Coherent channeling radiation of electron beam in optical lattice

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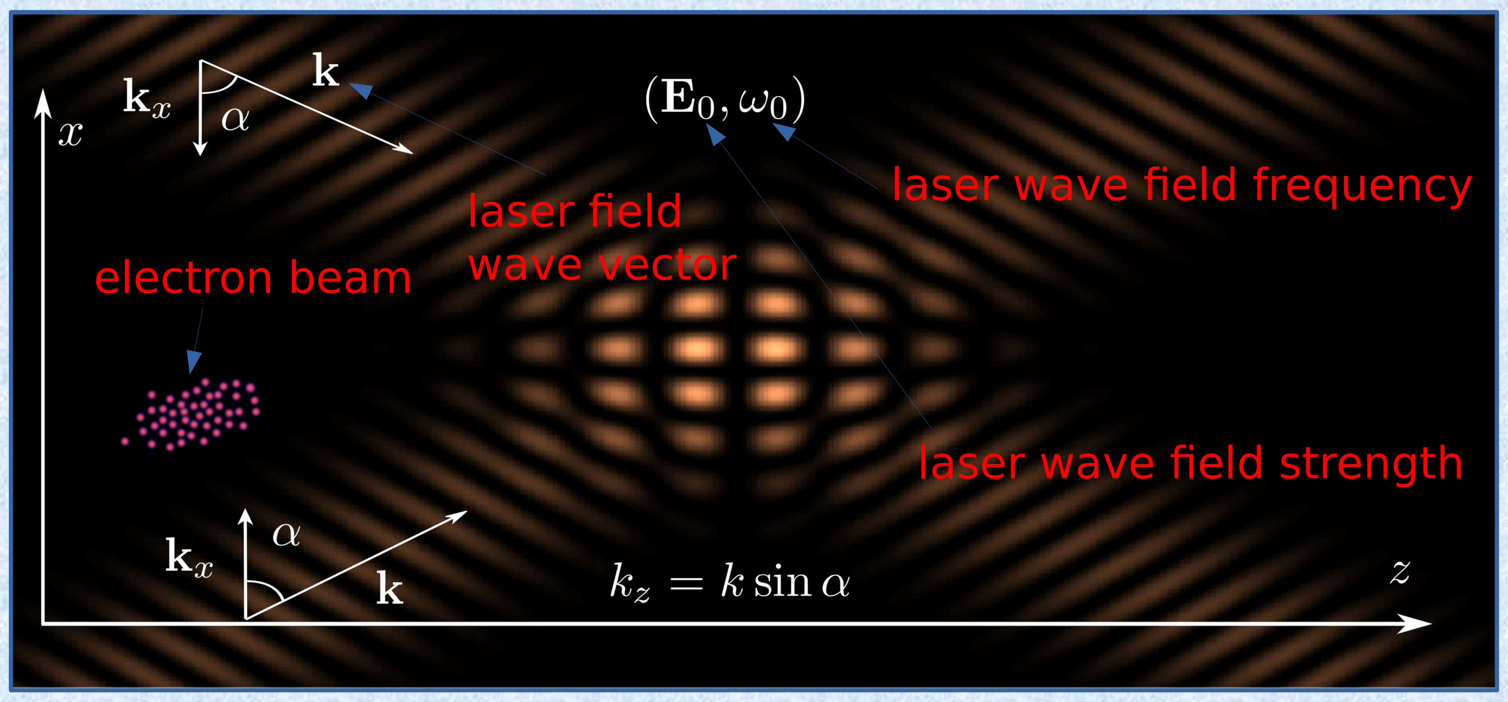
Electron beam in optical lattice

This work presents preliminary results on the electron beam radiation in the field of an optical lattice. The figure below shows a geometry of the considering task.



The optical lattice is formed by two plane p-polarized laser waves.

Coherent part of the spectrum

The spectral-angular distribution of EM radiation by the electron beam has the form:

$$\frac{d^2\mathcal{E}}{do d\omega} = \frac{\omega^2 R_0^2}{(2\pi)^2 c}\left(|\mathbf{A}_\omega|^2 - |\varphi_\omega|^2\right)$$

