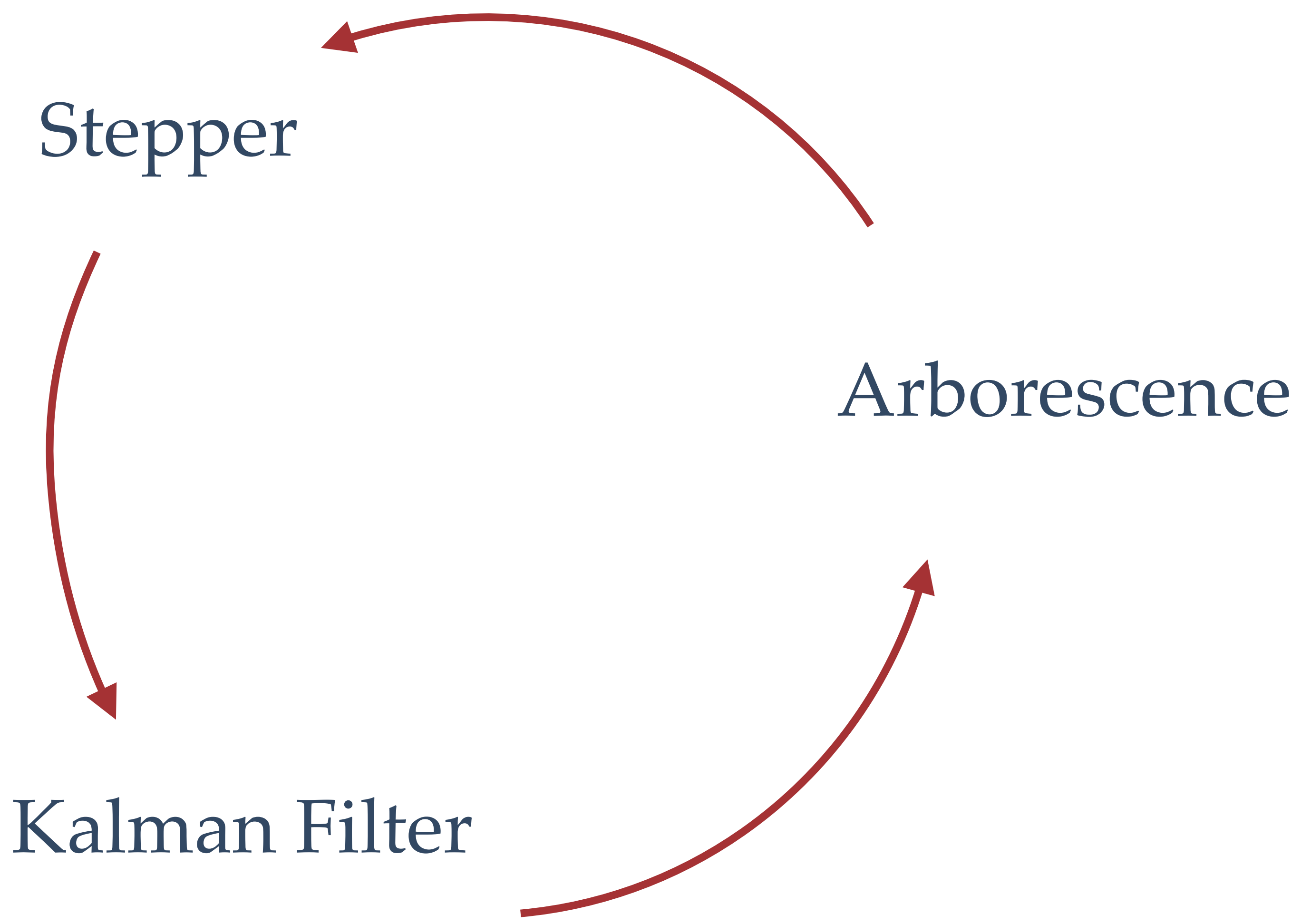
Global reconstruction with TOE

Alexandre Sécher on behalf of the FOOT collaboration

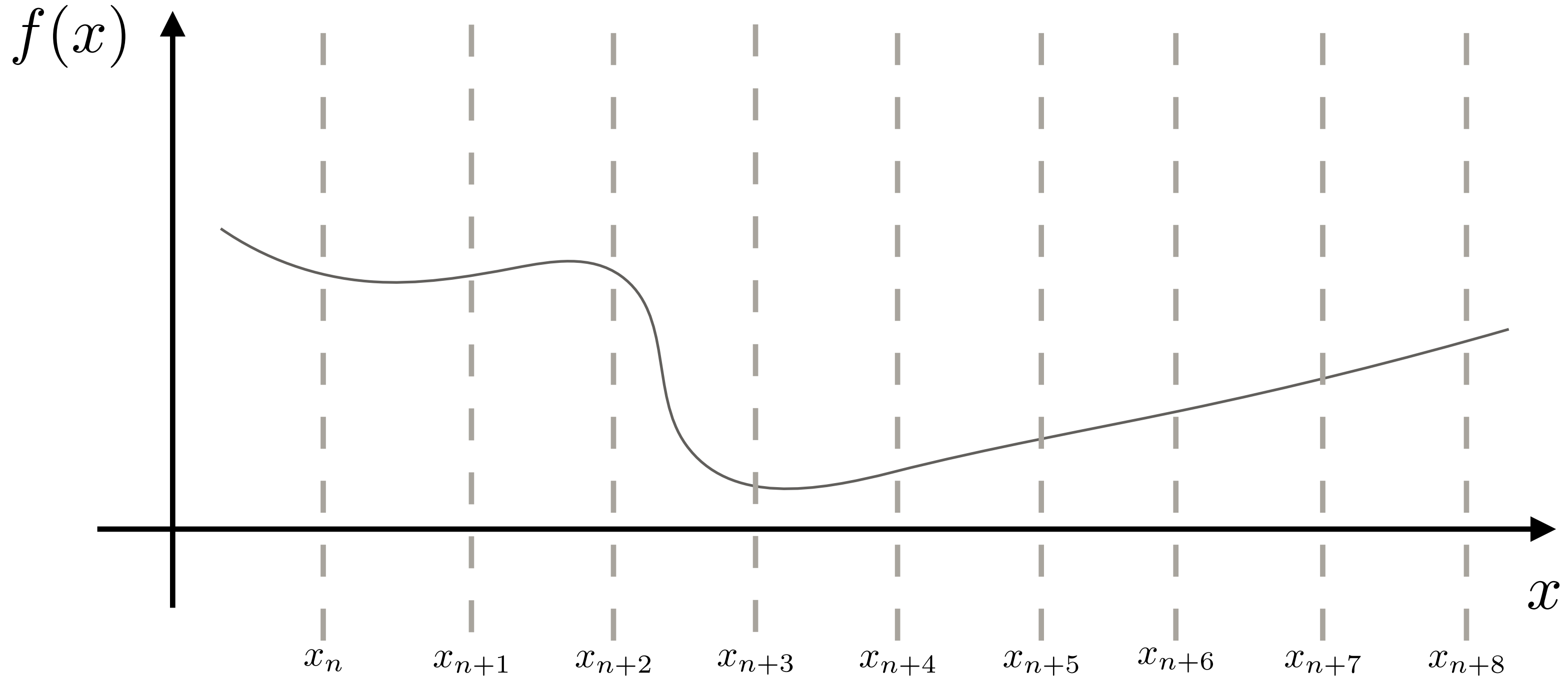
