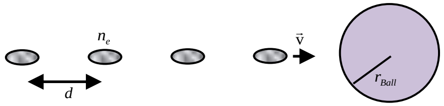

Spectral-Angular Distribution of Radiation Generated by Train of Electron Bunches Passing Through the Centre of a ball

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Introduction and formulation of the problem

The influence of matter on electromagnetic processes covers a wide range of phenomena that have found a number of important practical applications. Our work is devoted to this topic.

In it the spectral-angular distribution of radiation generated by train of electron bunches passing through the center of a ball is studied.

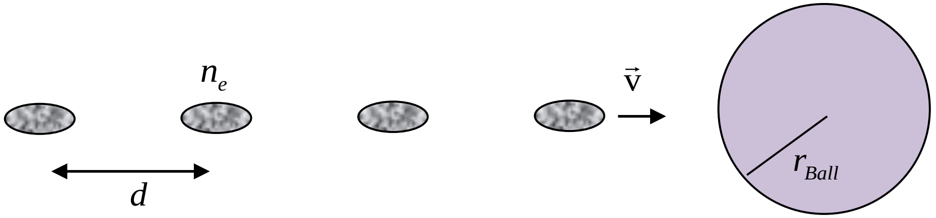
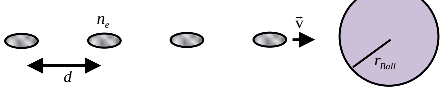


Fig.1

The ball can be dielectric, conductive, or can be made of a composite material.



Analytical calculations

The spectral-angular and spectral distributions of radiation generated by **a single electron** [1,2]

$$\frac{dJ_1}{d\omega d\theta} = 2\pi sin\theta \, \frac{c}{\sqrt{\varepsilon_1}} \left| \sum_l a_2^{(l)} \vec{X}_{l0}^{(2)} \right| \equiv I_1^*(\omega, \theta) \qquad (1)$$

$$\frac{dJ_1}{d\omega} = \frac{c}{\sqrt{\varepsilon_1}} \sum_l |a_2^{(l)}|^2 \equiv I_1(\omega)$$

Here $\vec{X}_{lm}^{(2)}$ is one of the spherical vectors:

$$\vec{X}_{lm}^{(2)} = \frac{\vec{\nabla}_{\vec{n}} Y_{lm}}{\sqrt{l(l+1)}}$$

[1] S.R. Arzumanyan, J. Phys.: Conf. Series, 357, (2012) 012008.

[2] L.Sh. Grigoryan, A.H. Mkrtchyan, H.F. Khachatryan, Proceedings of the Int. Conf. on "Electron, Positron, Neutron and X–Ray Scattering under External Influences", Armenia, Sept.14–20, 2015, Yerevan 2016, 47-52.

Radiation of a single electron

In 2012, 2015 the radiation of a single relativistic electron was studied, which, with constant velocity, passes through the center of a ball [1,2].

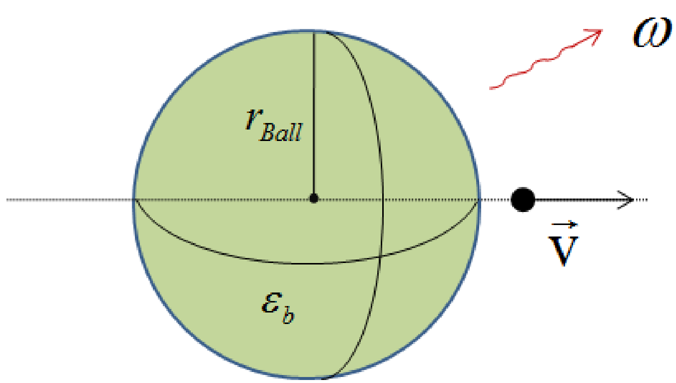
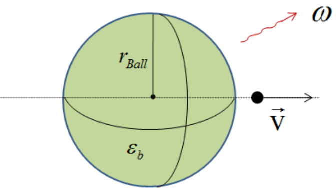


Fig. 2

An arbitrary dielectric function $\varepsilon(\omega)$, $\mu = 1.$

[1] S.R. Arzumanyan, J. Phys.: Conf. Series, 357, (2012) 012008.