We will consider the coherent part of the radiated spectrum of the non-divergent channeled electron beam symmetrically distributed relative to the bottom of the OL channel. And we will also assume that the argument of the Bessel functions that determine the radiation field is much less than unity. Then the coherent part of the spectral-angular distribution will take the form:

$$\frac{d^2\mathcal{E}}{do d\omega} = N(N-1)\frac{e^2\omega^6 d_{ch}^6 \sin^4\theta\cos^4\phi}{(8\pi)^3 c^5 \sigma_\perp^2 \Omega_0^2}\left(\frac{4\Omega_0^2}{\omega^2\sin^2\theta\cos^2\phi} - \gamma_\parallel^{-2}\right)\times$$
$$\times e^{-\frac{\omega^2}{c^2}\left(\sigma_\parallel^2\cos^2\theta + \sigma_\perp^2\sin^2\theta\sin^2\phi\right)}\left|F\left(\frac{\omega}{\Omega_0}(1-\beta_\parallel\cos\theta) - 2, \Omega_0 T, \frac{\sigma_\perp}{d_{ch}}\right)\right|^2$$

longitudinal beam size

transverse beam size

Electron dynamics in OL

We neglect the interaction between electrons in the beam. The trajectory of a channeled electron in the field of an optical lattice of the considered geometry has the form:

initial longitudinal coordinate

channeled oscillation amplitude

initial phase

$$\mathbf{r}(t) = (z_0 + c\beta_\parallel t)\mathbf{e}_z + b\sin(\Omega(b)t + \alpha)\mathbf{e}_x$$

$$\alpha = \mathrm{arctg}\,\frac{x}{v_x\Omega(b)}$$

initial transverse velocity

$$b = \sqrt{x^2 + \frac{v_x^2}{\Omega^2(b)}}$$

$$\Omega(b) = \Omega_0\left(1 - \left(\frac{\pi b}{2d_{ch}}\right)^2\right)$$

$$d_{ch} = \frac{\pi}{k_x}$$

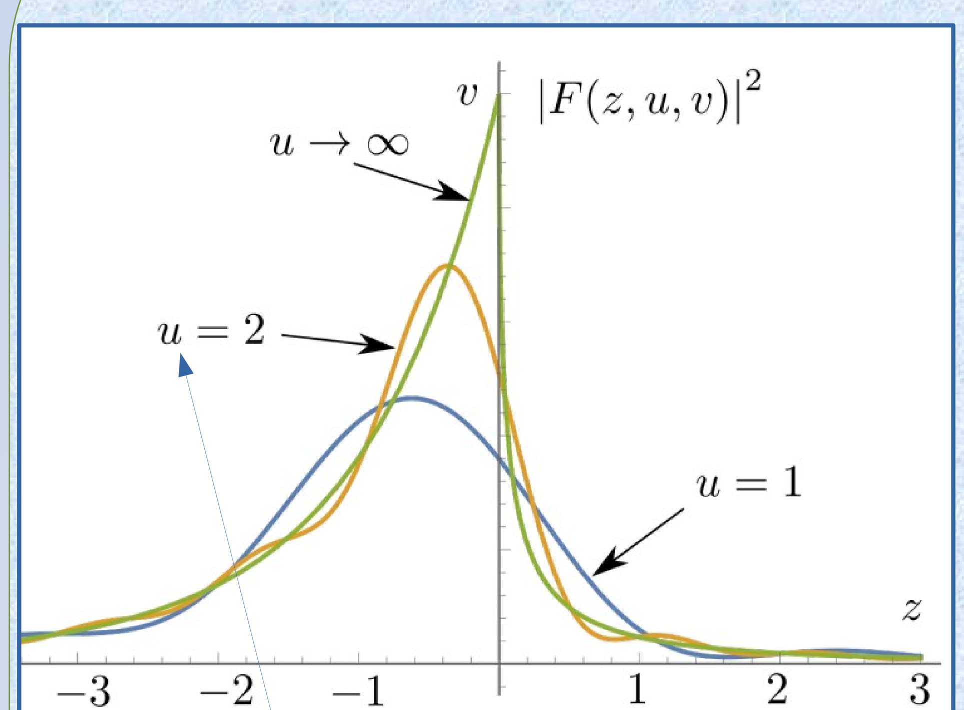
initial transverse coordinate

low-amplitude channeled oscillation frequency

channel width

The frequency of channeled oscillations of a particle depends on the amplitude. The above expression for the dependence of frequency on amplitude is a good approximation for not very large amplitude values. In the following, we will consider the EM radiation by $\boxed{N}$ electrons movement in the OL channel during the time $\boxed{T}$. Also, in follows, by $\boxed{\mathbf{k}}$ we denote the wave vector of the radiated EM wave.

Spectral-angular distribution of radiation



$$F(z,u,v) = v\left(1 - \frac{e^{izu}}{\sqrt{1 - i\pi^2 v^2 u}}\right) - \frac{\sqrt{z}}{\pi}e^{\frac{z}{\pi^2 v^2}}\times$$
$$\times\left[\mathrm{erf}\left(\sqrt{\frac{z}{\pi^2 v^2}(1 - i\pi^2 v^2 u)}\right) - \mathrm{erf}\left(\sqrt{\frac{z}{\pi^2 v^2}}\right)\right]$$

error function

The number of complete oscillations of a channeled particle moving near the bottom of the OL channel.