Tracking Of Ejectiles

➡Algorithm for concurrent track recognition and reconstruction

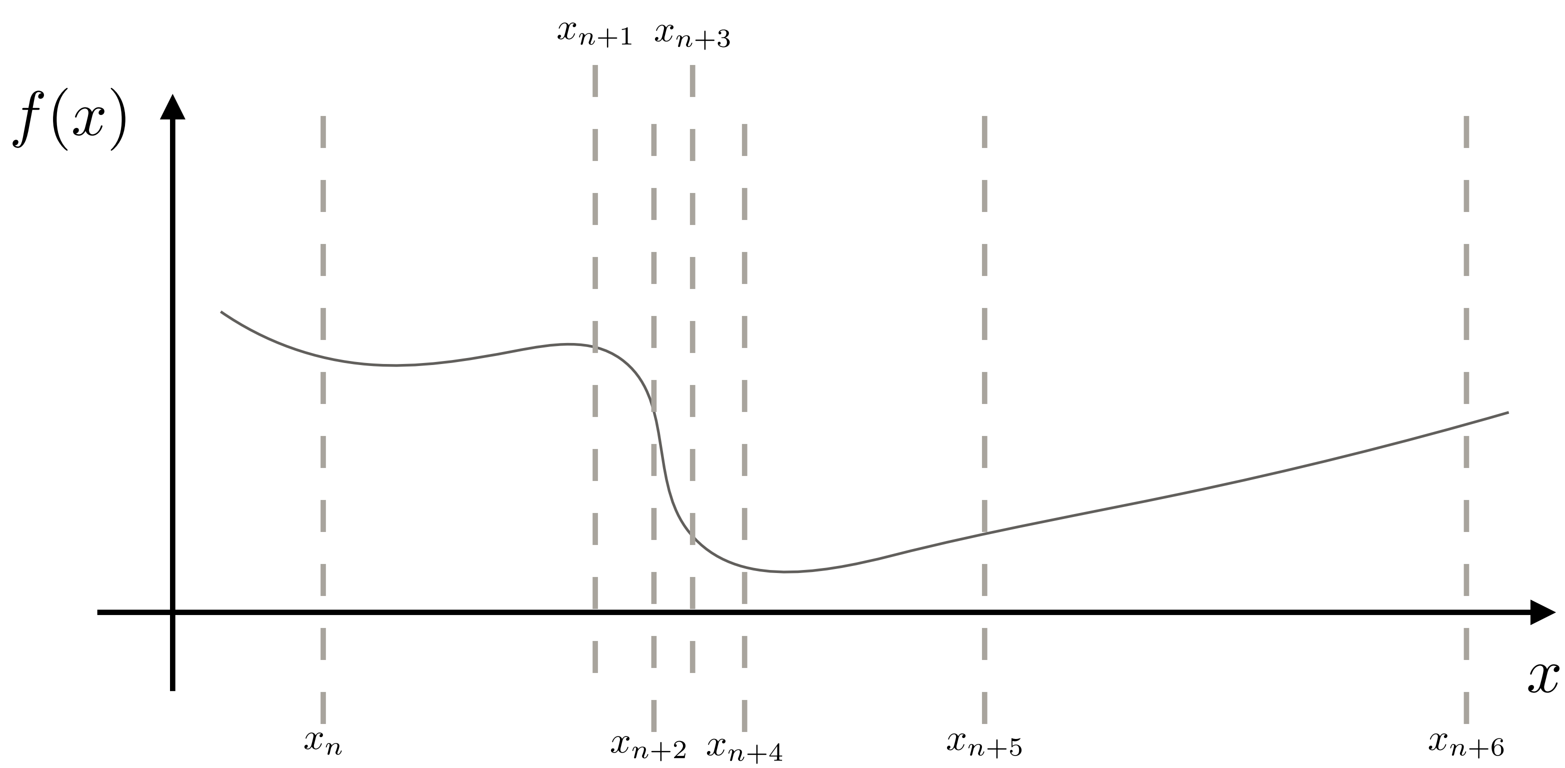


Stepper

GRKN4:



