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ABOUT FREQUENCIES OF RADIATION OF RELATIVISTIC PARTICLES IN PERIODICAL CRYSTALLINE STRUCTURE

VOLUME REFLECTION UNDULATOR

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Introduction

- Crystalline undulator based on channeling effect was proposed in [1,2]. Two experiments on 10 GeV positron [3] beam and 855 electron beam [4] were performed. The idea of the volume reflection undulator was briefly proposed in ref. [5]. The main advantage of the VRU is that it can work with both positrons and electrons
- Here we consider some properties of the volume reflection undulator. The expression for the undulator parameter as a function of the incident electron energy is derived. The frequencies of radiation emitted from the volume reflection undulator are considered. The threshold character of the radiation is discussed. This project has received funding through the MSCA4Ukraine project, which is funded by the European Union.

- [1] V.V. Kaplin, S.V. Plotnikov, S.A. Vorob'ev. Radiation of channeling particles in deformed crystals. Phys. Stat Sol. (b) 97 (1980) K89 – K91.
- [2] V.G. Baryshevsky, I.Ya. Dubovskaya, A.O. Grubich. Generation of gamma-quanta by channeling particles in the presence of a variable external field. Phys. Lett. A 77 (1980) 61-64.
- [3] V.T. Baranov et al., First results on investigation of radiation from positrons in a crystalline undulator. JETP Lett. 82 (2005) 562–564.
- [4] H. Backe, D. Krambrich, W. Lauth, K.K. Andersen, J. Lundsgaard Hansen, Ulrik I. Uggerhøj. Radiation emission at channeling of electrons in a strained layer $Si_{1-x}Ge_x$ undulator crystal. Nuclear Instruments and Methods in Physics Research Section B 309 (2013) 37-44.
- [5] U. Wienands et al., Channeling and radiation experiments at SLAC. Nucl. Instrum. Math. B 402 (2017) 11 – 15.

Two schemes of the volume refection undulator

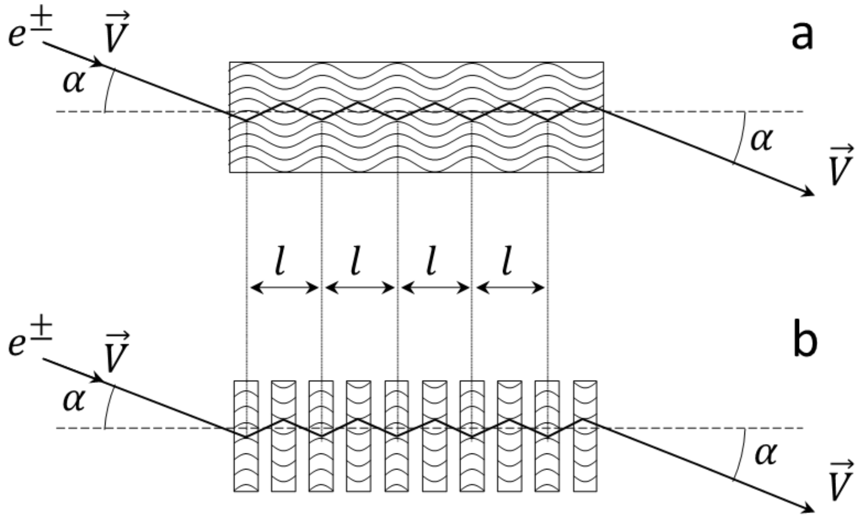


Fig.1. Two schemes of the crystalline undulators based on the volume reflection effect. Bent crystallographic planes are shown by curves. The layered volume reflection undulator is shown in Fig. 1a and the solid state volume reflection undulator is shown in Fig. 1b. The electron e^{-} or positron e^{+} moves with velocity $\vec{V}$ in almost straight lines between the reflecting points at angle α with respect to the undulator axis shown be dashed line.

Undulator parameter in volume reflection undulator

One of the key characteristics of an undulator devise is the undulator parameter K. The undulator parameter K can be estimated as the relation of the maximum angle α between the undulator axis and the electron velocity vector crossing the axis to the value of the inverse relativistic factor of the electron beam γ^{-1}

$$K = \alpha\gamma . \tag{1}$$

The undulator devise works in the undulator regime and emits coherent quasi-monochromatic radiation if $K < 1$ and the undulator devise works in the wiggler regime and emits incoherent radiation if $K > 1$. Below we will believe that the particle moves in almost straight lines between reflecting points and angle $\alpha = const$ in the both versions of the volume reflection undulator shown in Fig. 1.

