Giant Pairing Vibration in the Continuum Beyond RPA

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AGENCIA ESTATAL DE INVESTIGACIÓN

PID2020-114687GB-100

The Giant Pairing Vibration seminal paper

Volume 69B, number 2 PHYSICS LETTERS 1 August 1977

HIGH-LYING PAIRING RESONANCES★

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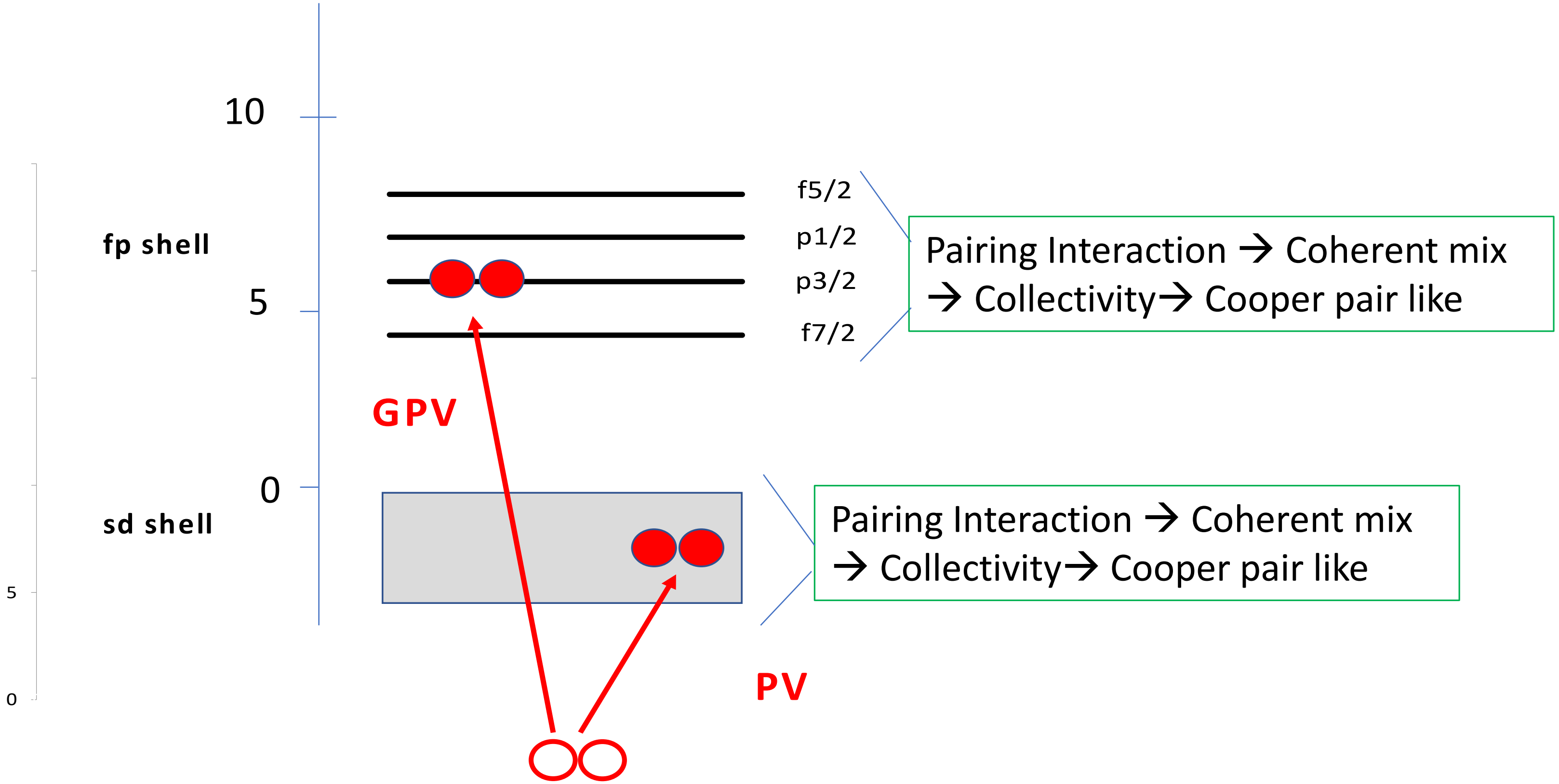
Received 1 April 1977

Pairing vibrations based on the excitation of pairs of particles and holes across major shells are predicted at an excitation energy of about $70/A^{1/3}$ MeV and carrying a cross section which is 20%–100% the ground state cross section.

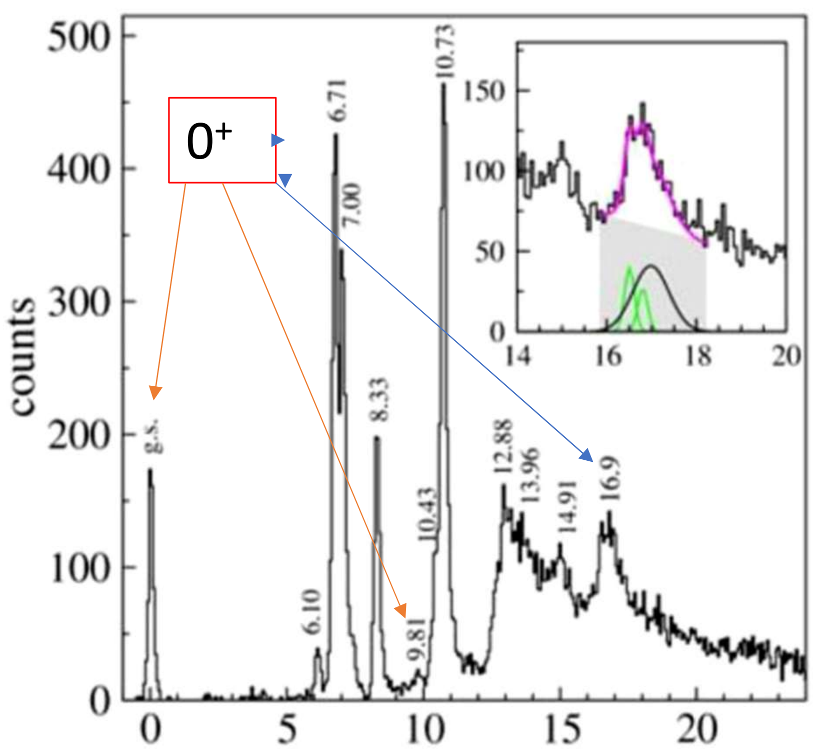
The Pairing Vibrations; schematics

A → A+2; for example ^{16}O → ^{18}O

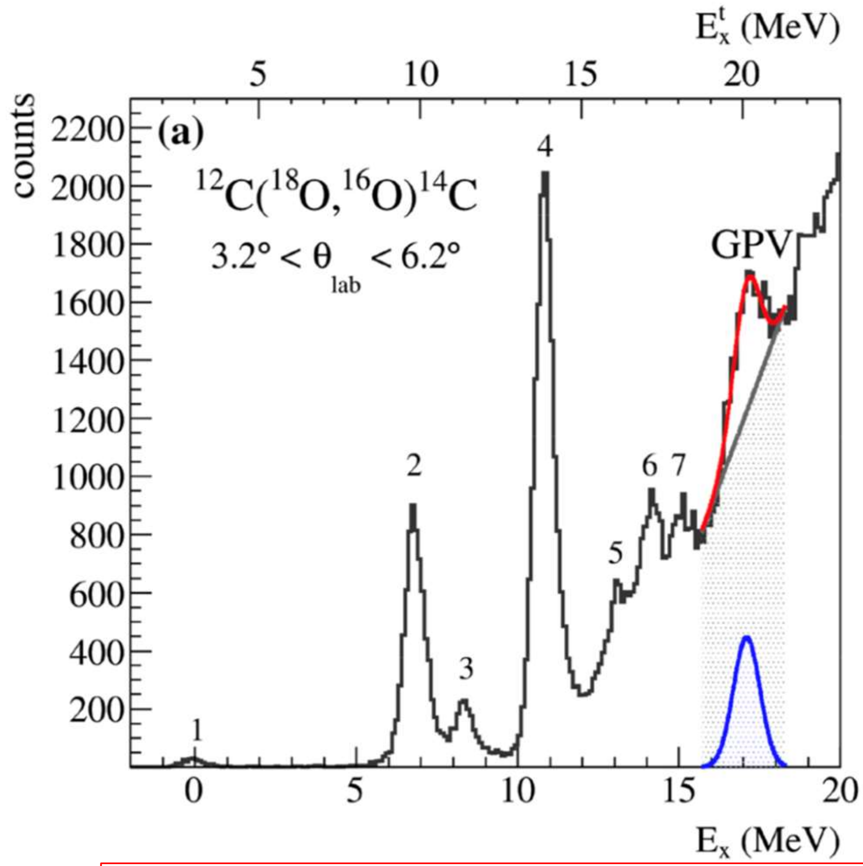


Pair Addition mode produced in two neutron transfer reactions:
A(t,p)A+2 for example

