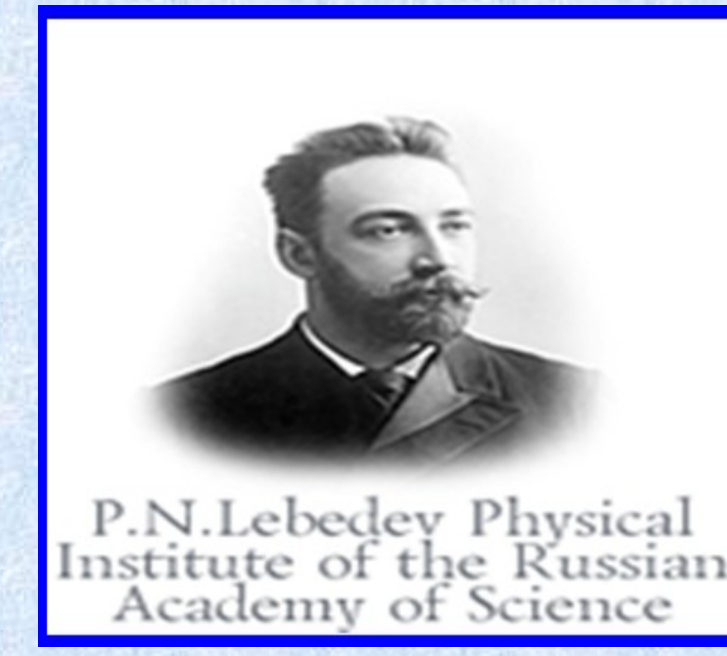# On charged particle dynamics near flat solid surface

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Effective potential of curved surface

ссылка на обзор?

As shown [*S.B. Dabagov, A.V. Dik. "Surface channeling of capillary-guided charged particles". **ArXiv: physics***], the elastic potential of a nonrelativistic but fast particle moving inside one solid cylindrical cavity can be presented in the form:

$$U_{eff}(r) = U_{at}(r) + U_{ind}(r)$$

the distance between the center of the cavity and the particle

the interaction potential of a charged particle with the field of surface excitations

the average interaction potential of a particle with surface atoms

$$U_{ind}(r) = -\frac{(eZ)^2 \omega_s}{2\pi} \mathcal{P} \int_{-q_0}^{q_0} \frac{dq}{q v_\parallel - \omega_s} \sum_{n=-\infty}^{\infty} \frac{(\epsilon_0-1)K_n'(qR_0)K_n(qR_0)}{I_n'(qR_0)K_n(qR_0) - \epsilon_0 K_n'(qR_0)I_n(qR_0)} I_n^2(qr)$$

where R_0 is a cavity radius, eZ is the charge of the particle, ω_s is the frequency of surface excitations, q_0 is a wave vector corresponding to the cutoff frequency, $v_\parallel$ is the particle longitudinal velocity (the speed parallel to the surface), $K_n(z), I_n(z)$ are the modified Bessel functions, ϵ_0 is the static permittivity of solid medium outside of the cavity, $\mathcal{P}$ means the principal value of the integral.

Effective Potential of flat surface

Evaluating the limiting case when a charged particle interacts with a surface of infinite radius of curvature $R_0 \to \infty$, i.e. a plane or flat surface, the potentials U_{at}, U_{ind} are determined as follows:

particle charge number   surface atomic number

$$U_{at}(z) = 2\pi n \int_0^\infty \frac{e^2 Z_1 Z_2}{\sqrt{z^2+r^2}} f\left(\frac{\sqrt{z^2+r^2}}{a}\right) r\, dr$$

$$U_{ind}(z) = \frac{(eZ_1)^2 \omega_s}{\pi v_\parallel} \frac{\epsilon_0-1}{\epsilon_0+1} G\left(2\frac{\omega_s}{v_\parallel}z\right)$$

