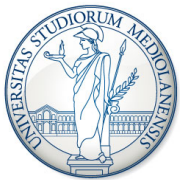


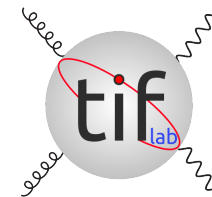


MACHINE LEARNING AN UNKNOWN PHYSICAL LAW: THE STRUCTURE OF THE PROTON

STEFANO FORTE
UNIVERSITÀ DI MILANO & INFN



UNIVERSITÀ DEGLI STUDI DI MILANO
DIPARTIMENTO DI FISICA



ROME PHYSICS ENCOUNTERS

FRASCATI, FEBRUARY 20, 2020

PHYSICS AT THE LHC AS PRECISION PHYSICS

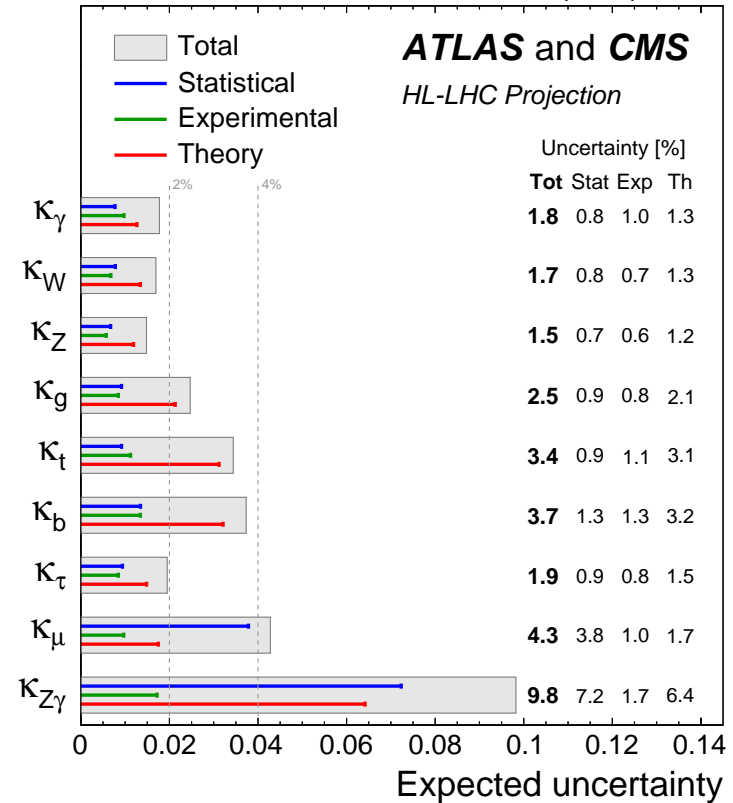
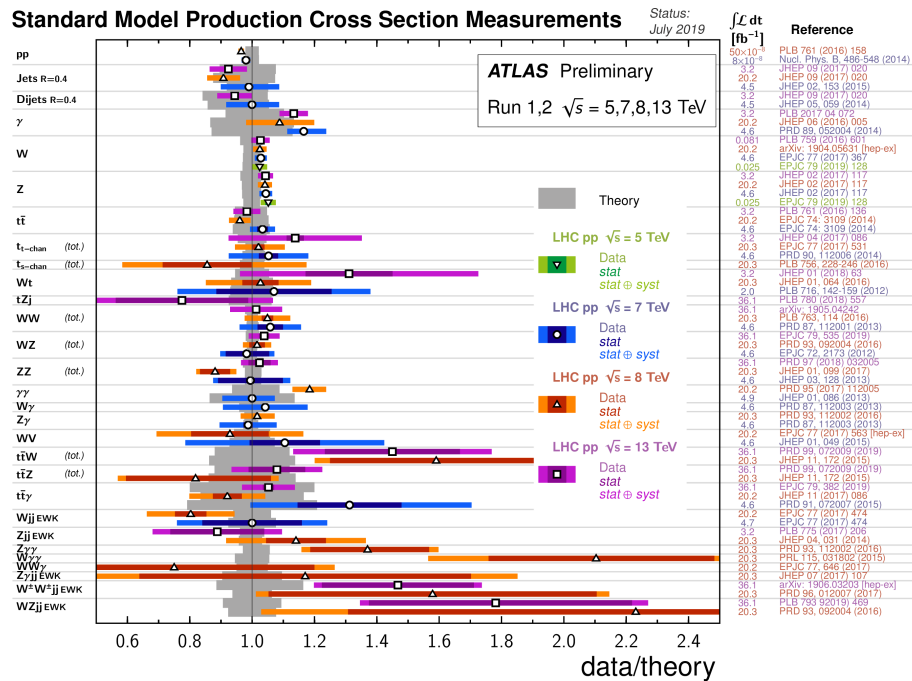
DEVIATIONS FROM SM

HL-LHC: 2024-2040

SM CROSS-SECTIONS TODAY:

TH. VS EXP.

$\sqrt{s} = 14 \text{ TeV}, 3000 \text{ fb}^{-1}$ per experiment



$$\kappa_j^2 = \sigma_j / \sigma^{\text{SM}}$$

- SM TESTED AT THE PERCENT LEVEL
- SEEING DEVIATIONS REQUIRES SUB-PERCENT ACCURACY

SUMMARY

PDFs: A RECAP SEQUENCE

- DETERMINING PDFs
- DISCOVERING NEW PHYSICS
- PDF UNCERTAINTIES, TOLERANCE AND ALL THAT

ARTIFICIAL INTELLIGENCE

- PDFs, AI AND ML
- THE NNPDF METHODOLOGY: IDEAS AND TESTS
- THE STATE OF THE ART: ACCOMPLISHMENTS AND CHALLENGES

MACHINE LEARNING PDFs

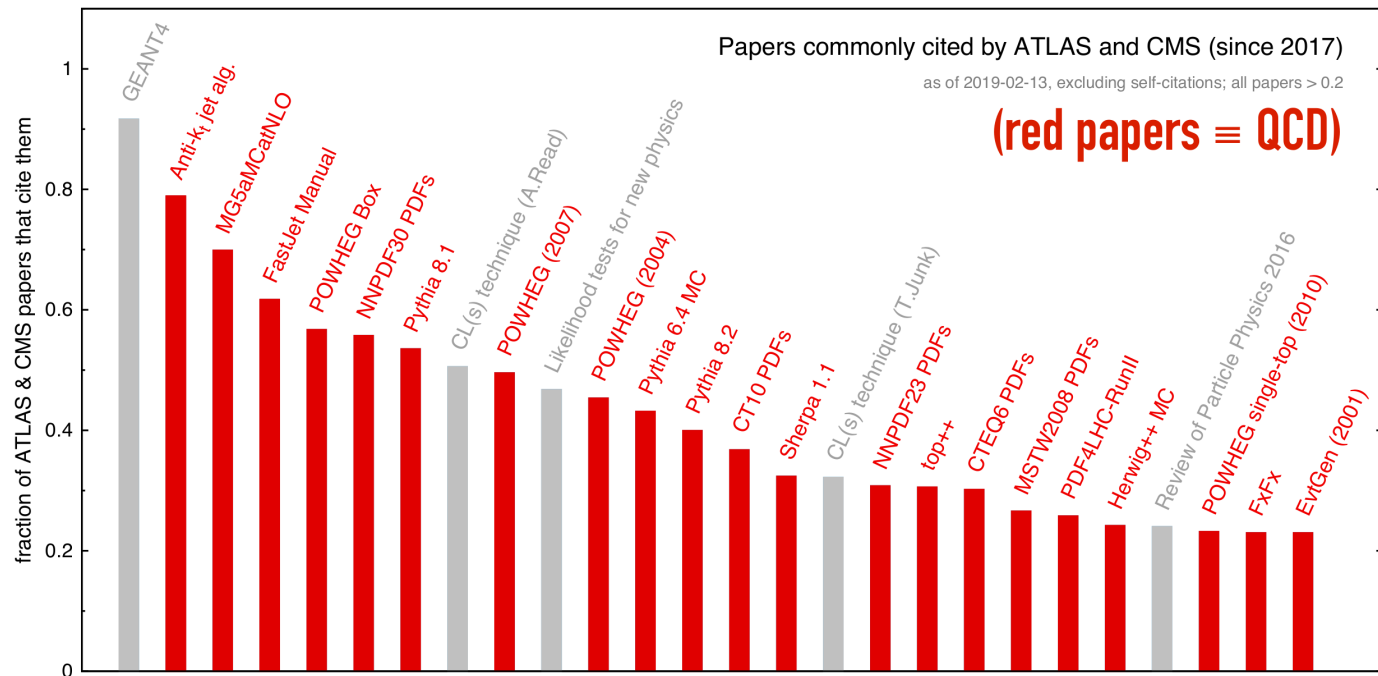
- OPTIMIZATION
- HYPEROPTIMIZATION
- INTO THE UNKNOWN