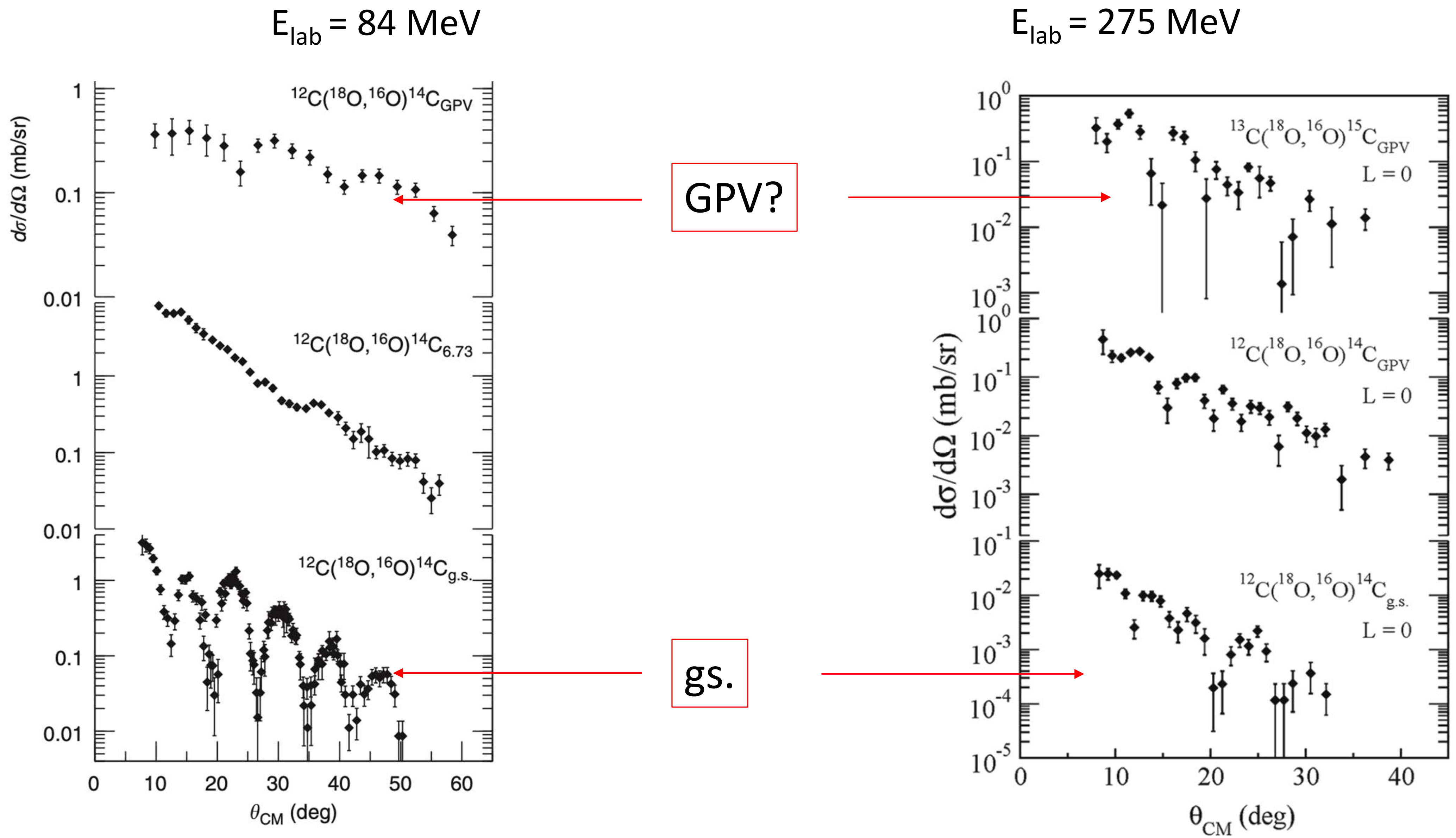
Several unsuccessful experimental searches have been carried out over the years , but recently a bump has been detected at E* ≈ 16 MeV in the reaction $^{12}C(^{18}O,^{16}O)^{14}C$ at E_{lab} = 84 and 275 MeV and intepreted as a signature of GPV



F. Cappuzzello et al., Nat. Comm. 6 (2015) 6743

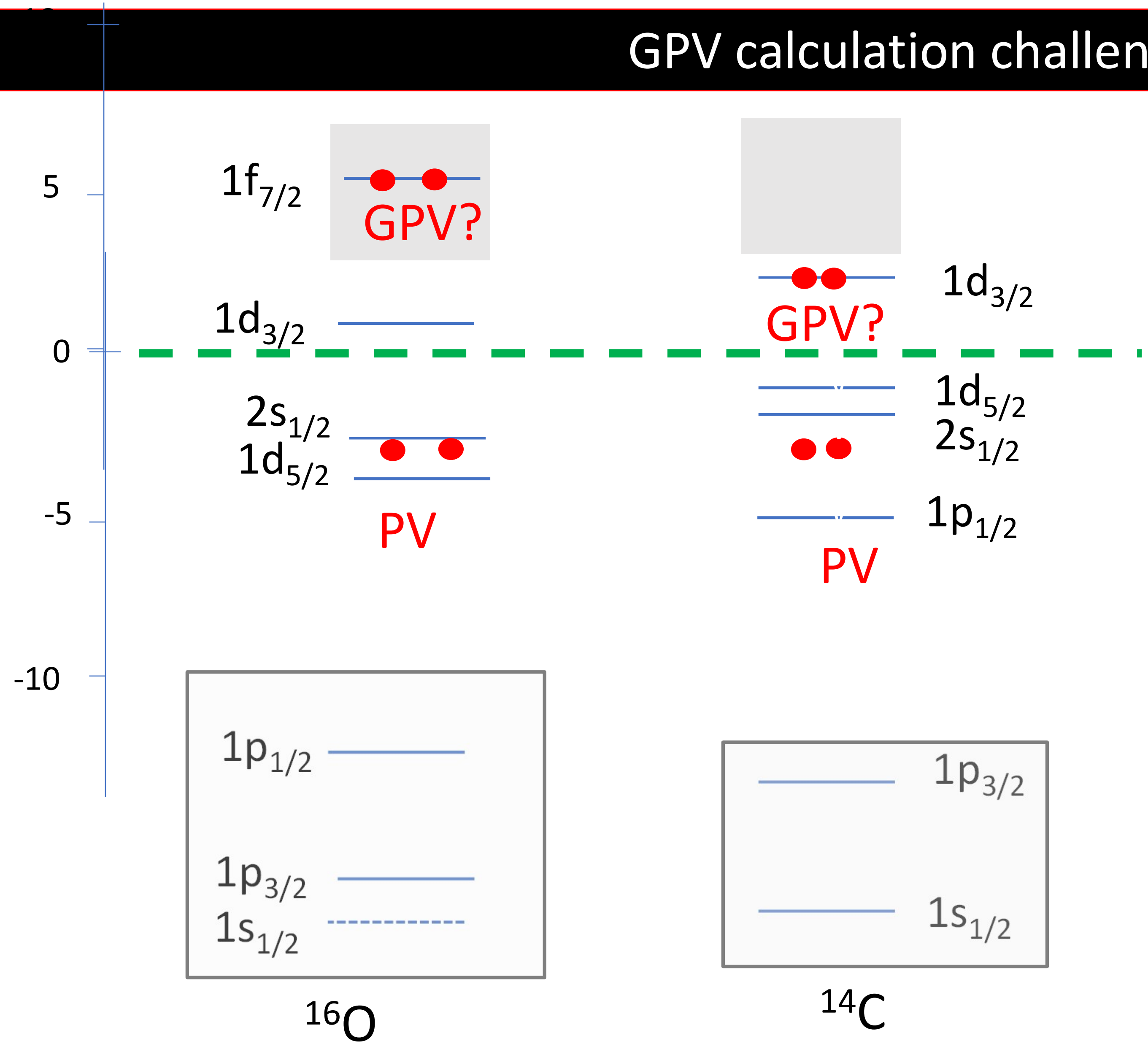


F. Cappuzzello et al., Eur. Phys. J. A 57 (2021) 34



E_lab = 84 MeV
E_lab = 275 MeV
dσ/dΩ (mb/sr)
12C(18O,16O)14C GPV
12C(18O,16O)14C 6.73
12C(18O,16O)14C g.s.
θ_CM (deg)
GPV?
gs.
13C(18O,16O)15C GPV
L = 0
12C(18O,16O)14C GPV
L = 0
12C(18O,16O)14C g.s.
L = 0
dσ/dΩ (mb/sr)
θ_CM (deg)

GPV calculation challenges



In the GPV **both** neutrons lie in the continuum → discretization method via hard wall boundary.

