Reduction of multiple scattering of positively charged ultra relativistic particles channelling in planar fields of single crystals

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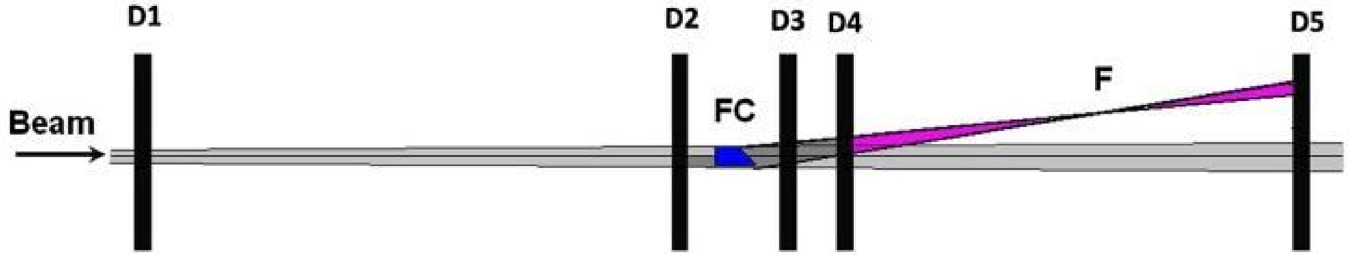
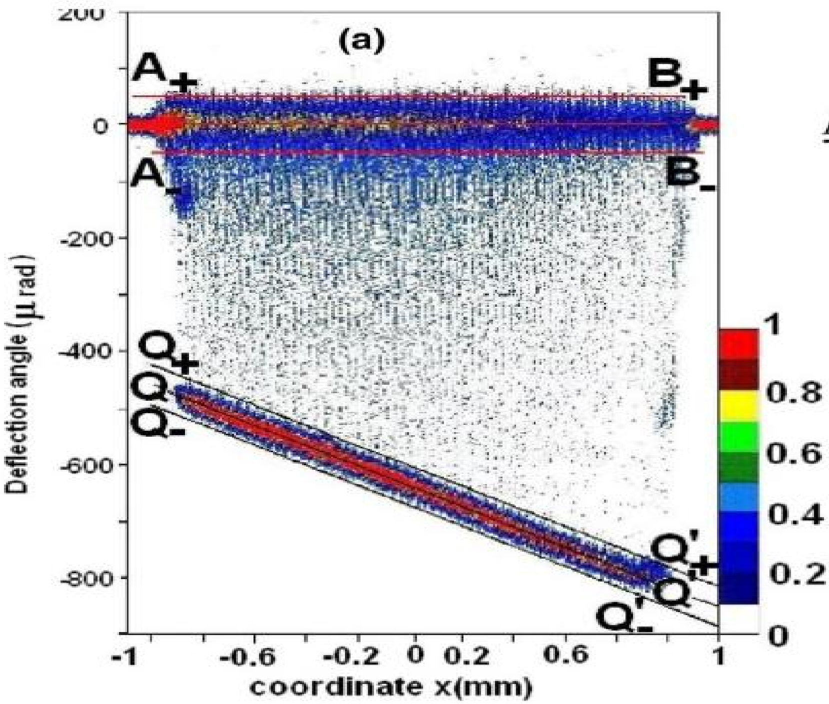
Recently the effect of reduction of multiple scattering of positively charged particles was observed at channeling in bent (111) and (110) planes of silicon[1]. The effect is observed in the plane orthogonal to the bending plane. The degree of reduction of rms scattering angle channeling particles (in comparing with equivalent amorphous media) reached up to 8 times. The analytical theory of observed phenomenon was proposed in the article[2].In this report we present the improved variant of the theory and give the results of calculation of reduction of multiple scattering angles for particles channeling in (111) and (110) planar electric fields of silicon and germanium single crystals. The increase in the angle of multiple scattering (in comparison with an equivalent amorphous medium) during planar channeling of negatively charged particles is also discussed.

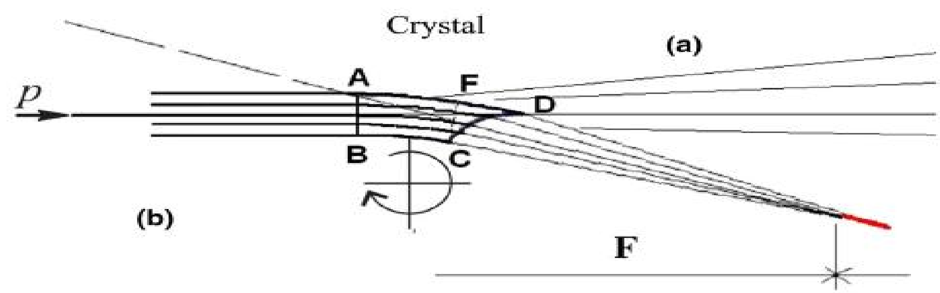
References

1. W. Scandale et.al.Eur. Phys. J. Plus (2022) 137:811 https://doi.org/10.1140/epjp/s13360-022-03034-
2. Yu.A. Chesnokov and V.A. Maisheev Nucl. Instr. Meth. B 486, 11 (2021)