PDFS AND PRECISION PHYSICS

UNCERTAINTIES AND QCD

- THE LHC IS A PROTON COLLIDER \Rightarrow ANY INTERACTION CONTAINS A STRONG INTERACTION
- QCD IS THE MAIN THEORETICAL PROBLEM
- .

PAPERS MOST CITED BY ATLAS (BY FRACTION)

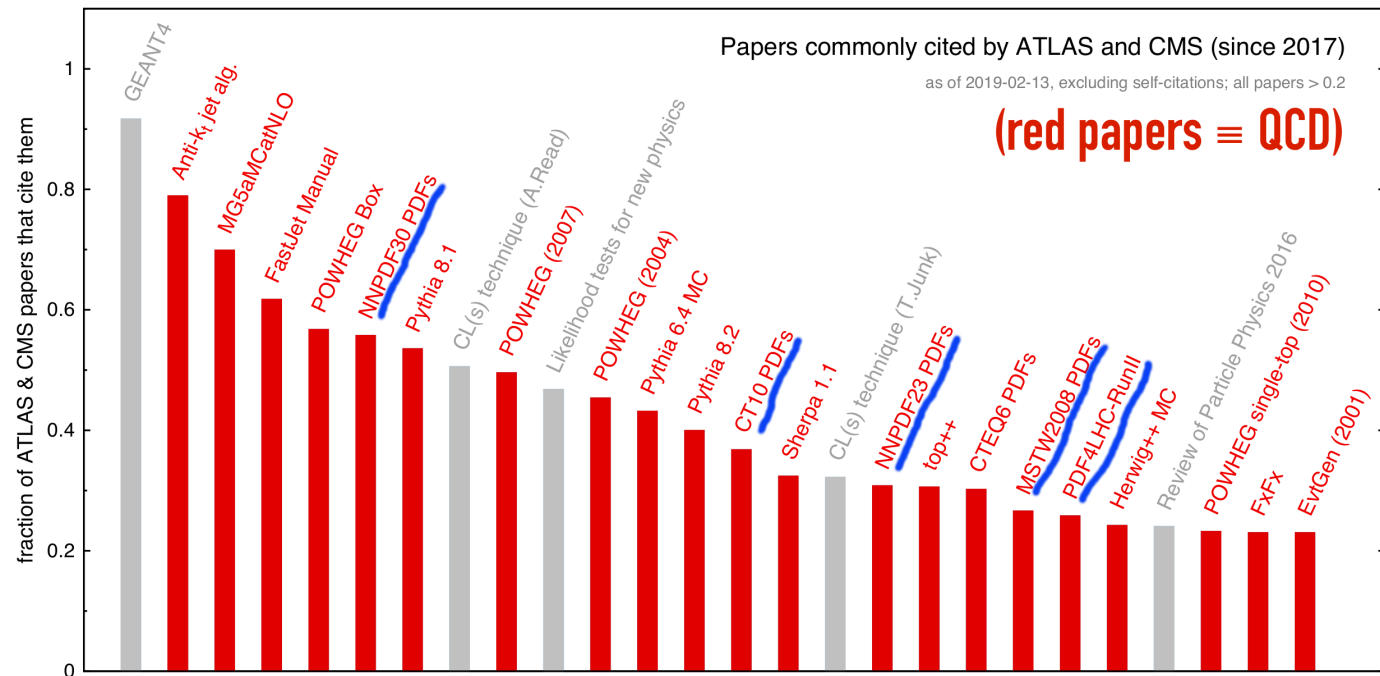


(G. Salam, 2019)

UNCERTAINTIES QCD, AND PDFs

- THE LHC IS A PROTON COLLIDER \Rightarrow ANY INTERACTION CONTAINS A STRONG INTERACTION
- QCD IS THE MAIN THEORETICAL PROBLEM
- PDFs ARE THE DOMINANT ISSUE

PAPERS MOST CITED BY ATLAS (BY FRACTION)

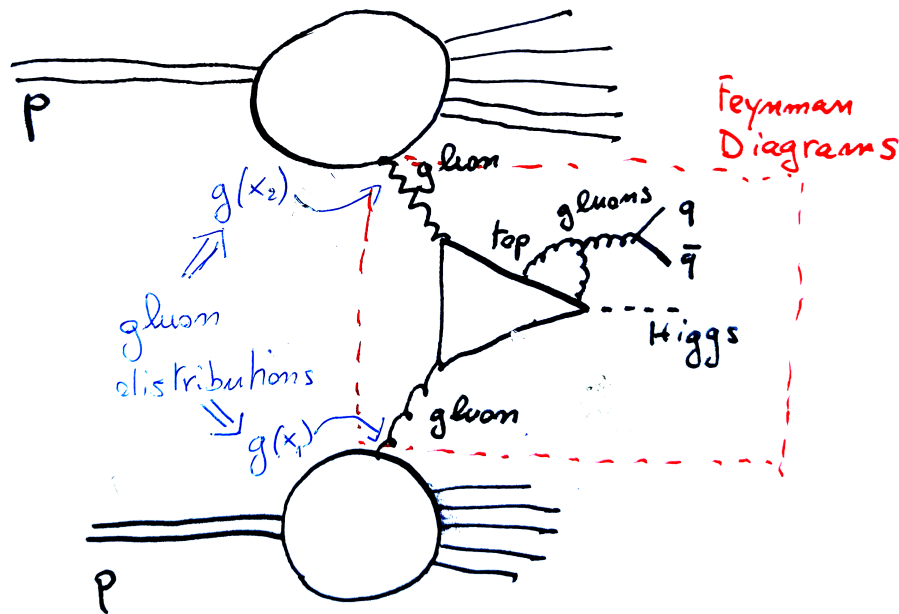


(G. Salam, 2019)

