Cross section evaluation and systematic study on the number of isotopes reconstructed, mass and its resolution depending on p, TOF, E_{kin}, E_{loss}

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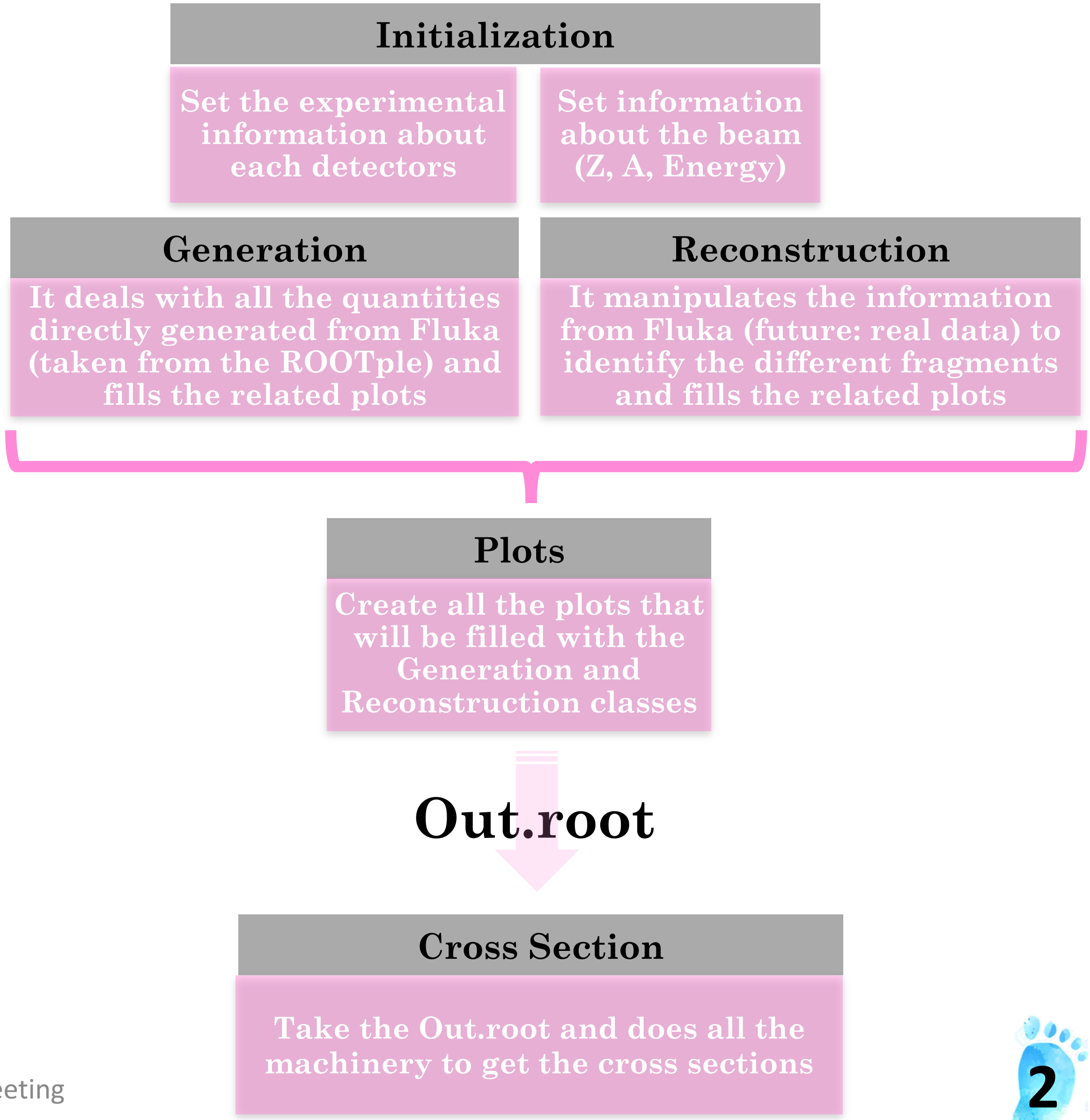
Introduction

- Stand-alone macros were available for the reconstruction of all the needed quantities to
  - identify different fragments
  - elaborate the information to get the different cross sections

- They had got bigger and bigger over time and were not optimized for current needs

- Need to rearrange the code in a more SHOE friendly way (so to be included in the future)

- Previous results were never double checked



Introduction

Simulation:

- ^{16}O 200 MeV/u on 5 mm C_2H_4 target
- Newgeom_v1.0
- $1x10^8$ primaries (2880551 interactions -> 2.88 %)
- Odd events = MC events & Even events = Real events

Selection:

- Cross all the detectors
- Tracks originated in the target
- 5.5 < Z reco < 6.5
- Fit $\chi^2 < 5$

Cross section reconstruction with FOOT

Differential cross sections of each produced fragment

$$\frac{d\sigma_f}{dE_{kin}} = \frac{(Y_f - Bkg_f)^U}{N_{Prim} \cdot N_t \cdot \Omega_{Ekin}\, \epsilon_f}$$

- f -> fragment: all Carbon Isotopes
- N_{prim} -> number of primary events
- N_t -> number of scattered center per unit area
- ε_f -> efficiency
- Ω_{Ekin} -> phase space

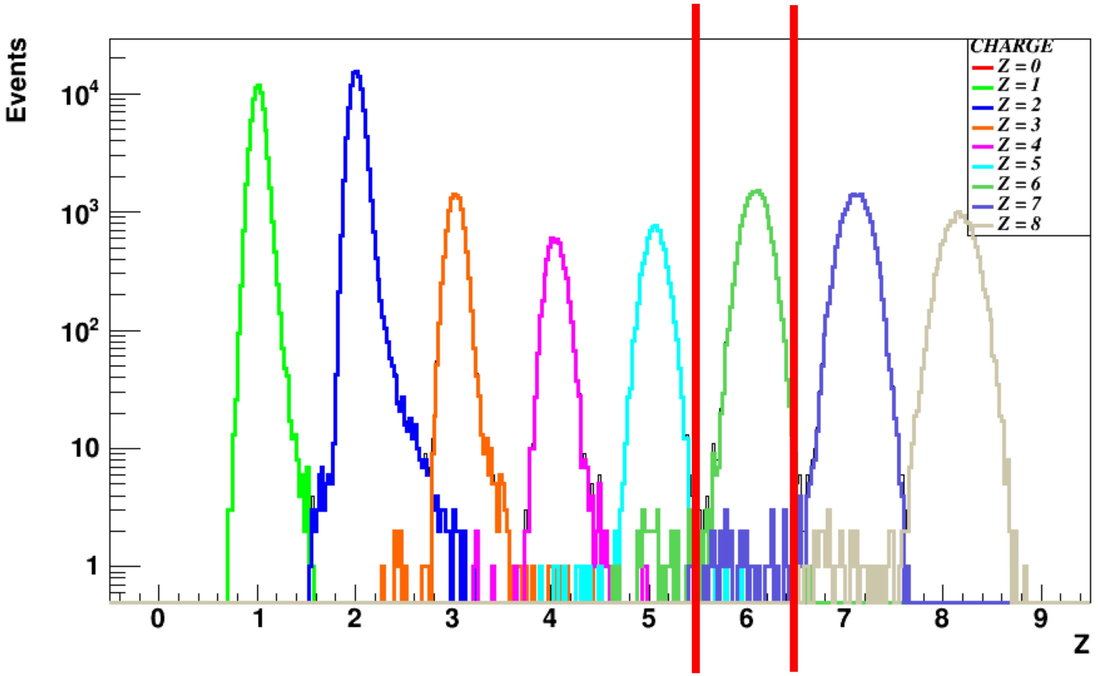
- Bkg -> Background : events counted with A=12, but generated with A≠ 12 (≈ 8%)
- U -> Unfolding : the reconstructed distribution must be corrected from the experimental effects

$(Y_f - Bkg_f)^u$ Unfolded (Yield – Bkg) of the fragment

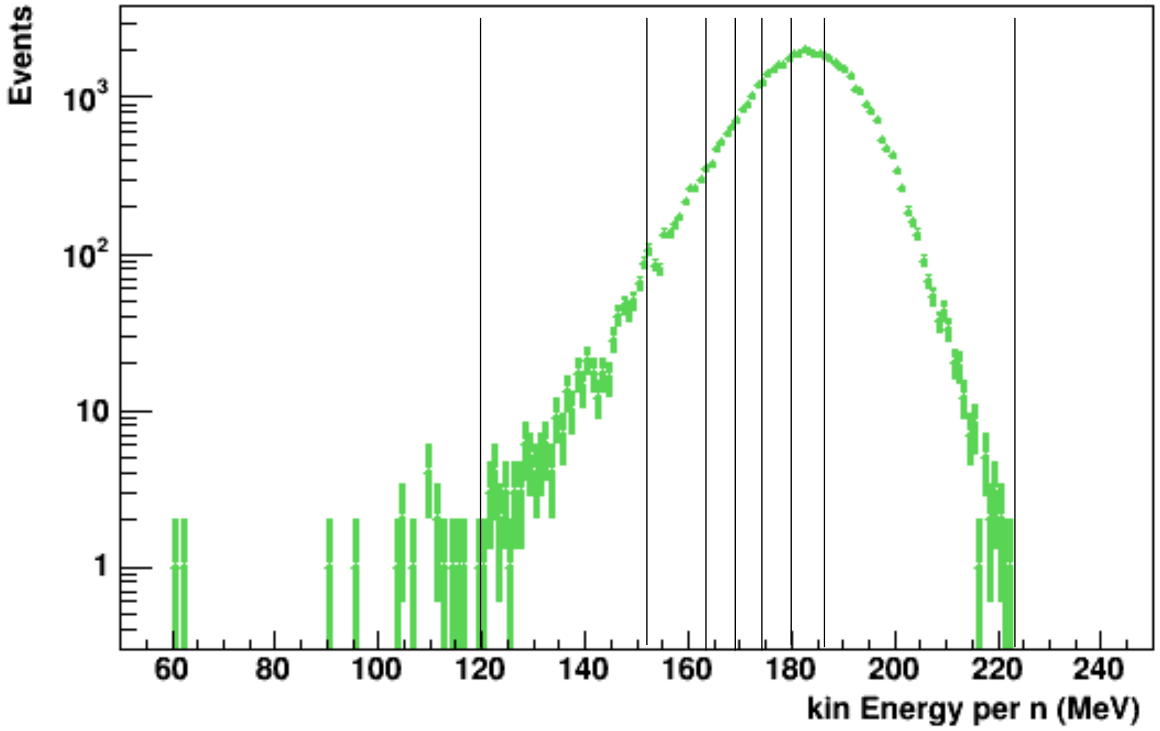
Fragments identification on *real events*

$$-\frac{dE}{dx} = \frac{\rho \cdot Z}{A} \frac{4\pi N_A m_e c^2}{M_U} \left(\frac{e^2}{4\pi\epsilon_0 m_e c^2}\right)^2 \frac{z^2}{\beta^2} \left[\ln\left(\frac{2m_e c^2 \beta^2}{I \cdot (1-\beta^2)}\right) - \beta^2\right]$$

