Spin dynamics in nonuniform electromagnetic wave fields

A V Dik[1], S B Dabagov[1,2,3]

[1]LPI RAS, Moscow, Russian Federation; [2]INFN-LNF, Frascati, Italy; [3]NRNU MEPhI, Moscow, Russian Federation

P.N.Lebedev Physical Institute of the Russian Academy of Science

A fermion in a nonuniform EM wave field

We consider the interaction of a fermion with an arbitrary inhomogeneous field of an EM wave completely described by the vector potential written in common form (using the normalization, at which the scalar potential equals to zero):

$$\mathbf{A}(\mathbf{r},t)=\sum_{\mathbf{q}}\left(\mathbf{a}_{\mathbf{q}}e^{i(\mathbf{qr}-\omega_{\mathbf{q}}t)}+\mathbf{a}^*_{\mathbf{q}}e^{-i(\mathbf{qr}-\omega_{\mathbf{q}}t)}\right)$$

We represent the wave function of a particle as an expansion in terms of plane longitudinal waves (by longitudinal direction we mean the direction of the particle motion before interaction with the EM wave field):

$$\Omega_{\parallel}(k_z)=c\sqrt{k_z^2+(mc/\hbar)^2}$$

Longitudinal energy

$$\Psi(\mathbf{r},t)=\frac{1}{\sqrt{L}}\sum_{k_z}\psi_{k_z}(\mathbf{r}_\perp,t)e^{i(k_z z-\Omega_{\parallel}(k_z)t)}$$

We examine the fields, for which the quantities $\mathbf{a}_{\mathbf{q}},\mathbf{a}^*_{\mathbf{q}}$ are nonzero in some range of the wave vector $\mathbf{q}_0-\triangle\mathbf{q}\le\mathbf{q}\le\mathbf{q}_0+\triangle\mathbf{q}$. Additionally, we assume that the longitudinal momentum of a particle much exceeds the momentum maximum $k_z\gg|\mathbf{q}_0\pm\triangle\mathbf{q}|$. In other words the longitudinal energy of a charged particle is much greater than the energy of any photon in the EM wave field.

Effective Potential

The bispinor ψ_{k_z} can be represented as a product of slowly evoluting in time bispinor $\bar{\psi}_{k_z}$ and rapidly oscillating in time operator matrix $\hat{S}(t)$. A bispinor slowly varying in time in the first order of the expansion in terms of the longitudinal energy satisfies the equation:

$$i\hbar\frac{\partial\bar{\psi}_{k_z}}{\partial t}=\left(-\frac{(c\hbar)^2}{2E_{\parallel}}\nabla^2_\perp+U_{eff}\right)\bar{\psi}_{k_z}$$

the dash-on-top means averaging over the period of fast oscillations

elastic part of the effective potential

$$U_{eff}=\frac{e^2\overline{\mathbf{A}^2}}{2E_{\parallel}}+\frac{(c\hbar)^2}{2E_{\parallel}}\overline{(\nabla_\perp\phi)^2}-\frac{\hbar^2}{2E_{\parallel}}\overline{\left(\frac{\partial\phi}{\partial t}\right)^2}-\frac{ec\hbar}{E_{\parallel}}\overline{(\mathbf{A}_\perp,\nabla_\perp\phi)}$$

The effective potential of the elastic interaction for the fermion completely coincides with that for a scalar particle (*A.V. Dik, E.N. Frolov, S.B. Dabagov, J. Instrum.* ***13****, (C0218), 2018*). Spin effects appear only in the second order of expansion in longitudinal energy. For more details, see *A.V. Dik, S.B. Dabagov, Phys. Lett. B.* ***839****, (137786), 2023.*

$$E_{\parallel}=\hbar\Omega_{\parallel}(k_z)\qquad \phi=-\frac{e\beta_z}{\hbar}\int\limits_{t_0}^{t}A_z d\tau$$

