2 neutrons

Building blocks:

$$B^\dagger_{\nu\tau} = \sum_{k=1}^{\Omega} \frac{1}{2\epsilon_k - E_\nu} P^\dagger_{k\tau} \qquad (E_\nu \equiv \text{“}pair\ energy\text{”})$$

Ansatz for the eigenstates:

$$|\Psi\rangle = [B^\dagger_1 B^\dagger_2]^{T=0}|0\rangle \qquad (\Rightarrow quartet)$$

One finds:

$$\begin{aligned} H^{(iv)}|\Psi\rangle &= (E_1 + E_2)|\Psi\rangle \\ &+\left(1 - g\sum_k \frac{1}{2\epsilon_k - E_1} - \frac{g}{E_2 - E_1}\right)[P^\dagger B^\dagger_2]^{T=0}|0\rangle \\ &+\left(1 - g\sum_k \frac{1}{2\epsilon_k - E_2} - \frac{g}{E_1 - E_2}\right)[P^\dagger B^\dagger_1]^{T=0}|0\rangle \end{aligned}$$

being $P^\dagger_\tau = \sum_k P^\dagger_{k,\tau}$.

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being $P^{\dagger}_{\tau} = \sum_k P^{\dagger}_{k,\tau}$.

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being $P^\dagger_\tau = \sum_k P^\dagger_{k,\tau}$.

Second case: 4 protons and 4 neutrons

Ansatz:

$$|\Psi\rangle = d_1[B_1^\dagger B_2^\dagger]^0[B_3^\dagger B_4^\dagger]^0|0\rangle + d_2[B_1^\dagger B_3^\dagger]^0[B_2^\dagger B_4^\dagger]^0|0\rangle + d_3[B_1^\dagger B_4^\dagger]^0[B_2^\dagger B_3^\dagger]^0|0\rangle$$

One finds:

$$\begin{aligned}
H^{(iv)}|\Psi\rangle &= (E_1+E_2+E_3+E_4)|\Psi\rangle \\
&+\left(d_1 - \sum_k \frac{g\cdot d_1}{2\epsilon_k - E_1} - \frac{g\cdot d_{123}}{E_2-E_1} - \frac{g\cdot d_{12}}{E_1-E_4} - \frac{g\cdot d_{13}}{E_1-E_3}\right)[P^\dagger B_2^\dagger]^0[B_3^\dagger B_4^\dagger]^0|0\rangle \\
&+\left(d_1 - \sum_k \frac{g\cdot d_1}{2\epsilon_k - E_2} - \frac{g\cdot d_{123}}{E_1-E_2} - \frac{g\cdot d_{12}}{E_2-E_3} - \frac{g\cdot d_{13}}{E_2-E_4}\right)[P^\dagger B_1^\dagger]^0[B_3^\dagger B_4^\dagger]^0|0\rangle \\
&+\left(d_1 - \sum_k \frac{g\cdot d_1}{2\epsilon_k - E_3} - \frac{g\cdot d_{123}}{E_4-E_3} - \frac{g\cdot d_{12}}{E_3-E_2} - \frac{g\cdot d_{13}}{E_3-E_1}\right)[P^\dagger B_4^\dagger]^0[B_1^\dagger B_2^\dagger]^0|0\rangle \\
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&+\cdots
\end{aligned}$$

being: $d_{ij} \equiv d_i + d_j, \quad d_{123} \equiv d_1 + d_2 + d_3$

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One finds:

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&+\cdots
\end{aligned}$$

being: $d_{ij} \equiv d_i + d_j, \quad d_{123} \equiv d_1 + d_2 + d_3$

The general recipe for an even-even $N = Z$ system

- Adopt the collective pairs

  $$B^\dagger_{\nu\tau} = \sum_{k=1}^{\Omega} \frac{1}{2\epsilon_k - E_\nu} P^\dagger_{k\tau}$$

  as building blocks.
- Construct the states $|s\rangle$, product of $B^\dagger_\nu$'s arranged into $T = 0$ quartets,

  $$|s\rangle = \prod_{q=1}^{N_q} [B^\dagger_{\nu(1,q,s)} B^\dagger_{\nu(2,q,s)}]^{T=0} |0\rangle,$$

  such that the space $\{|s\rangle\}$ be invariant under the interchange of any two pairs.
- Expand $|\Psi\rangle$ into this basis: $|\Psi\rangle = \sum_{s=1}^{N_s} d_s |s\rangle$
- Solve the set of equations