GRKN5(6):



Classical seventh-, sixth-, and fifth-order Runge-Kutta-Nyström formulas with stepwise control for general second-order differential equations, Felhberg E., NASA TR R-432, 1974

Kalman Filter

Principle:

Time update: prediction

Propagated state vector:

$$\hat{s}_{k|k-1} = f(\hat{s}_{k-1|k-1})$$

Propagated covariance:

$$S_{k|k-1}$$

Measurement update: correction

m_k measurement vector

M_k measurement covariance

H_k measurement matrix

Compute gain:

$$K_k = S_{k|k-1} H_k^T (H_k S_{k|k-1} H_k^T + M_k)^{-1}$$

Corrected vector:

$$\hat{s}_{k|k} = \hat{s}_{k|k-1} + K_k (m_k - H_k \hat{s}_{k|k-1})$$

Corrected covariance:

$$S_{k|k} = S_{k|k-1} - K_k H_k S_{k|k-1}$$

Unscented Kalman Filter

Time update: prediction

Form 2N+1 sigma points:

$$\mathcal{U}_0 = \hat{s}_{k-1|k-1}, \quad w_0 \in [0,1)$$

$$\mathcal{U}_i = \hat{s}_{k-1|k-1} \pm \sqrt{\frac{N}{1-w_0}} A_i, \quad AA^T = S, \quad w_i = \frac{1-w_0}{2N}$$

Propagate all of the sigma points:

$$\mathcal{V}_i = f(\mathcal{U}_i)$$

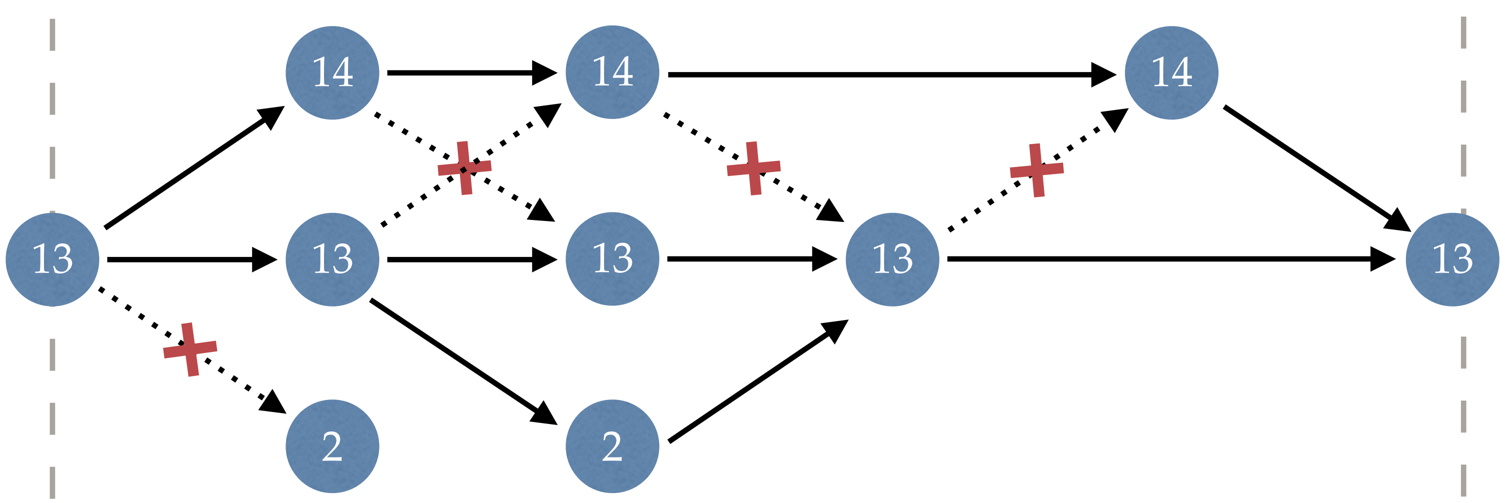
Form propagated state vector and covariance:

$$\hat{s}_{k|k-1} = \sum_{i=0}^{2N} w_i \mathcal{V}_i \qquad S_{k|k-1} = \sum_{i=0}^{2N} w_i \{\mathcal{V}_i - \hat{s}_{k|k_1}\}\{\mathcal{V}_i - \hat{s}_{k|k-1}\}^T$$

A new extension of the Kalman Filter for non-linear systems, Julier S. et Uhlmann K., 1997

Arborescence

- Basis for the combinatorial approach
- Holds the possible trajectories for an incident particle
- Each node contains a corrected state by UKF
- Splits along the way for each cluster meeting the insertion criterion



Insertion criterion:

Residuals vector and covariance:

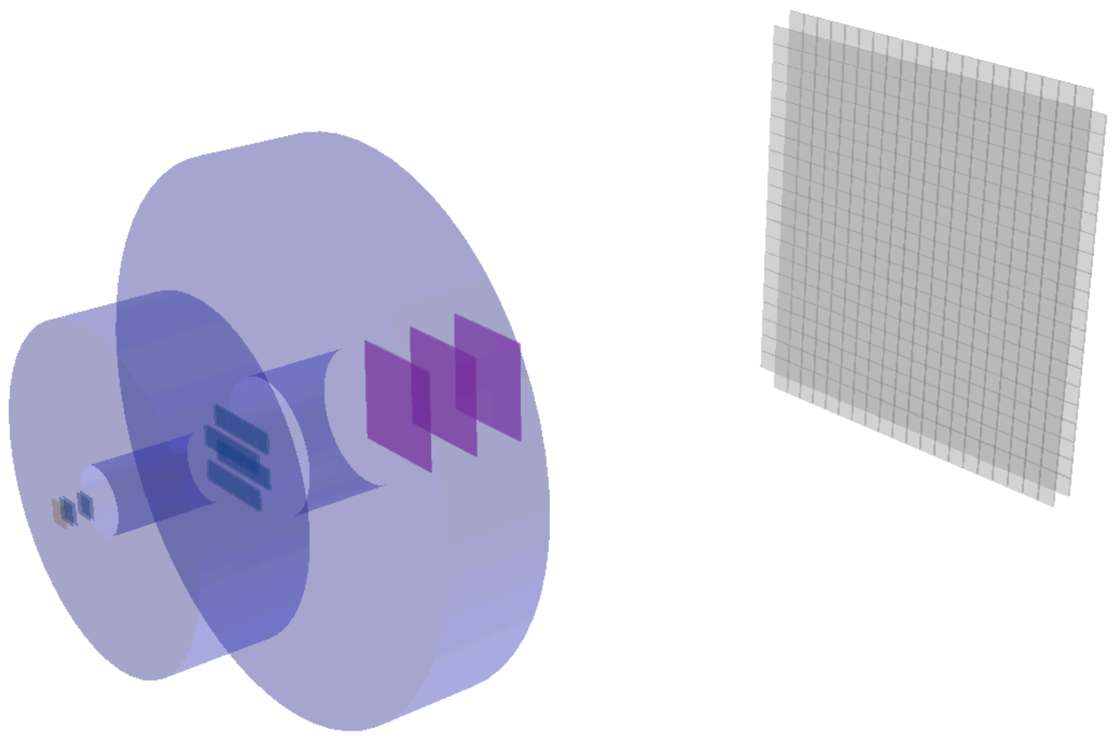
$$r_{k|k-1} = m_k - H_k \hat{s}_{k|k-1}$$

$$R_{k|k-1} = M_k + H_k S_{k|k-1} H_k^T$$

Chisquared update:

$$\chi^2 = r_{k|k-1}^T R_{k|k-1} r_{k|k-1}$$

Scheme



Forward propagation, layer based:

- Form hypothesis: static and dynamic
- Start propagation from root nodes
- Propagate from layer to layer, confront prediction with measurements
- Use tof wall as final cross-check

Propagation model

- Derived from Lorentz's force
- No additional effects: no multiple coulomb scattering, nor energy loss
- Propagation through the z variable

Propagation model

$$\begin{pmatrix} \frac{d^2x}{dz^2} \\ \frac{d^2y}{dz^2} \end{pmatrix} = \frac{qR}{p} \begin{pmatrix} \frac{dx\,dy}{dz^2} B_x - \left[1 + \frac{dx}{dz}^2\right] B_y + \frac{dy}{dz} B_z \\ \left[1 + \frac{dy}{dz}^2\right] B_x - \frac{dx\,dy}{dz^2} B_y - \frac{dx}{dz} B_z \end{pmatrix}$$

$$R = \sqrt{\frac{dx}{dz}^2 + \frac{dy}{dz}^2 + 1}$$

→

Propagated state:

$$s = \begin{bmatrix} x \\ y \\ dx/dz \\ dy/dz \end{bmatrix}$$

Static hypothesis

Propagation model:
$F(q, p, ...)$

Particle charge, number of nucleons:

- charge retrieved from the tof wall

$Z = 1$	p, d, t added
$Z = 2$	3He, α added
$Z > 2$	2 x Z added

Norm of momentum

- fragments emitted in the forward direction, energy close to the one of the beam

$$pc = \sqrt{E_k^2 - 2E_k mc^2}$$

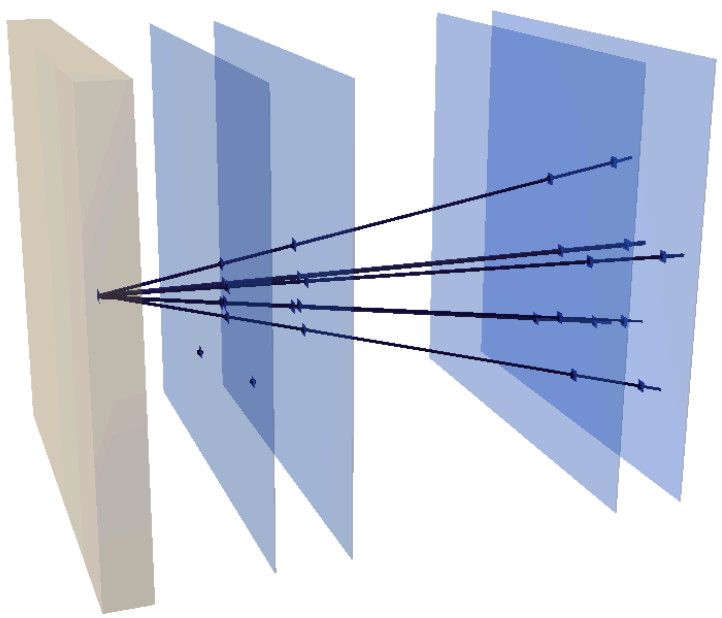
- Each static hypothesis leads to a different arborescence