The function $\boxed{F}$ included in the expression for the spectral-angular distribution is determined mainly by the ratio of the transverse beam size to the width of the OL channel and the radiation time. As the radiation time (the number of total channeled oscillations) increases, the behavior of the function $\boxed{F}$ rapidly tends to its limit value $\boxed{T \to \infty}$.

EM channeling radiation

At large distances, the squares of the Fourier moduli of the field potential transformations during the electron beam motion have the form

$$\delta_n^j = n\alpha^j + k_z z_0^j + k_y y_0^j$$

$$|\varphi_\omega|^2 = \frac{e^2}{R_0^2}\left[N\left|\sum_{n=-\infty}^{\infty} J_n(k_x x_j)e^{-i\delta_n^j}\frac{e^{i(\omega - k_z v_\parallel - n\Omega_j)T} - 1}{i(\omega - k_z v_\parallel - n\Omega_j)}\right|^2 + \right.$$
$$\left. + N(N-1)\left|\sum_{n=-\infty}^{\infty} J_n(k_x x_j)e^{-i\delta_n^j}\frac{e^{i(\omega - k_z v_\parallel - n\Omega_j)T} - 1}{i(\omega - k_z v_\parallel - n\Omega_j)}\right|^2\right]$$

Bessel function

$$|\mathbf{A}_\omega|^2 = \frac{e^2}{R_0^2}\left[N\left|\sum_{n=-\infty}^{\infty}\left(\beta_\parallel\mathbf{e}_z + \frac{n\Omega_j}{k_x c}\mathbf{e}_x\right)J_n(k_x x_j)e^{-i\delta_n^j}\frac{e^{i(\omega - k_z v_\parallel - n\Omega_j)T} - 1}{i(\omega - k_z v_\parallel - n\Omega_j)}\right|^2 + \right.$$
$$\left. + N(N-1)\left|\sum_{n=-\infty}^{\infty}\left(\beta_\parallel\mathbf{e}_z + \frac{n\Omega_j}{k_x c}\mathbf{e}_x\right)J_n(k_x x_j)e^{-i\delta_n^j}\frac{e^{i(\omega - k_z v_\parallel - n\Omega_j)T} - 1}{i(\omega - k_z v_\parallel - n\Omega_j)}\right|^2\right]$$

Spectral-angular distribution of radiation

Integration of the radiation energy over the polar $\boxed{\phi}$ angle gives the following expression:

$$\frac{d^2\mathcal{E}}{\sin\theta d\theta d\omega} = N(N-1)\frac{e^2\omega^6 d_{ch}^6\sin^4\theta}{(8\pi)^3 c^5\sigma_\perp^2\Omega_0^2}\left[\frac{4\Omega_0^2}{\omega^2\sin^2\theta}f_1\left(\frac{\omega^2\sigma_\perp^2}{2c^2}\sin^2\theta\right) - \right.$$
$$\left. - \gamma_\parallel^{-2} f_2\left(\frac{\omega^2\sigma_\perp^2}{2c^2}\sin^2\theta\right)\right] e^{-\frac{\omega^2}{c^2}\sigma_\parallel^2\cos^2\theta}\left|F\left(\frac{\omega}{\Omega_0}(1-\beta_\parallel\cos\theta) - 2, \Omega_0 T, \frac{\sigma_\perp}{d_{ch}}\right)\right|^2$$

$$f_2(\xi) = \pi e^{-\frac{\xi}{2}}\left[I_0\left(\frac{\xi}{2}\right) + \frac{\xi - 1}{\xi}I_1\left(\frac{\xi}{2}\right)\right]$$

$$f_1(\xi) = \pi e^{-\frac{\xi}{2}}\left[I_0\left(\frac{\xi}{2}\right) + I_1\left(\frac{\xi}{2}\right)\right]$$

modified Bessel functions

As can be seen from the expression above, the contribution to the coherent part of the spectrum comes from both the longitudinal and transverse dimensions of the beam. Moreover, the contribution of each of the beam parameters can be distinguished separately. This expression was obtained by neglecting the beam divergence and assuming the smallness of the argument of the Bessel functions determining the radiation field.

Conclusion

In the presented work the expression for the spectral-angular distribution of the coherent part of EM radiation by a channeled electron beam was obtained. We considered an electron beam symmetrically distributed with respect to the axis of the OL channel, and we also assumed that the beam was not divergent. The presented expression for the spectral-angular distribution is valid under the condition that the argument of the Bessel functions determining the radiation field is small, which corresponds to not very high intensities of the laser field forming the OL. At the moment, the work is not finished and will continue. In particular, the case of high intensities of the laser field forming the OL is considered.