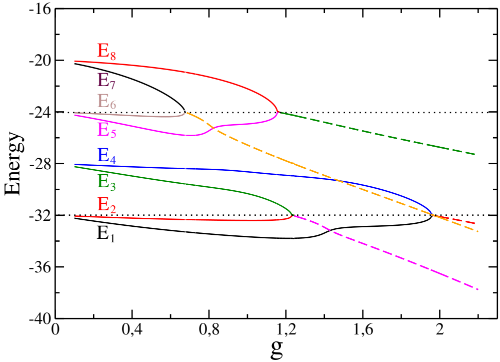
  $$\frac{d_s}{g} - \sum_{k=1}^{\Omega} \frac{d_s}{2\epsilon_k - E_\nu} - \sum_{\nu' \neq \nu}^{(1,2N_q)} \frac{S_{\nu'\nu}(s)}{E_{\nu'} - E_\nu} = 0, \qquad S_{\nu'\nu}(s) = \sum_t I(t, \nu', \nu, s) d_t$$

  This guarantees that

  $$H^{(iv)} |\Psi\rangle = (\sum_\nu E_\nu) |\Psi\rangle$$

An example of numerical results

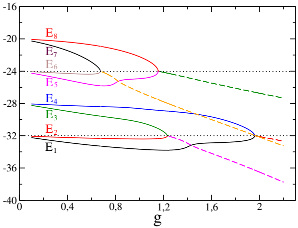
Pair energies for the ground state of a system of
8 protons and 8 neutrons over 8 equispaced levels



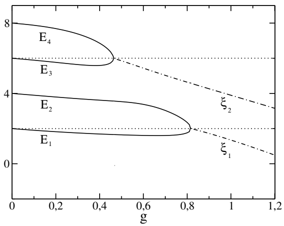
M.S. and N. Sandulescu, J. Phys. G: Nucl. Phys. 47 (2020) 045112

Comparison: isovector vs like-particle pairing

Pair energies for the ground state of a system of 8 protons and 8 neutrons over 8 equispaced levels



Pair energies for the ground state of a system of 8 like particles over 8 equispaced levels



R.W. Richardson,
Phys. Lett. **3**, 277 (1963)
R.W. Richardson and N. Sherman,
Nucl. Phys. **52**, 221 (1964)

Conclusions (1st part)

- The exact $T = 0$ seniority-zero eigenstates of a constant-strength isovector pairing Hamiltonian are linear superpositions of products of quartets.
- Quartets are the distictive features of these eigenstates.
- The isovector pairing Hamiltonian favours the formation of α-like structures in $N = Z$ nuclei.

Previous exact treatments of the isovector pairing Hamiltonian

- R.W. Richardson, Phys. Rev. **144**, 874 (1966)
  H.-T. Chen and R.W. Richardson, Phys. Lett. B **34**, 271 (1971)
  H.-T. Chen and R.W. Richardson, Nucl. Phys. A **212**, 317 (1973)
- Feng Pan and J.P. Draayer, Phys. Rev. C **66**, 044314 (2002)
- J. Links, H.-Q. Zhou, M.D. Gould, and R.H. McKenzie, J. Phys. A **35**, 6459 (2002)
- J. Dukelsky, V.G. Gueorguiev, P. Van Isacker, S. Dimitrova, B. Errea, and S. Lerma H., Phys. Rev. Lett. **96**, 072503 (2006)