PDF papers underlined

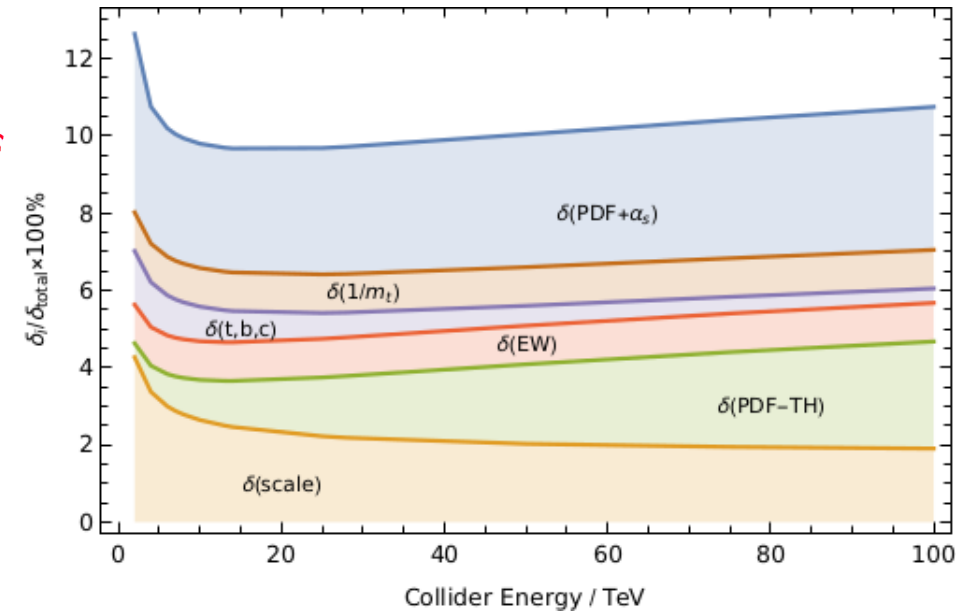
UNCERTAINTIES AND PDFs

QCD FACTORIZATION



UNCERTAINTIES:

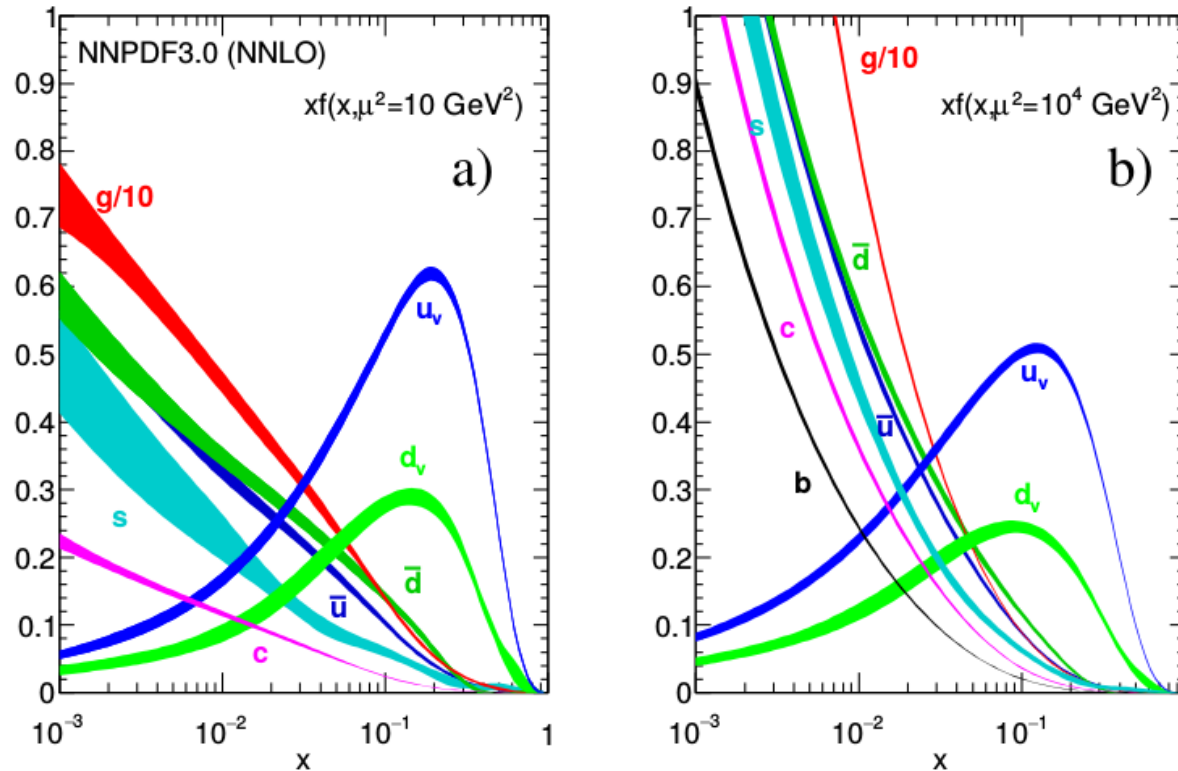
HIGGS IN GLUON FUSION



(HL-LHC Higgs WG report, 2019)

- PDF EXPRESS THE LIKELIHOOD OF A QUARK OR GLUONS (PARTONS) TO ENTER A COLLISION
- THEIR KNOWLEDGE IS A DOMINANT SOURCE OF UNCERTAINTY

A PORTRAIT OF THE PROTON AS SEEN FROM A HIGGS BOSON



(PDG 2018)

- **PARTON DISTRIBUTIONS:** MOMENTUM FRACTION DISTRIBUTIONS FOR EACH TYPE OF QUARK, ANTIQUARK & THE GLUON
- **EXTRACTED FROM DATA,** COMPARING PDF-DEPENDENT PREDICTION & INVERTING
- **MUST DETERMINE A PROBABILITY DISTRIBUTION OF FUNCTIONS FROM A DISCRETE SET OF DATA**

HOW DID WE GET HERE?

DISCOVERY AT A HADRON COLLIDER AND PDFs

THE DISCOVERY OF THE W (1984)

EXPERIMENTAL DISCOVERY



EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN-EP/85-108
11 July 1985

W PRODUCTION PROPERTIES AT THE CERN SPS COLLIDER

UA1 Collaboration, CERN, Geneva, Switzerland

Aachen¹ - Amsterdam (NIKHEF)² - Annecy (LAPP)³ - Birmingham⁴ - CERN⁵ -
Harvard⁶ - Helsinki⁷ - Kiel⁸ - London (Imperial College⁹ and Queen Mary College¹⁰) - Padua¹¹ -
Paris (Coll. de France)¹² - Riverside¹³ - Rome¹⁴ - Rutherford Appleton Lab.¹⁵ -
Saclay (CEN)¹⁶ - Victoria¹⁷ - Vienna¹⁸ - Wisconsin¹⁹ Collaboration

The corresponding experimental result for the 1984 data at $\sqrt{s} = 630$ GeV is

$$(\sigma \cdot B)_W = 0.63 \pm 0.05 (\pm 0.09) \text{ nb.}$$

This is in agreement with the theoretical expectation [14] of $0.47^{+0.14}_{-0.08}$ nb. We note that the 15%

THEORETICAL PREDICTION

42

G. Altarelli et al. / Vector boson production

TABLE 2
Values (in nb) of the total cross sections for W^\pm and Z^0 production

\sqrt{s} (GeV)	$W^+ + W^-$		Z^0			$\frac{\sigma(W^+ + W^-)}{\sigma(Z^0)}$	$\frac{\sigma(W^+ + W^-)}{\sigma(Z^0)}$	$\frac{\sigma(W^+ + W^-)}{\sigma(Z^0)}$	
	GHR	DO1	DO2	GHR	DO1	DO2	GHR	DO1	DO2
540	4.2	4.3	4.1	1.3	1.3	1.2	3.1	3.4	3.5
700	6.2	6.3	6.1	2.0	1.9	1.8	3.1	3.3	3.4
1000	9.5	9.5	9.6	3.1	3.0	2.9	3.1	3.2	3.3
1300	12.5	12.5	12.9	4.0	3.9	3.9	3.1	3.2	3.3
1600	15.5	15.6	16.5	5.0	4.8	5.0	3.1	3.2	3.3