Longitudinal velocity

$$\beta_z(k_z)=\frac{ck_z}{\Omega_{\parallel}(k_z)}$$

Spin dynamics in OL

We have considered the dynamics of the polarization vector $\boldsymbol{\zeta}$ of a particle in the field of an OR formed by two plane p-polarized waves crossed at an angle $\pi/2-\alpha$ in the semiclassical approximation. Based on the Bargmann-Michel-Telegdi equation and representing the polarization vector as a sum of smoothly varying $\bar{\boldsymbol{\zeta}}$ and rapidly oscillating $\boldsymbol{\xi}$ vectors, the following equations were obtained:

channeling trajectory

$$\frac{d\bar{\boldsymbol{\zeta}}}{dt}=\left[\bar{\boldsymbol{\Omega}}(t),\bar{\boldsymbol{\zeta}}\right]\qquad \bar{\boldsymbol{\Omega}}(t)=\mathbf{e}_y\bar{\Omega}_{amp}\sin\left(2k\bar{x}(t)\cos\alpha\right)\qquad \boldsymbol{\xi}(t)=-\left[\mathbf{a}(t),\bar{\boldsymbol{\zeta}}\right]$$

magnetic moment   anomalous magnetic moment

$$\omega=\omega_0(1-\beta_z\sin\alpha)$$

$$\bar{\Omega}_{amp}=2\frac{(\mu-\mu')eE_0^2\cos\alpha}{\hbar mc\omega_0(1-\beta_z\sin\alpha)}\left(\frac{\beta_z}{\gamma_{\parallel}^2}-\frac{\beta_z^2\sin\alpha}{(\gamma_{\parallel}+1)^2}+\frac{\beta_z+(\beta_z^2-2)\sin\alpha}{\gamma_{\parallel}^2(\gamma_{\parallel}+1)(1-\beta_z\sin\alpha)}\right)$$

$$b_1=2\frac{\mu+\mu'(\gamma_{\parallel}-1)}{\gamma_{\parallel}}$$

laser field frequency   longitudinal Lorentz factor

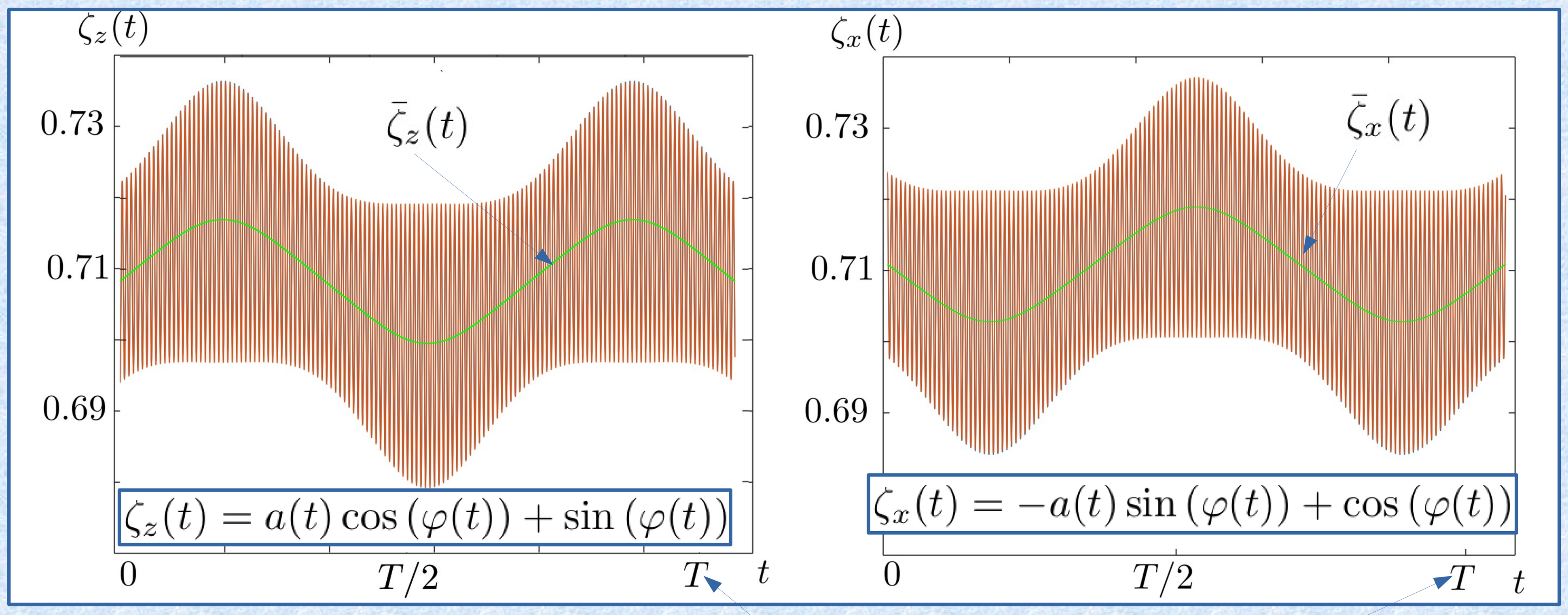
laser field strength   initial longitudinal coordinate of the particle