Realistic pairing interactions is needed → Gogny, DDDI, V-14,….

An accurate description of s-p levels is crucial → PVC

Incorporate all this in the pp-RPA eq.'s

Basic Tool: The pp-RPA equations

$$|A+2,\tau\rangle=\Big(\sum_{m<n} X^{\tau}_{mn}a^{+}_{m}a^{+}_{n} - \sum_{i<j} Y^{\tau}_{ij}a^{+}_{j}a^{+}_{i}\Big)|A,0\rangle$$

Pairing Interaction → Coherent mix
→ Collectivity→ Cooper pair like

$$\begin{pmatrix} A & B \\ B^{+} & C \end{pmatrix}\begin{bmatrix} R_p^{\tau,\lambda} \\ R_h^{\tau,\lambda} \end{bmatrix}=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{bmatrix} R_p^{\tau,\lambda} \\ R_h^{\tau,\lambda} \end{bmatrix}\cdot\hbar\Omega_{\tau,\lambda},$$

$$A_{mnm'n'}=\delta_{mm'}\delta_{nn'}(\epsilon_m+\epsilon_n)+\bar{v}_{mnm'n'},$$
$$C_{iji'j'}=-\delta_{ii'}\delta_{jj'}(\epsilon_i+\epsilon_j)+\bar{v}_{iji'j'},$$
$$B_{mnij}=-\bar{v}_{mnij},$$

$$(R_p^{\tau})_{mn}=X^{\tau}_{mn}, \qquad (R_p^{\lambda})_{mn}=Y^{\lambda}_{mn},$$
$$(R_h^{\tau})_{ij}=Y^{\tau}_{ij}, \qquad (R_h^{\lambda})_{ij}=X^{\lambda}_{ij}.$$

From The Nuclear Many Body Problem by Ring and Schuck

ppRPA using BOX boundary conditions

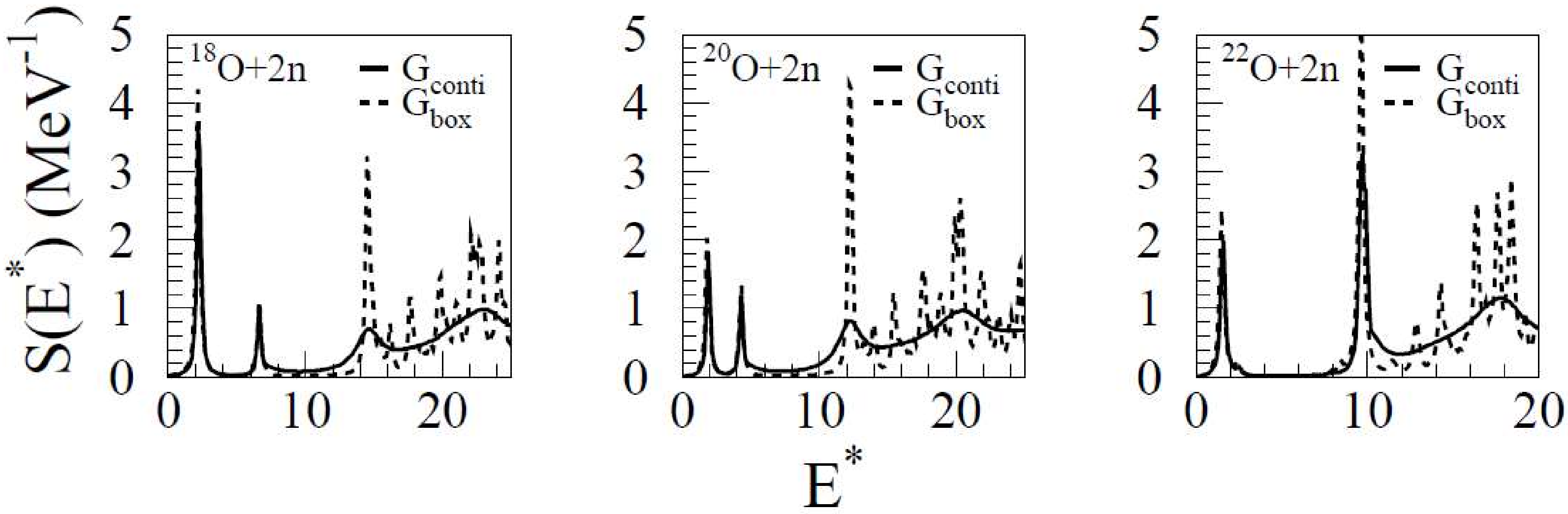
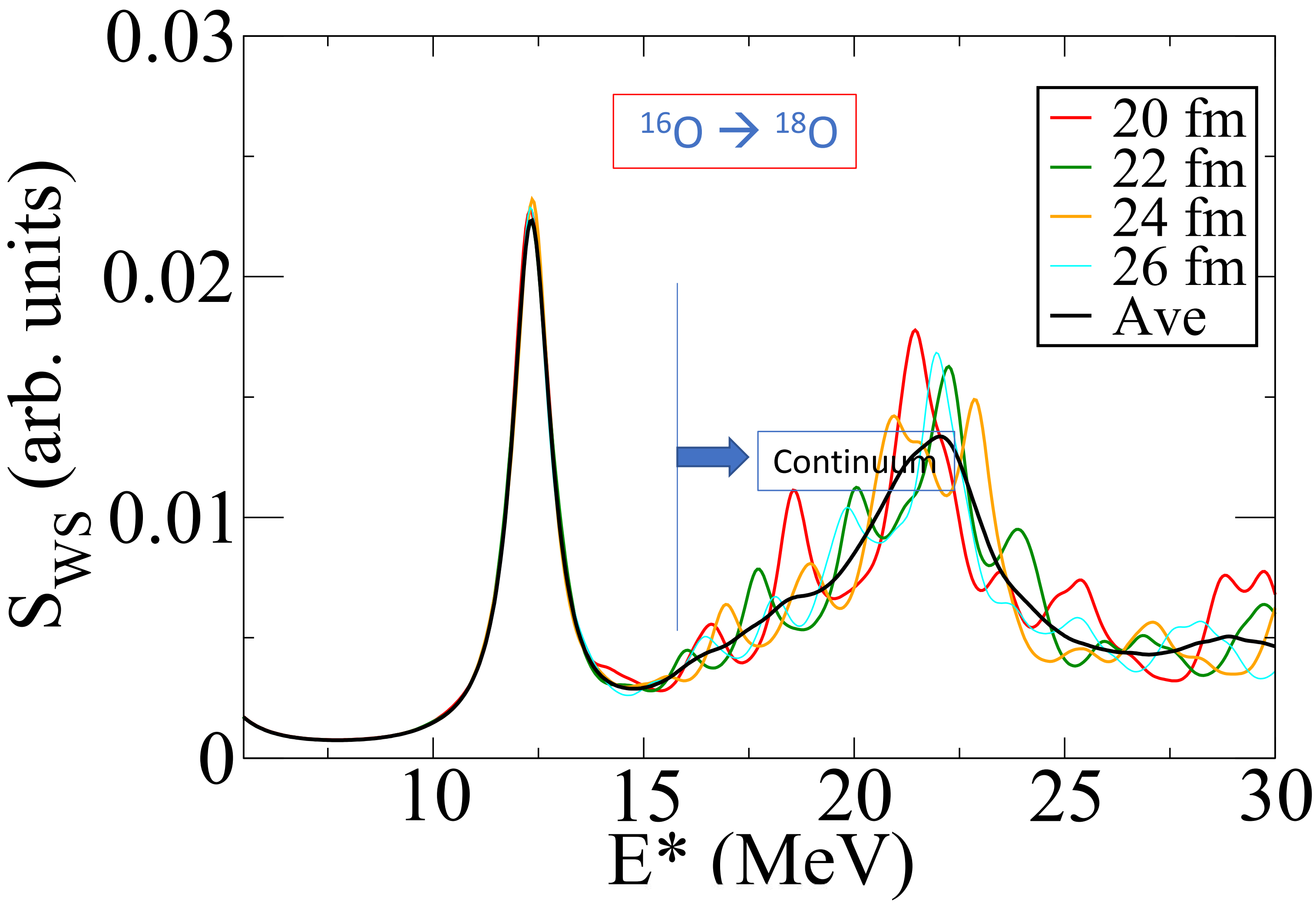


FIG. 1. The QRPA response for the two-neutron transfer on $^{18,20,22}O$. The exact continuum calculations are in solid lines whereas the calculations with box boundary conditions are in dashed lines. The results are displayed as functions of E*, the excitation energy with respect to the parent nucleus ground state. Rbox = 22.5 fm

E. Khan et al PRC69 (2004) 014314

ppRPA using BOX boundary conditions



pp-RPA with the Gogny(pairing) force.