ALTARELLI, ELLIS, GRECO, MARTINELLI, 1984

- AGREEMENT AND UNCERTAINTIES AT 20% CONSIDERED TO BE SATISFACTORY
- RESULTS FROM DIFFERENT PDF SETS DIFFER BY AT LEAST 5%
- NO WAY TO ESTIMATE PDF UNCERTAINTIES

DISCOVERY AT A HADRON COLLIDER AND PDFs

THE DISCOVERY OF THE W (1984)

PDFs IN 1984

THEORETICAL PREDICTION

42

G. Altarelli et al. / Vector boson production

TABLE 2
Values (in nb) of the total cross sections for W^\pm and Z^0 production

\sqrt{S} (GeV)	$W^+ + W^-$			Z^0			$\frac{\sigma(W^+ + W^-)}{\sigma(Z^0)}$		
	GHR	DO1	DO2	GHR	DO1	DO2	GHR	DO1	DO2
540	4.2	4.3	4.1	1.3	1.3	1.2	3.1	3.4	3.5
700	6.2	6.3	6.1	2.0	1.9	1.8	3.1	3.3	3.4
1000	9.5	9.5	9.6	3.1	3.0	2.9	3.1	3.2	3.3
1300	12.5	12.5	12.9	4.0	3.9	3.9	3.1	3.2	3.3
1600	15.5	15.6	16.5	5.0	4.8	5.0	3.1	3.2	3.3

ALTARELLI, ELLIS, GRECO, MARTINELLI, 1984

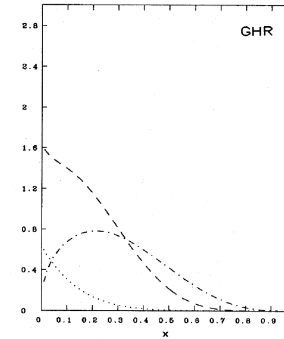


FIG. 25. Parton distributions of Glück, Hoffmann, and Reya (1982), at $Q^2=5 \text{ GeV}^2$: valence quark distribution $x[u_v(x)+d_v(x)]$ (dotted-dashed line), $xG(x)$ (dashed line), and q_v (dotted line).

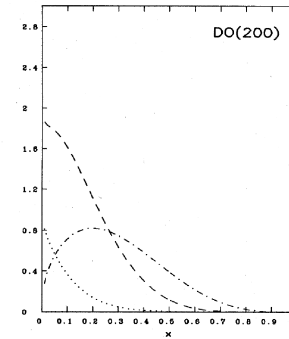


FIG. 27. "Soft-gluon" ($\Lambda=200 \text{ MeV}$) parton distributions of Duke and Owens (1984) at $Q^2=5 \text{ GeV}^2$: valence quark distribution $x[u_v(x)+d_v(x)]$ (dotted-dashed line), $xG(x)$ (dashed line), and $q_v(x)$ (dotted line).

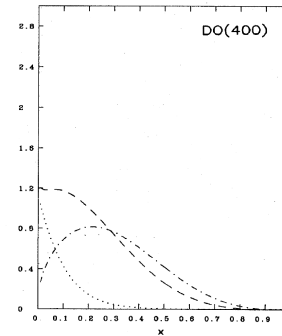


FIG. 26. "Hard-gluon" ($\Lambda=400 \text{ MeV}$) parton distributions of Duke and Owens (1984) at $Q^2=5 \text{ GeV}^2$: valence quark distribution $x[u_v(x)+d_v(x)]$ (dotted-dashed line), $xG(x)$ (dashed line), and $q_v(x)$ (dotted line).

Rev. Mod. Phys., Vol. 56, No. 4, October 1984

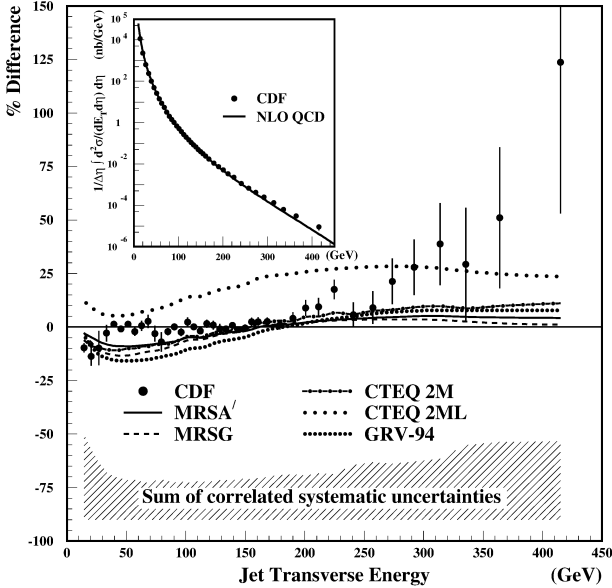
GHR VS DUKE-OWENS

- AGREEMENT AND UNCERTAINTIES AT 20% CONSIDERED TO BE SATISFACTORY
- RESULTS FROM DIFFERENT PDF SETS DIFFER BY AT LEAST 5%
- NO WAY TO ESTIMATE PDF UNCERTAINTIES

DISCOVERY AT A HADRON COLLIDER AND PDFs

THE DISCOVERY OF QUARK COMPOSITENESS (1995)

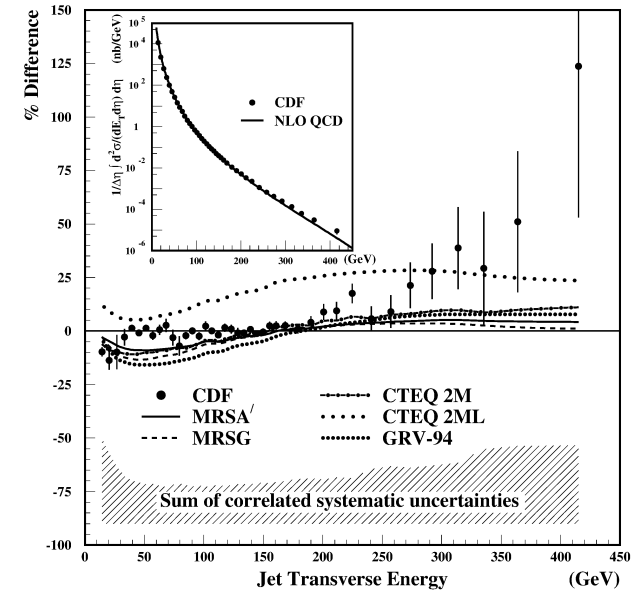
- DISCREPANCY BETWEEN QCD CALCULATION AND CDF JET DATA (1995)
- EVIDENCE FOR QUARK COMPOSITENESS
- .