screening function

$$G(\xi) = \int_0^\infty \left(\left[\text{Ei}\left(\xi\left(\cosh t - \frac{q_0 v_\parallel}{\omega_s}\right)\right) - \text{Ei}(\xi\cosh t)\right] e^{-\xi\cosh t} \right.$$
$$\left. - \left[\text{Ei}\left(-\xi\left(\cosh t + \frac{q_0 v_\parallel}{\omega_s}\right)\right) - \text{Ei}(-\xi\cosh t)\right] e^{\xi\cosh t} \right) dt$$
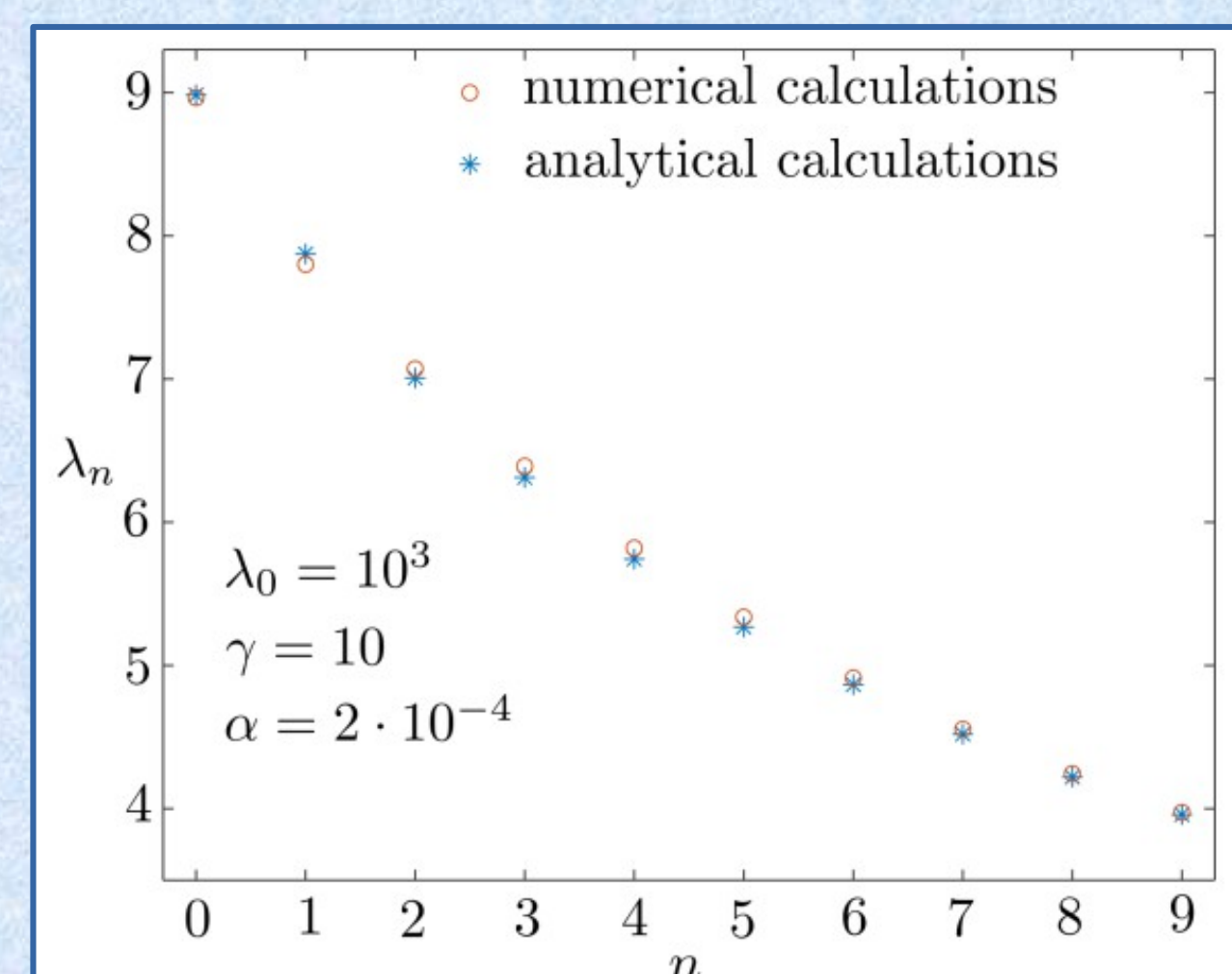
exponential integral

Critical angle

We are interested in the case of sufficiently large particle velocities $v_\parallel$, since it is for them that the continuous potential approximation is valid, so we will consider the case $q_0 v_\parallel/\omega_s \gg 1$, for which the expression for $U_{ind}(z)$ becomes much simpler

modified Bessel function   modified Struve function

$$U_{ind}(z) \approx -\pi\frac{(eZ_1)^2 \omega_s}{4v_\parallel}\frac{\epsilon_0-1}{\epsilon_0+1}\left(I_0\left(2\frac{\omega_s}{v_\parallel}z\right) - \boldsymbol{L}_0\left(2\frac{\omega_s}{v_\parallel}z\right)\right)$$

the effective potential describing the interaction of a charged particle moving at a small angle to a flat surface will take the form (here we use the screening function in Bohr approximation):

$$U_{eff}(z) = \frac{2\pi e^2 Z_1 Z_2 a}{\sigma_0} F\left(2\frac{\omega_s}{v_\parallel}z\right)$$

$$\gamma = \frac{v_\parallel}{2\omega_s a}$$

$$\alpha = \frac{Z_1 \omega_s \sigma_0}{8 v_\parallel Z_2 a}\frac{\epsilon_0-1}{\epsilon_0+1}$$