The angle α in the volume reflection effect is close to the Lindhard angle for both positively and negatively charged particle, i.e. electrons and positrons. The Lindhard angle θ_L is

$$\theta_L = \sqrt{\frac{2U}{pV}} , \tag{2}$$

where U is the depth of the potential well between crystallographic planes, p and V are momentum and velocity of the incident particle respectively. The angle α is

$$\alpha = \xi\theta_L , \tag{3}$$

where the coefficient $\xi \approx 1$. Inserting (2,3) to (1), we obtain

$$K = \xi\sqrt{\frac{2U\gamma}{mV^2}} , \tag{4}$$

One can see from Eq. (4) that the undulator parameter monotonically increases on the energy of incident relativistic particle. Equating (1) to unity, we obtain the full particle energy E_u at which $K = 1$

$$E_u = \frac{(mVc)^2}{2U\xi^2} , \tag{5}$$

where m is electron mass, $V \to c$ for ultra-relativistic particles. At typical values $U \approx 20$ eV between crystallographic planes (111) and (220) in Si single-crystal [5], $E_u \approx 6.25$ GeV for $\xi = 1$. This means that the undulator devise based on (111) or (220) bent crystallographic planes of silicon singlecrystal should work in the undulator regime and emit quasi-monochromatic radiation at the incident electron or positron beam energy below 6.25 GeV or in the wiggler regime at the incident particle beam energy exceeding 6.25 GeV. The behavior of the angles γ^{-1}, and α, and the undulator parameter K as functions of the incident electron or positron energy E at $\xi = 1$ is shown in Fig. 2.