Dynamic hypothesis

- Corresponds to the state to be propagated, requires reasonable initial values

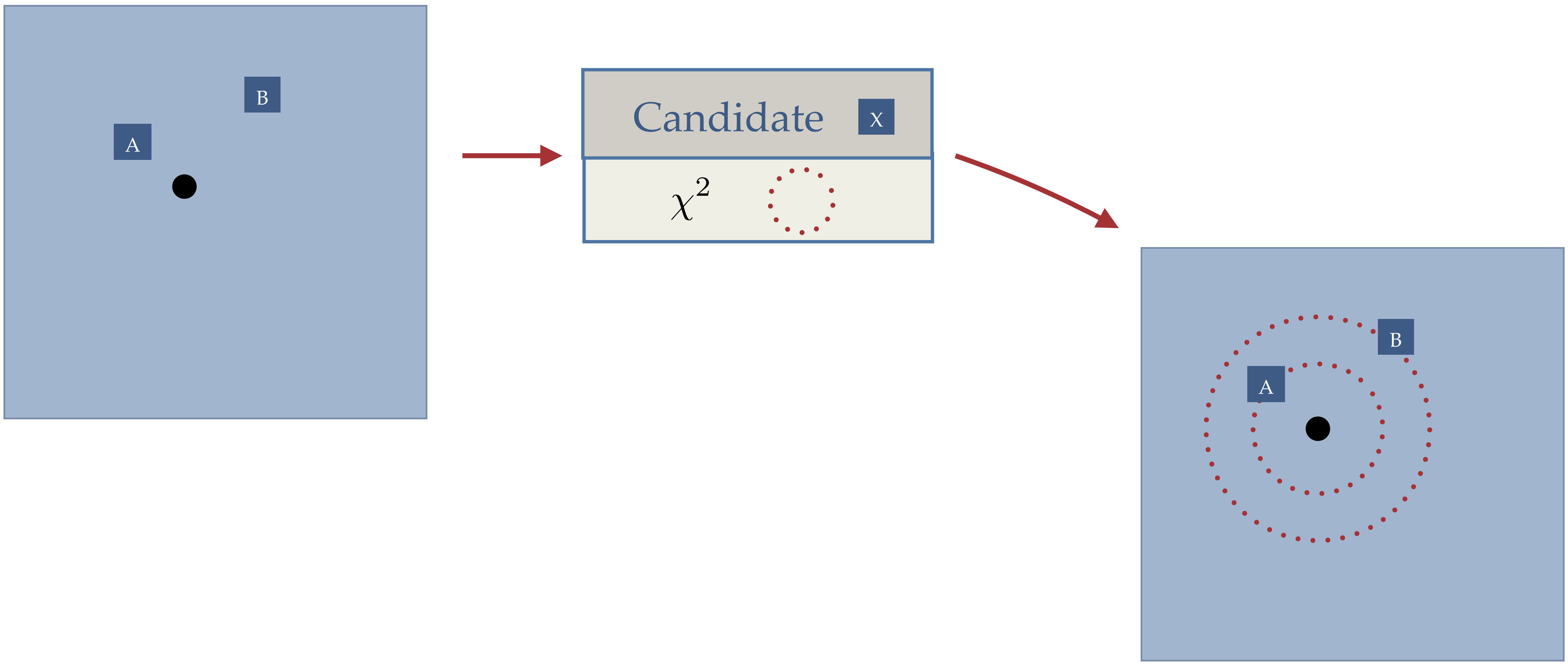
Propagated state:

$$s = \begin{bmatrix} x \\ y \\ \mathrm{d}x/\mathrm{d}z \\ \mathrm{d}y/\mathrm{d}z \end{bmatrix}$$