Averaging details:

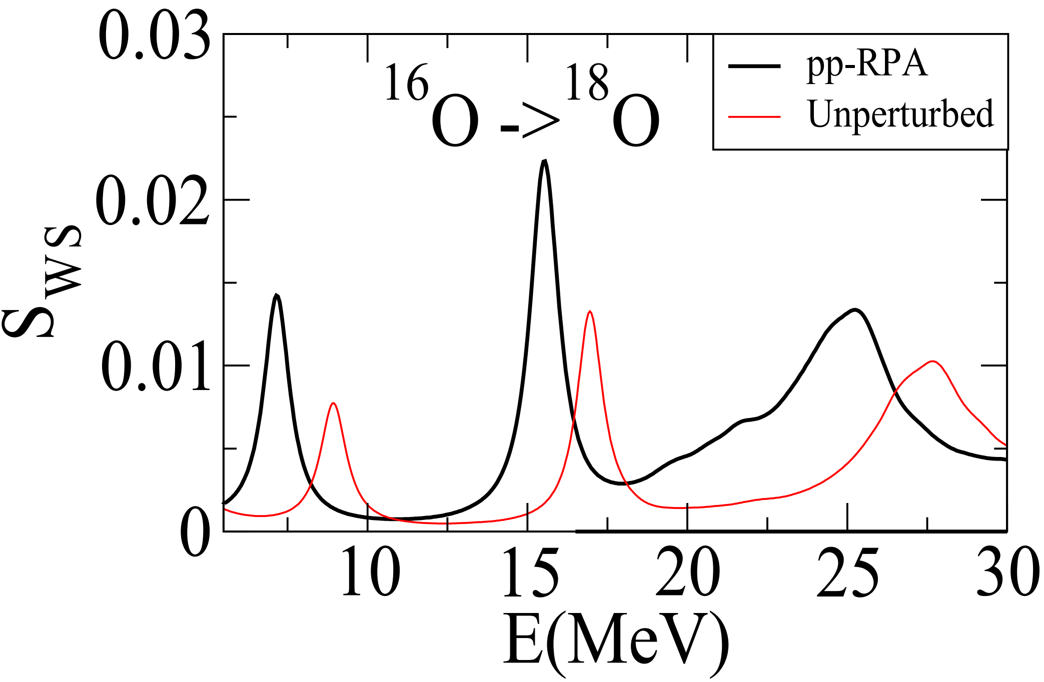
1. Box sampling
2. Small Gaussian convolution.

$$S^i_{WS} = \sum_{nn'lj} [X^i_{nn'lj} + Y^i_{nn'lj}] \int dr G(r) \psi_{nlj}(r) \psi_{n'lj}(r).$$

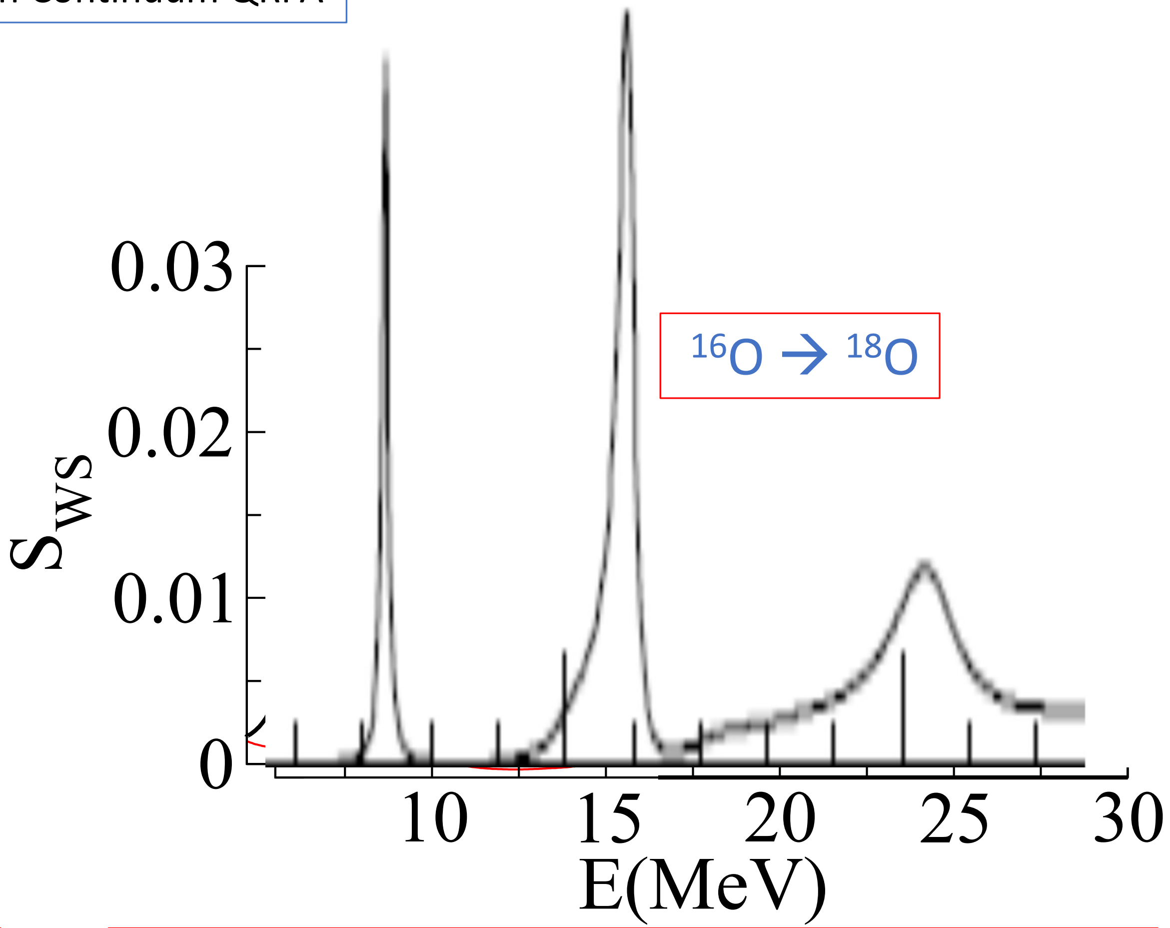
$$G(r) \equiv (1 + exp[(r - R_S)/a_S]$$

ppRPA using BOX boundary conditions

Comparison with Continuum QRPA



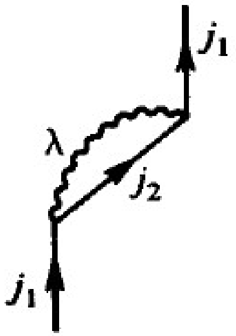
Using BOX boundary conditions. This work.

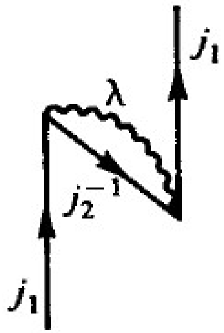


Continuum QRPA by Matsuo using Sly4 mean field plus DDDI pairing force (private communication)

Position of the Single Particle Levels and NFT/PVC: Self-energy

We will not use a "standard" mean field but a new one fitted on data after including beyond mean field





$$\delta\varepsilon(j_1) = \begin{cases} \dfrac{h^2(j_1,j_2\lambda)}{\varepsilon(j_1)-\varepsilon(j_2)-\hbar\omega_\lambda} & \varepsilon(j_2) > \varepsilon_F \\ -\dfrac{h^2(j_1,j_2\lambda)}{\varepsilon(j_2)-\varepsilon(j_1)-\hbar\omega_\lambda} & \varepsilon(j_2) < \varepsilon_F \end{cases}$$