Undulator parameter K

$$K = \xi\sqrt{\frac{2U\gamma}{mV^2}}$$

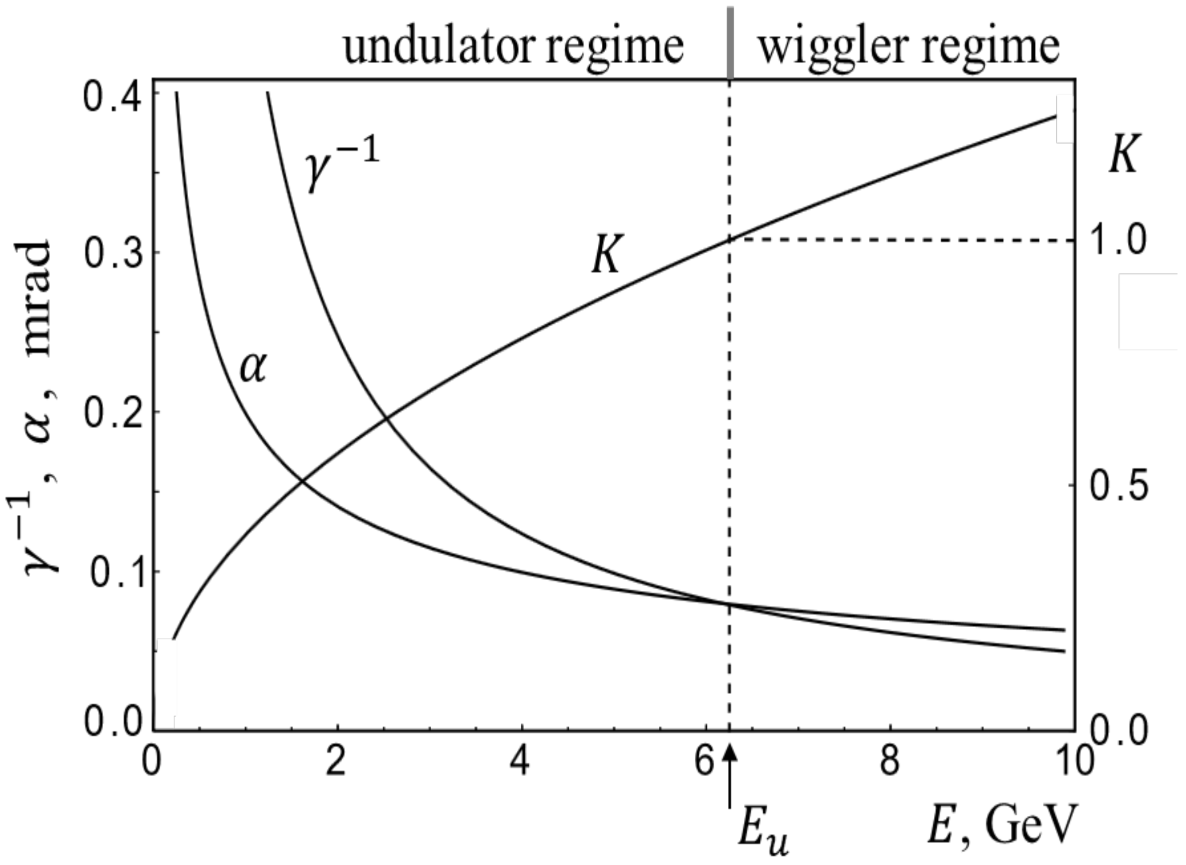
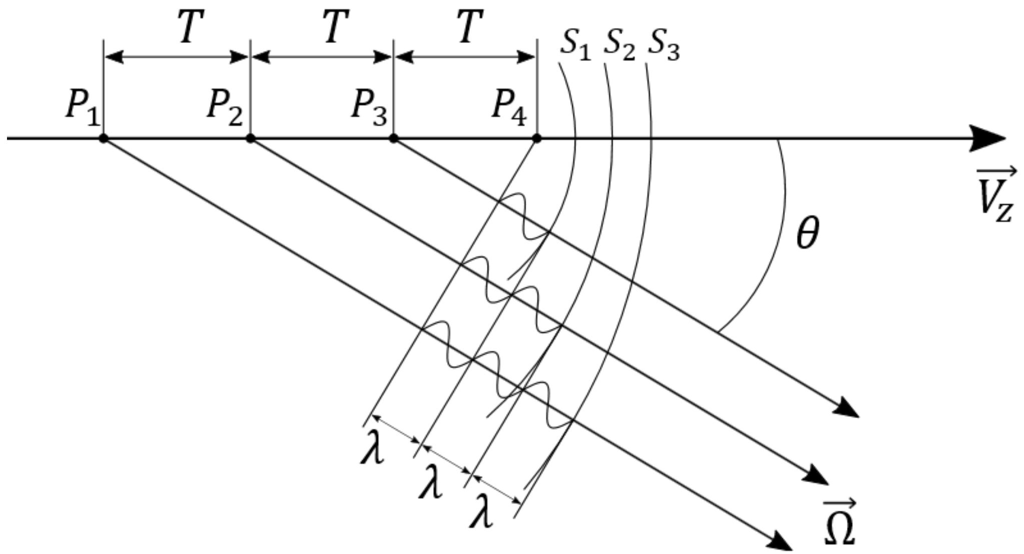


Fig. 2. Angles γ^{-1}, α, and the undulator parameter K as functions of the full electron or positron energy E at $U = 20$ eV and $\xi = 1$. Calculations were performed by formulae (2-5).

What frequencies of radiation are possible from a periodic structure?

The equation for radiation frequency was obtained from the Huygens construction. This cinematic consideration of coherent radiation is independent of the mechanism of the radiation. The mechanism may be, e.g. coherent transition radiation or coherent bremsstrahlung.

$$\hbar\omega << E \qquad \varepsilon = 1 - \left(\frac{\omega_p}{\omega}\right)^2$$



$$\omega = \frac{2\pi}{T} \frac{V_z}{1 - \frac{\sqrt{\varepsilon}}{c} V_z \cos\theta}$$

This is quadratic equation

The solution of the quadratic equation (9,10) is

$$\omega = \frac{\omega_0}{2} \pm \sqrt{\left(\frac{\omega_0}{2}\right)^2 - \frac{\left(\gamma\omega_p\right)^2}{1+K^2+\left(\gamma\theta\right)^2}}\,. \tag{11}$$

where

$$\omega_0 = \frac{4\pi nc}{l}\gamma^2 \frac{1}{\left[1+K^2+\left(\gamma\theta\right)^2\right]} \tag{12}$$

is the solution of the of Eq. (10) if the term $\left(\gamma\omega_p\right)^2$ is neglected.