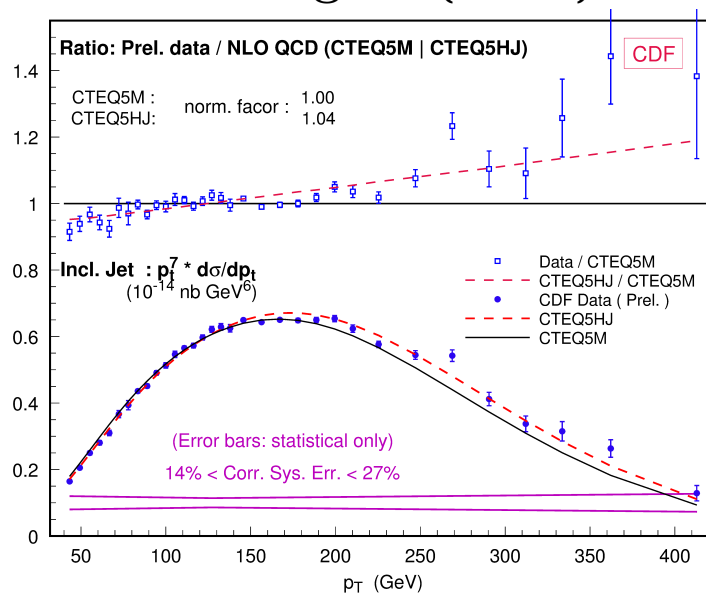
DISCOVERY AT A HADRON COLLIDER AND PDFs

A BETTER DETERMINATION OF THE GLUON PDF (1995)

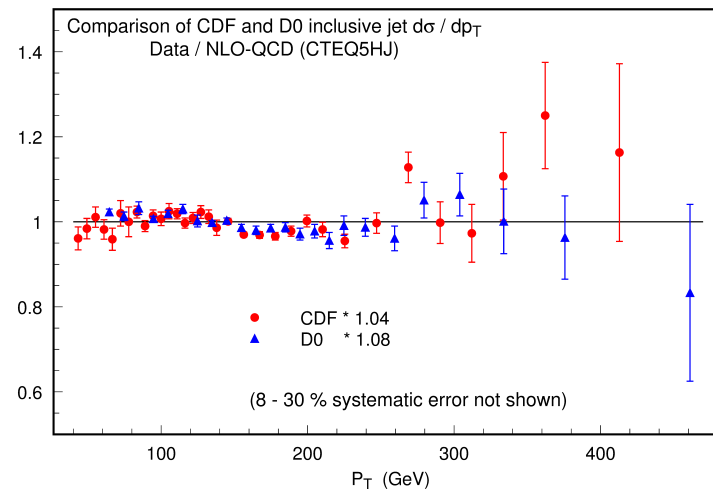
- DISCREPANCY BETWEEN QCD CALCULATION AND CDF JET DATA (1995)
- ~~EVIDENCE FOR QUARK COMPOSITENESS~~
- NO INFO ON PARTON UNCERTAINTY \Rightarrow RESULT STRONGLY DEPENDS ON GLUON AT $x \gtrsim 0.1$



DISCREPANCY REMOVED IF JET DATA INCLUDED IN THE FIT NEW CTEQ FIT (1996)



FINAL CTEQ FIT (1998)



WHAT'S THE PROBLEM ~ 2000

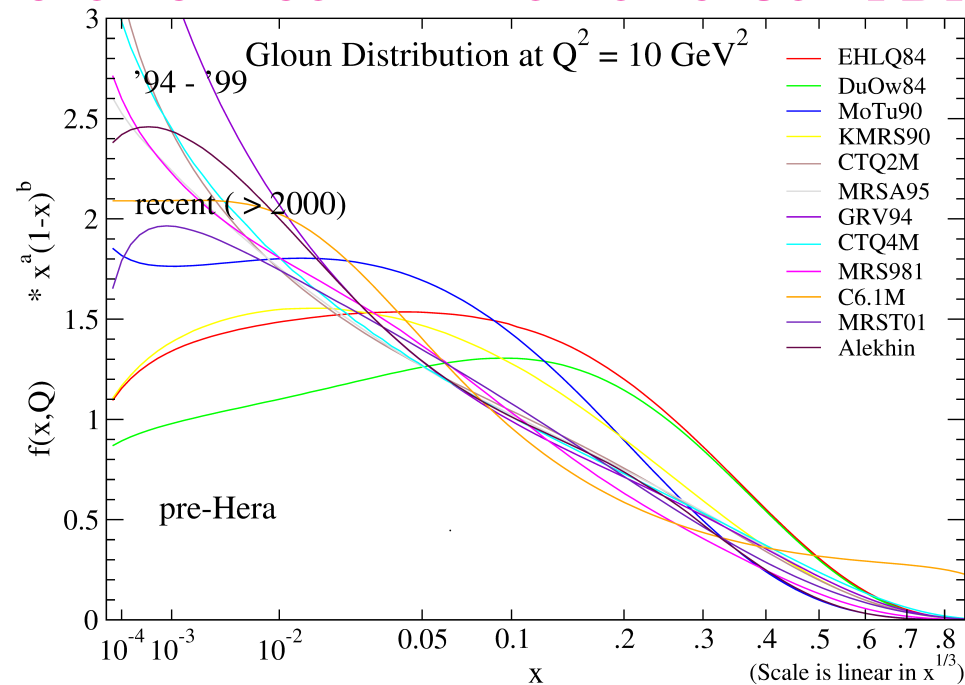
PDFs DETERMINED FITTING A **MODEL-INSPIRED FUNCTIONAL FORM**

gluon parametrization (MRST 2004)

$$xg(x, Q_0^2) = A_g(1-x)^{\eta_g}(1 + \epsilon_g x^{0.5} + \gamma_g x)x^{\delta_g} - A_-(1-x)^{\eta_-}x^{-\delta_-}$$

- PROBLEM **REDUCED** TO **FINITE-DIMENSIONAL**
- **WHO PICKS** THE FUNCTIONAL FORM?