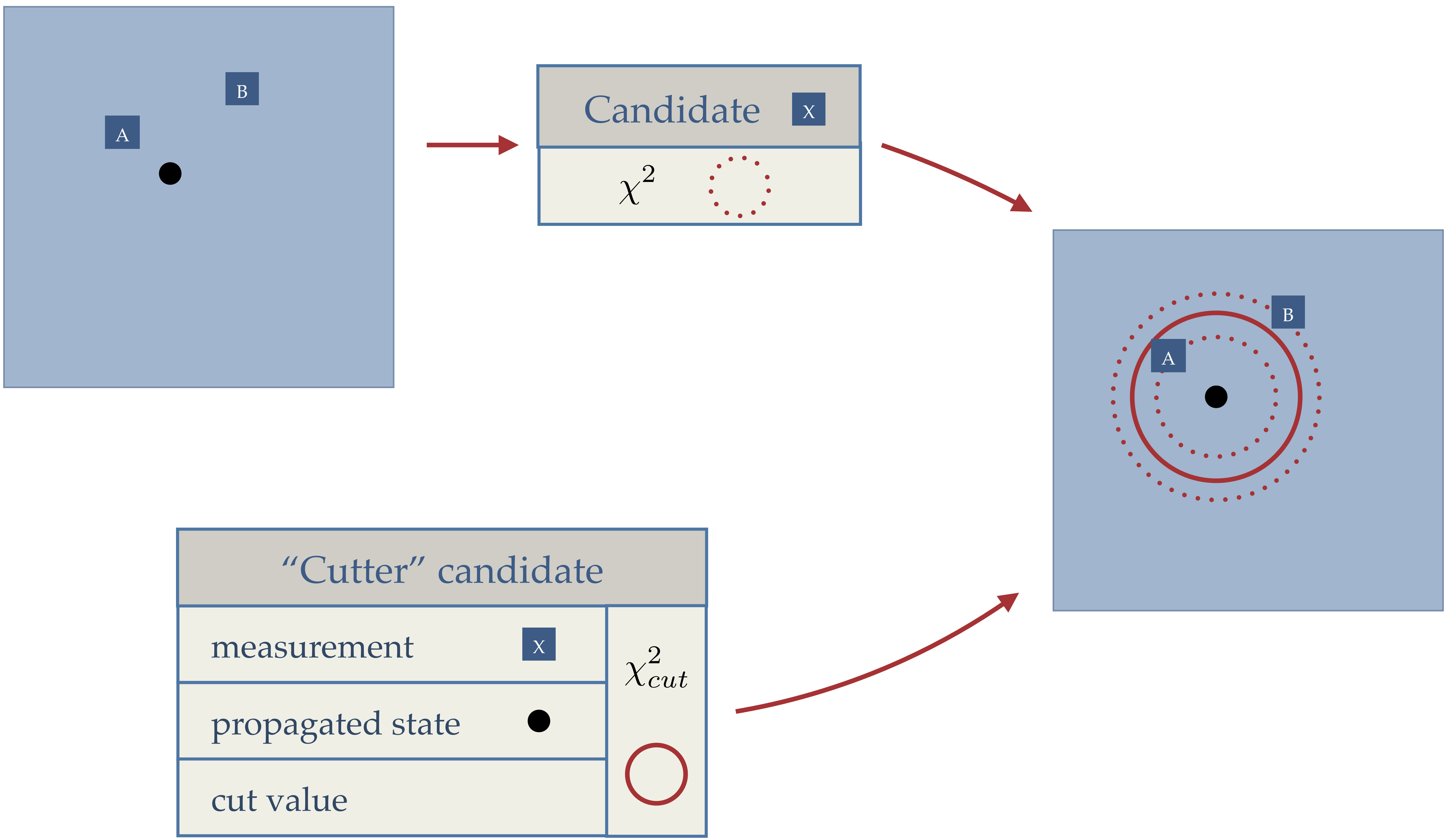


- Local reconstruction in vertex detector provides the necessary informations
- Each dynamic hypothesis will correspond to a root node in the arborescence