Each charge has a different kinetic energy range



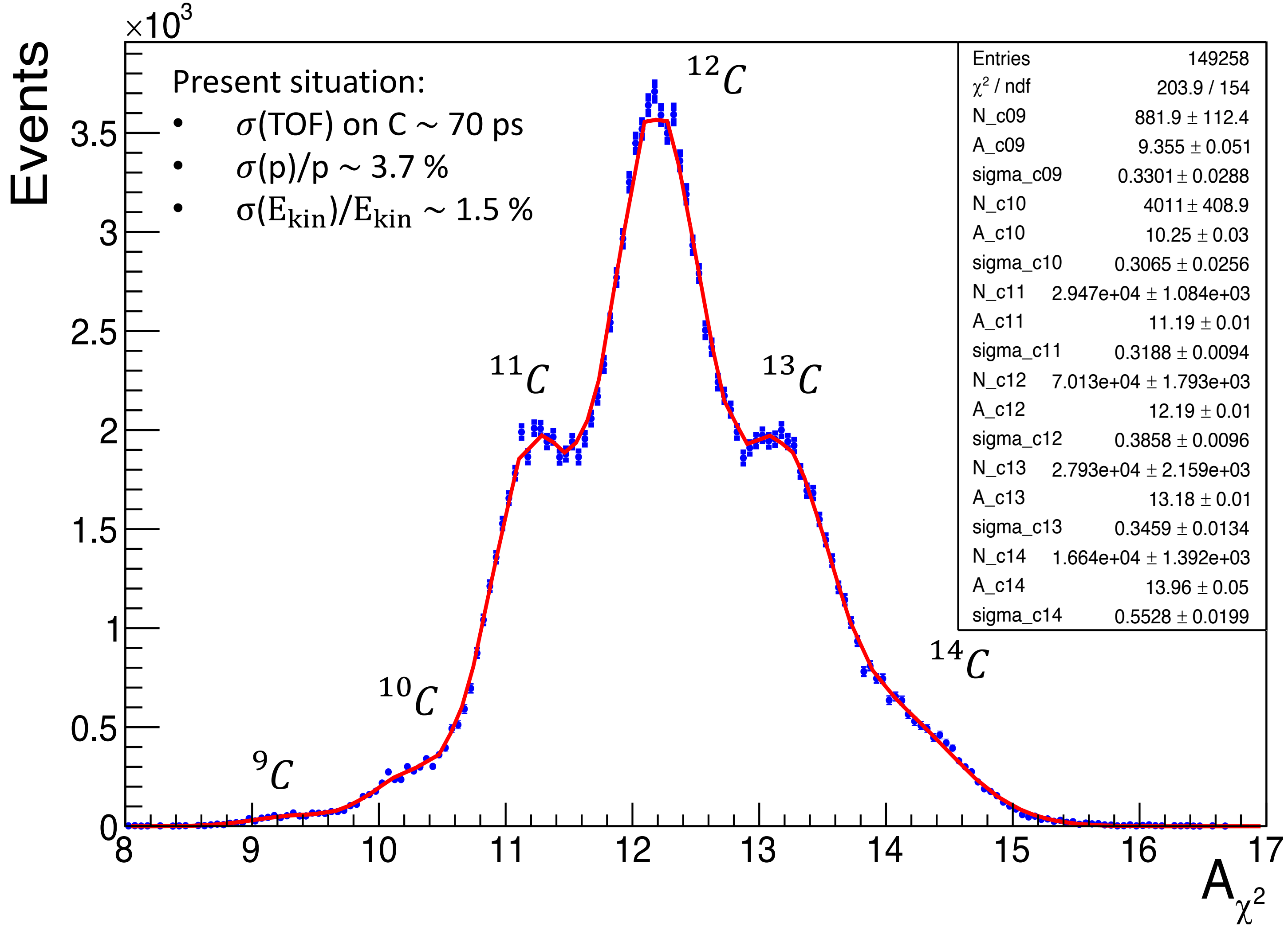
5.5 < Zreco < 6.5



Different energetic bins in order to have about the same amount of events in each bin

The total reconstructed Carbon data are fitted with

$$f = gauss_{C9} + gauss_{C10} + gauss_{C11} + gauss_{C12} + gauss_{C13} + gauss_{C14}$$

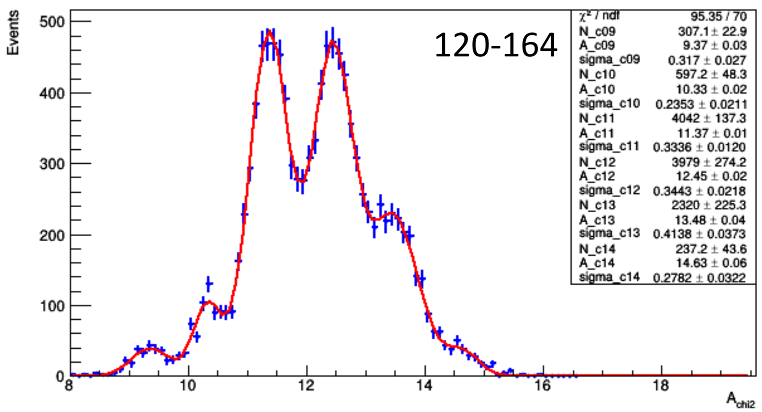


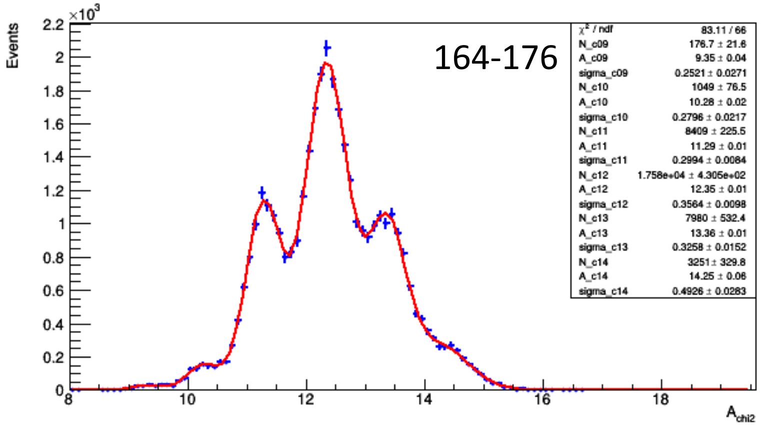
- N -> number of events; free parameter
- μ -> input from MC with 4% range variation
- σ -> input from MC with 8% range variation

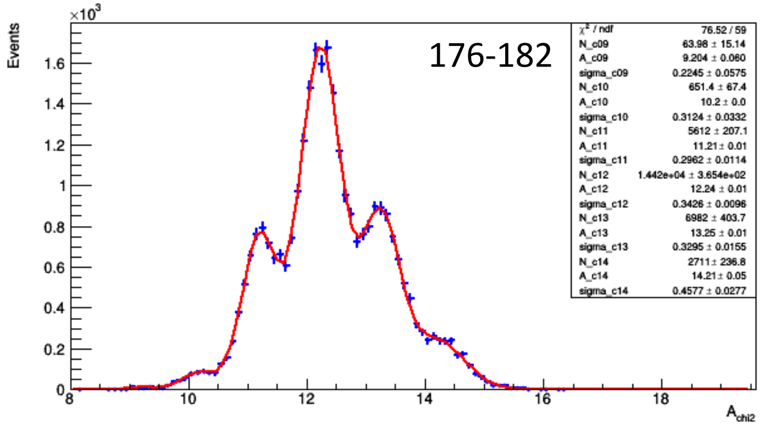
Number of different Carbon isotopes (wrt total carbon):

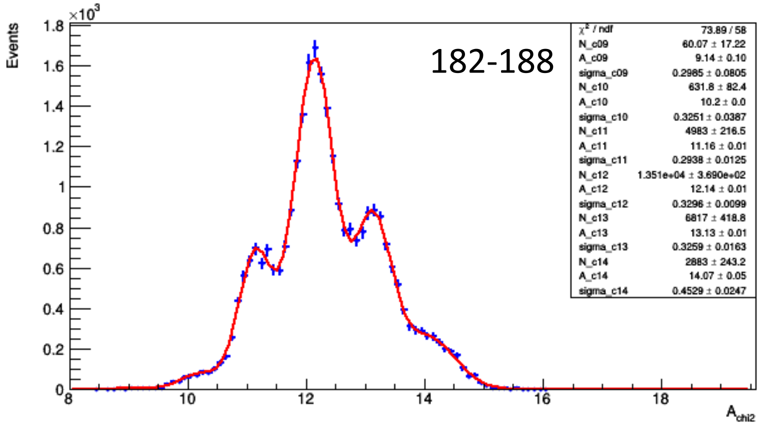
- ^{9}C : 0.6%
- ^{10}C : 2.7%
- ^{11}C : 19.7 %
- ^{12}C : 47 %
- ^{13}C : 18.71%
- ^{14}C : 11.14 %