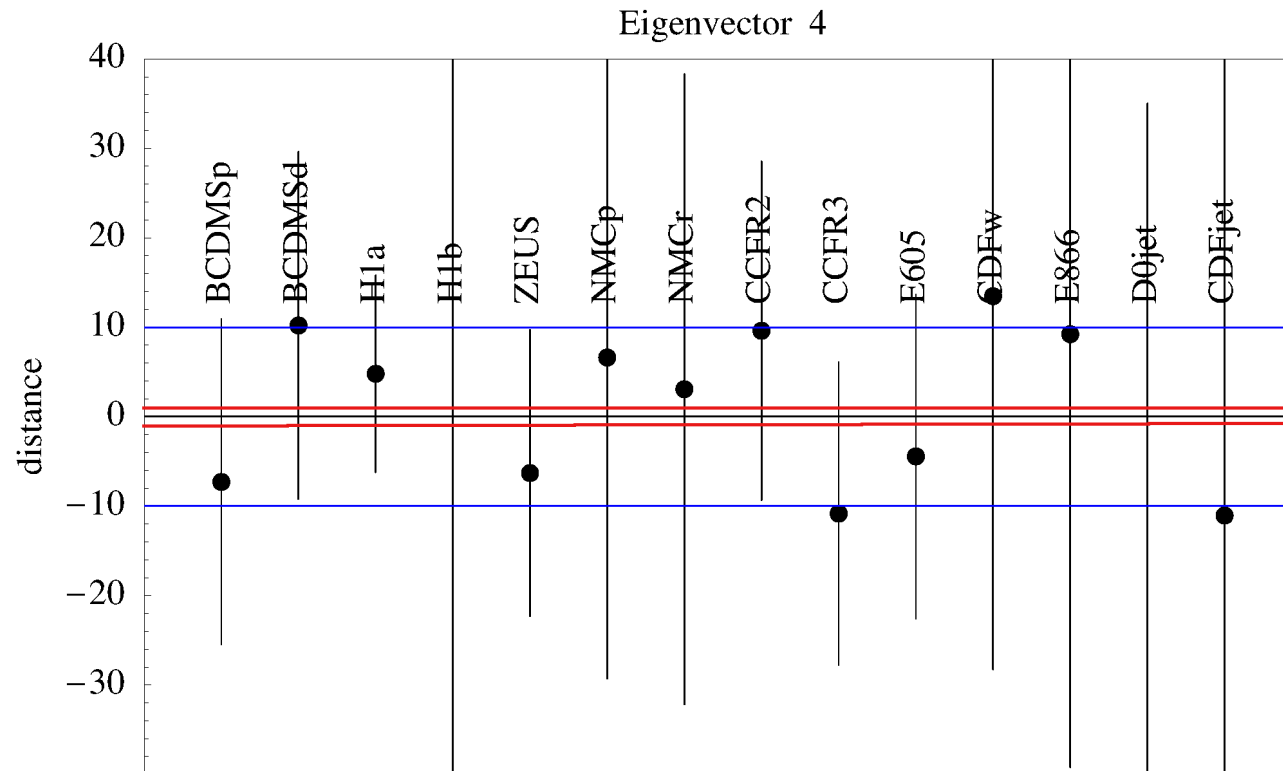
HISTORICAL COMPILATION OF GLUON PDFs



FIRST PDFs WITH UNCERTAINTIES (2002) “TOLERANCE”

one sigma & ten sigma intervals for typical
covariance matrix eigenvalue

vs best value and uncertainty from individual experiments

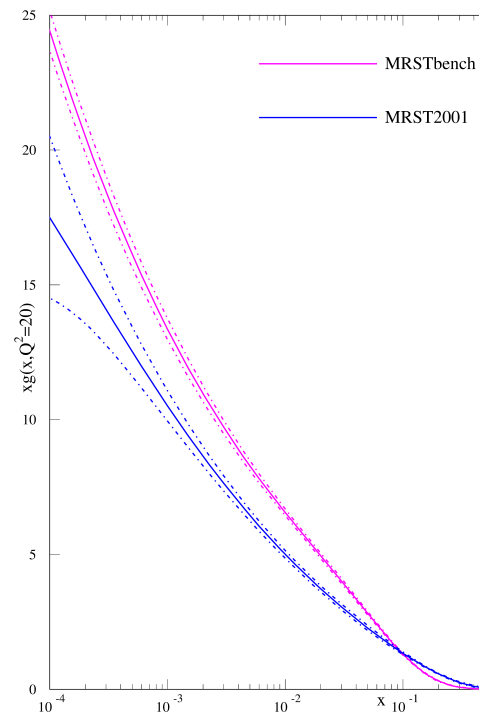


- SPREAD OF BEST-FIT FROM DIFFERENT DATA HUGE W.R. TO TEXTBOOK UNCERTAINTIES
- PDF UNCERTAINTIES RESCALED BY “TOLERANCE” $T \sim 10$

THE HERA-LHC BENCHMARK (2005)