[2] L.Sh. Grigoryan, A.H. Mkrtchyan, H.F. Khachatryan, Proceedings of the Int. Conf. on "Electron, Positron, Neutron and X–Ray Scattering under External Influences", Armenia, Sept.14–20, 2015, Yerevan 2016, 47-52.



Ball made of fused quartz

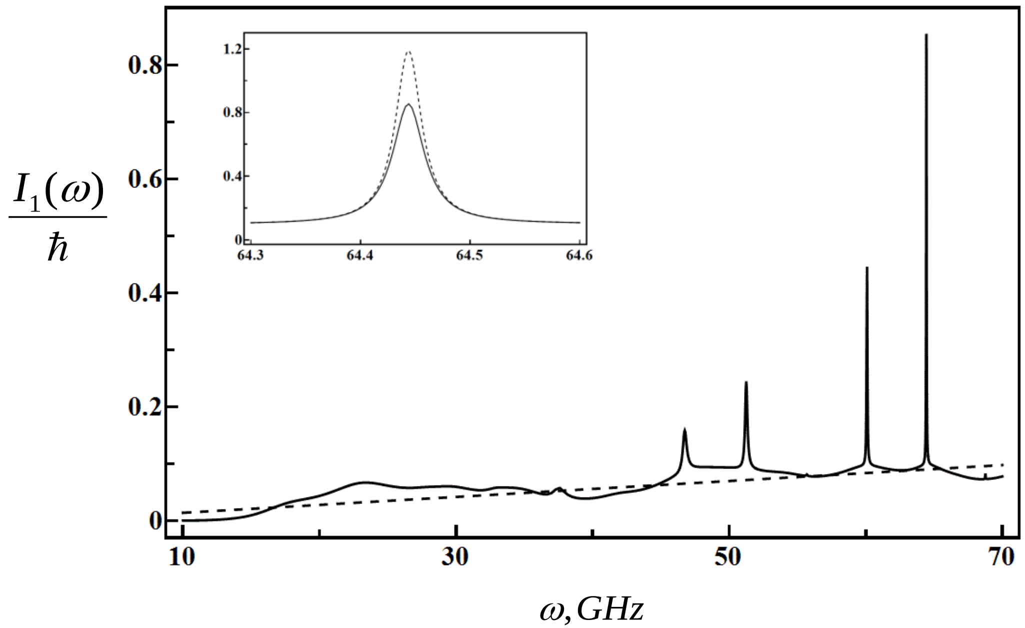
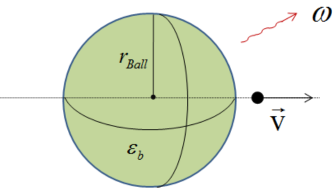


Fig.3 Spectral distribution of the radiation energy of a single electron with 2 MeV energy flying through the center of a ball with a dielectric permittivity

(fused quartz) $$\varepsilon_0 = \varepsilon_0' + i\varepsilon_0'' = 3.78(1 + 0.0001i)$$

and radius $r_{Ball} = 4$ cm (solid curve). The dotted curve corresponds to the motion of an electron in an infinite medium with $\varepsilon_0 = \varepsilon_0'$ and the condition that the radiation is accumulated along the path length $2r_{Ball}$. Cherenkov condition is satisfied:

$$v > c/\sqrt{\varepsilon_0'}$$



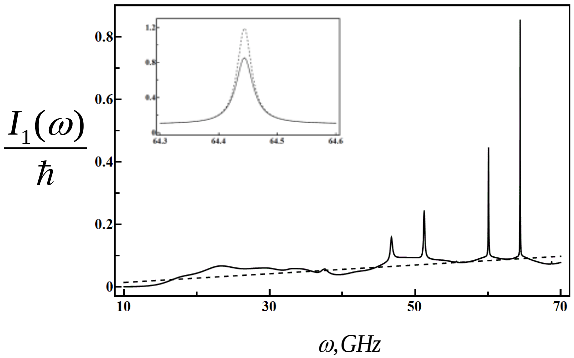


Fig.3

Summary

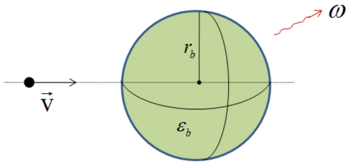
1.

On certain "resonant” frequencies ω_{res} with $\lambda_{res} \sim r_{Ball}$

sharp peaks are observed, whose height is almost an order of magnitude higher than that at neighboring frequencies.