Quartet condensation in $N = Z$ nuclei

The Quartet Condensation Model (QCM) assumes that

$$|\Psi_{gs}\rangle = (Q^+)^{n_q}|0\rangle$$

For the isovector pairing Hamiltonian discussed above

$$Q^+ = \sum_{ij} x_{ij}[P_i^\dagger P_j^\dagger]^{T=0}$$

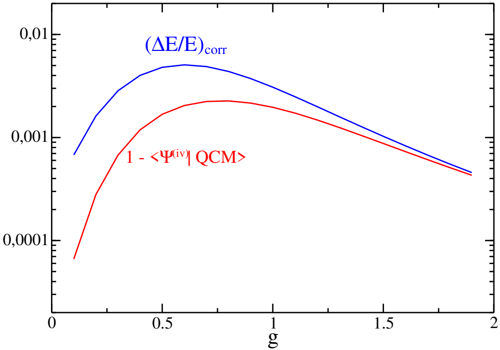
with

$$P^\dagger_{iM_T} = [a^\dagger_{i+} a^\dagger_{i-}]^{T=1}_{M_T}$$

N. Sandulescu et al., PRC 85 (2012) 061303(R)

Validity of the QCM approximation for the IV pairing Hamiltonian

System of 6 protons and 6 neutrons over 6 equispaced levels



M.S. and N. Sandulescu, J. Phys. G: Nucl. Phys. 47 (2020) 115101

QCM based excited states of the IV pairing Hamiltonian: formalism

Excited states of the isovector pairing Hamiltonian are built on the QCM ground state as follows:

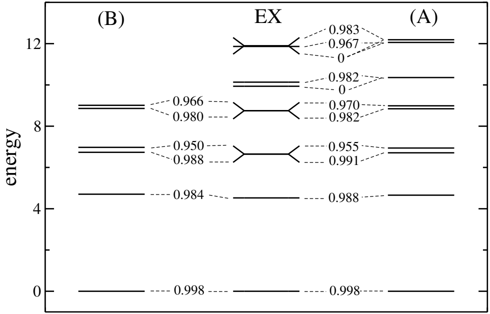
$$|\Phi_\nu\rangle = \tilde{Q}_\nu^+ (Q^+)^{n_q - 1} |0\rangle$$

with

$$\tilde{Q}_\nu^+ = \sum_{ij} y_{ij}^{(\nu)} [P_i^+ P_j^+]^{T=0}$$

QCM based excited states of the IV pairing Hamiltonian: application

Spectrum of the isovector pairing Hamiltonian for a system
of 6 protons and 6 neutrons over 6 equispaced levels
$(g = 1)$



M.S. and N. Sandulescu, Phys. Lett. B 820 (2021) 136476

The case of a general isovector-isoscalar pairing Hamiltonian

$$H = \sum_i \epsilon_i N_i + \sum_{i,j} V_{J=0}^{T=1}(i,j) \sum_{T_z} P^+_{i,T_z} P_{j,T_z} + \sum_{i\leq j, k\leq l} V_{J=1}^{T=0}(ij,kl) \sum_{J_z} D^+_{ij,J_z} D_{kl,J_z}$$

$$P^+_{i,T_z} = \sqrt{\frac{2j_i+1}{2}} [a_i^+ a_i^+]^{T=1,J=0}_{T_z}, \qquad D^+_{j_1 j_2 J_z} = \frac{1}{\sqrt{1+\delta_{j_1 j_2}}} [a^+_{j_1} a^+_{j_2}]^{J=1,T=0}_{J_z}$$

- The QCM ground state:

$$|\Psi_{gs}\rangle = (Q^+_{ivs})^{n_q}|0\rangle, \qquad Q^+_{ivs} = Q^+_{iv} + Q^+_{is}$$

$$Q^+_{iv} = \sum_{ij} x_{ij} [P_i^+ P_j^+]^{T=0}, \qquad Q^+_{is} = \sum_{j_1 j_2 j_3 j_4} y_{j_1 j_2 j_3 j_4} [D^+_{j_1 j_2} D^+_{j_3 j_4}]^{J=0}$$

