Deblurring for Nuclei:
3D Characteristics of Heavy-Ion Collisions

Pawel Danielewicz

Facility for Rare Isotope Beams
Michigan State University

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Talk on flow by Mizuki Nishimura

Should flow be studied as before??

Teaching optics. . .

Possible nuclear & high-energy applications beyond heavy ions

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Paradigm: Triple-Differential Yields from Data

Distributions for *Fixed Direction of Reaction Plane*
from Theory and Experiment

no control over plane — some control, v_n — full control, $\frac{d^3N}{dp^3}$

Claim: You can go from center to right panel
through deblurring *for data*

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Deblurring by Example

Budd, *Crime Fighting Math*, plus.maths.org magazine

Blurred Photo of Moving Car

Deblurred

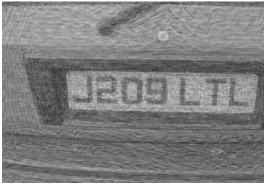
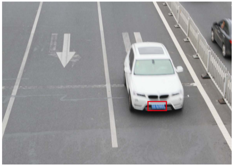


Photo of Parked Car

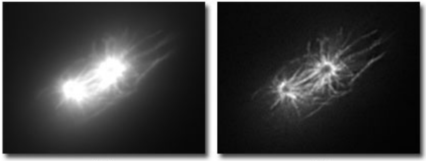
Fast Moving



Lu *et al.*, IEEE Trans Image Processing 25, 2311 (2016)

Deblurring in Optical Microscopy

Before and After Nearest Neighbor Deconvolution Analysis



(a) **Figure 1** (b)

https://micro.magnet.fsu.edu/primer/digitalimaging/deconvolution

Correcting f/Distortions Due to Apparatus or Method

Detector efficiency ϵ, n measured ptcle number, N actual number

$$N \simeq \frac{1}{\epsilon}\, n$$

Typical energy loss in thick target $\overline{\Delta E}$ for detected particle

$$E_{\text{prod}} \simeq E_{\text{det}} + \overline{\Delta E}$$

General problem stated probabilistically, with $P(\zeta|\xi)$ - probability to measure ptcle characteristic to be ζ when it is actually ξ

$$n(\zeta) = \int \mathrm{d}\xi \, P(\zeta|\xi)\, N(\xi)$$

For small distortions, P finite only when ζ little different from ξ.
Optical terminology: P - blurring or transfer function.

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Bayesian Deblurring

Distorted $n(\zeta)$ measured, while pristine $N(\xi)$ sought:

$$n(\zeta) = \int \mathrm{d}\xi \, P(\zeta|\xi)\, N(\xi)$$

$P(\zeta|\xi)$ - probability that ptcle with ζ detected while it really has characteristic ξ, understood given the method/apparatus, can be simulated (Geant4) & can depend on N

$Q(\xi|\zeta)$ - complementary probability that ptcle has characteristic ξ while measured at ζ - unknown.

Bayesian relation: number of times ptcle has characteristic in dξ while measured in dζ is

$$P(\zeta|\xi)\, N(\xi)\, \mathrm{d}\xi\, \mathrm{d}\zeta = Q(\xi|\zeta)\, n(\zeta)\, \mathrm{d}\xi\, \mathrm{d}\zeta$$

Hence $$N(\xi) = \frac{\int \mathrm{d}\zeta\, Q(\xi|\zeta)\, n(\zeta)}{\int \mathrm{d}\zeta'\, P(\zeta'|\xi)}\,, \quad Q(\xi|\zeta) = \frac{P(\zeta|\xi)\, N(\xi)}{\int \mathrm{d}\xi'\, P(\zeta|\xi')\, N(\xi')}$$

Richardson-Lucy method solves eqs iteratively till stabilization

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Richardson-Lucy (RL) Method from Astronomy