laser wave vector

$$b_3=2\frac{\mu+\mu'\gamma_{\parallel}}{\gamma_{\parallel}+1}$$

$$\mathbf{a}(t)=\mathbf{e}_y 2E_0\frac{b_1-b_3\beta_z\sin\alpha}{\hbar\omega_0(1-\beta_z\sin\alpha)}\cos\left(k\bar{x}(t)\cos\alpha\right)\cos\left(\omega t-kz_0\sin\alpha\right)$$

Dynamics of the polarization vector



channeled oscillation period

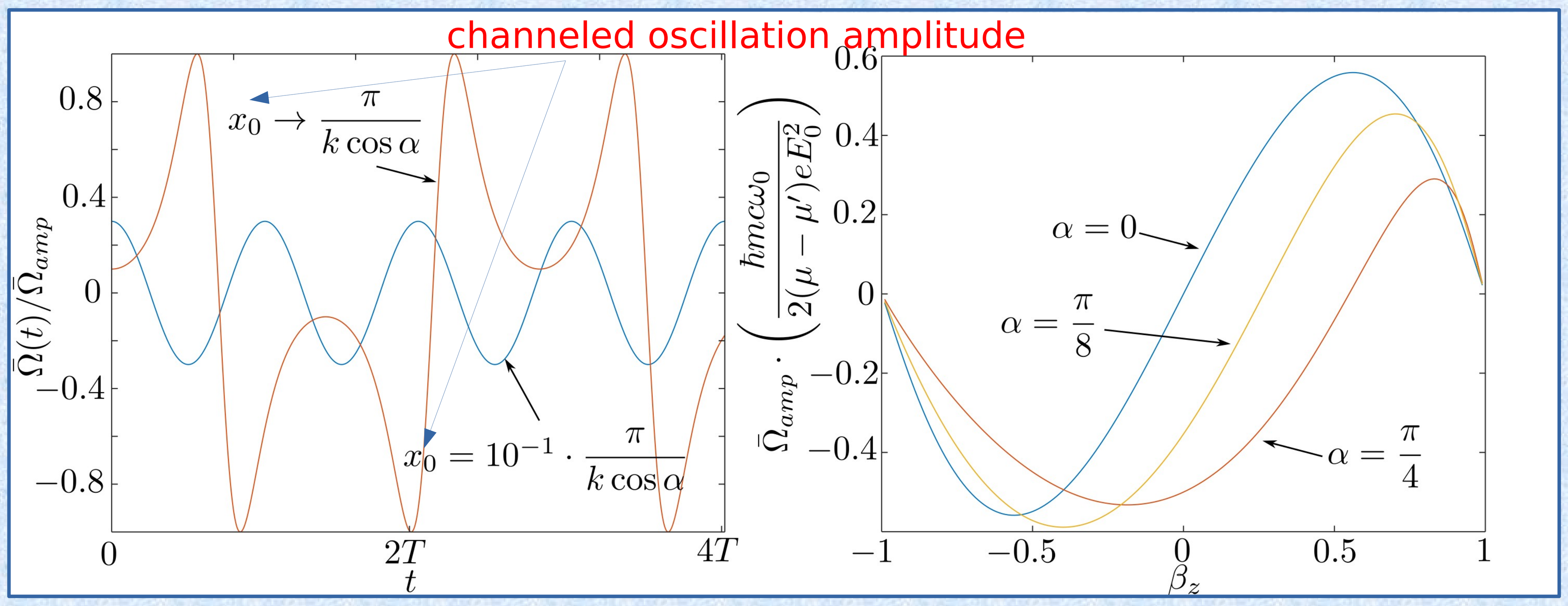
$$\varphi(t)=\varphi_0-\int\limits_0^t\bar{\Omega}(\tau)d\tau$$