2.

Numerical calculations show that allowance for the dielectric energy losses in the ball material practically does not affect the radiation intensity, except for the neighborhoods of the "resonant" frequencies. In the vicinity of these allocated frequencies, even small losses of the radiation energy in the ball material (as, for example, in fused quartz) noticeably reduce the radiation intensity. This fact is reflected in the graph shown in the upper left corner of Fig.3, where the dashed curve corresponds to the case of absence of dielectric losses.



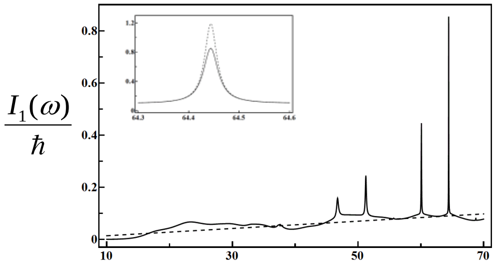


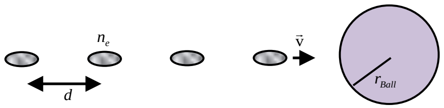
Fig.3

Summary

3. The presence of peaks is due to the constructive superposition of electromagnetic waves generated by a charged particle inside the ball and repeatedly reflected from its internal surface.

In the work [3] it is proposed to use the presence of narrow resonance peaks in the Cherenkov Radiation (CR) spectrum of a relativistic electron for the generation of coherent CR by a train of electron bunches passing through the center of the dielectric ball.

[3] A.R. Mkrtchyan, P.A. Aleksandrov, L.Sh. Grigoryan, A.H. Mkrtchyan, H.F. Khachatryan, M.V. Kovalchuk, Proceedings of the Int. Conf. on "Electron, Positron, Neutron and X–Ray Scattering under External Influences", Part 1, Armenia, Oct.21–26, Yerevan, 2019, 40-48.



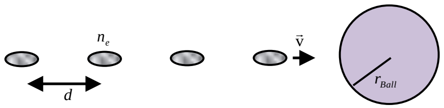
Train of bunches

The energy of radiation generated by a train of bunches

$$\int F(\omega) I_1(\omega) d\omega \equiv \int I(\omega) d\omega \qquad (1)$$

$I_1(\omega)$ is the spectral density of energy emitted by a single charge

$F(\omega)$ is the structural factor of a train of electron bunches.



Structural factor

$$F = n_e[1 - f_e(\omega) f_{tr}(\omega)]n_b + n_e^2 f_e(\omega) n_b^2 f_{tr}(\omega) \qquad (2)$$

is determined by the coherence factor of the radiation of electrons inside the bunches:

$$f_e = \exp(-\omega^2 \sigma^2 / \mathrm{v}^2) \qquad (3)$$

(Gaussian distribution of electrons with root-mean-square deviation is assumed σ)

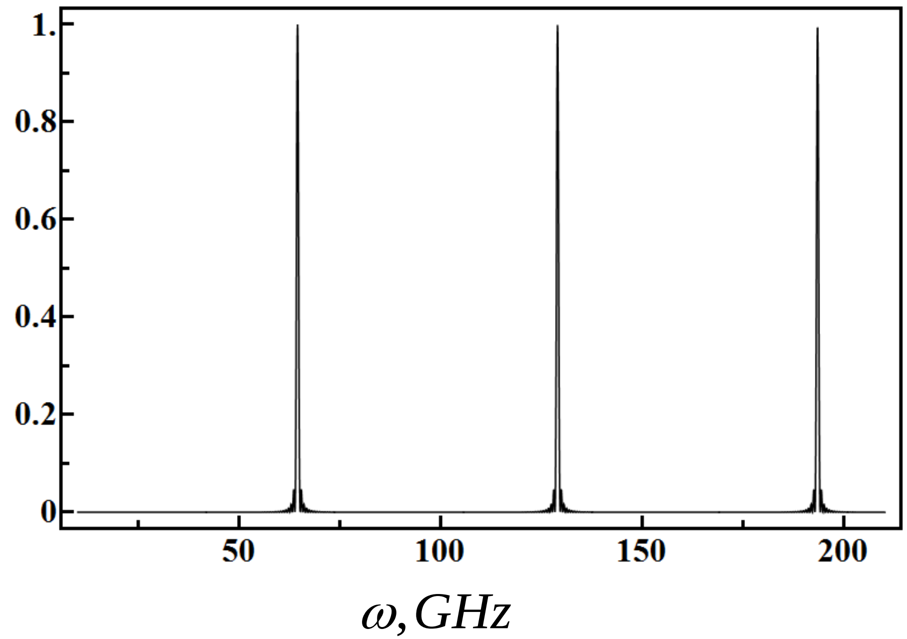
and the coherence factor of the radiation of the bunches inside the train:

$$f_{tr} = \frac{\sin^2(\omega d\, n_b / 2\mathrm{v})}{n_b^2 \sin^2(\omega d / 2\mathrm{v})} \qquad (4)$$

d is the distance between bunches, n_e is the number of electrons in the bunch,

n_b is the number of bunches in the train.

$$\frac{\sin^2(\omega d n_b / 2\mathrm{v})}{n_b^2 \sin^2(\omega d / 2\mathrm{v})} = f_{tr}$$



$n_b = 100,$
$d \approx 2.8cm,$
$E_e = 2MeV$

Fig. 4 Dependence of coherence factor of radiation of bunches inside the train on the cyclic frequency of the emitted wave.

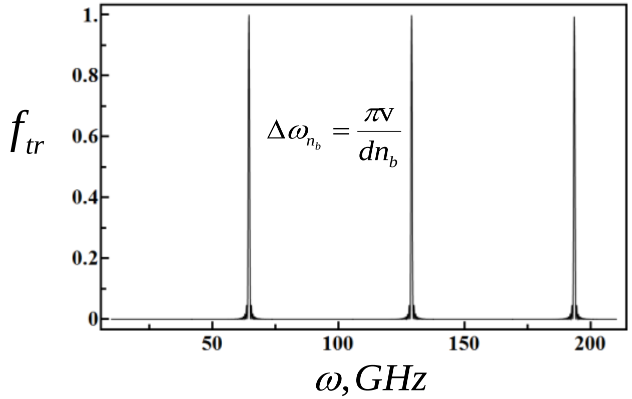
The train of bunches radiates coherently $f_{tr}(\omega) = 1$ at discrete frequencies

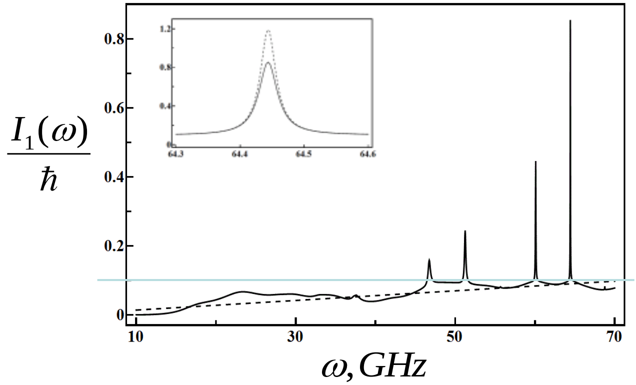
$$\omega = \frac{2\pi\mathrm{v}}{d} m, \quad m = 1,2...$$

(the frequency of the emitted electromagnetic waves is proportional to the frequency of succession of bunches).

and quasi-coherently $0.5 < f_{tr}(\omega) \le 1$ in the vicinity of these frequencies with the width:

$$\Delta\omega_{n_b} = \frac{\pi\mathrm{v}}{d n_b} \sim \frac{1}{n_b}$$





Comparing the data in figures one comes to the conclusion that

there is a unique situation when

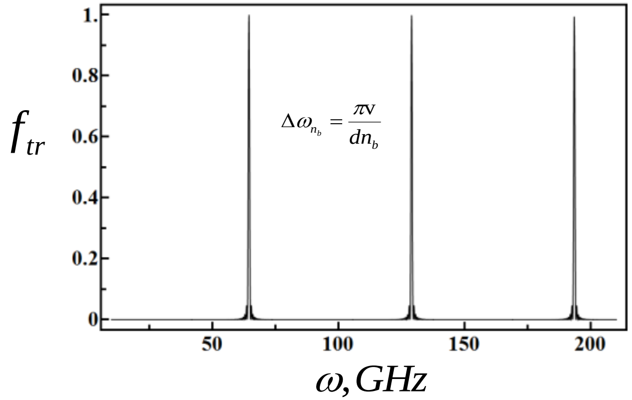
(a) one of the resonant frequencies of the ball turns out to be equal to the frequency of succession of the bunches