- The excited states built on the QCM ground state

$$|\Phi_{\nu,JJ_z}\rangle = \tilde{Q}_{\nu,JJ_z} (Q^+_{ivs})^{n_q-1}|0\rangle$$

$$\tilde{Q}^+_{\nu,JJ_z} = \sum_{T'} \sum_{J_1(i_1 j_1)} \sum_{J_2(i_2 j_2)} Y^{(\nu)}_{JJ_z}(T', J_1(i_1 j_1), J_2(i_2 j_2)) [P^+_{J_1,T'}(i_1,j_1) P^+_{J_2,T'}(i_2,j_2)]^{J,T=0}_{J_z}$$

The case of a general isovector-isoscalar pairing Hamiltonian

$$H = \sum_i \epsilon_i N_i + \sum_{i,j} V^{T=1}_{J=0}(i,j) \sum_{T_z} P^+_{i,T_z} P_{j,T_z} + \sum_{i \le j, k \le l} V^{T=0}_{J=1}(ij,kl) \sum_{J_z} D^+_{ij,J_z} D_{kl,J_z}$$

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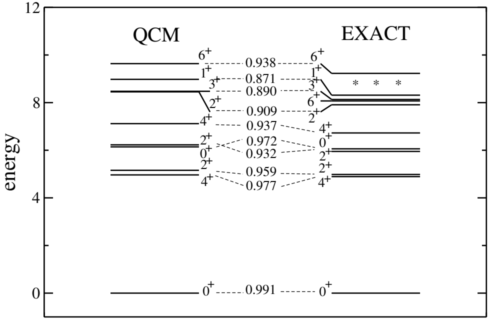
QCM based approximation for a general IV + IS pairing Hamiltonian

Spectrum of the isovector + isoscalar pairing Hamiltonian
for a system of 6 protons and 6 neutrons with
s.p.e.'s and $V_{J=0}^{T=1}(i,j)$, $V_{J=1}^{T=0}(i,j)$ from USDB



M. S. and N. Sandulescu, PLB 820 (2021) 136476

Conclusions (2nd part)

- The ground state of an isovector plus isoscalar pairing Hamiltonian in $N = Z$ systems can be well described as a condensate of a $T = 0, J = 0$ quartet built by isovector plus isoscalar pairs.
- The low-lying excited states of this Hamiltonian can be constructed by promoting one of the quartets of the condensate to an excited ($T = 0$) configuration.
- The proton-neutron pairing favours the formation of α-like quartet condensates in $N = Z$ systems.

Spectra of $N = Z$ nuclei in a formalism of quartets

Three basic problems.

1) How to define the quartets

$$q^+_{JM} = \sum_{i_1 j_1 J_1} \sum_{i_2 j_2 J_2} \sum_{T'} q_{i_1 j_1 J_1, i_2 j_2 J_2, T'} [[a^+_{i_1} a^+_{j_1}]^{J_1 T'} [a^+_{i_2} a^+_{j_2}]^{J_2 T'}]^{JT=0}_M$$

2) How to fix them

- "statically": from the nearest $T = 0$ one-quartet system
  (M.S. and N.Sandulescu, PRL 115 (2015) 112501, PRC 91 (2015) 064318)
- "dynamically": from intrinsic states

3) What to do with them

- Configuration interaction calculations in the space of quartets
- Projecting the intrinsic states

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- “dynamically”: from intrinsic states

3) What to do with them

- Configuration interaction calculations in the space of quartets
- Projecting the intrinsic states

Spectra of $N = Z$ nuclei in a formalism of quartets

Three basic problems.

1) How to define the quartets

$$q^+_{JM} = \sum_{i_1 j_1 J_1} \sum_{i_2 j_2 J_2} \sum_{T'} q_{i_1 j_1 J_1, i_2 j_2 J_2, T'} [[a^+_{i_1} a^+_{j_1}]^{J_1 T'} [a^+_{i_2} a^+_{j_2}]^{J_2 T'}]^{JT=0}_M$$

2) How to fix them

- "statically": from the nearest $T = 0$ one-quartet system
  (M.S. and N.Sandulescu, PRL 115 (2015) 112501, PRC 91 (2015) 064318)
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3) What to do with them

- Configuration interaction calculations in the space of quartets
- Projecting the intrinsic states