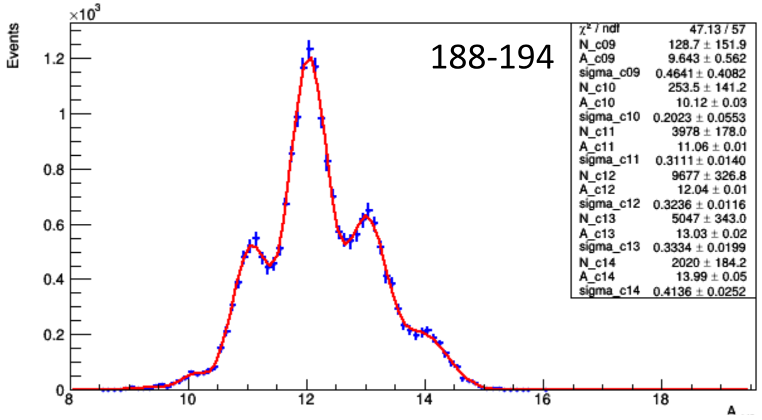
Same fit function, but applied to each defined energetic range

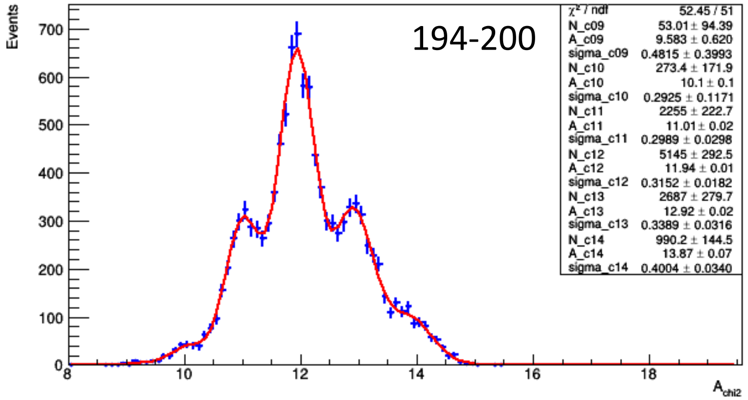


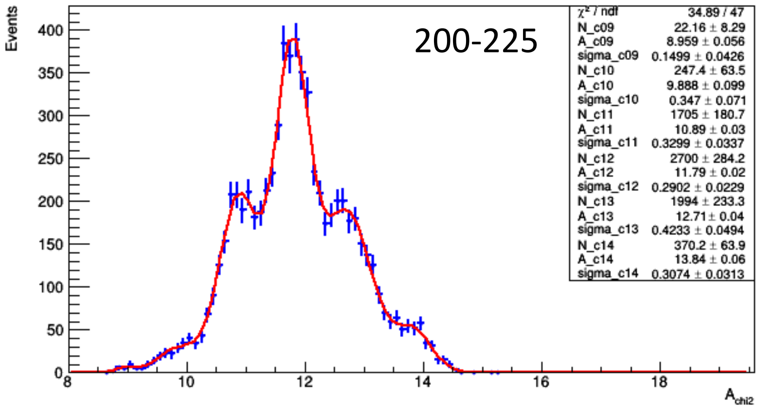








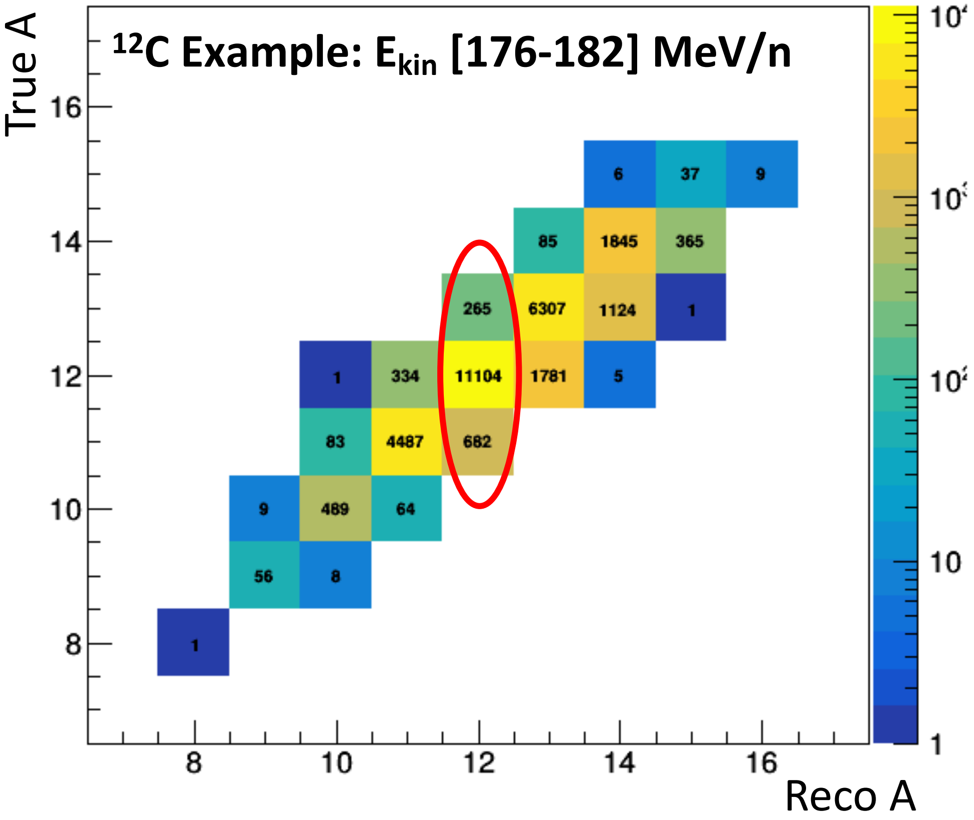




$$\frac{d\sigma_f}{dE_{kin}} = \frac{(\boldsymbol{Y_f} - \mathbf{Bkg_f})^{\mathbf{U}}}{N_{Prim} \,.\, N_t \,.\, \Omega_{Ekin}\, \epsilon_f}$$

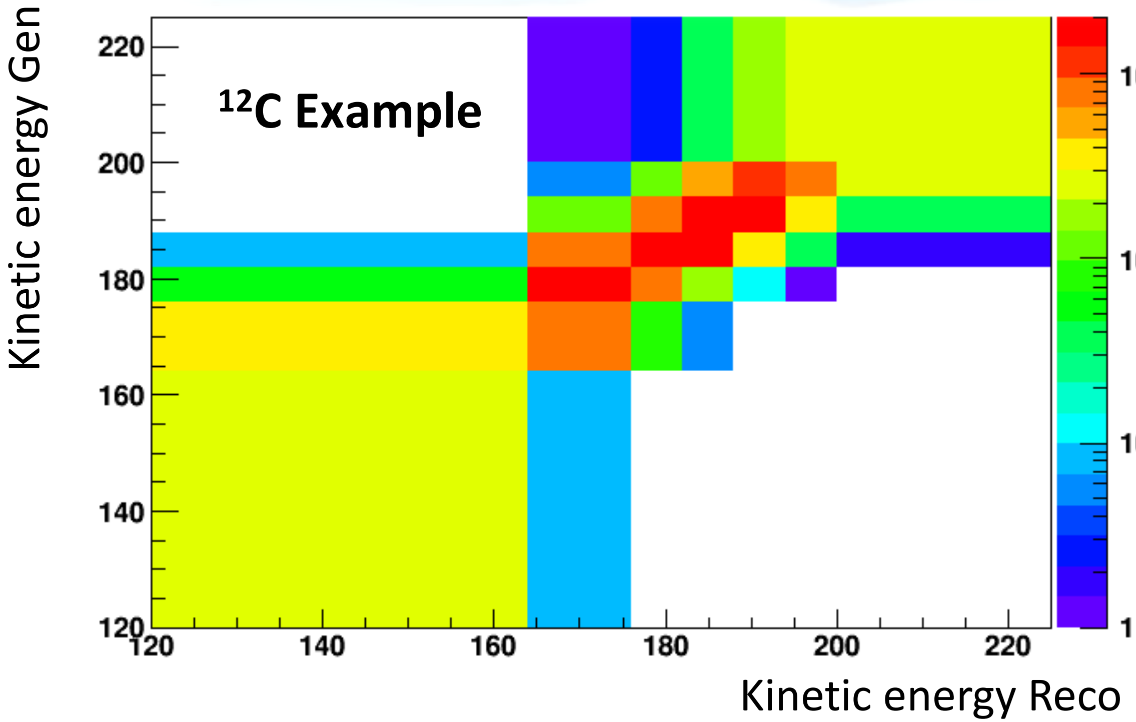
$$\frac{d\sigma_f}{dE_{kin}} = \frac{(Y_f - \text{Bkg}_f)^U}{N_{Prim} \cdot N_t \cdot \Omega_{Ekin} \, \epsilon_f}$$

Background evaluation & Unfolding process



- Bkg: events counted with A=12 but generated with A≠12

- 944/12048 = 8%
  $(Y_f - Bkg_f) = Y_f - 0.08 * Y_f = 0.92 * Y_f$



- How many events generated in a kinetic energy bin are reconstructed in a different one

$$AX = Y \quad \text{->} \quad X = A^{-1}Y$$

Correction matrix | **True distribution** | **Reconstructed distribution (bkg subtracted)**

$$\frac{d\sigma_f}{dE_{kin}} = \frac{(Y_f - Bkg_f)^U}{N_{Prim} \cdot N_t \cdot \Omega_{Ekin}\, \epsilon_f}$$