Iterative method, r - iteration index

$$n^{(r)}(\zeta) = \int \mathrm{d}\xi \, P^{(r)}(\zeta|\xi) \, N^{(r)}(\xi)$$

$$A^{(r)}(\xi) = \frac{\int \mathrm{d}\zeta \, \frac{n(\zeta)}{n^{(r)}(\zeta)} \, P^{(r)}(\zeta|\xi)}{\int \mathrm{d}\zeta' \, P^{(r)}(\zeta'|\xi)}$$

$$N^{(r+1)}(\xi) = A^{(r)}(\xi) \, N^{(r)}(\xi)$$

ξ & ζ are binned (pixelated), n & N are arrays and P transformation (transfer) matrix from the method/apparatus. Deblurring amounts to iterative multiplication of arrays by matrices + matrix reconstruction. Typical start: $N^{(1)}(\xi) = n(\xi)$

Richardson JOSA 62(1972)55 ; Lucy AJ 79(1974)745

https://en.wikipedia.org/wiki/Richardson-Lucy_deconvolution

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Other methods include Fourier transformation

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3D Nature of Collisions of Heavy Nuclei

Transport simulation of 2 GeV/nucl Au + Au at $b = 6$ fm

PD *et al.* Science298(2002)1592

z - beam, x - reaction plane

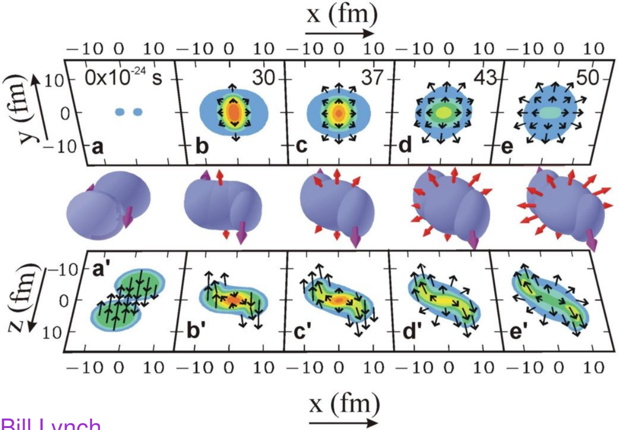
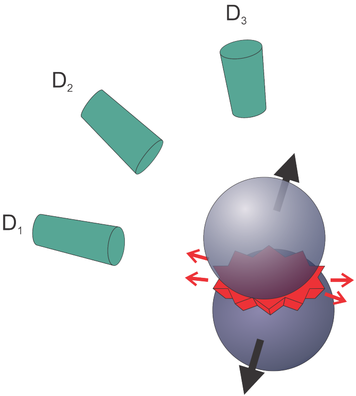


figure by Bill Lynch

Rich 3D structure, but no control over the reaction-plane direction in experiment

Estimating Reaction-Plane Direction



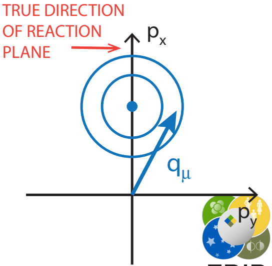
Plane direction estimated with

$$\mathbf{q}_\mu = \frac{1}{N} \sum_{\nu \neq \mu} \omega_\nu \, \mathbf{p}_\nu^\perp \qquad \omega_\nu = \begin{cases} +1, & \text{if } p_\nu^z > 0 \\ -1, & \text{if } p_\nu^z < 0 \end{cases}$$

N - measured particle multiplicity; other ptcles in the event used as reference for μ

PD&Odyniec
PLB157(85)146

Problem: Reference vector $\mathbf{q}_\mu$ Gaussian fluctuates around true plane direction



Any direct record of 3D characteristics will be blurred!