λ^π : 2^+ most relevant

Part of the S-P strength goes to the intermediate state-> Fragmentation

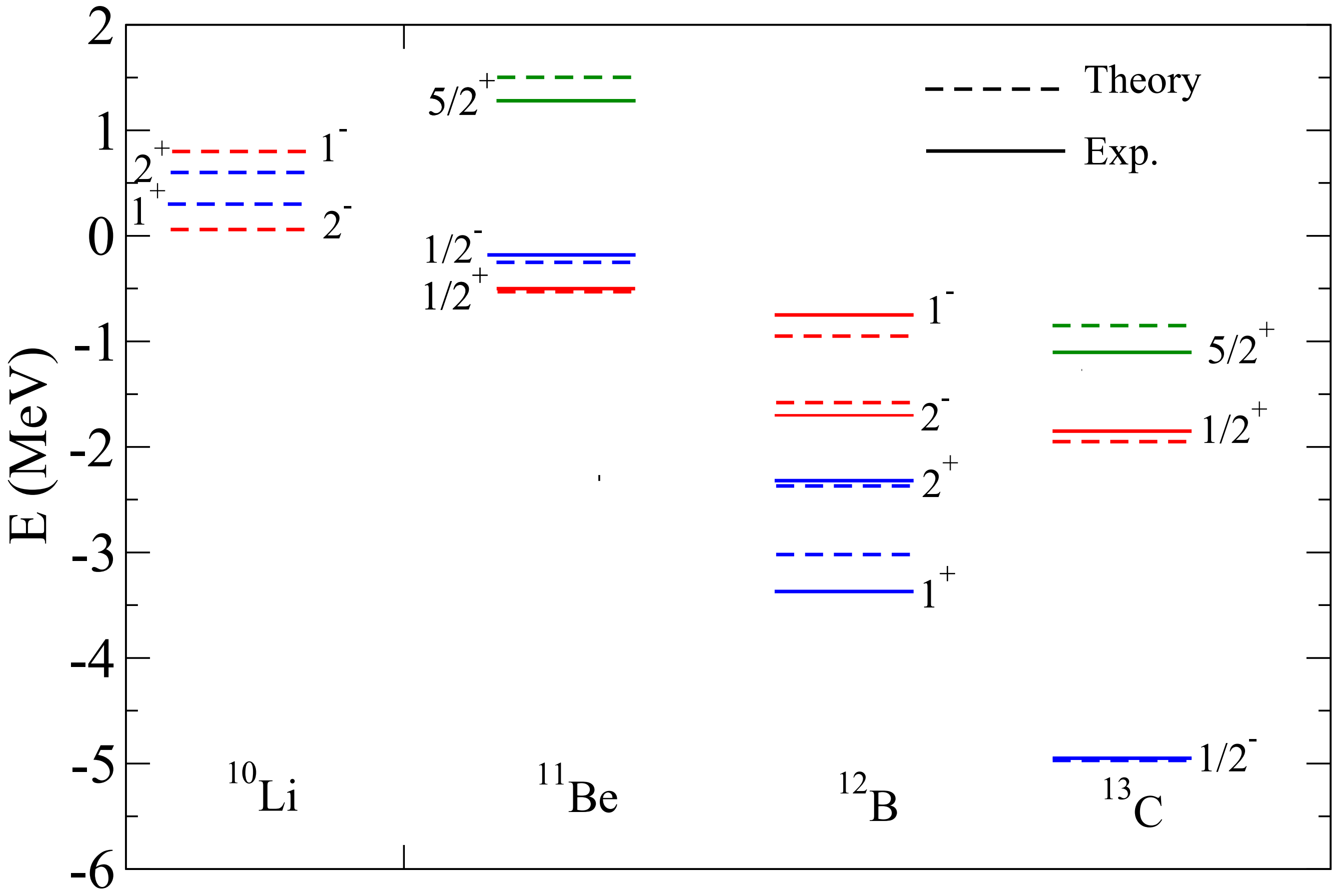
PVC vertex

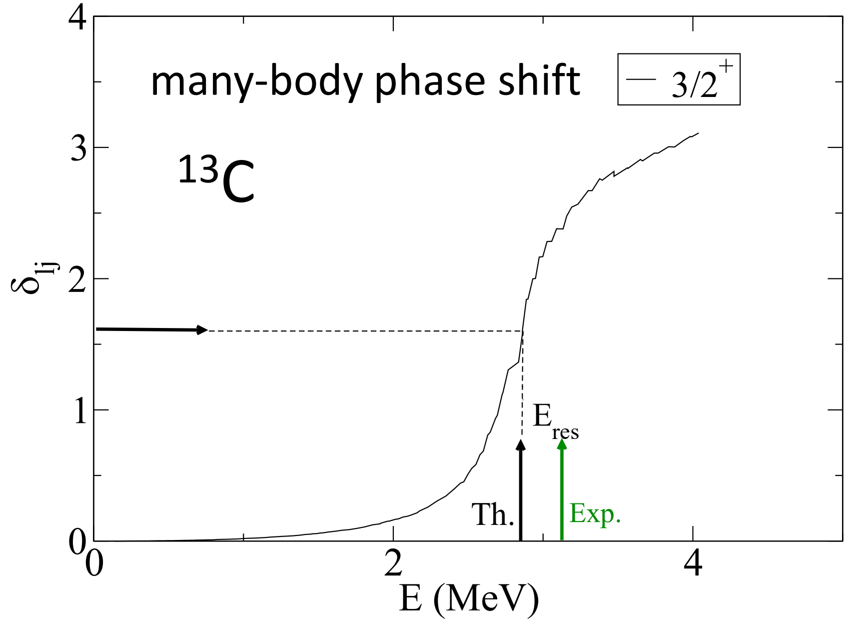
$$h(j_1,j_2\lambda) \equiv \langle j_2, n_\lambda = 1; I = j_1, M = m_1 | H' | j_1 m_1 \rangle$$

$$= (-1)^{j_1+j_2}(2j_1+1)^{-1/2}(2\lambda+1)^{-1/2}\langle j_2 \| k_\lambda Y_\lambda \| j_1 \rangle \langle n_\lambda = 1 \| \alpha_\lambda \| n_\lambda = 0 \rangle$$

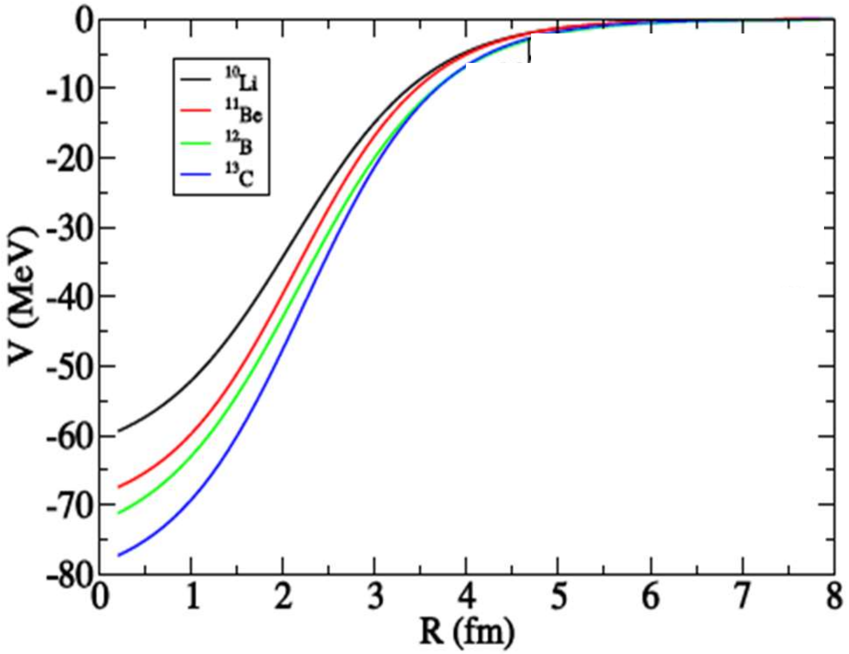
Position of the Single Particle Levels and NFT/PVC: Self-energy

Many-body states in N=7 isotones arising from quadrupole coupling with single-particle states calculated in a common mean-field potential





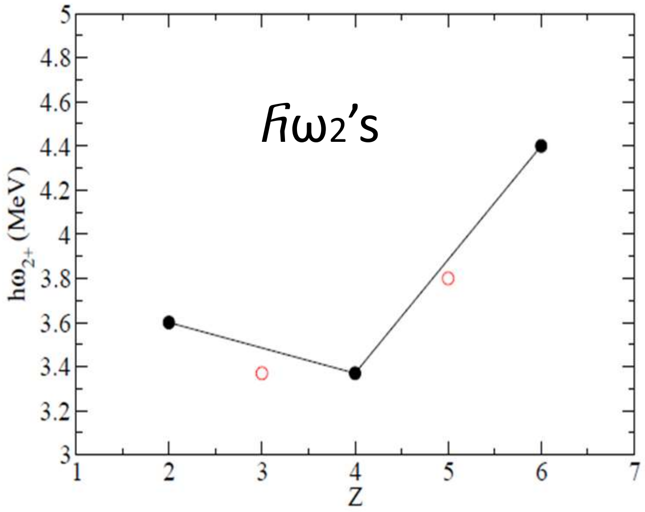
Position of the Single Particle Levels and NFT/PVC: Self-energy

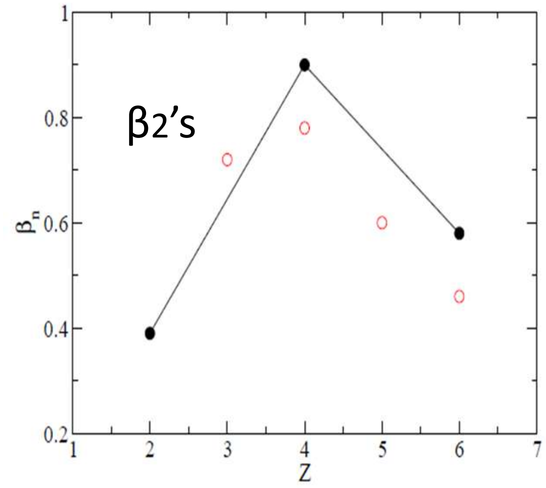


Simple parametrization:

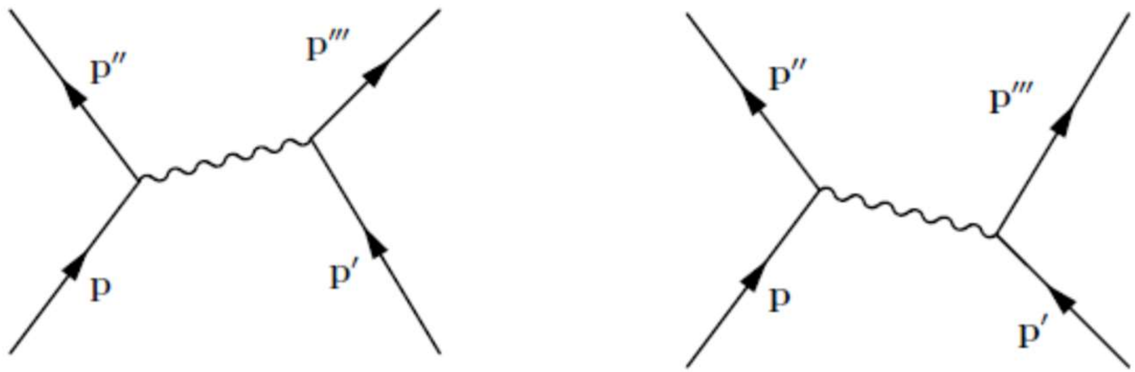
V_{WS}=-82+54(N-Z)/A *MeV*
a = 0.75 *fm;* R_{WS}=0.99$A^{1/3}$ *fm*
V_{LS}=0.0082V_{WS}

Bare mean field potential for N=7 isotones





Extended Role of Vibrations in the PV and GPV: The Phonon Exchange Induced Interaction, V^{ind}



$$V^{ind}_{pp'p''p'''} = \sum_{\lambda\nu} \left[\frac{h_{pp''\lambda\nu} h_{p'''p'\lambda\nu}}{E - (\epsilon^{emp}_{p''} + \epsilon^{emp}_{p'} + \hbar\omega_{\lambda\nu})} + \frac{h_{p''p\lambda\nu} h_{p'p'''\lambda\nu}}{E - (\epsilon^{emp}_{p} + \epsilon^{emp}_{p'''} + \hbar\omega_{\lambda\nu})} \right]$$

Present in every nucleus!!

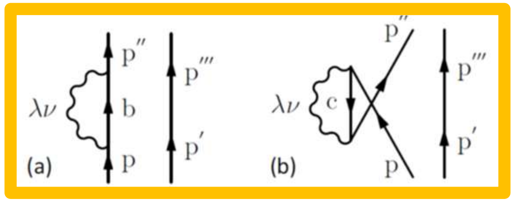
As a consequence: pairing V^{bare}(p,p';p'',p''') must leave room to V^{ind}→ reduced V^{bare} by 20%

EXTENDED pp-RPA (E-dependent)

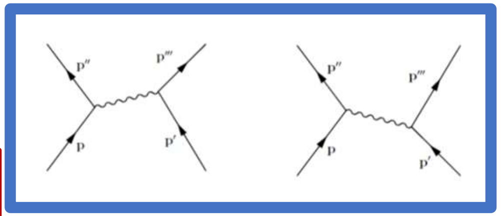
$$\begin{pmatrix} A_{pp'p''p'''} & B_{pp'h''h'''} \\ -B_{p''p'''hh'} & -A_{hh'h''h'''} \end{pmatrix} \begin{pmatrix} X_{p''p'''} \\ Y_{h''h'''} \end{pmatrix} = E \begin{pmatrix} X_{pp'} \\ Y_{hh'} \end{pmatrix}$$

Incorporating Self-energy and Induced Interaction

$$A_{pp'p''p'''} = [(\epsilon_p + \epsilon_{p'}) + \Sigma_{pp''(p')}(E)\delta_{p'p'''} + \Sigma_{p'p'''(p)}(E)\delta_{pp''} + V^{bare}_{pp'p''p'''} + V^{ind}_{pp'p''p'''}(E) + Exch(p,p')]N_{pp'p''p'''}$$

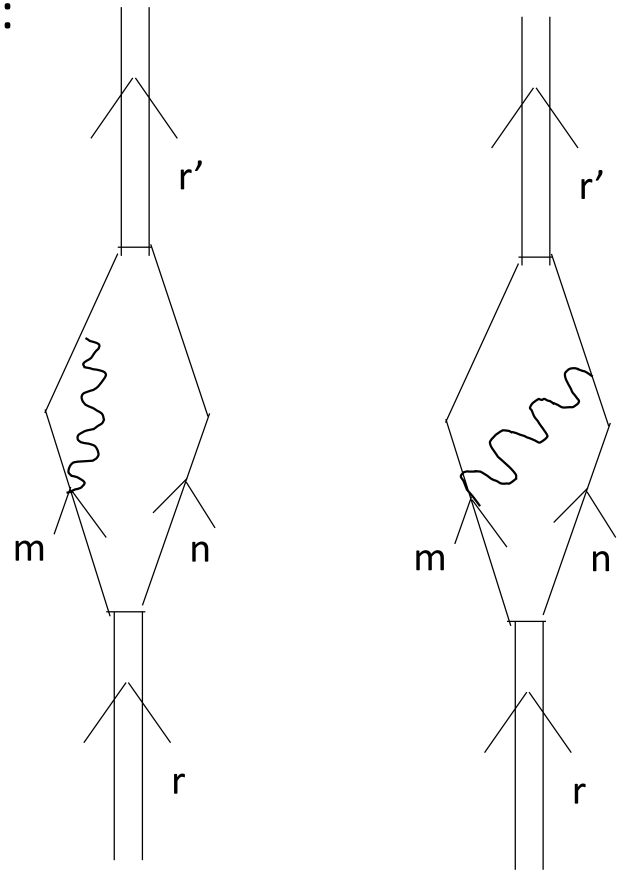


$$B_{pp'hh'} = [V^{bare}_{pp'hh'} + V^{ind}_{pp'hh'}(E) + Exch(p,p')]\, N_{pp'p''p'''}$$



EXTENDED pp-RPA (E-dependent)

Technical note: This extended pp-RPA is comparable to the NFT treatment: In fact, If self-energy and Vind are included perturbatively in a second diagonalisation, the following “well known” NFT diagrams for the matrix elements appear:



EXTENDED pp-RPA (E-dependent)

Technical note II:

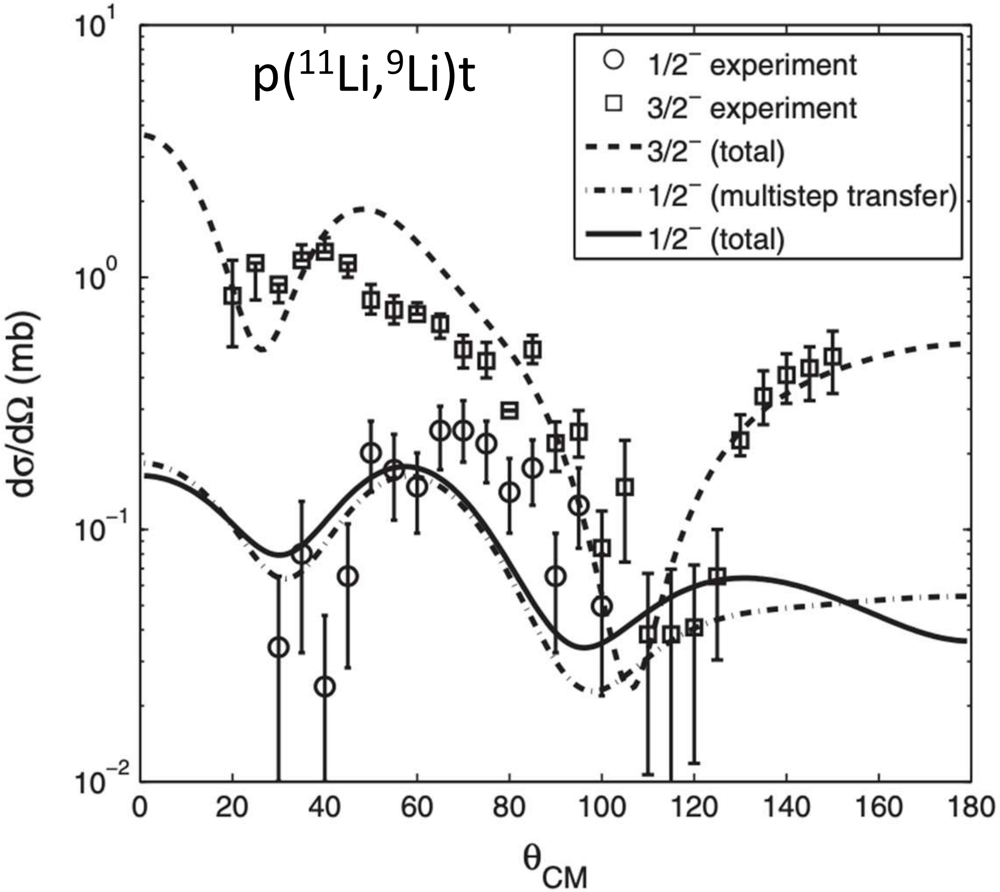
The self-energy and the induced interaction are energy-dependent, thus it is possible to reconstruct the amplitudes of the resulting 0+ states on the intermediate 2p-1phonon configurations, so that they can be written:

$$|0^+_n> \; = \; \sum_{pp'} (X_{pp'} \; (n) \; |pp'(0^+)> + \; Y_{hh'} \; (n) \; |h\,h'(0^+)>) \; + \sum_{pp'\nu} R_{pp'\nu} \; (n) \, |pp'(2^+)\nu(2^+)>$$

Can also be obtained by diagonalizing an energy independent matrix in the extended basis including them.