The asymptote of the low-frequency decision ω_1^{as} can be found from (11)

$$\omega_1^{as} = \frac{l\omega_p^2}{4\pi nc}\,. \tag{16}$$

The asymptote (16) is independent of the relativistic factor and harmonics frequencies are inversely proportional to the harmonics number. The asymptote of the high-frequency decision ω_2^{as} is

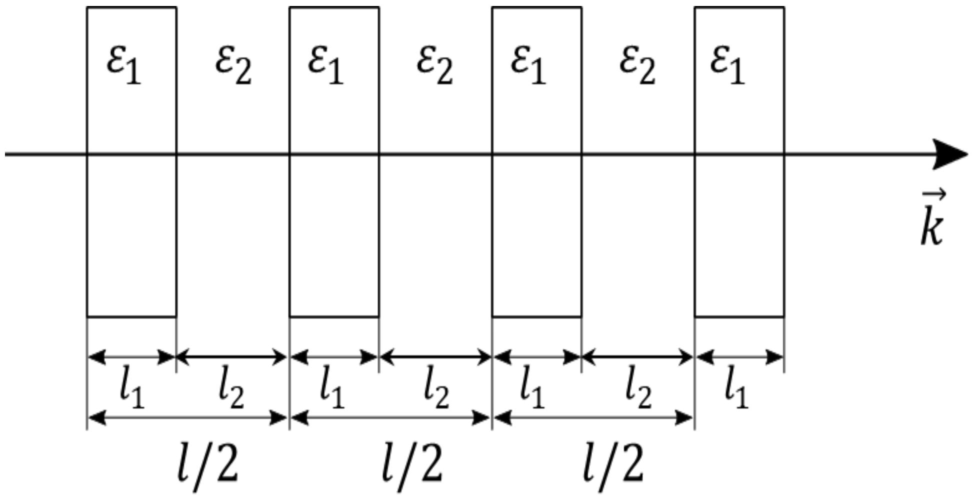
$$\omega_2^{as} = \omega_0 - \omega_1^{as} = \frac{4\pi nc}{l}\gamma^2 \frac{1}{\left[1+K^2+\left(\gamma\theta\right)^2\right]} - \frac{l\omega_p^2}{4\pi nc} \tag{17}$$

The asymptote (17) approaches to the solution (12) without account of the term $\left(\gamma\omega_p\right)^2$ in Eq. (10) at increasing of the harmonic number.

Similar asymptotes were obtained by Baier and Katkov in [i], where they considered properties of transition radiation. Their results were developed by Potylitsyn in his book [ii].

i V.N Baier, V.M Katkov. Transition radiation as a source of quasi-monochromatic X-rays. Nuclear Instruments an Methods A 439 (2000) 189-198.
ii A.P. Potylitsyn. Electromagnetic Radiation of Electrons in Periodic structures. Springer - Verlag Berlin Heidelberg, 2011

Plasma frequency in layered undulator



The wavefront passes the period during the time

$$,\frac{2l_1\sqrt{\varepsilon_1}}{c}+\frac{2l_2\sqrt{\varepsilon_2}}{c}=\frac{l\sqrt{\varepsilon_{eff}}}{c}, \tag{18}$$

where $\varepsilon_{eff}=1-\left(\frac{\omega_{peff}}{\omega_l}\right)^2$ and ω_{peff} are the effective values of the permittivity and plasma frequency in the layered undulator. The effective value of the plasma frequency can be found from Eq. (18)

$$\omega_{peff}^2=\frac{2l_1\omega_{p1}^2+2l_2\omega_{p2}^2}{l} \tag{19}$$

In practically important case of vacuum gap at $\omega_{p2}=0$ or air gap at $\omega_{p2}\approx 0$ we obtain

$$\omega_{peff}^2=\frac{2l_1}{l}\omega_{p1}{}^2 \tag{20}$$

and corresponding expression for effective permittivity

$$\varepsilon_{eff}=1-\frac{2l_1}{l}\left(\frac{\omega_{p1}}{\omega_l}\right)^2 \tag{21}$$

Below we will use these expressions (20,21) for calculations of the frequencies of undulator radiation from a layered undulator installed in vacuum or air.