A schematic view of the layout of the experiment for the measurement of multiple scattering of protons in the crystal deflectors. D1-D5 are the silicon microstrip detectors, FC is a bent focusing silicon crystal. Part of the beam after the bent crystal is deflected and focused at point F (focal point)

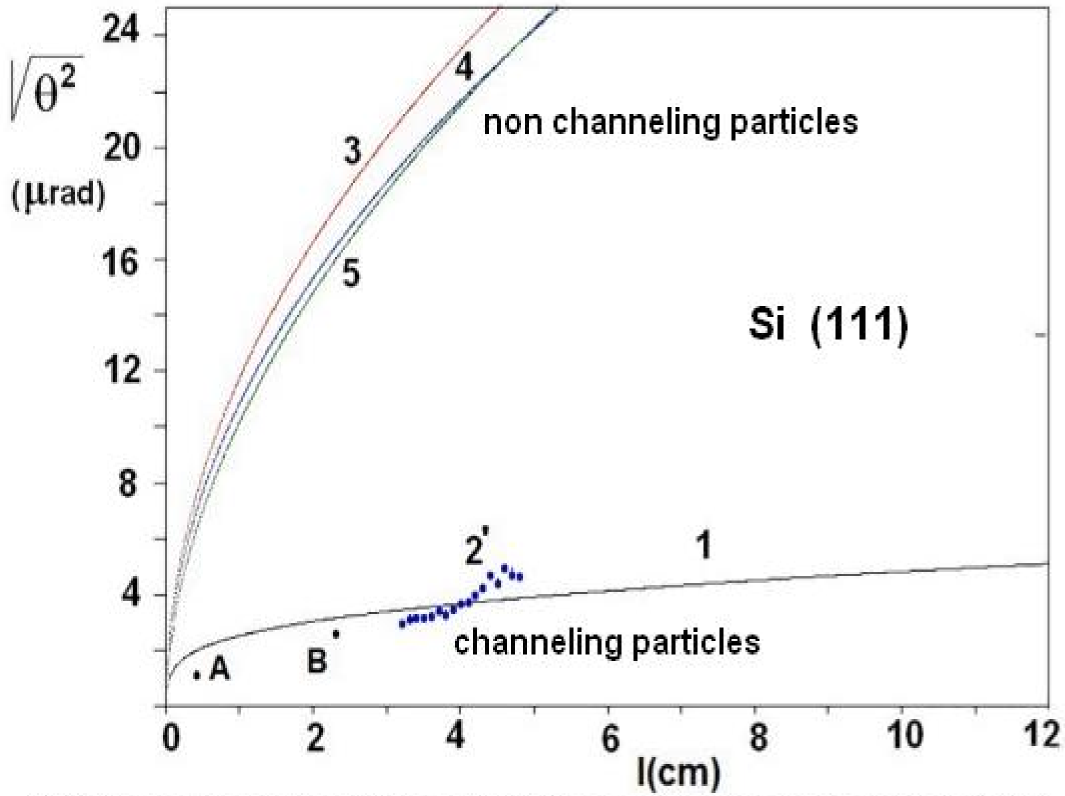




Focusing crystal: operating principle

the measured 2 dimensional plot

deflection angle versus tansverse coordinate



First measurements of rms of multiple scattering angle (blue points) as functions of the thickness of crystal.

The curve 1 is calculation to our simulation model.
The curves 3,4,5 are prediction for equavalent amorphous medium They are close to measurements (within 10-15%)

Model for simulation of the process

Schematic representation of the scattering process of a particle channeling in a crystal.

In our further calculation we will use the equation

$$\overline{\theta}^2 = N_n l \int_{b_{min}}^{b_{max}} \theta^2\left(b\right) 2\pi b db = \frac{8\pi N_n Z^2 e^4 l}{p^2 v^2} \ln \frac{b_{max}}{b_{min}} \qquad (14)$$