Intrinsic states of deformed $N = Z$ nuclei in a formalism of quartets

- “ground” intrinsic state

$$|\Theta_g\rangle = \mathcal{N}_g (Q_g^+)^n |0\rangle, \qquad Q_g^+ = \sum_J \alpha_{g,J} (q_g^+)_{J0}$$

$$(q_g^+)_{J0} = \sum_{i_1 j_1 J_1} \sum_{i_2 j_2 J_2} \sum_{T'} q^{(g)}_{i_1 j_1 J_1, i_2 j_2 J_2, T'} [[a^+_{i_1} a^+_{j_1}]^{J_1 T'} [a^+_{i_2} a^+_{j_2}]^{J_2 T'}]^{JT=0}_0$$

- “excited” intrinsic states

$$|\Theta_k\rangle = \mathcal{N}_k Q_k^\dagger (Q_g^\dagger)^{(n-1)} |0\rangle, \qquad Q_k^\dagger = \sum_J \alpha_{k,J} (q_k^\dagger)_{Jk}$$

$$(q_k^+)_{Jk} = \sum_{i_1 j_1 J_1} \sum_{i_2 j_2 J_2} \sum_{T'} q^{(k)}_{i_1 j_1 J_1, i_2 j_2 J_2, T'} [[a^+_{i_1} a^+_{j_1}]^{J_1 T'} [a^+_{i_2} a^+_{j_2}]^{J_2 T'}]^{JT=0}_k$$

$$k = 0 \to \text{“}\beta\text{”},\ k = 2 \to \text{“}\gamma\text{”}, \cdots$$

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- "ground" intrinsic state

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$$k = 0 \rightarrow \text{"}\beta\text{"}, \; k = 2 \rightarrow \text{"}\gamma'', \cdots$$

Generating spectra of $N = Z$ nuclei

Two ways of proceedings:

- Via configuration interaction calculations in the space of quartets q^+_{JM}

$$|\Psi^{(n)}_{\overline{M}}, \{N_{JM}\}\rangle = \prod_{J\in(0,J_{max});M\in(-J,J)} (q^+_{JM})^{N_{JM}}|0\rangle$$

$$\sum_{JM} N_{JM} = n, \qquad \sum_{JM} M N_{JM} = \overline{M}$$

M. S. and N. Sandulescu, PLB 827 (2022) 136987

- Via projection
  - 1) projecting states of good angular momentum from the intrinsic states:

$$\hat{P}_J|\Theta_g\rangle$$

$$\hat{P}_J|\Theta_k\rangle, \qquad k = 0, 2, \cdots$$

  - 2) diagonalizing the Hamiltonian in the space of the projected states:

$$\{\hat{P}_J|\Theta_g\rangle, \hat{P}_J|\Theta_k\rangle\}$$

M. S. and N. Sandulescu, EPJA (2023), in press

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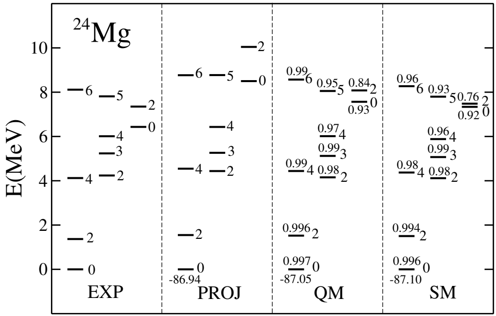
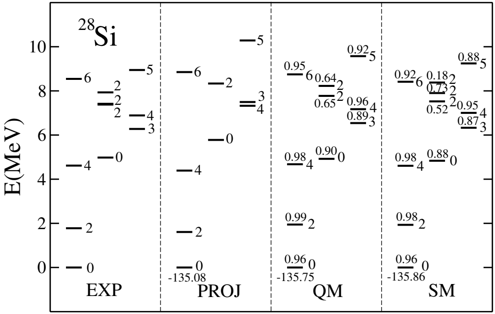
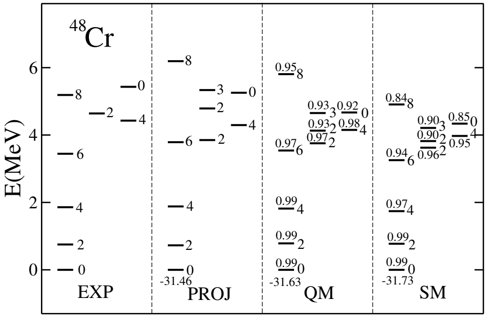
QM vs projection from intrinsic states: ^{24}Mg