EXTENDED pp-RPA (E-dependent)

Role of phononic components in direct reactions



G. Potel et al, PRL 105 (2010) 172502

EXTENDED pp-RPA (E-dependent)

Similar theoretical schemes

Second RPA; Subtraction problem (Exact GS!!)

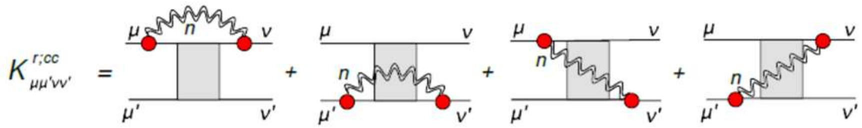
We (NFT) don't have such constrain

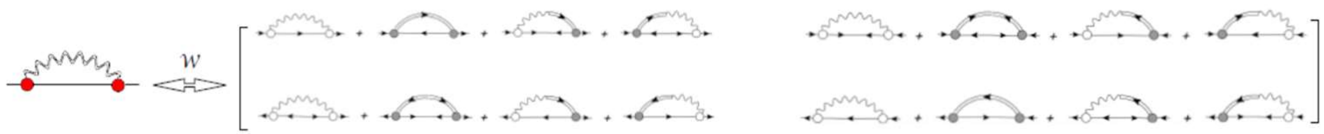
PHYSICAL REVIEW C **92**, 034303 (2015)

Subtraction method in the second random-phase approximation: First applications with a Skyrme energy functional

D. Gambacurta,[1] M. Grasso,[2] and J. Engel[3]

E. Litvinova and Y. Zhang
(arXiv 2208.07843v1)

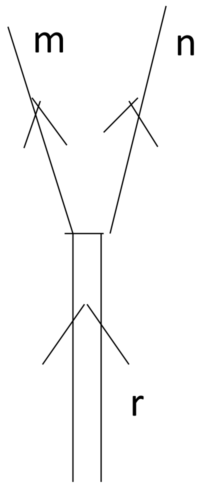




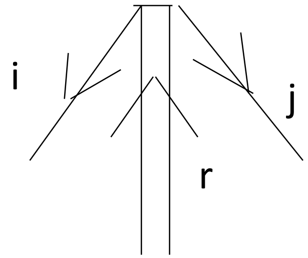
EXTENDED pp-RPA (E-dependent)

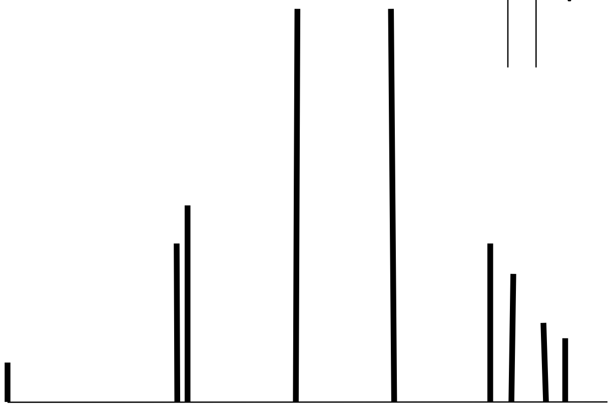
Summarizing: Pairing correlations , Decay width to the continuum and PVC effects

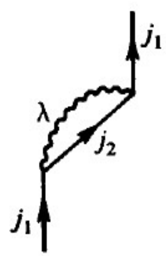
$X_{mn}(r) =$

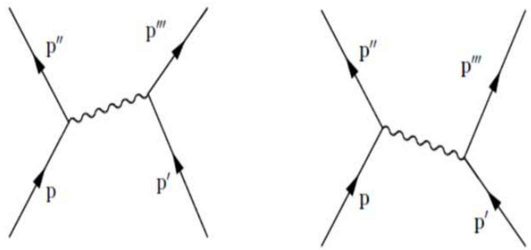


$Y_{ij}(r) =$

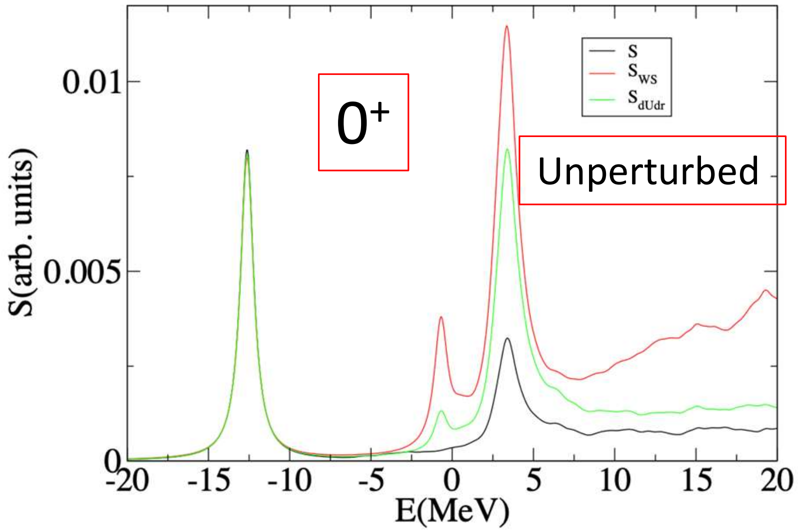








EXTENDED pp-RPA RESULTS



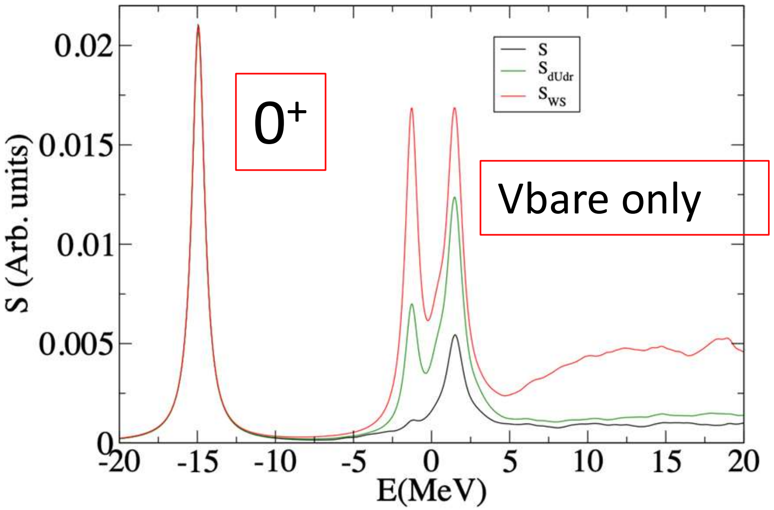
14C

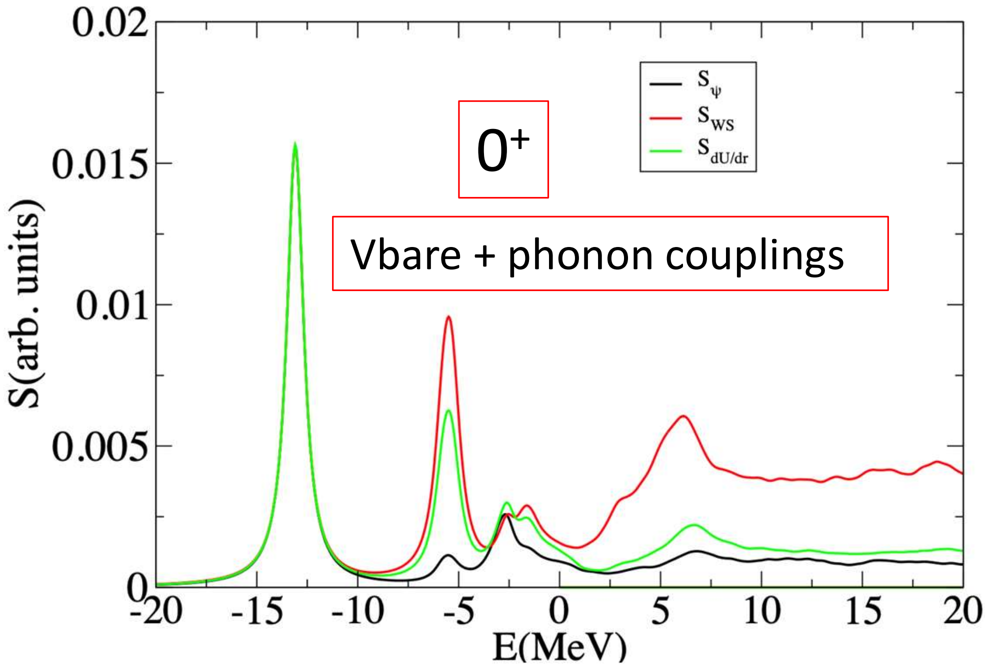
Form factors:

volume

density

surface





EXTENDED pp-RPA RESULTS

Xpp' ; Ypp' and Rpp'2+ amplitudes for 12C + 2n (0+) states: Bound states

	E_{gs}= -13.09 MeV $R^{2_1^+}=0.130$			$E_{0_2^+}$= -5.96 MeV $R^{2_1^+}=0.382$			$E_{0_3^+}$= -3.47 MeV $R^{2_1^+}=0.348$	
l_j	X^2_{lj}	Y^2_{lj}		X^2_{lj}	Y^2_{lj}		X^2_{lj}	Y^2_{lj}
$s_{1/2}$	0.006	0.003		0.283	-		0.376	-
$p_{1/2}$	0.833	-		0.050	-		0.043	-
$p_{3/2}$	-	0.002		0.001	-		-	-
$d_{3/2}$	0.003	-		0.005	-		-	-
$d_{5/2}$	0.046	-		0.327	-		0.256	-

Table 4: Main 0-phonon components of the wavefunctions of the ground state and of the two lowest excited 0^+ states calculated with a constant effective mass, $m_{eff} = m_{red} = 0.92m$ ($R_{box} = 28$ fm).

	$R^{2_1^+}_{ljl'j'}$						
l_j / $l'_{j'}$	$s_{1/2}$	$p_{1/2}$	$p_{3/2}$	$d_{3/2}$	$d_{5/2}$	$f_{5/2}$	$f_{7/2}$
$s_{1/2}$	-	-	-	-	0.003	-	-
$p_{1/2}$	-	-	0.105	-	-	0.0146	-
$p_{3/2}$	-	0.105	-	0.004	-	-	-
$d_{3/2}$	-	-	0.004	-	-	-	-
$d_{5/2}$	0.003	-	-	-	0.005	-	-
$f_{5/2}$	-	0.0146	-	-	-	-	-
$f_{7/2}$	-	-	-	-	-	-	-

Table 5: Phonon components $R^{2_1^+}_{ljl'j'}$ larger than 0.001, calculated in the wavefunction of the ground state of ^{14}C calculated with a constant effective mass, $m_{eff} = m_{red} = 0.92m$ ($R_{box} = 28$ fm).

EXTENDED pp-RPA RESULTS

Xpp' and cumulative Rpp'2+ amplitudes for 12C + 2n (0+;GPV)

	$E= 6.87$ $R_{box}=20$ $R^{2^+_1} = 0.623$	$E= 6.91$ $R_{box}=22$ $R^{2^+_1} = 0.729$	$E= 7.14$ $R_{box}=24$ $R^{2^+_1} = 0.728$	$E= 6.96$ $R_{box}=26$ $R^{2^+_1} = 0.613$	$E= 7.11$ $R_{box}=28$ $R^{2^+_1} = 0.785$
l_j	X^2_{lj}	X^2_{lj}	X^2_{lj}	X^2_{lj}	X^2_{lj}
$s_{1/2}$	0.06	0.041	0.03	0.04	0.012
$p_{1/2}$	0.112	0.004	0.001	0.005	0.012
$p_{3/2}$	0.029	0.003	0.056	0.005	0.05
$d_{3/2}$	0.006	0.019	0.007	0.003	0.007
$d_{5/2}$	0.154	0.195	0.179	0.279	0.111
$f_{5/2}$	-	-	-	-	-
$f_{7/2}$	-	-	-	-	-

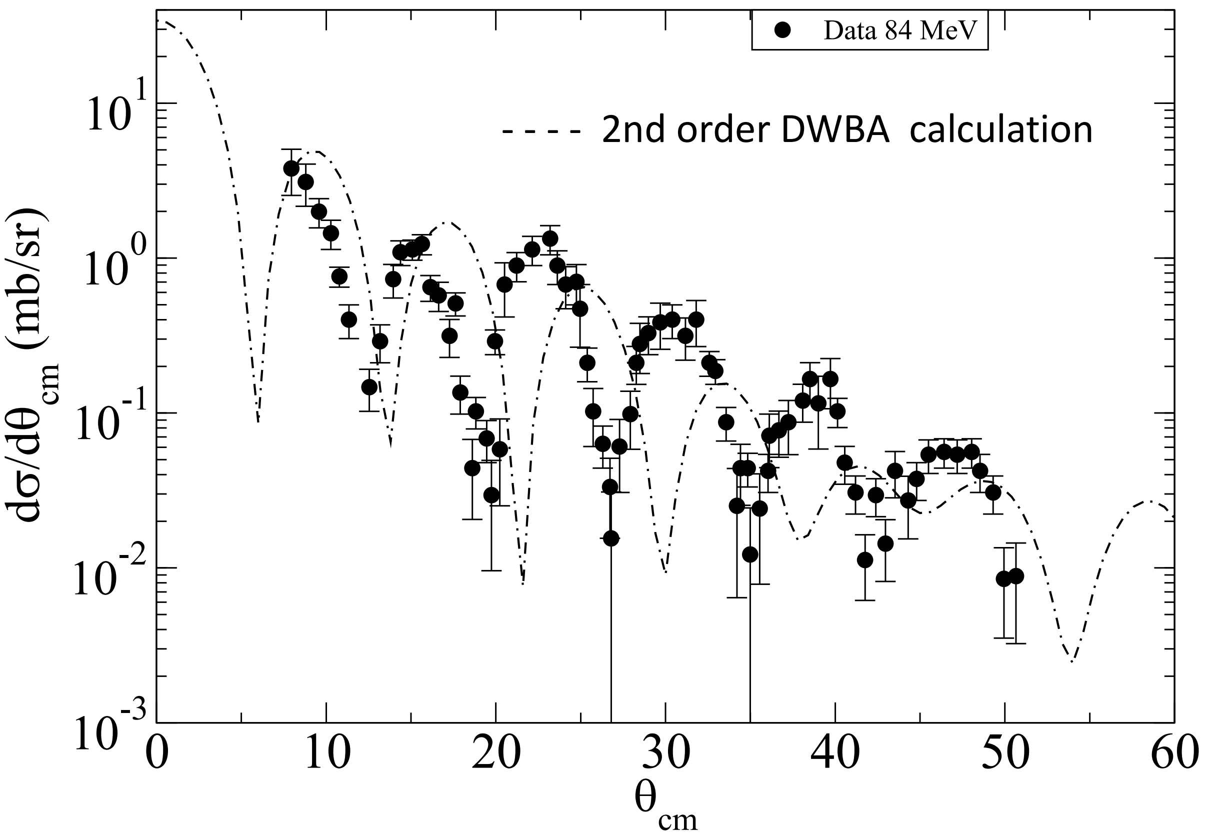
Table 23: Main 0-phonon components of the wavefunctions of the excited state of ^{14}C carrying the largest S_{dUdr} strength around $E = 7$ MeV for a series of boxes (R_{box} = 20-28 fm).

Note: About 70% on the phononic side!!

$|0^+_n\rangle = \sum_{pp'} (X_{pp'}(n) \, |pp'(0^+)\rangle + Y_{hh'}(n) \, |hh'(0^+)\rangle) +$
$\sum_{pp'\nu} R_{pp'\nu}(n) \, |pp'(2^+)\nu(2^+)\rangle$

EXTENDED pp-RPA RESULTS

$^{12}C(^{18}O,^{16}O)^{14}C(gs)$ at E_{lab} = 84 MeV



CONCLUSIONS

We have computed the 2n-transfer strength to populate 0+ states in the continuum of 14C and made the first steps to compute the absolute cross section of the reaction $^{12}C(^{18}O,^{16}O)^{14}C$. The theoretical model is based on particle-particle RPA extended to include the effects of coupling to collective quadrupole vibrations, in keeping with previous calculations of weakly-bound systems.

The aim is to compare our results with the bump and the associated angular distribution revealed in the excitation spectrum and attributed to the Giant Pairing Vibration.