Here N_n is the mean nuclear density, b is the impact parameter of a particle and $\theta(b) = 2Ze^2/(pbv)$. According to Ref. [17] $b_{max} = 137^2 r_e Z^{-1/3}$ and $b_{min} = 0.57 r_e Z^{1/3}$, where r_e is the electron radius. For silicon $Z = 14$and $N_n = 8/a_{Si}^3$ (8 is the number atoms in an elementary cell $a_{Si} = 5.43\text{Å}$ is the size of cube of elementary cell) and hence we get $b_{min} = 3.87 \times 10^{-13} cm$, $b_{max} = 2.2 \times 10^{-9} cm$ $\ln(b_{max}/b_{min}) = 8.64$ and we obtain the equation for the mean square angle of multiple scattering of positively charged particles channeling in crystallographic planes

$$\left\langle \theta^2\left(L\right)\right\rangle = \frac{4\pi N_a Z^2 e^4}{p^2 c^2}\int_0^L \int_0^{\xi_{max}} \frac{dN}{d\xi}\left(\xi, l\right) F_m\left(\xi\right) d\xi dl$$

In this equation N_a is the effective atomic density. In Eq. (14) the value N_n is the mean atomic density of amorphous medium. The next subsection is devoted to finding this value for our case. It is important the distribution functions $\frac{dN}{d\xi}(\xi, l\Big)$ should be normalized on unit for every l.

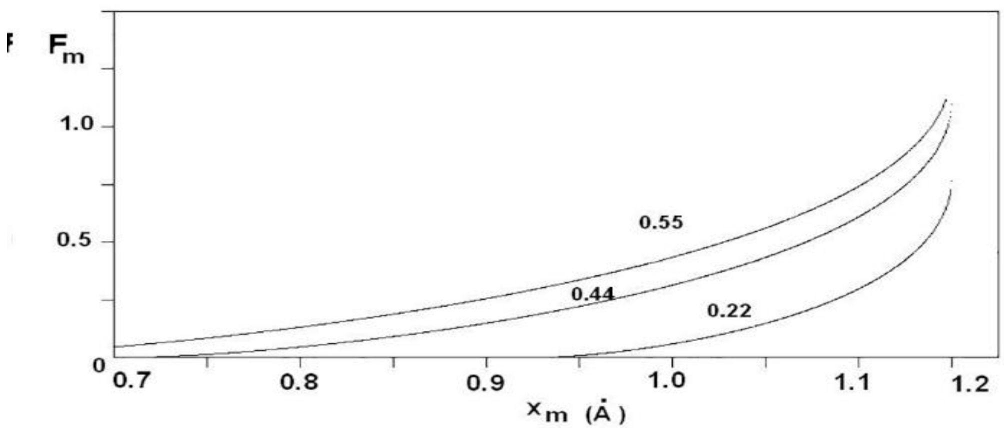
$$\frac{dN}{d\xi_m}\left(\xi_m\right) = \frac{\pi c\,\mathscr{E}(x_m)}{2N\theta_c E_0 \omega(\xi_m)}$$

Here we present the initial distribution over amplitudes x_m of particle motion and $\xi = 2x_m/d$, where d is interplanar distance, $\mathscr{E}(x_m)$ is the planar electric field, $\omega(\xi_m)$ is the frequency of particle motion.

It can be seen that the magnitude of the scattering angle is determined by the value of the logarithm of the ratio of the minimum and maximum impact factors. For a channeling particle, this ratio at each moment of time depends on the position of the particle in the interplanar space. So if a particle is located at approximately the same distance from neighboring planes, then it practically does not scatter. If a particle reaches a maximum deviation (i.e., is located near nuclei) then its scattering will be maximum. It follows that in order to properly account for the factor mentioned here, it is necessary to take the value of the logarithm averaged by the period of movement.

$$F_m\left(x_m\right) = \frac{1}{2\pi}\int_0^{2\pi} \ln\left\{\frac{d/2 - x_m|\sin t|}{b_{max}}\right\}\tau\left(d\Big/2 - x_m\Big|\sin t\Big|\right)dt, \qquad (17)$$

where function $\tau(x)$ is equal to 1 if $|x| < b_{max}$, and 0 if $|x| > b_{max}$. Fig. illustrates the behaviour of $F_m(x_m)$ functions at different amplitudes. At $x_m = d/2$ these functions tend to infinity. However, this does not matter because particles are practically absent on these coordinates