Efficiency evaluation

$$\epsilon_f = \frac{N_{Sel,f}}{N_{All,f}}$$

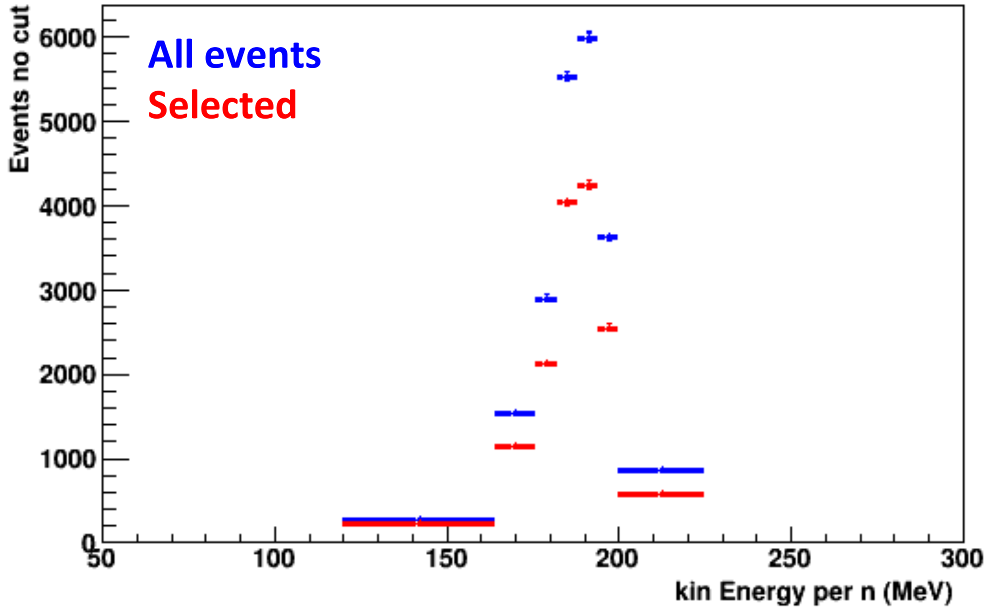
$N_{Sel,f}$ → $A_{gen} = 12$ & all selection (slide 3)

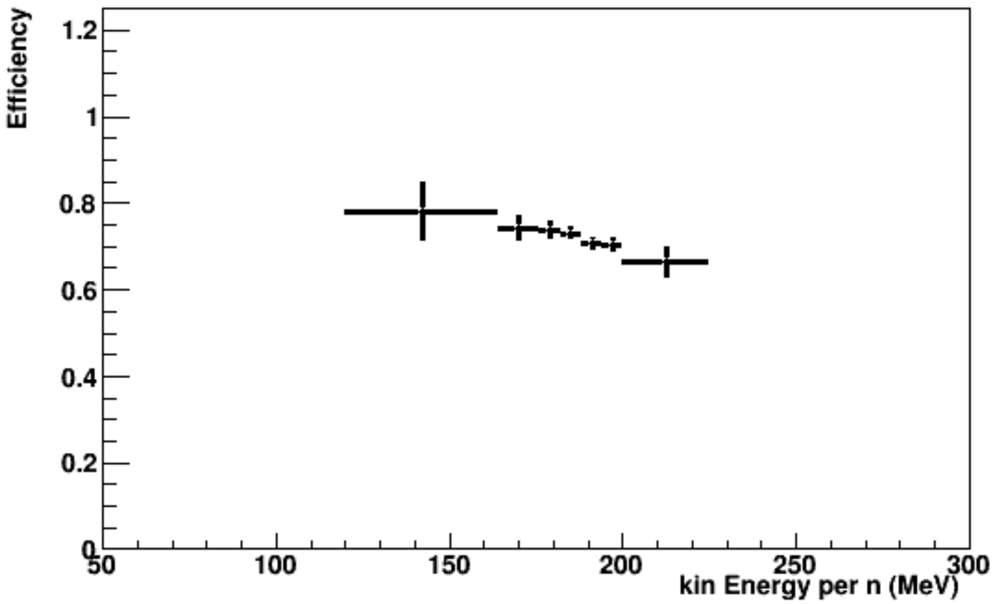
$N_{All,f}$ → $A_{gen} = 12$

At present, ϵ_f is evaluated on MC events (odd)

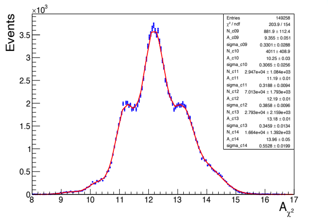
↓

In the future will be real data

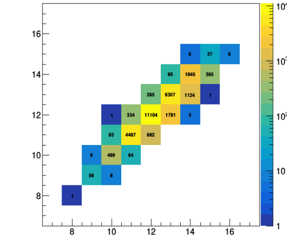




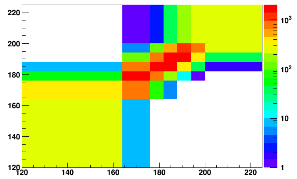
Cross section reconstruction with FOOT



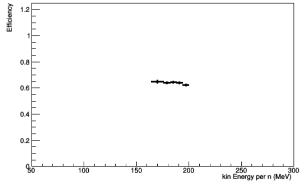
Y_f



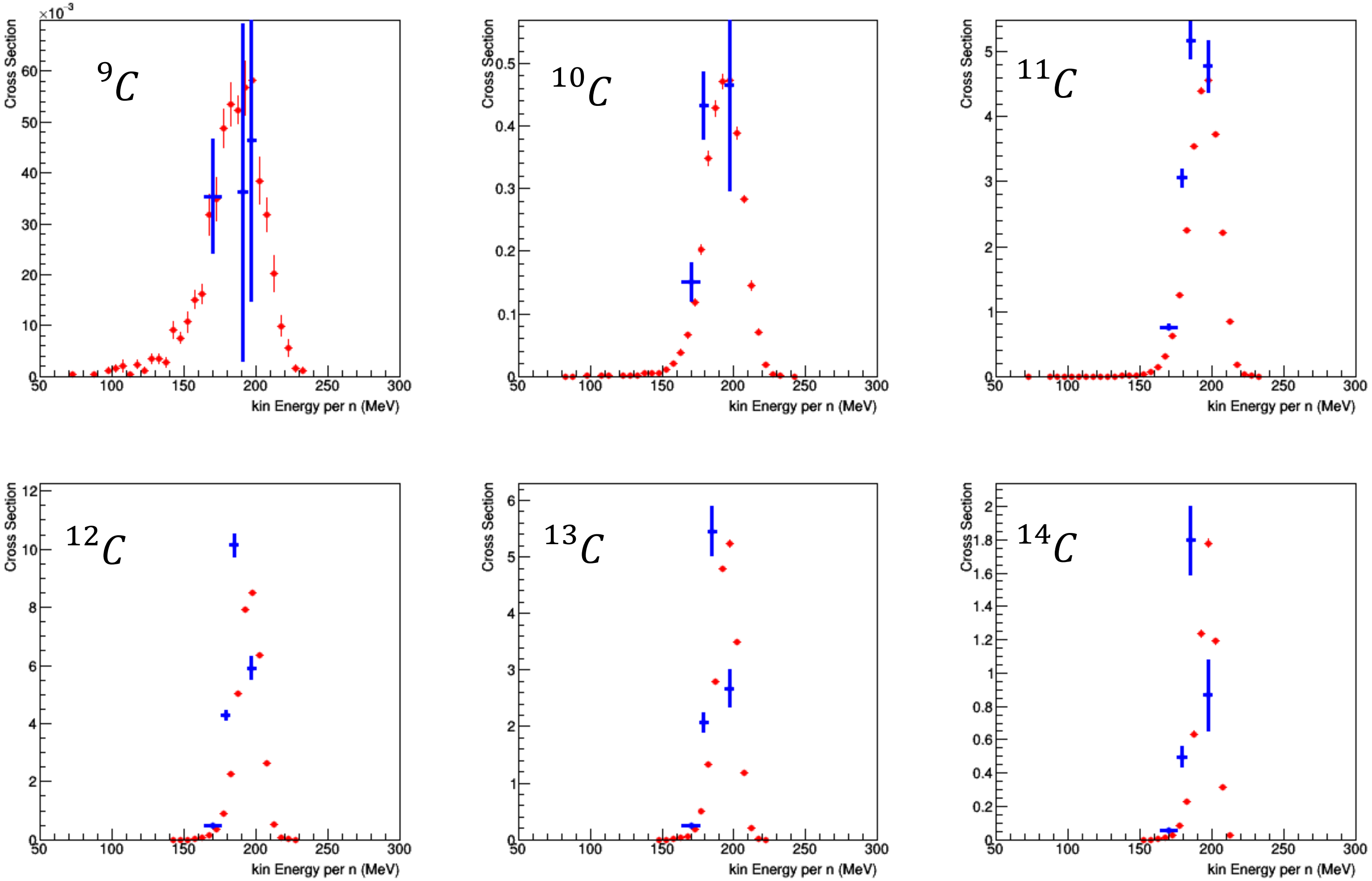
$Y_f\text{-}Bkg_f$



$(Y_f - Bkg_f)^U$



$$\frac{(Y_f - Bkg_f)^U}{\varepsilon_f}$$



1 $$\frac{d\sigma_f}{dE_{kin}} = \frac{(Y_f - Bkg_f)^U}{N_{Prim} \,.\, N_t \,.\, \Omega_{Ekin}\, \epsilon_f}$$

total Cross Section

$$\sigma_f = \frac{(Y_f - Bkg_f)^U}{N_{Prim} \,.\, N_t \,.\, \epsilon_f}$$

Cross section reconstruction with FOOT

The same procedure can be applied also to the simulation of

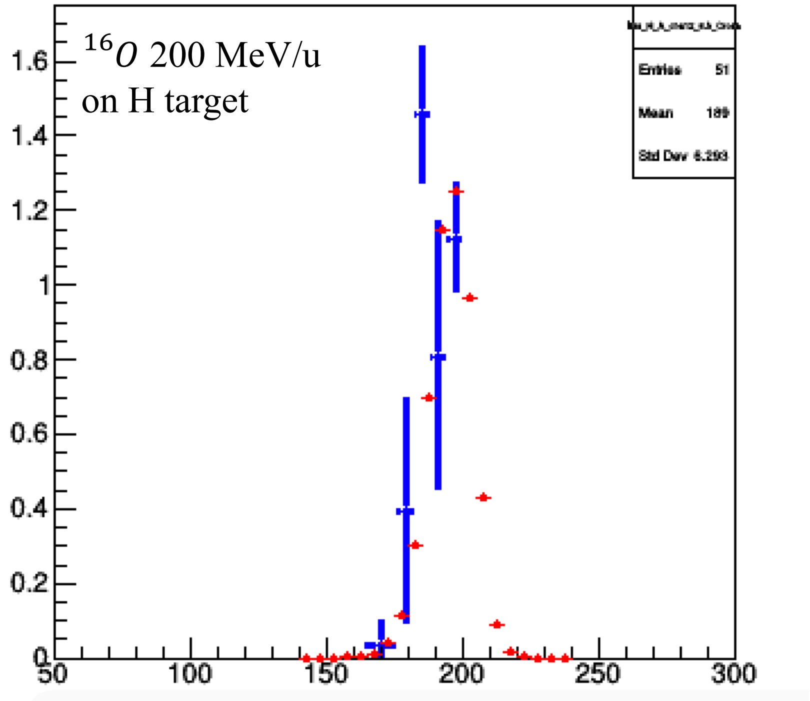
- ^{16}O 200 MeV/u on 5 mm **C target**
- Newgeom_v1.0
- $1x10^7$ primaries (377877 interactions -> 3.78 %)