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Current Solution: Angular Moments of Distributions

Solution: average angular moments (azimuthal Fourier coefficients)

$$v_n = \langle \cos n\phi \rangle$$

ϕ - angle relative to true reaction plane

Voloshin&Zhang ZfPhC70(1996)665

v_n derived from average scalar products/contractions, e.g.,

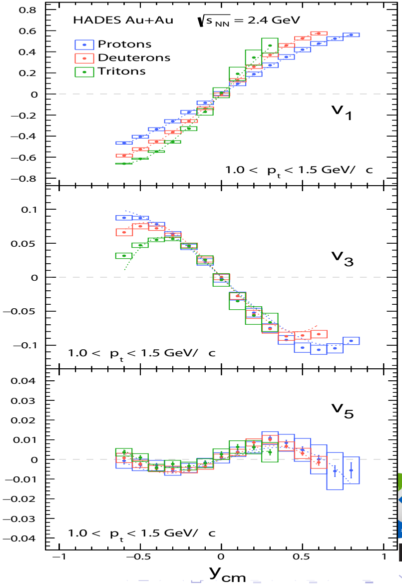
$$\langle \mathbf{p}_\mu^\perp \cdot \mathbf{q}_\mu \rangle \simeq p^\perp \langle q^x \rangle \langle \cos\phi \rangle$$

for different $p^\perp$, y and ptcle ID

Problem: unclear physics in v_n especially for higher n

1.23 GeV/nucl Au + Au $b \simeq 6$ fm

HADES PRL125(2020)262301



Schematic 1D Model

Proposition: Carry out as good determination of 3D info as you can & refine with deblurring. ~~V_n~~?

First 1D deblurring test.

Projectile at unknown velocity V deexcites emitting $N = 10$ ptcles distributed with box-like $\mathrm{d}N/\mathrm{d}v$ in projectile cm. Task: Measuring ptcles in lab, determine $\mathrm{d}N/\mathrm{d}v$. Cm velocity V' estimated from remaining ptcles, so V' & $\mathrm{d}N/\mathrm{d}v'$ smeared:

$$\frac{\mathrm{d}N}{\mathrm{d}v'} = \int \mathrm{d}V' \, \frac{\mathrm{d}P}{\mathrm{d}V'} \, \frac{\mathrm{d}N}{\mathrm{d}v}$$

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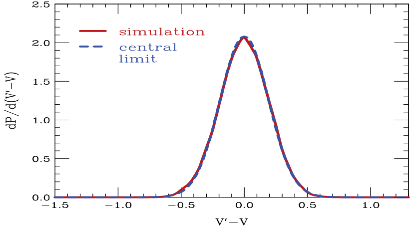
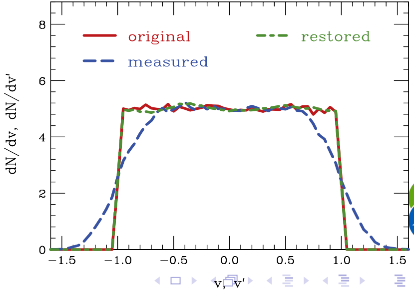
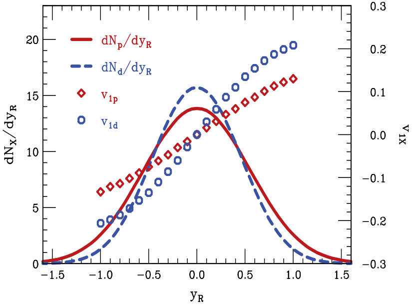
→Central-limit smear + RL deblur





3D Model for Collisions

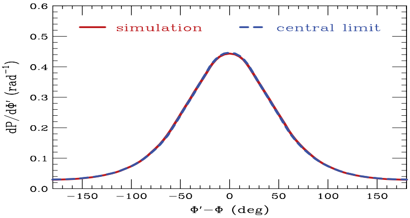
Customary thermal model with flow, N, d, t, ^{3}He, ^{4}He. $\langle Z_{Tot} \rangle = 50$
Rapidity dstr, temperature & flow typical for semicentral collisions at 300 MeV/nucl