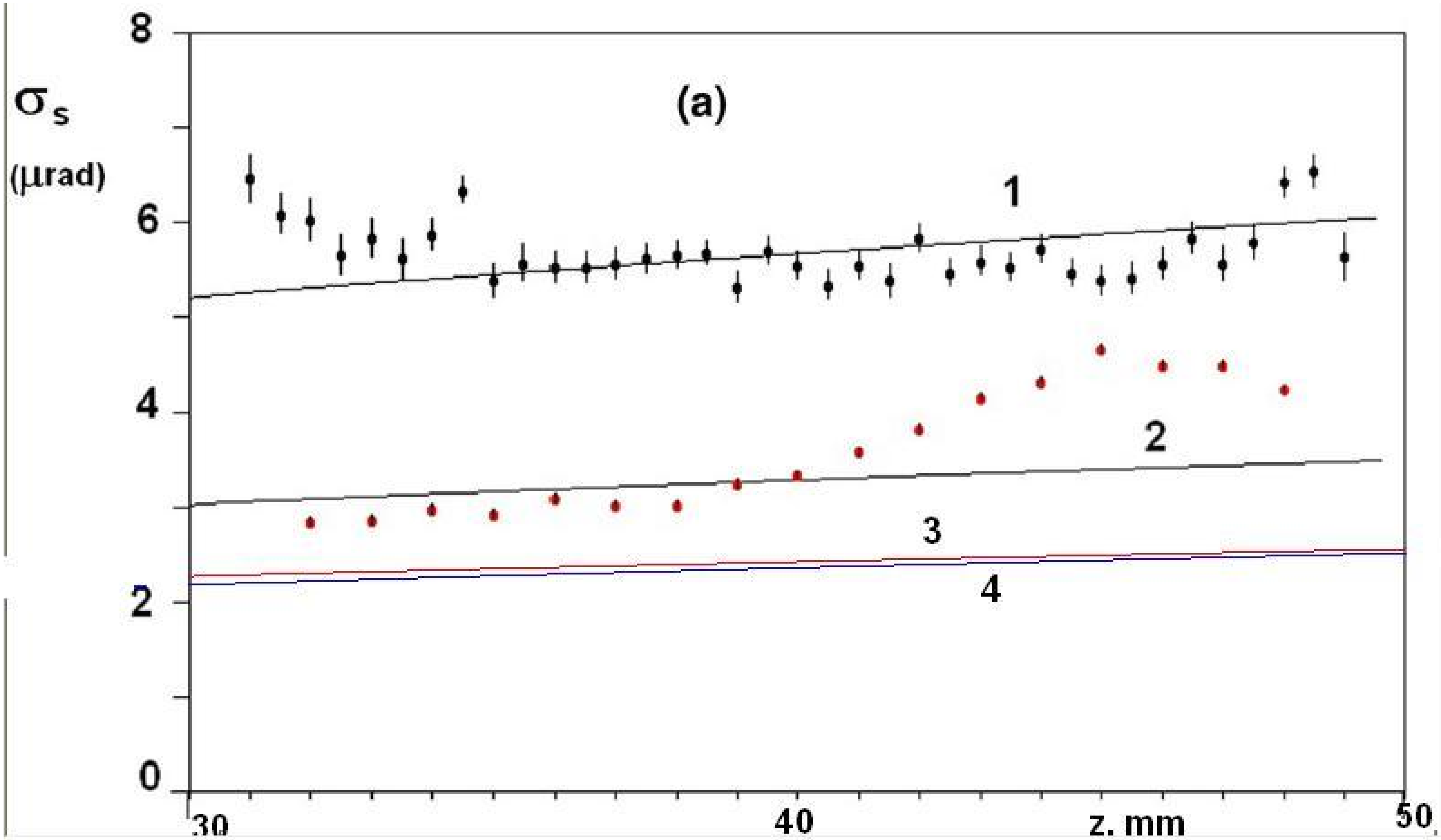
Amplitude dependences of $F_m(x_m)$-functions (see Eq. (17)) at different screening raadii (number near the curves(angstroms)).

Let us assume that we know the distribution function of channeled particles on the distance l(from entrance) Then we can find the distribution function on the distance $l+\delta l$ with help of the following equation:

$$\frac{dN}{dE}\left(E^{'},l+\delta l\right)=\int_0^{U_0}\frac{dN}{dE}\left(E,l\right)F\left(E,E^{'},\delta l\right)dE \qquad (9)$$

Thus, if the initial distribution function $dN/dE(E,0)$ is known we can find the distribution function on a distance equal to l by integration step by step in accordance with Eq. (9).The similary consideration of stohastic process one can find in literature

$F\left(E,E^{'},\delta l\right)$ is approximate function for short distance/



RMS of multiple scattering as function of cystal thickness.

1 Calculation for (111) Si plane and beam energy = 180 GeV.

2 Calculation for (111) Si plane and beam energy = 400 GeV,

3 Calculation for (110) Ge plane and energy = 400 GeV.

4 Calculations for (110) Si plane and energy = 400 GeV.

CONCLUSIONS

1) A method is proposed for calculating multiple scattering during particle channeling in crystals.

2) The calculation results are in good agreement with the measurements of the root-mean-square scattering angle particles with a momentum of 180 and 400 GzV channeling in the (111) planes of silicon.

3) the rms scattering angles for the (110) plane of silicon and germanium at a momentum of 400 GeV also were calculated.

4) The results obtained can be used to simplify the theory of the process.

5) The effect considered here is of great practical importance.

Thank you for your attention