Cross section (mbarn) of

^{16}O 200 MeV/u on H target

evaluated by difference

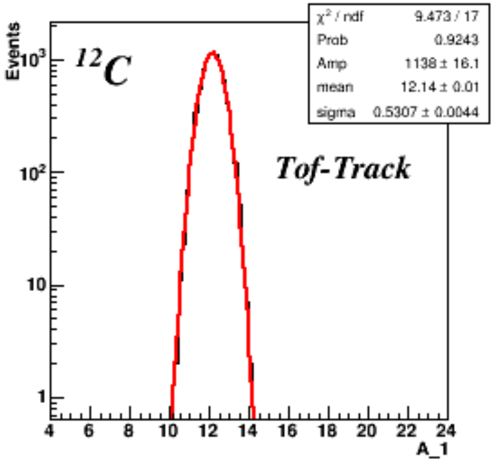
$$\frac{d\sigma}{dE_{kin}}(H) = \frac{1}{4}\left(\frac{d\sigma}{dE_{kin}}(C_2H_4) - 2\frac{d\sigma}{dE_{kin}}(C)\right)$$

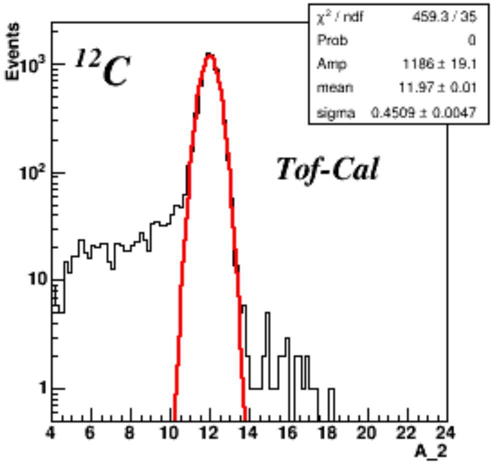


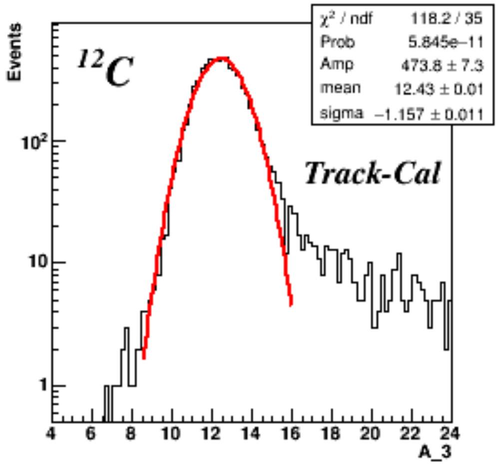
Fluka [mbarn]	Our [mbarn]	Diff
$25.33 \pm 0.1\%$	$23.14 \pm 14.1\%$	8.6%

Systematic study depending on ToF, p, Ekin

$$A_1 = \frac{p}{U\beta\gamma c} \qquad A_2 = \frac{E_k}{Uc^2(1-\gamma)} \qquad A_3 = \frac{pc^2 - {E_k}^2}{2Uc^2 E_k}$$





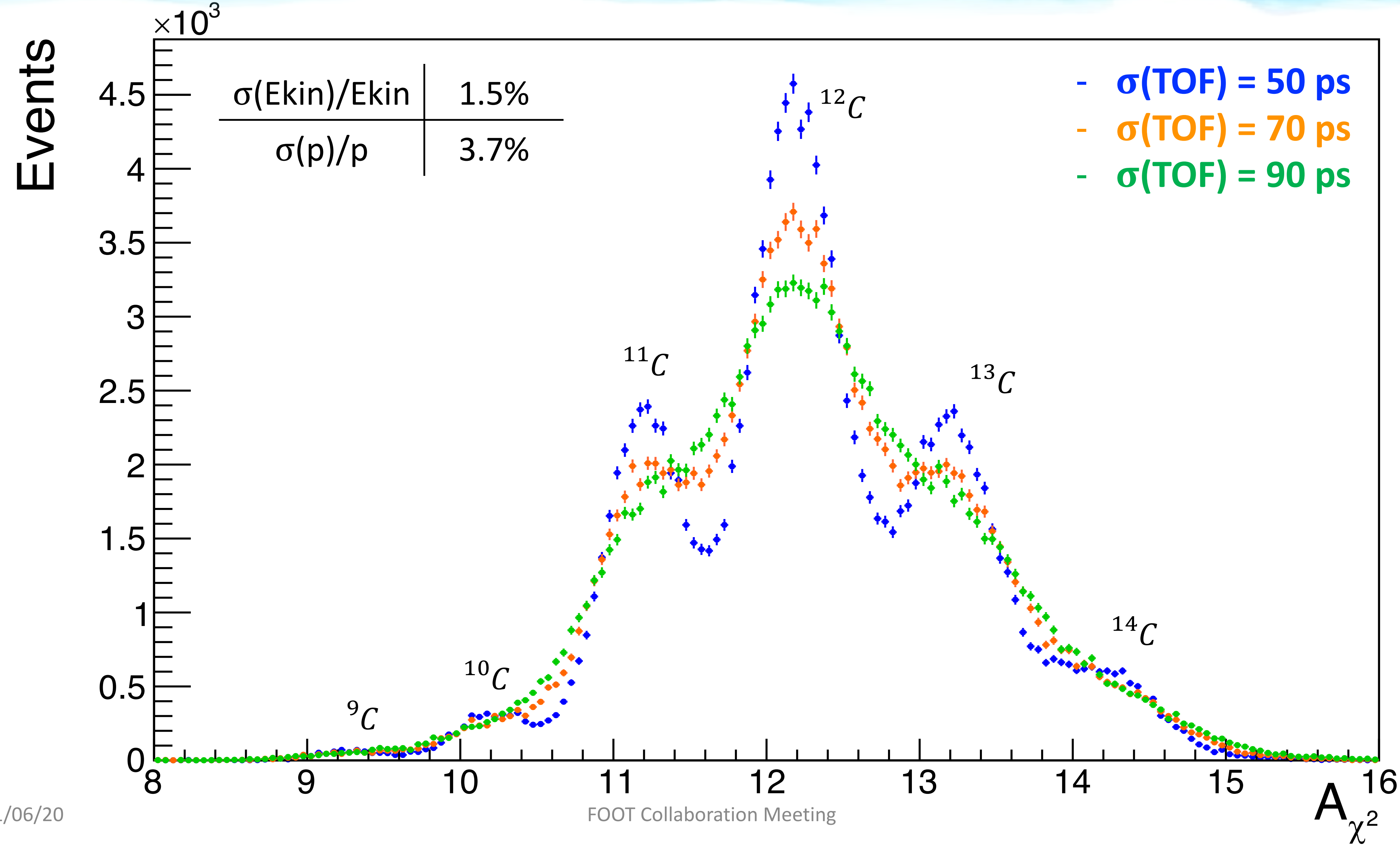


Best determination of A throught a standard χ^2 minimization

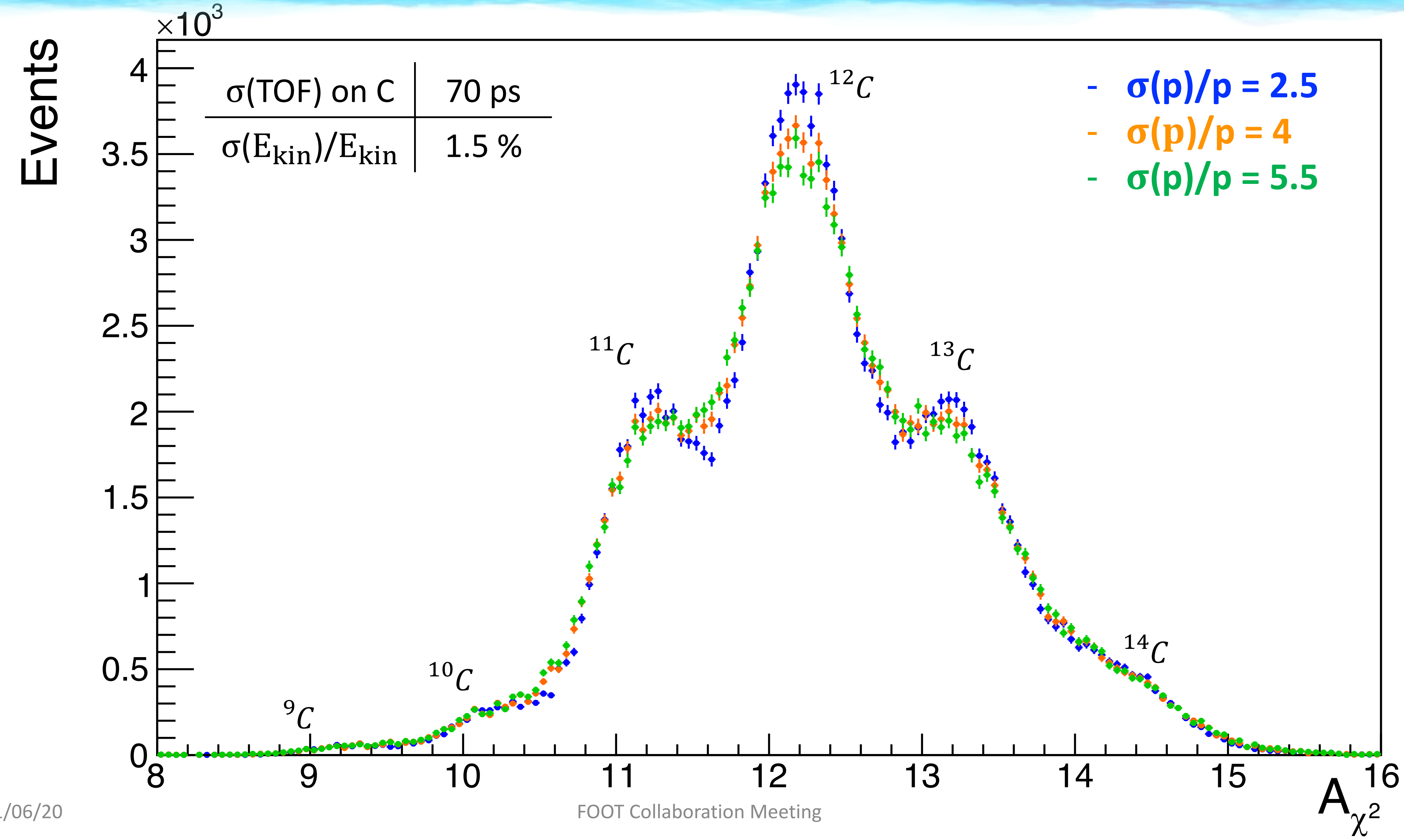
2 parameters fixed and 1 changed from the ideal to the worse scenario:

- 40 ps < TOF resolution < 100 ps
- 2.5 % < momentum resolution < 5.5 %
- 1 % < kinetic energy resolution < 2.8 %

Changing σ(TOF) on C

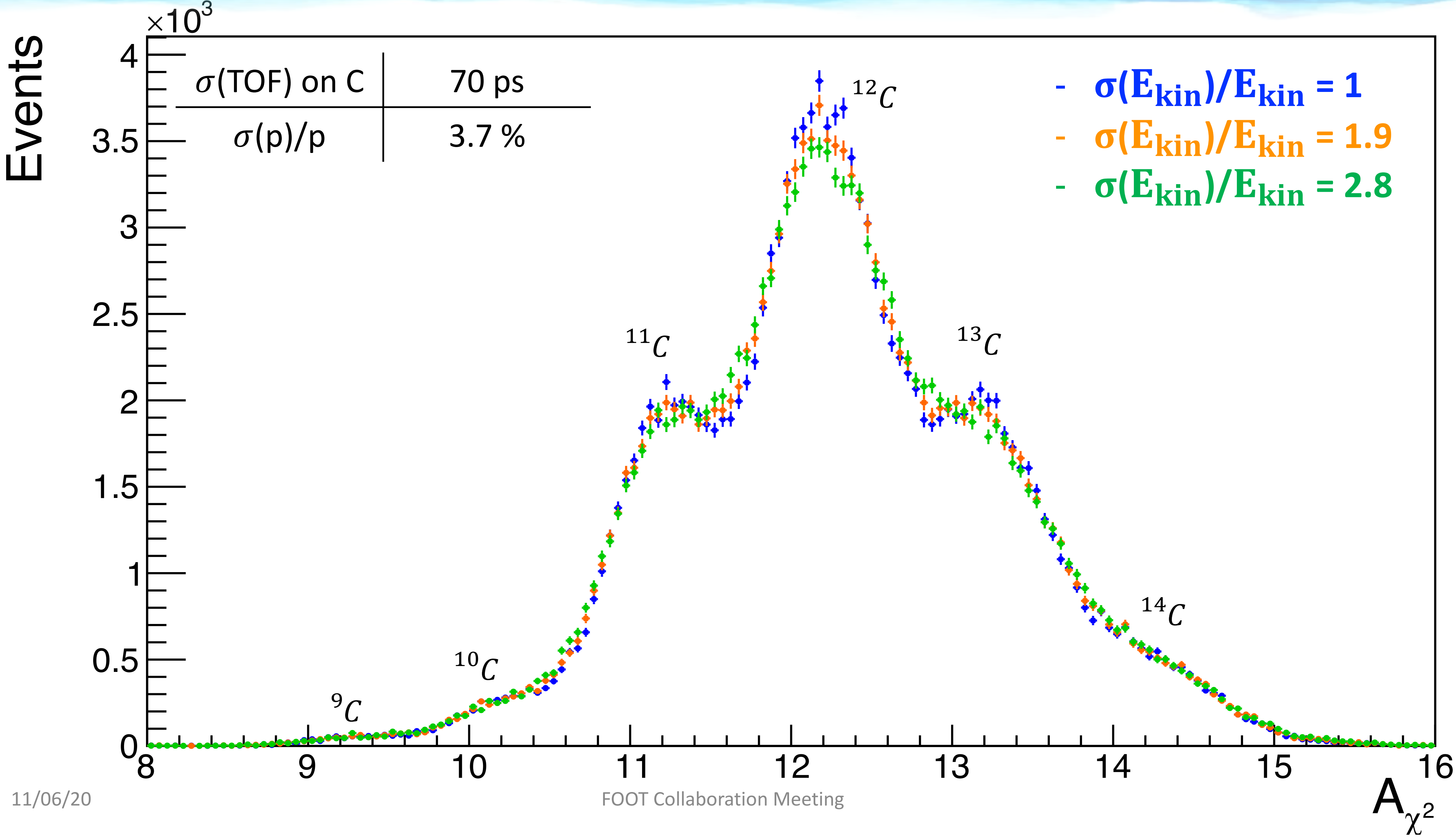


Changing $\sigma(p)/p$

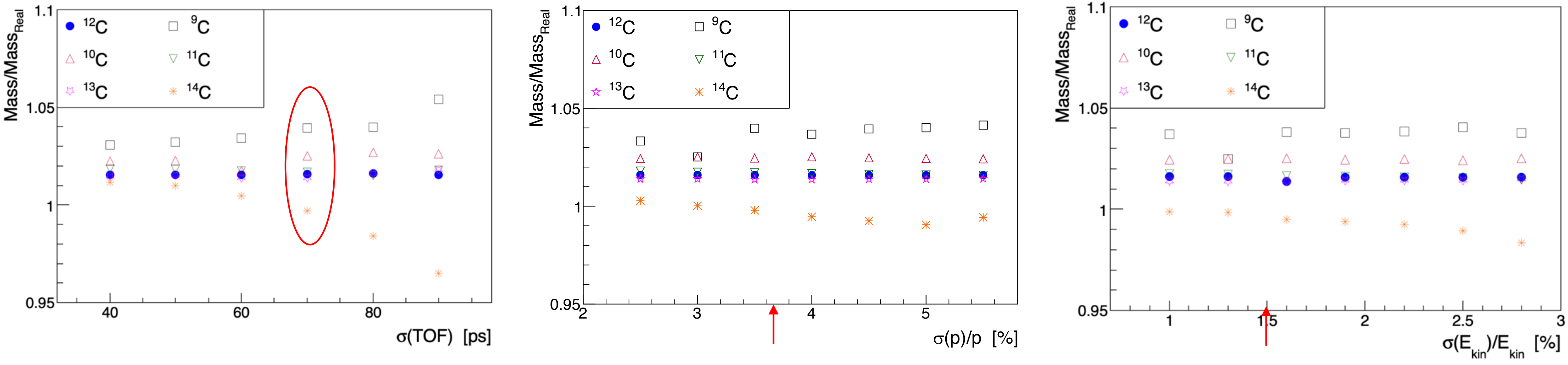


σ(TOF) on C	70 ps
$\sigma(E_{kin})/E_{kin}$	1.5 %

Changing $\sigma(E_{kin})/E_{kin}$



Mass (wrt real mass)

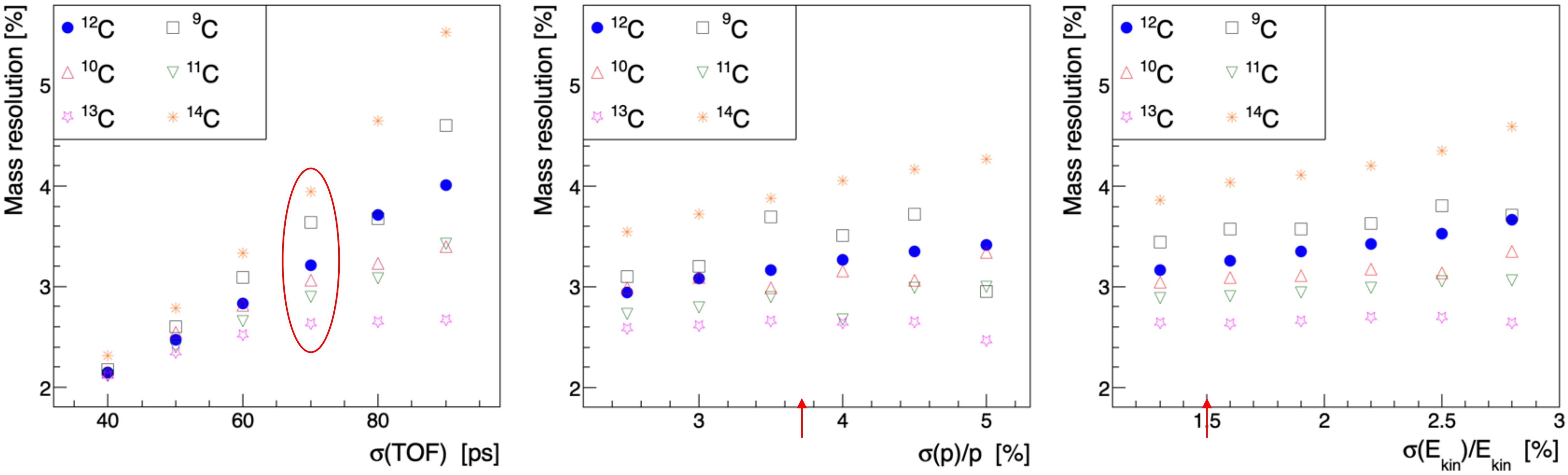


- ^{12}C : it represents ~50 % of the statistics. Constant mass overestimation of ~1.5% with an uncertainty of ~1%.