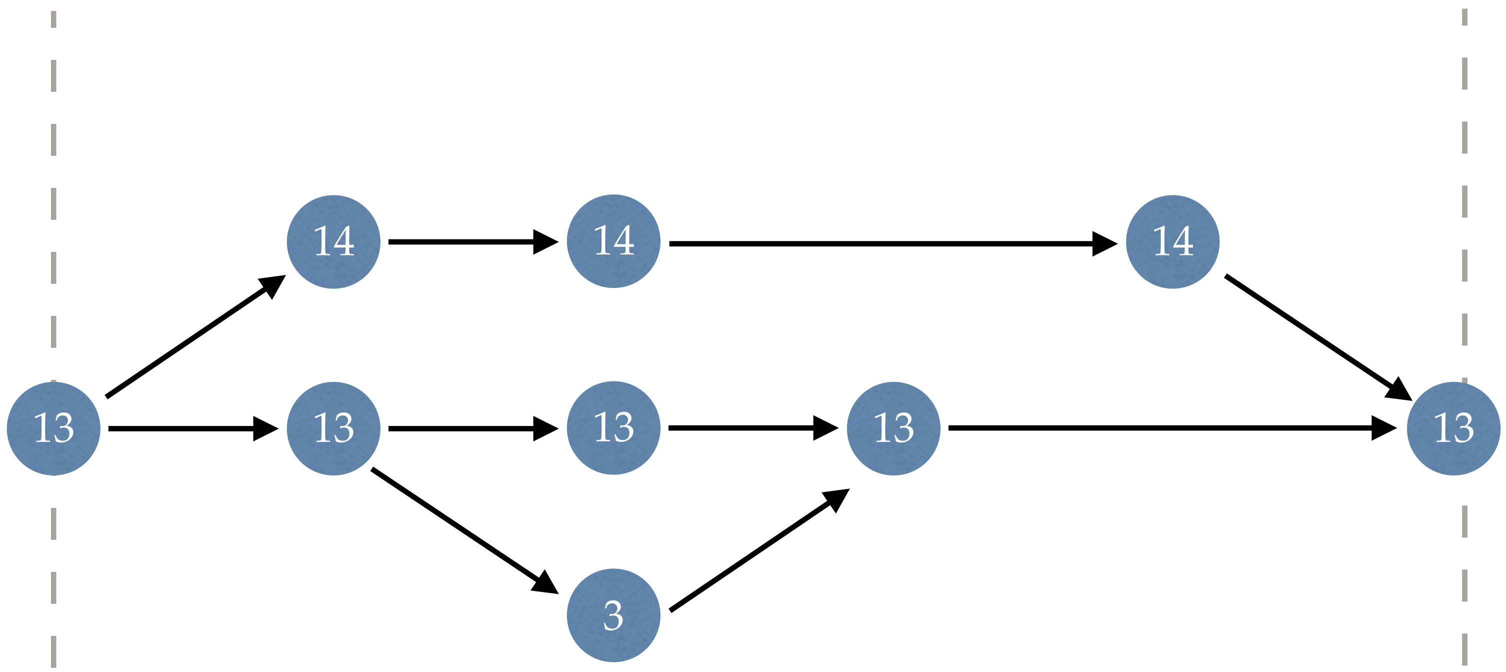
Confrontation



Confrontation



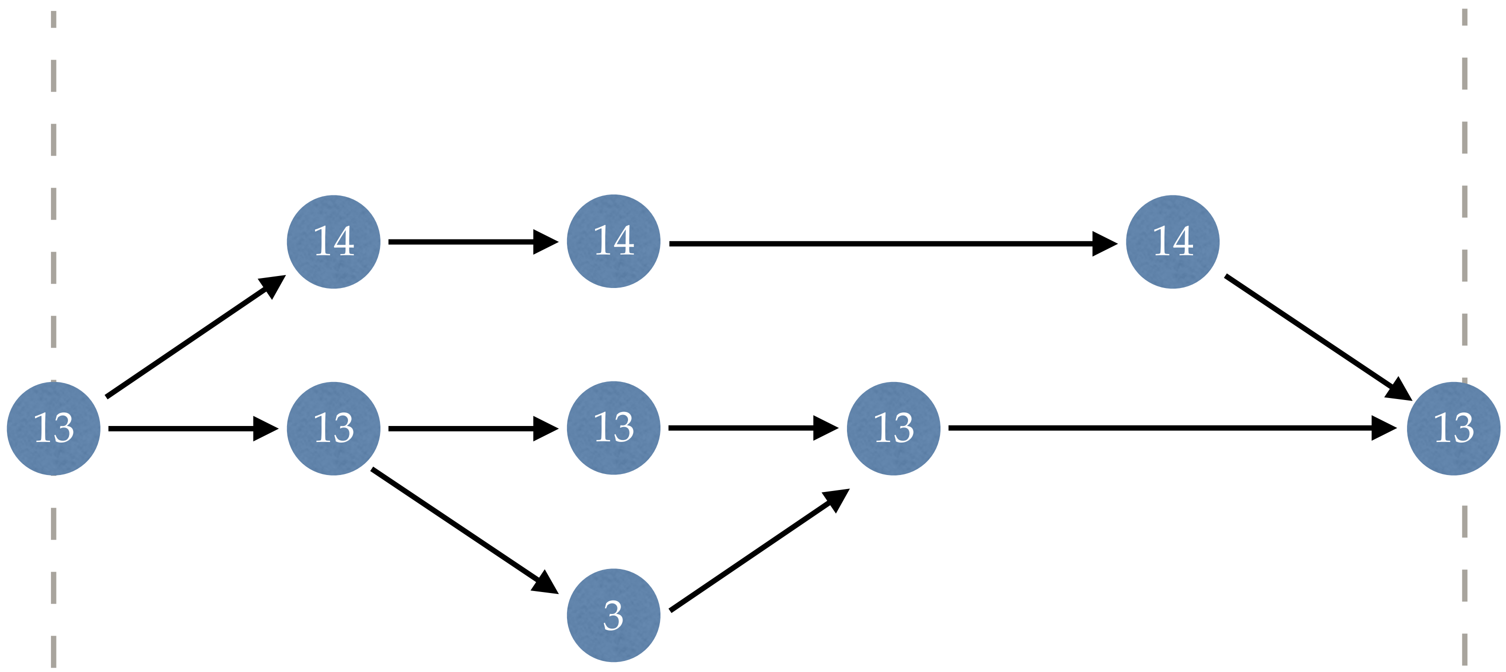
Shearing



Trajectories :

- 13, 14, 14, 14, 13
- 13, 13, 13, 13, 13
- 13, 13, 3, 13, 13

Shearing



Trajectories :

- 13, 14, 14, 14, 13
- 13, 13, 13, 13, 13
- 13, 13,   3, 13, 13

Insertion criterion:

$$\chi^2 = r^T_{k|k-1} R_{k|k-1} r_{k|k-1}$$

$$\chi^2_k = \sum_{I=0}^{k} \chi^2_i$$

→

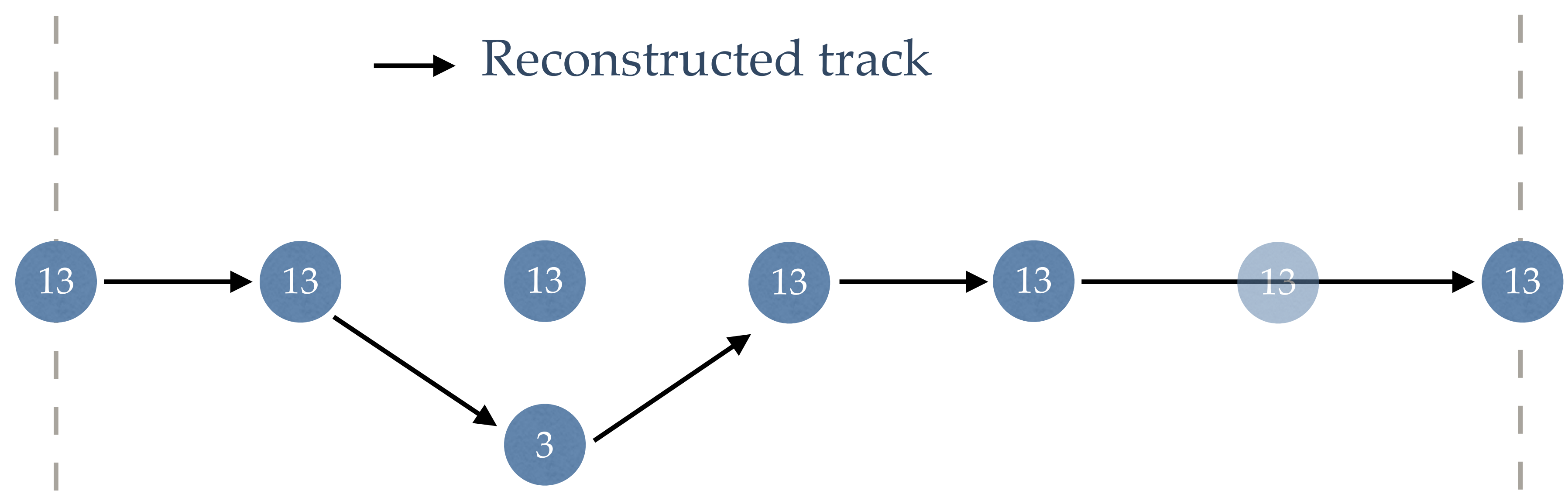
Shearing factor:

$$\frac{\chi^2_N}{N}$$

Efficiency, fake yield, clone number

- **Reference set (of reconstructible tracks)**: should be reconstructed by an ideally performing algorithm
- **Efficiency**: ratio between correctly reconstructed tracks and the reference set
- **Fake yield**: ratio between reconstructed fake tracks and the correctly reconstructed tracks
- **Clone number**: average number of candidate trajectories for a reconstructed track