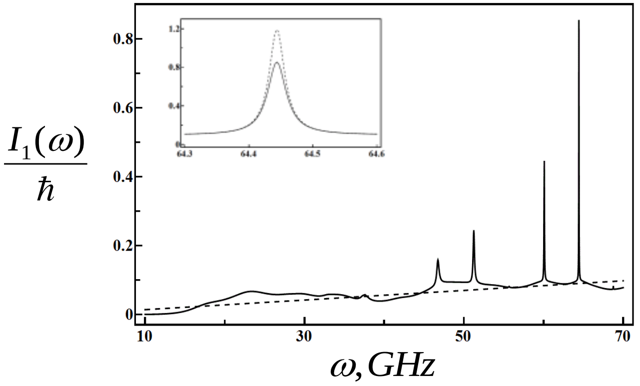
$$\omega_{res} = \frac{2\pi v}{d},$$

since in this case the train of bunches will radiate coherently at this resonant frequency of the ball, with the possible greatest spectral density:

and when

(b) $\Delta\omega << \Delta\omega_{n_b}$ i.e. $\boxed{n_b << \frac{\pi v}{d \cdot \Delta\omega}}$





since in this case the train of bunches will radiate coherently

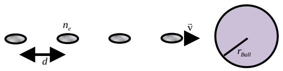
$$I(\omega) \approx n_e^2 f_e(\omega_{res}) n_b^2 I_1(\omega_{res}) \qquad f_{tr}(\omega_{res}) = 1$$

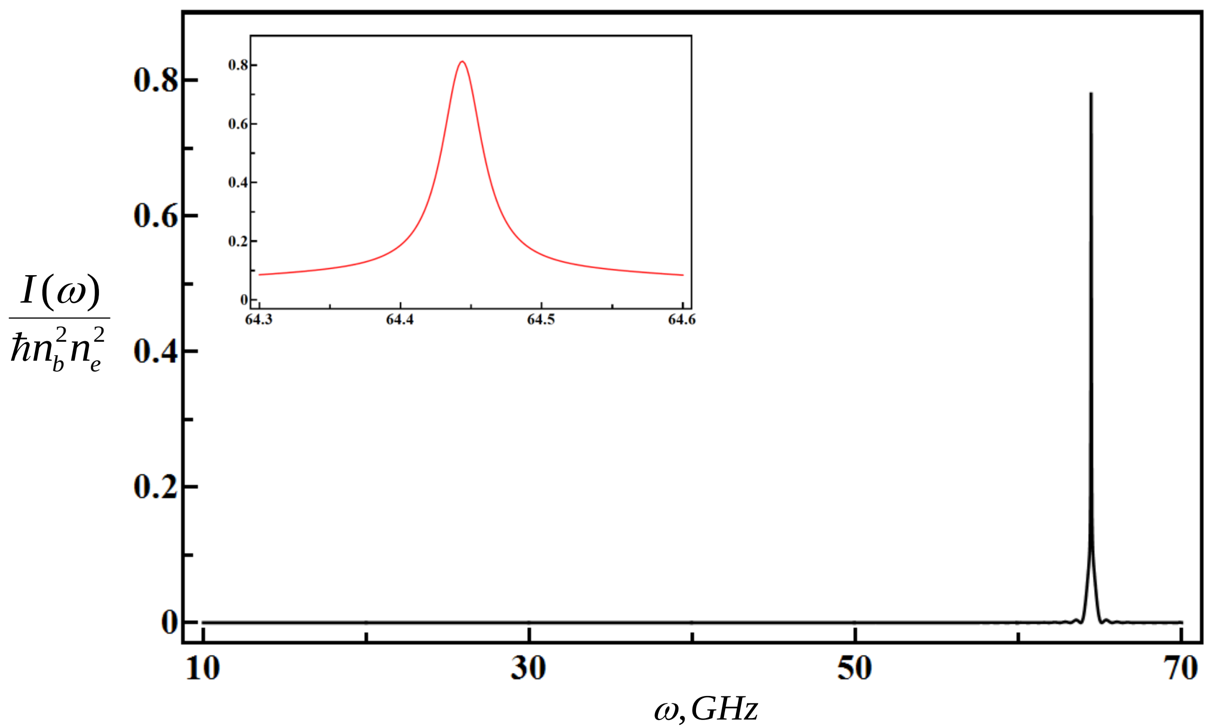
over the entire frequency range

$$\omega \in [\omega_{res} - \Delta\omega/2, \omega_{res} + \Delta\omega/2] \tag{5}$$