- ^{9}C, ^{14}C : a general worsening increasing the resolutions is evident, but TOF is the most effective. Uncertainty in these cases can vary from 1% (for small resolutions on TOF, p, Ekin) up to 10%.

- ^{9}C, ^{11}C, ^{13}C : mass reconstructed within a precision of +2-3% with an uncertainty of ~2%

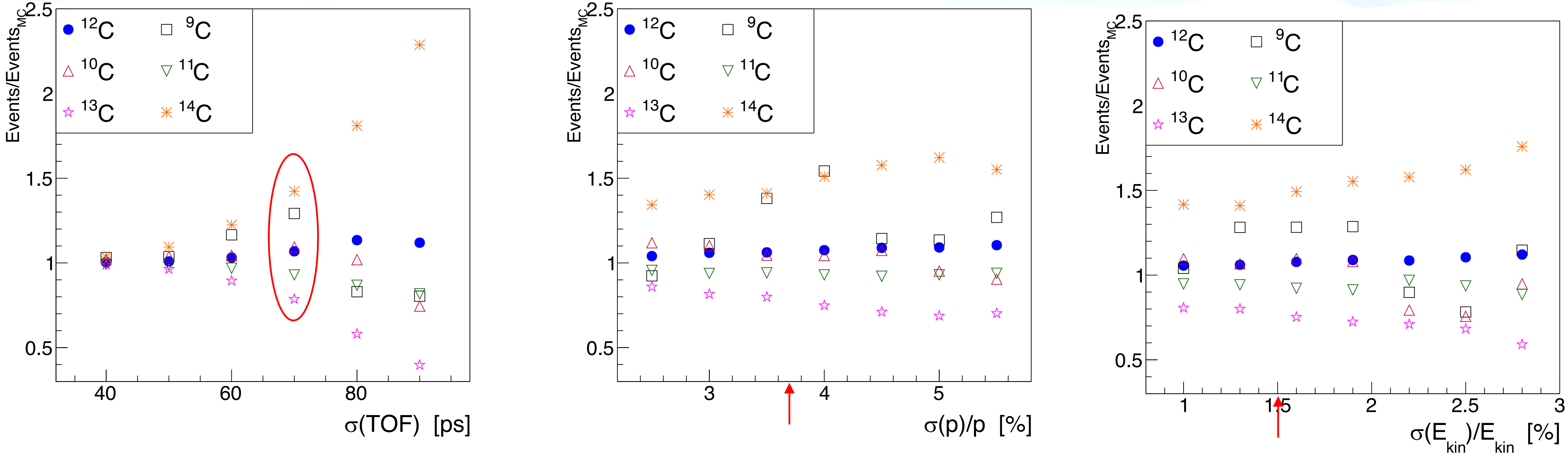
Mass resolution



- As expected the mass resolution increases with the increase of the resolutions on p, TOF, Ekin. TOF has the major impact.

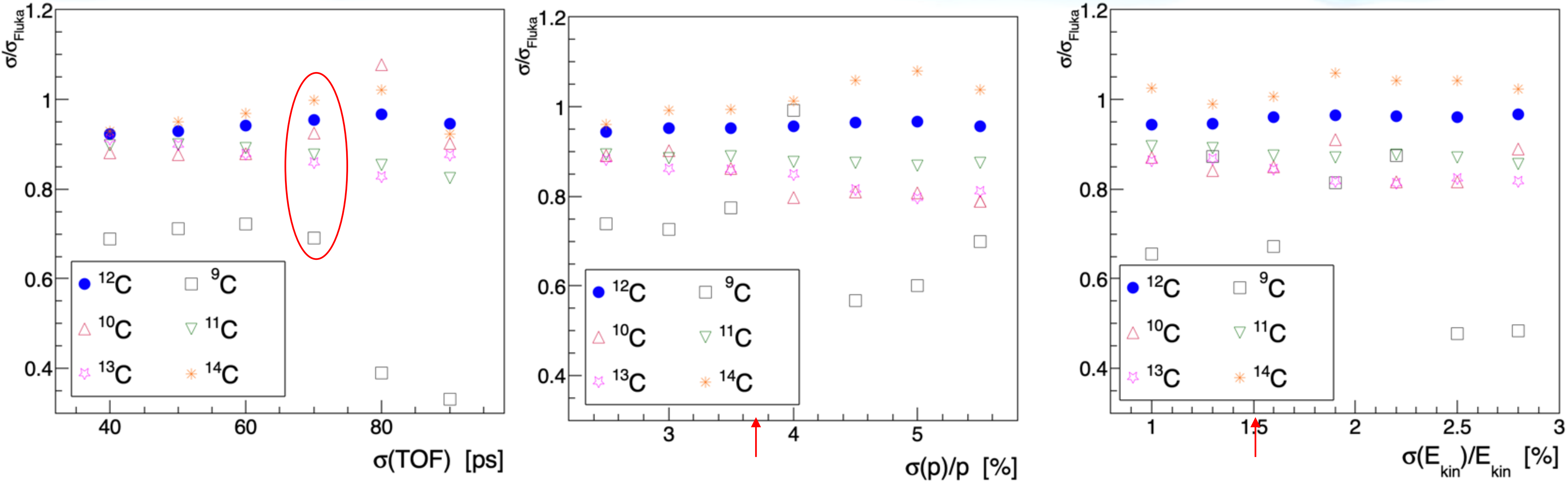
- At present (σ(TOF) ~70 ps) a resolution between 2.2% and 2.8% is guaranteed for all the isotopes of carbon.

Fitted events (wrt MC events)



- ^{12}C : the number of events reconstructed with the fit overestimates the MC events of ~1%.
- Strange behaviour of ^{13}C and ^{14}C when changing σ(TOF): more events are identified as ^{14}C than ^{13}C .
- ^{13}C has the worst behaviour: 10-20% of the events are always lost due to the fit.

Cross section (wrt MC cross section)



- ^{12}C : even if the events are almost the same of the MC, a constant underestimation of ~5% is always evident.
- ^{9}C : the statistics is not enough to precisely estimate the CS.

Conclusions

- The presented machinery for the cross section evaluation has been studied by referring at the current available information from MC simulations. In the future, something could change in order to work with real data.
- Even with a good statistics (1E+08) with respect to the real one, ^{9}C has a statistics too poor to precisely estimate its cross section.
- σ(TOF) is the most sensible parameter in terms of isotope identification.
- When the statistics is good, changing σ(p), σ(TOF), σ(Ekin) doesn't affect the fragments mass estimation, but the major impact is on the mass resolution.
- Even if the number of ^{12}C fragments reconstructed is almost the same as MC, cross section measurement always underestimates of ~5 %.

Thank you!