Cluster purity, cluster coverage



$\mathcal{N}_0 = 7$

$\mathcal{N} = 6$

$\mathcal{N}_{mc} = 5$

- **Cluster purity**: quantify how many clusters are correctly associated to the reconstructed track $\frac{\mathcal{N}_{mc}}{\mathcal{N}} \approx 83\%$
- **Cluster coverage**: quantify how many clusters were used by the algorithm $\frac{\mathcal{N}}{\mathcal{N}_0} \approx 86\%$

Results

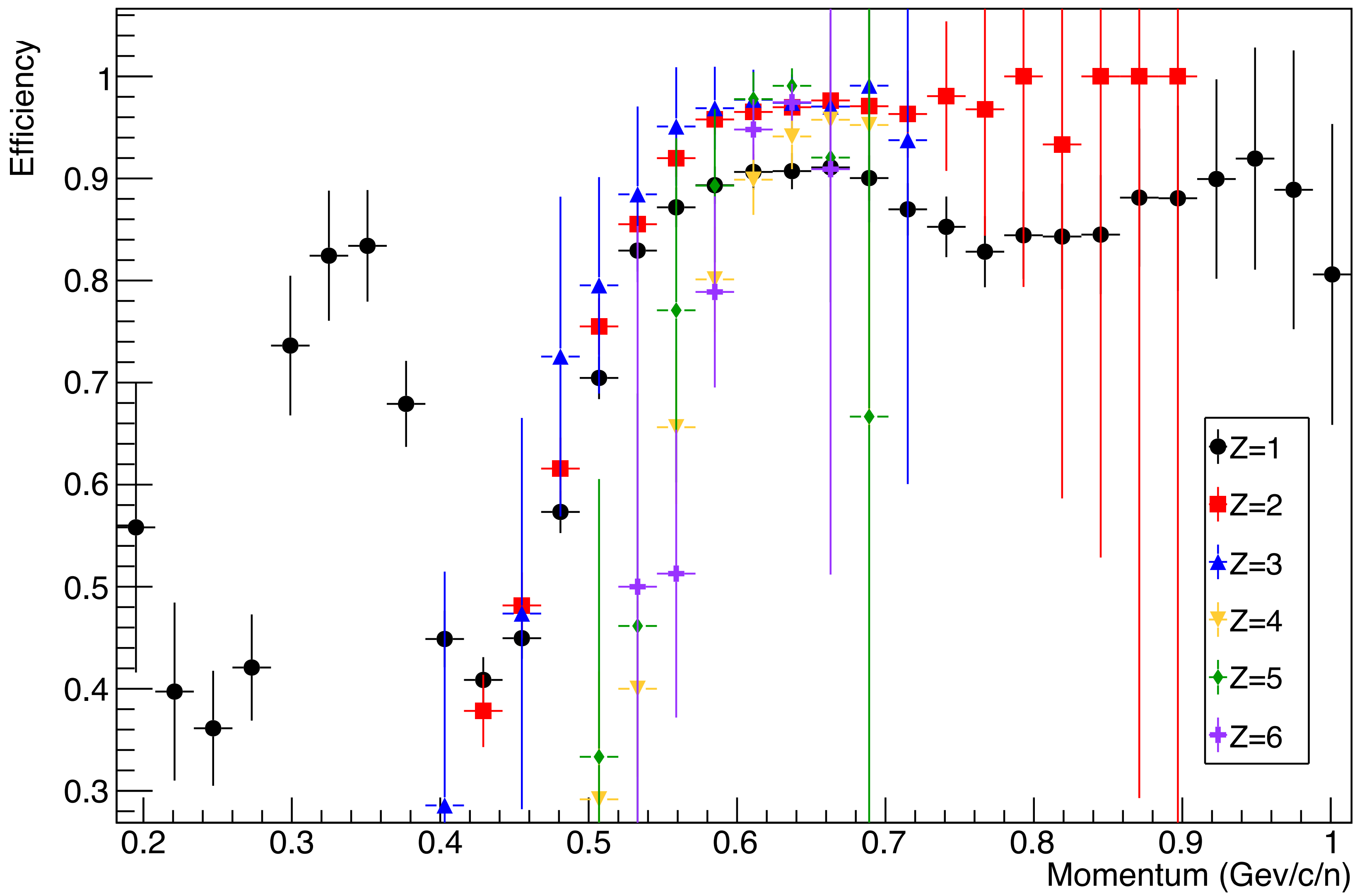
Simulation file: 12_C_200_1.root
Beam: 12C at 200 MeV/u
Target: C, 3 mm with density 1.83 g/cm^2
Number of processed events: 100 000

Efficiency	89.4 ± 0.4 %
Cluster purity	97.8 ± 0.1 %
Cluster coverage	83.2 ± 0.1 %
Fake yield	10.4 ± 0.1 %
Clone number	4.49 ± 0.01

Efficiency by charge

Average efficiency

$Z = 1$	$82.2 \pm 0.5\%$
$Z = 2$	$92.7 \pm 0.6\%$
$Z = 3$	$96 \pm 2\%$
$Z = 4$	$88 \pm 2\%$
$Z = 5$	$98 \pm 1\%$
$Z = 6$	$96 \pm 2\%$



Work remaining

- Re-computation of momentum, based on track length and time of flight
- Development of a tool to optimise free parameters deduction
- Add calorimeter information as the basis for momentum used in reconstruction
- Removal of clusters if in previous successful reconstruction
- Addition of new static hypothesis upon failure rather than adding them all at the start

Thank you for your attention