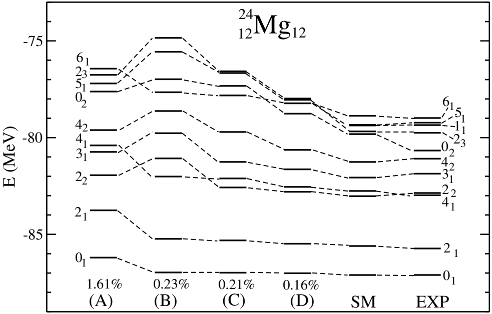
QM vs projection from intrinsic states: ^{28}Si



QM vs projection from intrinsic states: ^{48}Cr



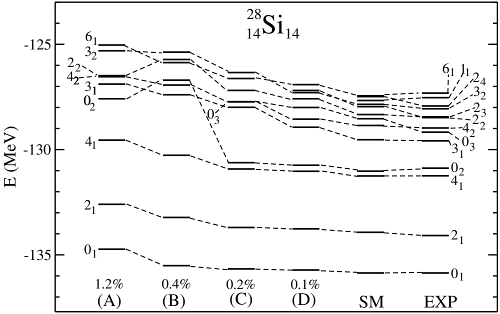
QM results: ^{24}Mg



(A)→ *static* : $J = 0, 2, 4$ (B)→ $|\Theta_g\rangle$: $J = 0, 2, 4$

(C)→ $|\Theta_2\rangle$: $J = 2, 3, 4$ (D)→ $|\Theta_0\rangle$: $J = 0, 2, 4$

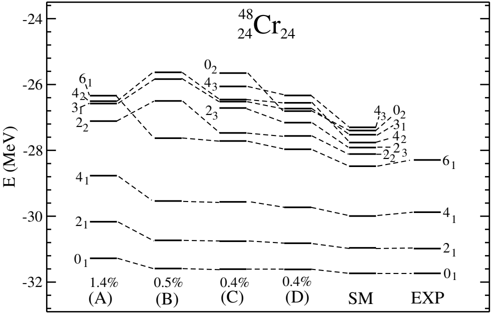
QM results: ^{28}Si



$(A) \rightarrow \textit{static} : J = 0, 2, 4$ $(B) \rightarrow |\Theta_g\rangle : J = 0, 2, 4$

$(C) \rightarrow |\Theta_0\rangle : J = 0, 2, 4$ $(D) \rightarrow |\Theta_3\rangle : J = 3, 4$

QM results: ^{48}Cr



$(A) \rightarrow \textit{static} : J = 0, 2, 4, 6 \qquad (B) \rightarrow |\Theta_g\rangle : J = 0, 2, 4, 6$

$(C) \rightarrow |\Theta_2\rangle : J = 2, 3, 4 \qquad (D) \rightarrow |\Theta_0\rangle : J = 0, 2, 4$

Conclusions (3rd part)

- Quartet-based intrinsic states provide a framework to understand in a conceptually simple and intuitive manner the emergence of band-like structures in $N = Z$ nuclei.

QCM results in the sd shell

Using the USDB Hamiltonian:

	E_{corr}(SM)	E_{corr}(QCM)	$\langle SM \vert QCM \rangle$
^{20}Ne	24.77	24.77 (-)	1
^{24}Mg	55.70	53.04 (4.77%)	0.85
^{28}Si	88.75	86.52 (2.52%)	0.86
^{32}S	122.51	122.02 (0.40%)	0.98

M.S. and N. Sandulescu, EPJA 53, 47 (2017)