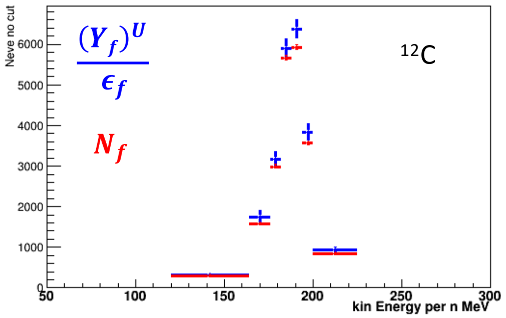
BACKUP SLIDES

Closure test

Real events: true quantities without cuts

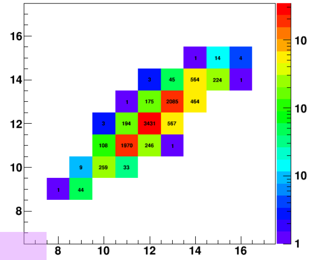
N_f

If all correct: Statistically equal



Real events: reco quantities with cuts

Y_f

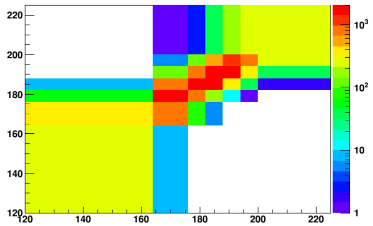


Bkg subtraction

$Y_f - Bkg_f$

Reco quantities with cuts

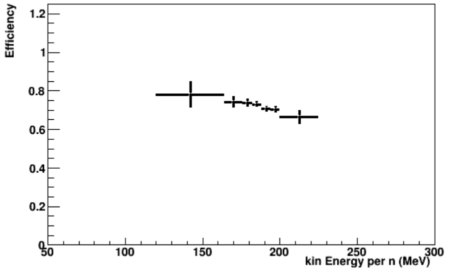
Unfolding



$(Y_f - Bkg_f)^U$

«true quantities» with cuts

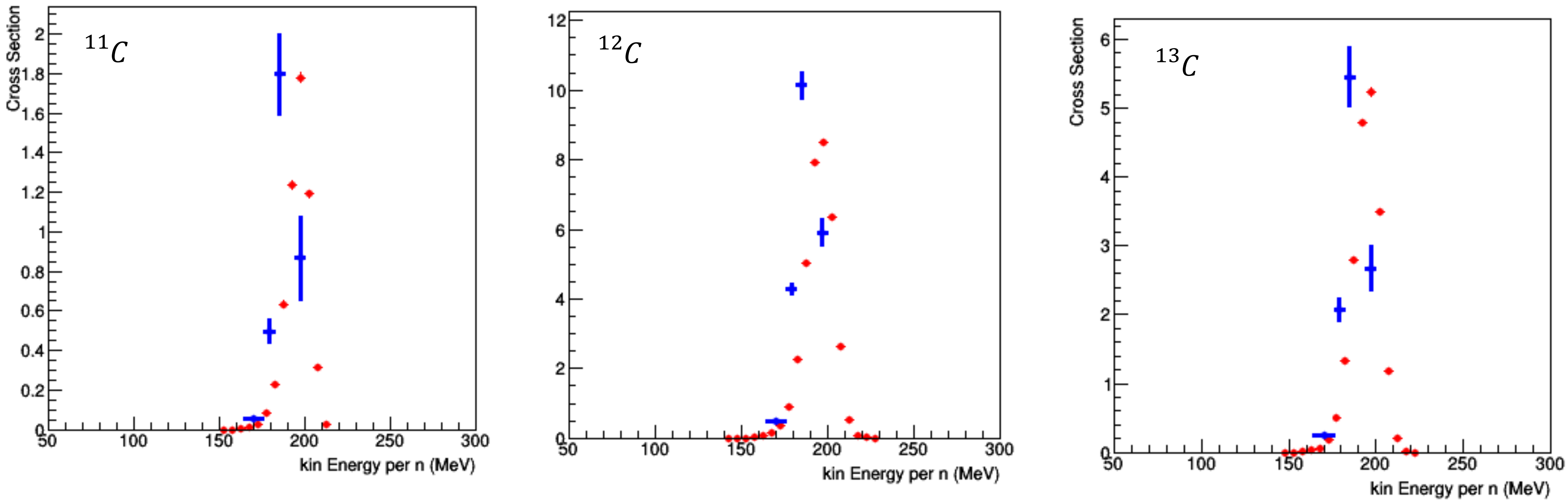
Efficiency



$$\frac{(Y_f - Bkg_f)^U}{\epsilon_f}$$

true quantities without cuts

Systematic shift in the Ekin recostruction



Mass reconstruction and fit

TOF (β) – TRACKER (p)

$$A_1 = \frac{m}{U} = \frac{p}{U\,\beta\,\gamma}$$

TOF (β)– CALO (E_{kin})

$$A_2 = \frac{m}{U} = \frac{E_{kin}}{U(\gamma - 1)}$$

TRACKER (p) – CALO (E_{kin})

$$A_3 = \frac{m}{U} = \frac{p^2 - E_{kin}{}^2}{2E_{kin}}$$

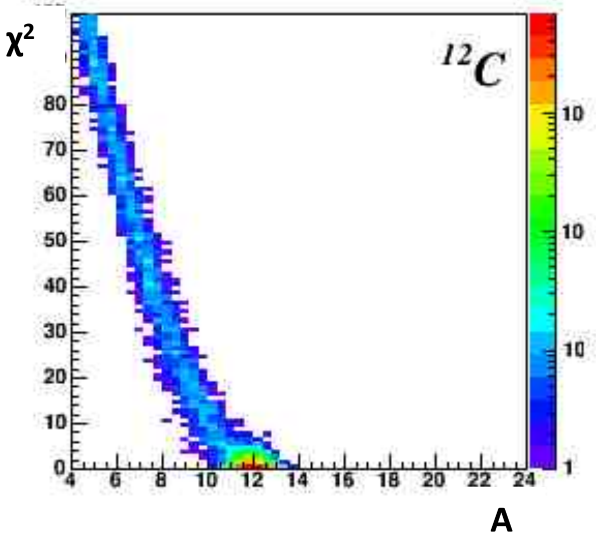
- **Standard χ^2 Fit**
  - Taking into account the correlation between A_1, A_2 and A_3 (reconstructed quantities)
  - Minimization method based on a function *f* defined by:

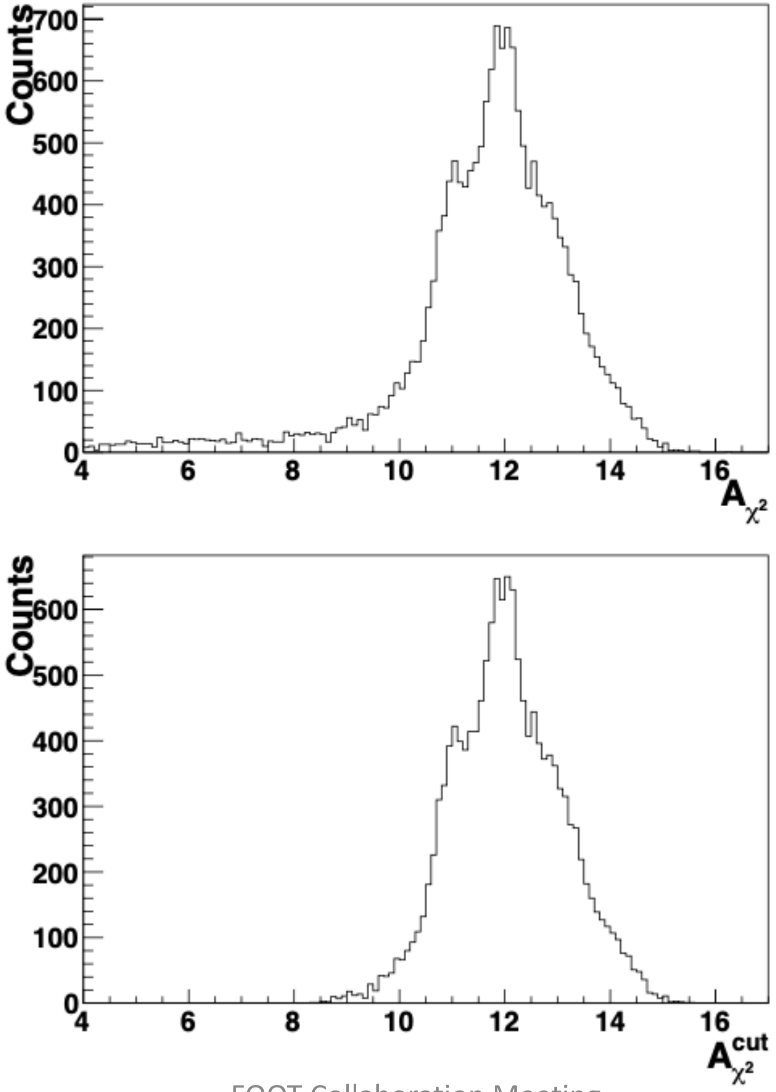
$$f = \left(\frac{(tof_{reco} - t)}{\sigma tof_{reco}}\right)^2 + \left(\frac{(p_{reco} - p)}{\sigma p_{reco}}\right)^2 + \left(\frac{(E_{kin,reco} - E_{kin})}{\sigma E_{kin,reco}}\right)^2 + (A_1 - A \quad A_2 - A \quad A_3 - A)\begin{pmatrix} C_{00} & C_{01} & C_{02} \\ C_{10} & C_{11} & C_{12} \\ C_{20} & C_{21} & C_{22} \end{pmatrix}\begin{pmatrix} A_1 - A \\ A_2 - A \\ A_3 - A \end{pmatrix}$$

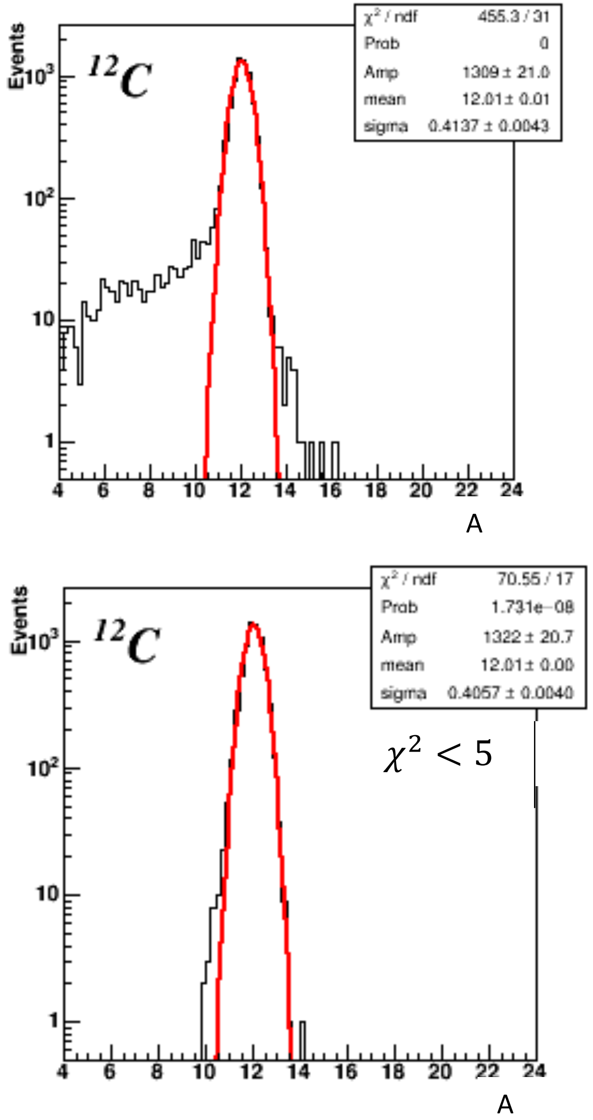
$C = (A \cdot A^T)^{-1}$ Correlation matrix →

$$A = \begin{pmatrix} \frac{\partial A_1}{\partial t} dt & \frac{\partial A_1}{\partial p} dp & 0 \\ \frac{\partial A_2}{\partial t} dt & 0 & \frac{\partial A_2}{\partial E_{kin}} dE_{kin} \\ 0 & \frac{\partial A_3}{\partial p} dp & \frac{\partial A_3}{\partial E_{kin}} dE_{kin} \end{pmatrix}$$

$\chi^2 < 5$ cut


χ²
¹²C
A


Counts
A_{χ^2}
Counts
$A^{cut}_{\chi^2}$


¹²C
χ² / ndf 455.3 / 31
Prob 0
Amp 1309 ± 21.0
mean 12.01 ± 0.01
sigma 0.4137 ± 0.0043
Events
A
¹²C
χ² / ndf 70.55 / 17
Prob 1.731e−08
Amp 1322 ± 20.7
mean 12.01 ± 0.00
sigma 0.4057 ± 0.0040
$\chi^2 < 5$
Events
A