Next, we choose certain values of the parameters of the radiating system and estimate the integrated radiation power inside the highest peak in the spectral distribution in Fig. 4, namely in the frequency range (5).

Spectral distribution in the case of **train of bunches**





$n_b = 100$

$n_e = 10^9$

$\sigma = 0.1 cm$

$E_e = 2 MeV$

$\varepsilon = 3.78(1 + 0.0001i)$

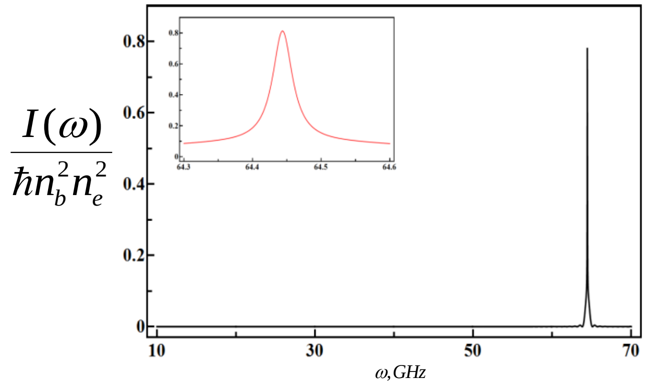
$d = 2.8258\, cm$

$r_{Ball} = 4\, cm$

Fig.5. Spectral distribution of radiation energy generated by a train of electron bunches flying through a dielectric ball made of fused quartz

$E_e = 2MeV \quad \sigma = 0.1cm$

$\varepsilon = 3.78(1 + 0.0001i)$



$n_e = 10^9$

$n_b = 100$

$d = 2.8258\,cm$

The power of a narrow-band quasi-coherent radiation, generated by a train of bunches in the range

$$\omega_{res} \pm \Delta\omega/2 \qquad \omega_{res} \approx 64.45GHz \qquad \Delta\omega \sim 50MHz$$

is equal to

$$\boxed{P \cong \frac{\mathrm{v}}{l} \int_{\omega_{res}-\Delta\omega/2}^{\omega_{res}+\Delta\omega/2} F(\omega) I_1(\omega) d\omega \cong \frac{\mathrm{v}}{l} \Delta W \cdot n_e^2 \cdot n_b^2 \sim 1kWt}$$

Here

$$\Delta W = \int_{\omega_0-\Delta\omega/2}^{\omega_0+\Delta\omega/2} I_1(\omega) d\omega \sim 1.3 \cdot 10^{-8} eV, \qquad l \cong n_b d$$