Executing the calculations similar to ones in previous section, we obtain the frequency

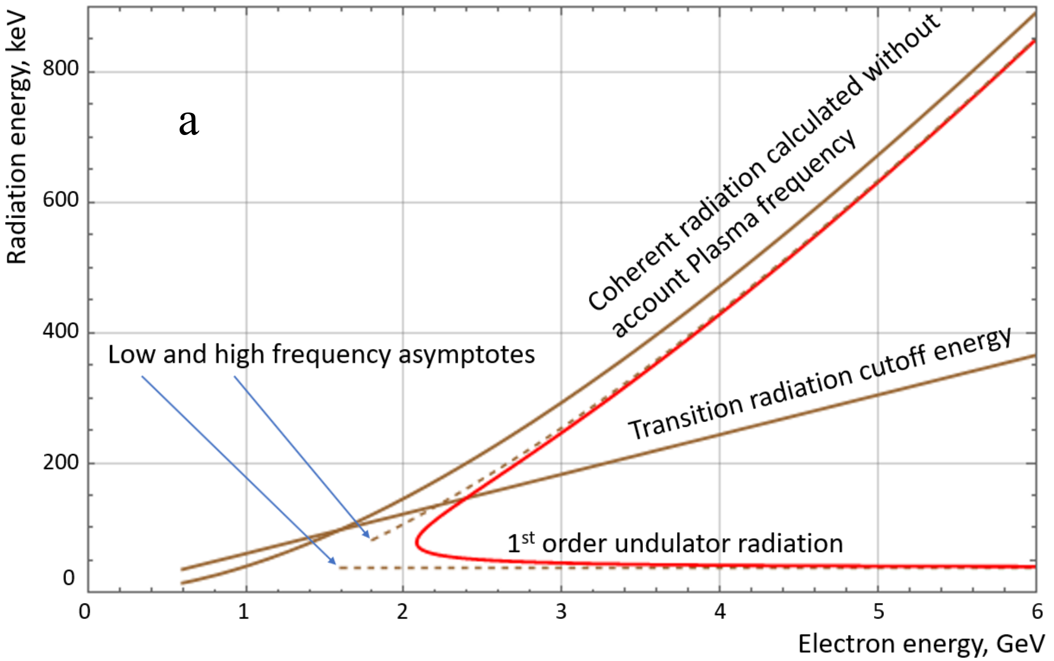
$$\omega_l=\frac{\omega_0}{2}\pm\sqrt{\left(\frac{\omega_0}{2}\right)^2-\frac{2l_1\left(\gamma\omega_{p1}\right)^2}{l\left[1+K^2+\left(\gamma\theta\right)^2\right]}}. \tag{22}$$