critical angle for particle capture

$$\theta_{cr} = \theta_{cr}^0 \sqrt{-F\left(2\frac{\omega_s}{v_\parallel}z_m\right)}$$

$$F(\xi) = e^{-\gamma\xi} - \tilde{\alpha}\left[I_0(\xi) - \boldsymbol{L}_0(\xi)\right]$$

$$\theta_{cr}^0 = \sqrt{\frac{2\pi e^2 Z_1 Z_2 a}{\sigma_0 E_\parallel}}$$

effective potential minimum coordinate

$$z_m \approx -a \ln\left(\frac{Z_1 \omega_s^2 \sigma_0(\epsilon_0-1)}{2\pi^2 v_\parallel^2 Z_2(\epsilon_0+1)}\right)$$

Channeled particle dynamics

Due to the non-simple form of the function $F(\xi)$, even the use of the WKB method does not allow one to find a solution of the equation describing channeled motion in an analytical form. For this reason, we replace the function $F(\xi)$ with the approximate function

$$F_{app}(\xi) = \frac{\alpha}{\pi}\left(\frac{\xi_m - c}{(\xi-c)^2} - \frac{2}{\xi-c}\right)$$

$$c = \xi_m + \frac{\alpha}{\pi F(\xi_m)}$$

$$\xi_m = 2\frac{\omega_s}{v_\parallel}z_m$$

With an approximate function $F_{app}(\xi)$, the dimensionless equation describing the channeled motion of a particle near the surface of a solid body takes the form

$$\lambda^2 = -\frac{E_\perp}{\delta\hbar\omega_s}$$

particle transverse energy

$$\delta = \frac{\hbar\omega_s}{E_\parallel}$$

$$\frac{d^2\psi}{d\xi^2} - \left(\lambda^2 + \lambda_0^2 F_{app}(\xi)\right)\psi(\xi) = 0$$

$$\lambda_0^2 = \frac{2\pi e^2 Z_1 Z_2}{\delta\hbar\omega_s}$$

particle longitudinal energy

Energy levels

Finite solutions of the dimensionless equation describing the channeled motion of a particle exist under the condition:

$$\lambda_n = \frac{\alpha\lambda_0^2}{\pi}\left[n + \frac{1}{2}\left(1 + \sqrt{1 - \left(\frac{2\alpha\lambda_0}{\pi}\right)^2 \frac{1}{F(\xi_m)}}\right)\right]^{-1}$$

Where n = 0,1,2... In the figure presents a numerical solution of the equation for transverse motion with the exact function $F(\xi)$ for some values of the parameters $\lambda_0, \alpha, \gamma$ and an approximate solution calculated analytically. The analysis shows that the difference between λ_n calculated using the analytical formula and the exact values does not exceed a few percent.

Energy levels

Thus, the levels of transverse motion of a channeled particle near the surface of a solid body take the form

particle mass   scrining parameter

$$E_\perp^n = -\frac{(eZ_1)^2 m(\epsilon_0-1)^2}{32\hbar^2(\epsilon_0+1)^2\left[n + \frac{1}{2}\left(1 + \sqrt{1 - \frac{1}{F(\xi_m)}\frac{a e^2 Z_1^3 m(\epsilon_0-1)^2}{16\pi\hbar^2 Z_2(\epsilon_0+1)^2}}\right)\right]^2}$$

As can be seen from the expression above, the dependence of the transverse energy levels on the longitudinal energy of the particle is contained in the quantity $F(\xi_m)$. For example, for an electron with a longitudinal energy in the range $E_\parallel = 10 \div 70$ keV interacting with a flat surface of a solid body (the characteristic frequency of surface excitations is $\omega_s = 10^{15}\text{sec}^{-1}$, and charge number Z_2=30) the transverse energy of ground state is $E_\perp^0 = 16 \div 6.5$ meV. As can be seen from the expressions transverse energy levels, with an increase of the longitudinal energy of the particle (the value $F(\xi_m)$ decreases), the depth of the potential well is decrease and, consequently, the modules energy of transverse states are also decrease.

Conclusion

In this work, the problem of the dynamics of a quantum charged particle in the field of the effective potential formed by the scattering averaged atomic potential of a flat surface of a solid body and the potential describing the elastic interaction of a particle with surface excitations of a solid body was considered. It is shown that a not deep potential well is formed in the effective potential which is capable of capturing particles in the channeling mode. The position of the minimum of the potential well depending on the longitudinal velocity of the particle is determined. The critical angle that determines the conditions for the capture of a particle in the channeling mode is found. It is shown that this angle is rather small and decreases with increasing energy. An approximate expression for the transverse energy levels is found. It is shown that the found approximate energy levels are in good agreement with those calculated exactly numerically.