Spectral-angular distribution in the case of single electron

Over a wide frequency range

In the vicinity of the "resonant" frequency

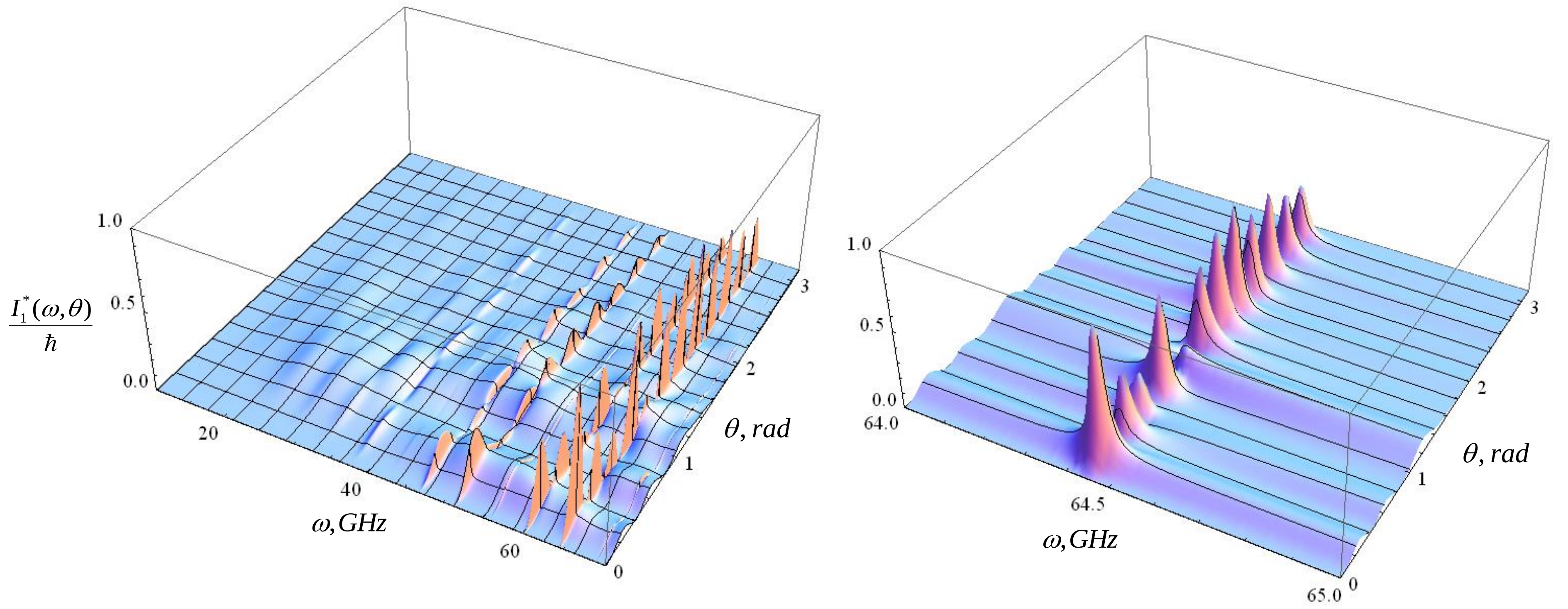
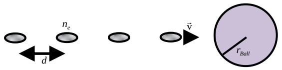
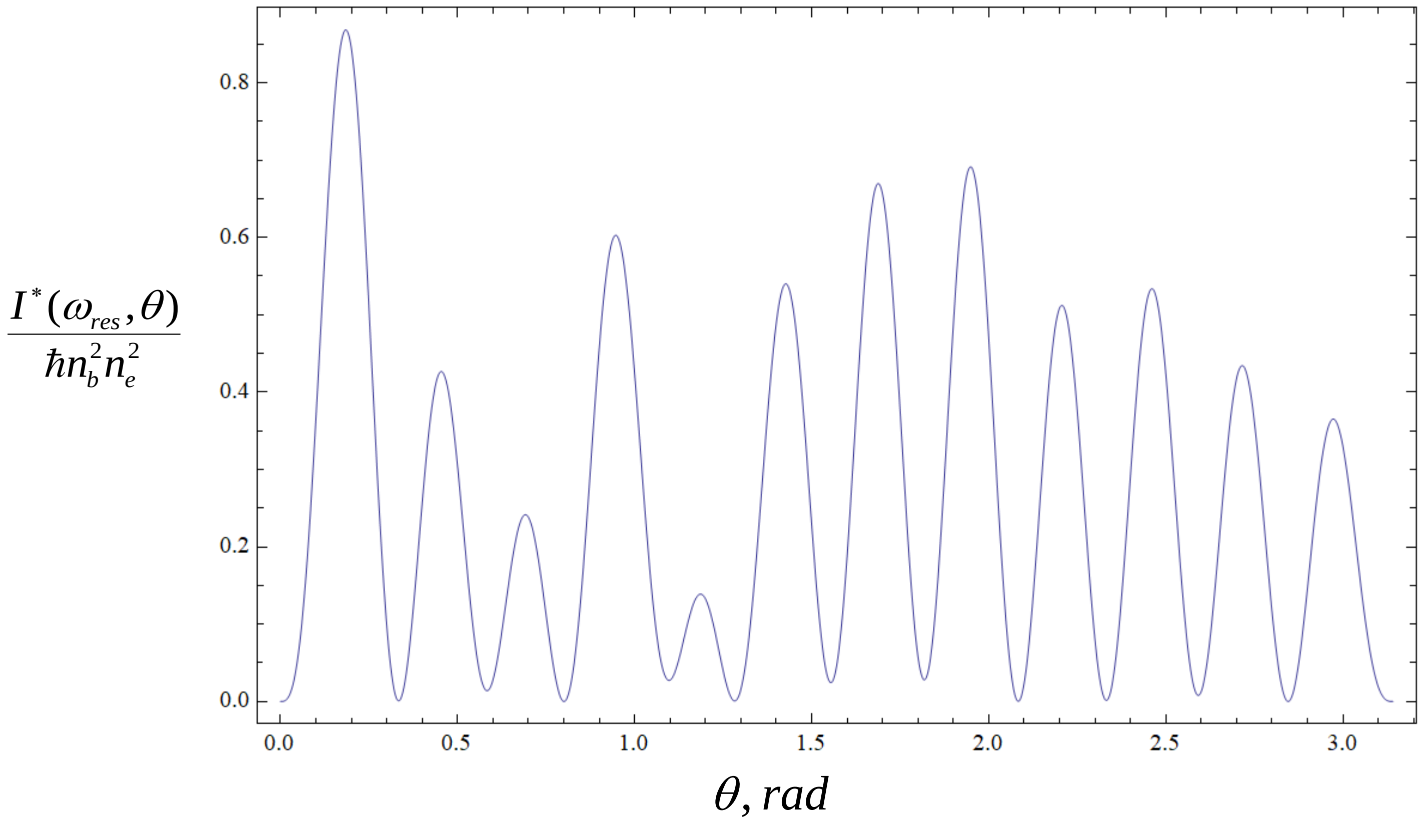