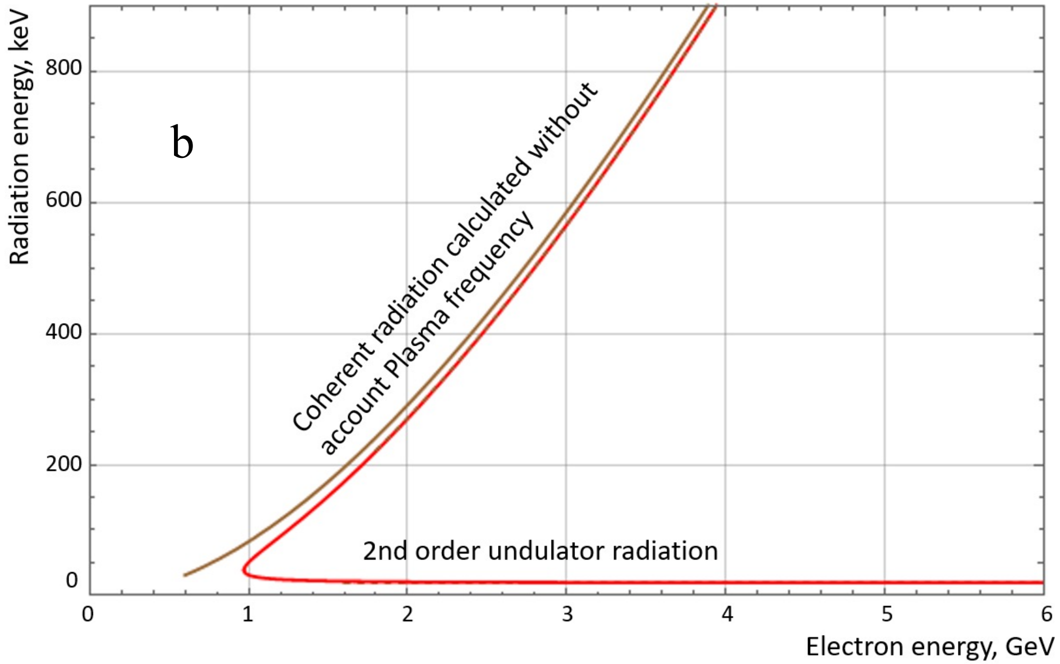
and asymptotes

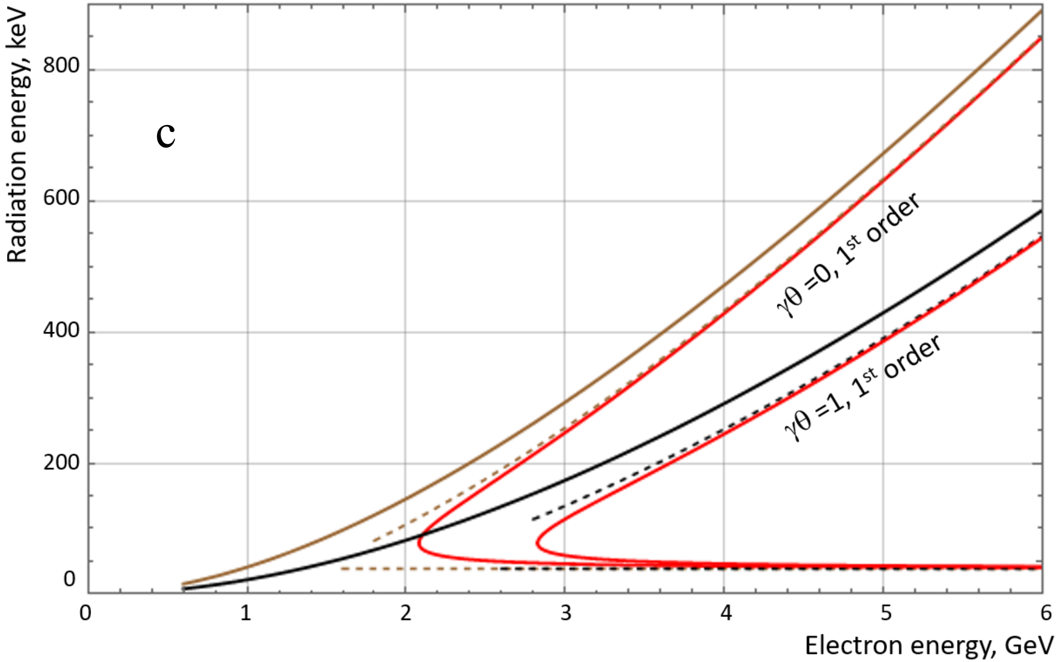
$$\omega_{l1}=\frac{2l_1\omega_p^2}{4\pi nc}. \tag{25}$$

and

$$\omega_{l2}=\omega_0-\omega_{l1}=\frac{4\pi nc}{l}\gamma^2\frac{1}{\left[1+K^2+\left(\gamma\theta\right)^2\right]}-\frac{2l_1\omega_p^2}{4\pi nc} \tag{26}$$







Energies of radiation from the layered volume reflection undulator as functions of incident electron energy. Reflections from Si (110) 0r (111) planes in bent crystalline plates. Undulator period in 200um, bent crystalline plates are of thickness 50um. Undulator radiation is shown by red curves.

a) 1st order $\gamma\theta = 0$

b) 2nd order $\gamma\theta = 0$

c) 1st order $\gamma\theta = 0$, $\gamma\theta = 1$

The threshold of radiation is at electron energies about 1-2 GeV

$$\gamma_{tr} = \frac{\omega_p \sqrt{1+K^2}}{\sqrt{\frac{(2\pi nc)^2}{2 l_1 l} - (\theta\omega_p)^2}}$$

Discussion

Here, we considered only the undulator parameter and frequencies of radiation from the volume reflection undulator. The important questions remain about stability of the motion of particles in volume reflection undulator. The experimental research can be performed at existing [i] or planned [ii] extracted electron/positron beams of energies up to 6 GeV in DESY.

i R. Diener et al, The DESY II test beam facility. Nucl. Instrum. Methods Phys. Res. A 922 (2019) 265–286

ii A. Sytov. G. Kube, L, Bandiera et al., First design of a crystal-based extraction of 6 GeV electrons for the DESY II Booster Synchrotron. The European Physical Journal C volume 82, Article number: 197 (2022).

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- Thank you for attention.