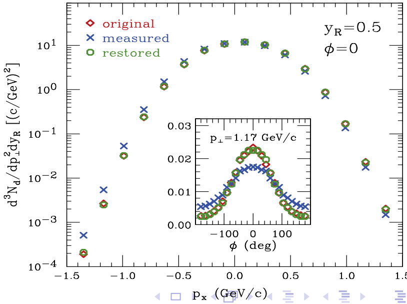
$$\frac{dN}{d\phi'} = \int d\Phi' \, \frac{dP}{d\Phi'} \, \frac{dN}{d\phi}$$

$$\phi' + \Phi' = \phi + \Phi$$

Uncertainty in reaction plane:



Triple differential spectrum in reaction plane:



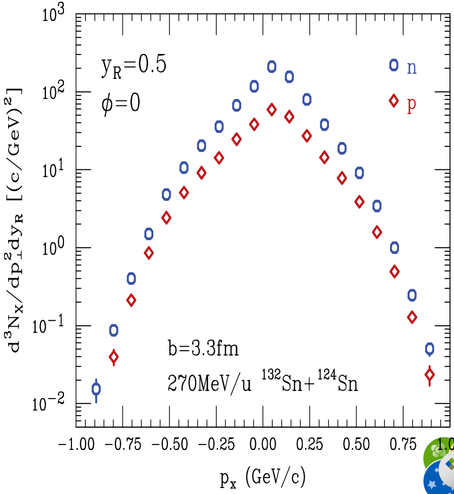
RL deblur + central-limit
Strong anisotropies restored!

Why 3D Characteristics?

Transport-model simulation of 270 MeV/nucl ^{132}Sn + ^{124}Sn collision at $b = 3.3$ fm

3-differential spectrum for the same conditions as in thermal model, but looks very different. Not parabolic, i.e., Gaussian, cusps, different left-right slopes, knees. Steeper slope on spectator side, softer on participant. Physics??

Averaged over ϕ, the spectrum would look thermal and no obvious sign in v_n...



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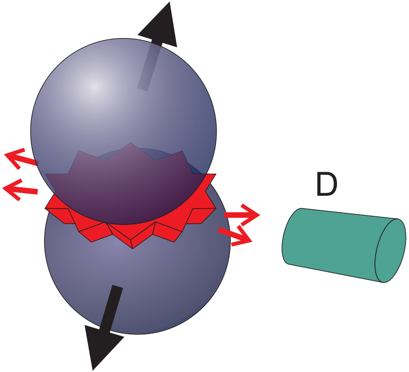
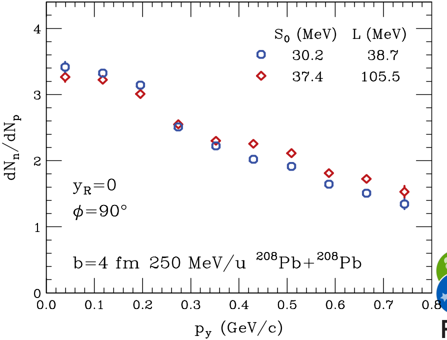
Symmetry energy at $\rho > \rho_0$?

Deblurring allows to effectively look into the heart of matter, unobscured high-density central region in the collisions



Transport-model simulations of 250 MeV/nucl ^{208}Pb + ^{208}Pb collisions w/medium-soft & stiff symmetry energy. n/p yield ratio at $\phi = 90°$, perp to reaction plane, and $y_{cm} = 0$.



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Conclusions

- Deblurring: strong record in optics & fields that heavily rely on optics: forensic science, astronomy & microscopy
- Deblurring can expand the reach of measurement ahead of any comparison to theory
- No reason for deblurring to confine to photons and not extend to other particles - its domain are probabilities
- Nuclear applications likely to involve significant inefficiencies and possible nonlinearities, but deblurring can still work
- Deblurring should effectively allow to control reaction plane in energetic heavy-ion collisions, hopefully expand horizons

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