- RESTRICTED AND VERY CONSISTENT DATASET USED
- RESULTS COMPARED TO THEN-BEST RESULT FROM FULL DATASET

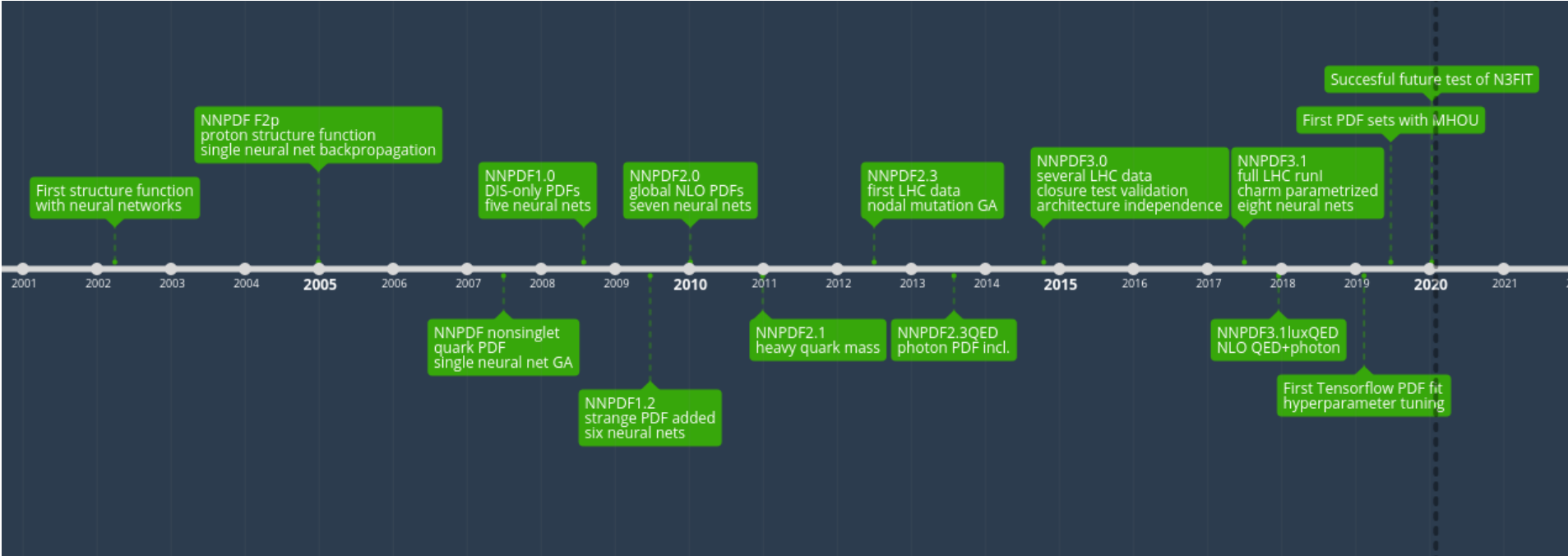
BENCHMARK VS DEFAULT GLUON



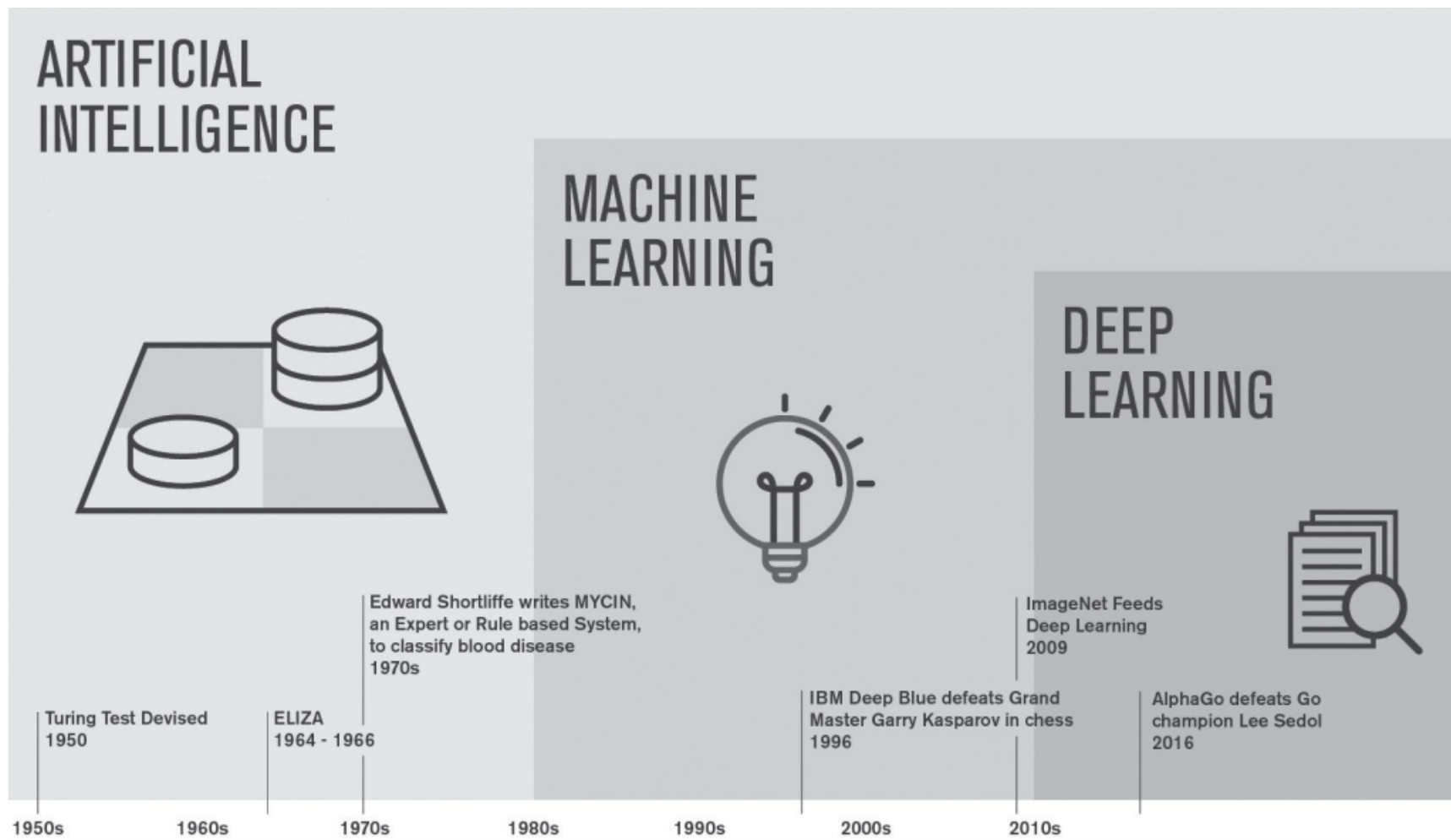
“...the partons extracted using a very limited data set are completely incompatible, even allowing for the uncertainties, with those obtained from a global fit with an identical treatment of errors...The comparison illustrates the problems in determining the true uncertainty on parton distributions.” (R.Thorne, HERALHC, 2005)

PDFS AND AI: NNPDF

PROTON STRUCTURE AS AN AI PROBLEM: NNPDF



FROM AI TO ML



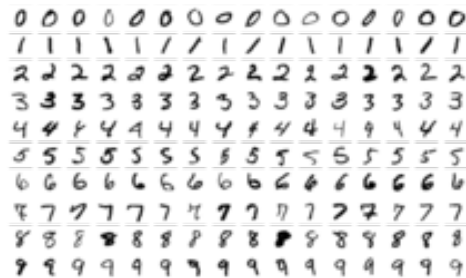
SHIFTING OF PARADIGMS

“KNOWLEDGE BASED” AI

- LEARN AND IMPLEMENT A SET OF RULES
- GOOD FOR CHESS, BAD FOR REAL LIFE



MACHINE LEARNING

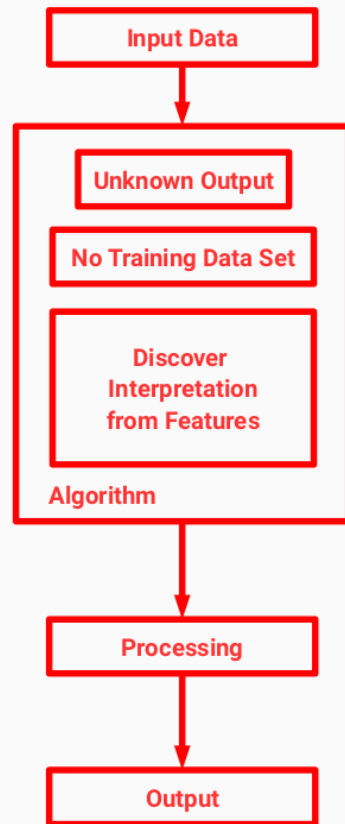


- “INTUITIVE” REPRESENTATION
- THE AI AGENT BUILD UP ITS OWN KNOWLEDGE



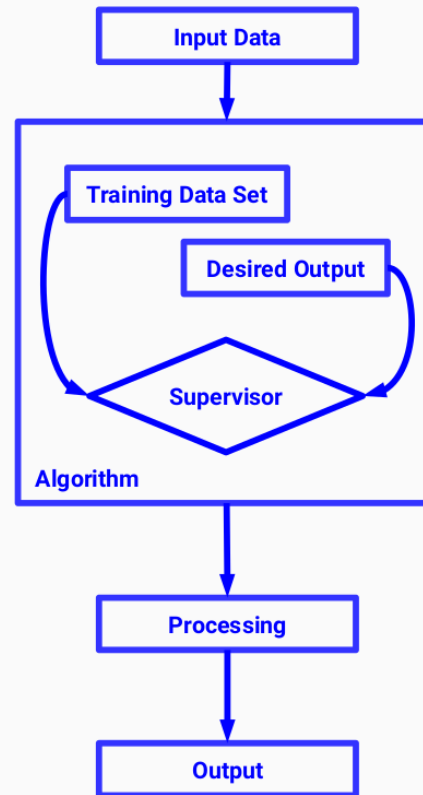
MACHINE LEARNING ALGORITHMS

Unsupervised learning



EXTRACT AND OPTIMIZE
DATA FEATURES

Supervised learning



OPTIMIZE A PROPERTY
LEARNING FROM DATA

Reinforcement learning



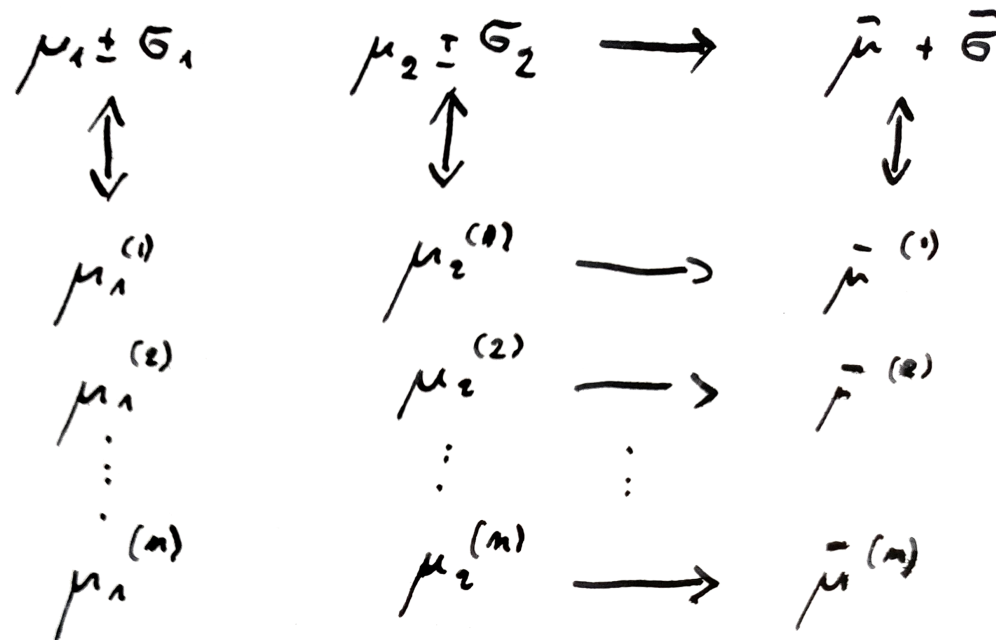
LEARN FROM DATA
THE LEARNING STRATEGY

THE NNPDF APPROACH COMBINING DATA BY MONTE CARLO

TWO MEASUREMENTS: $\mu_1 \pm \sigma_1$; $\mu_2 \pm \sigma_2$

COMBINATION: $\bar{\mu} \pm \bar{\sigma}$; $\bar{\mu} = \frac{\frac{\mu_1}{\sigma_1^2} + \frac{\mu_2}{\sigma_2^2}}{\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}}$; $\bar{\sigma}^2 = \frac{1}{\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}}$