initial phase

Time dependence of the projections of the electron polarization vector with the longitudinal Lorentz factor equal 50 moving in the field of the OL formed by plane waves with intensity 10^{15} W/cm^2 and frequency 10^{15} s^{-1}.

Angular velocity of smooth precession

channeled oscillation amplitude



Time dependence of the dimensionless average spin precession angular velocity for two real trajectories of channeled particle

Dependence of the dimensionless amplitude of the average spin precession angular velocity on the longitudinal particle velocity for 3 different angles

Possible application

The maximum angular velocity of smooth precession can reach 10^9 s^{-1}. Despite such a large value of the maximum value of the averaged angular velocity, one could not expect a rotation of the polarization vector through large angles, due to the periodicity of the phase phi caused by periodic oscillations of the particle itself in the direct OL. However, the situation should be changed in the case of a curved optical lattice similar to that of a bent crystal. Below are estimates of the rotation angle of the polarization vector for some short-lived particles in a curved microwave laser field with intensity 10^{23} W/cm^2.

particle	lifetime τ 10^{-12} s	mass mc^2 MeV	rotation angle $\triangle\varphi$ rad, for $\gamma=10$
Λ_c^+	0.2	2286.46	0.11
Ξ_c^+	0.44	2467.8	0.208
Ξ_c^0	0.1	2470.88	0.047
Ω_b^-	1.1	6071	0.086
Λ_b^0	1.425	5619.4	0.13
Ξ_b^0	1.49	5788	0.128
Ξ_b^-	1.56	5791	0.134

laser field intensity

elementary charge

rotation angle of the polarization vector

$$\triangle\varphi\sim 8\pi\frac{e^2}{\omega_0\gamma m^2c^3}I\mathcal{T}$$

laser field frequency   particle lifetime

particle mass

particle Lorentz factor

C o n c l u s i o n

In this work we show that for a fermion interacting with the field of arbitrary nonuniform electromagnetic wave the equation describing the dynamics of a bispinor slowly varying in time coincides with that for a scalar particle in the first order of the expansion in terms of the longitudinal energy. The spin effects in the equation for a slowly varying bispinor become essential only in the second order of expansion in longitudinal energy for ultrarelativistic particles, while for nonrelativistic ones in the third order of expansion in longitudinal energy. Analyzing the dynamics of an arbitrary spin for a charged particle in an OL field, we have shown that the dynamics of the polarization vector reveals a smooth precession, over which fast small oscillations are superimposed. The rate of smooth precession is proportional to the difference between the magnetic moment of the particle and its anomalous part. An estimate of the angular velocity of the spin vector precession is also found, which can reach rather large values ~10^9 s^{-1}.