Fig.5. Spectral-angular distribution of radiation energy generated by an electron flying through a dielectric ball made of fused quartz

$E_e = 2MeV$ $\quad \varepsilon = 3.78(1+0.0001i)$ $\quad r_{Ball} = 4\,cm$

Angular distribution in the case of train of bunches





$n_b = 100$

$n_e = 10^9$

$\sigma = 0.1 cm$

$E_e = 2 MeV$

$\varepsilon = 3.78(1+0.0001i)$

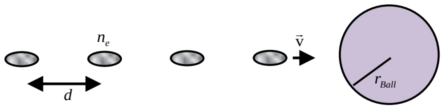
$d = 2.8258\,cm$

$r_{Ball} = 4\,cm$

$\omega_{res} \approx 64.45 GHz$

$\frac{F(\omega_{res})}{n_b^2 n_e^2} = 0.95$

Fig.6. Angular distribution of radiation energy generated by a train of electron bunches flying through a dielectric ball made of fused quartz



Conclusions

1. The possibility of generation of coherent CR from the train of equidistant one-dimensional electron bunches flying through the center of a ball made of a dielectric, a conductor, or of a composite material is studied.

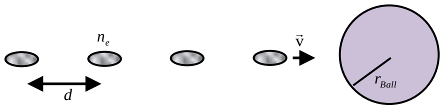
2. In the case of a ball of fused quartz, it was shown that for a special choice of the distance between bunches, $d \cong 2.8$ cm

resonant coherent CR of 100 bunches is formed in the neighborhood

of resonance frequency $\omega_0 \approx 64.45 GHz$ in a narrow frequency band $\Delta\omega \cong 50 MHz$.

3. The spectral-angular distribution in the case of a single particle and

the angular distribution of radiation at the resonant frequency in the case of a train of bunches are also presented.



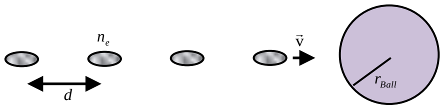
Conclusions

4. In a real situation (a) a train of not one but three-dimensional bunches are generated, and (b) this train must move along a hollow channel cut inside the ball (to reduce ionization losses, see e.g. [4])

The influence of these factors will be insignificant if the radius of the channel is much smaller than the wavelength and larger than the transverse dimensions of the bunch.

5. One can use this phenomenon for the development of powerful and narrow-band sources of electromagnetic waves in the Giga-Terahertz frequency range.

[4] Tyukhtin, A.V., Galyamin, S.N., Vorobev, V.V., Journal of the Optical Society of America B: Optical Physics, 2021, 38(3), pp. 711–718



Thank you

for your attention !