MONTE CARLO REPRESENTATION

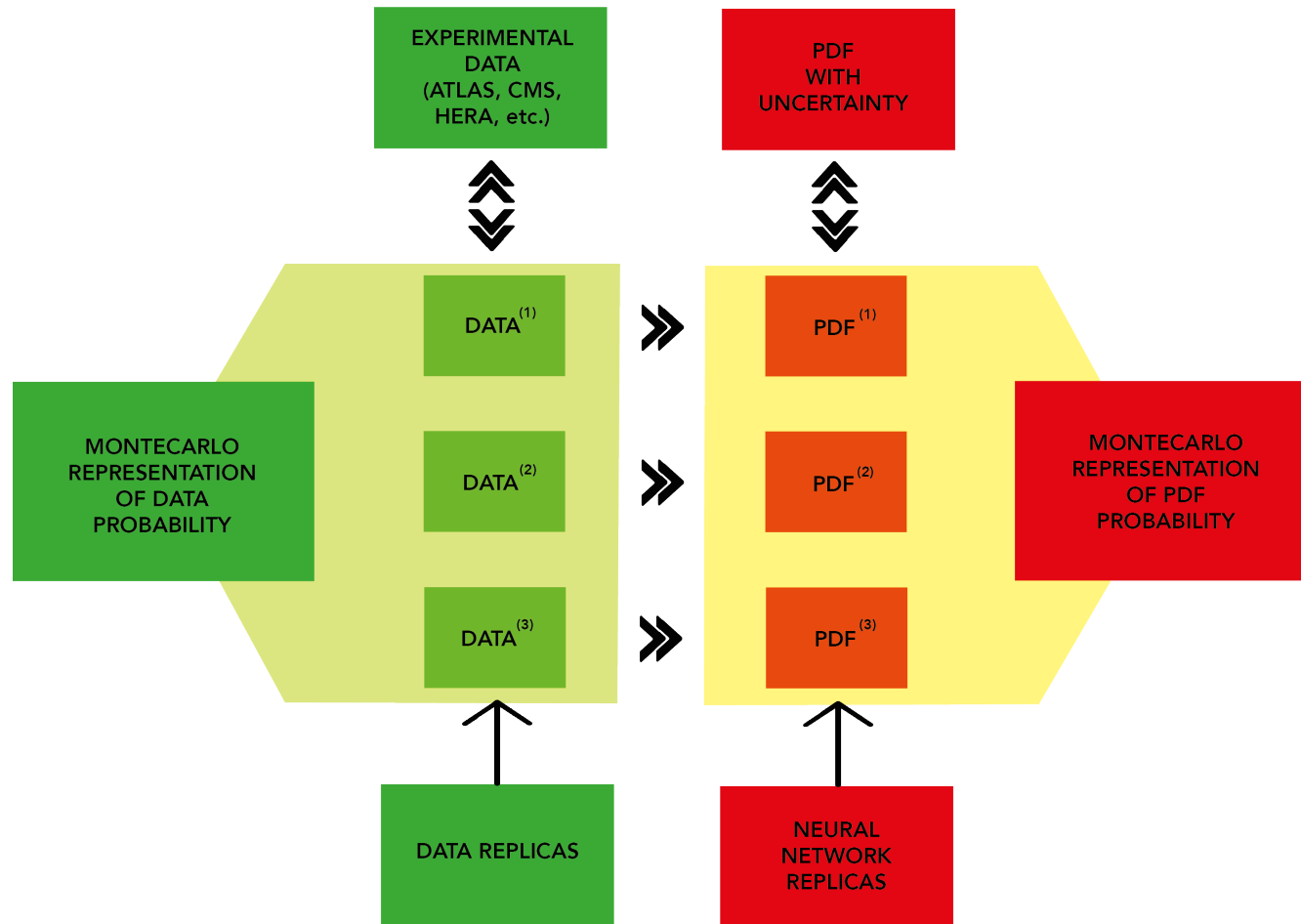


$\mu^{(i)}$ \Leftrightarrow **REPLICA SAMPLE** \Leftrightarrow **REPRESENTATION OF PROBABILITY DISTRIBUTION**
NEED ONLY TO KNOW HOW TO COMBINE CENTRAL VALUES

AI FOR PDFS: THE NNPDF APPROACH

THE FUNCTIONAL MONTE CARLO

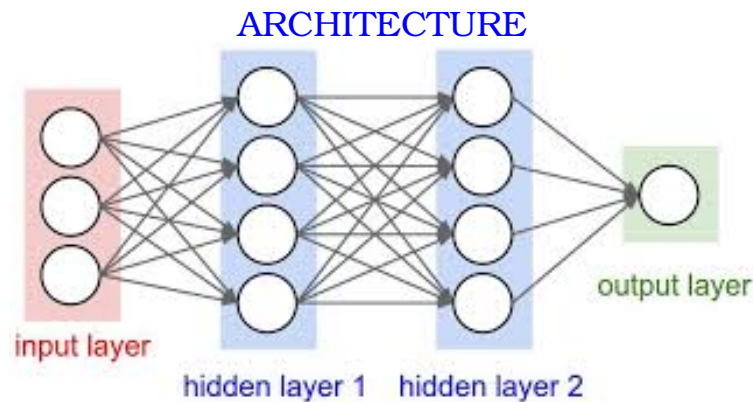
REPLICA SAMPLE OF FUNCTIONS \Leftrightarrow PROBABILITY DENSITY IN FUNCTION SPACE
 KNOWLEDGE OF LIKELIHOOD SHAPE (FUNCTIONAL FORM) NOT NECESSARY



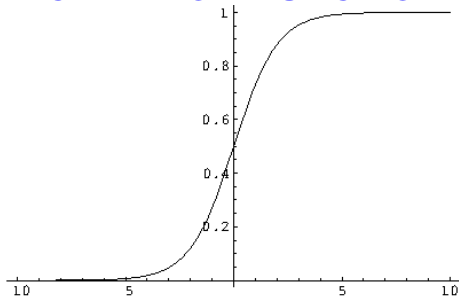
FINAL PDF SET: $f_i^{(a)}(x, \mu)$;

$i = \text{up, antiup, down, antidown, strange, antistrange, charm, gluon}; j = 1, 2, \dots, N_{\text{rep}}$

ARTIFICIAL INTELLIGENCE NEURAL NETWORKS



ACTIVATION FUNCTION



PARAMETERS

- **WEIGHTS** ω_{ij}
- **THRESHOLDS** θ_i

$$F_{\text{out}}^{(i)}(\vec{x}_{\text{in}}) = F \left(\sum_j \omega_{ij} x_{\text{in}}^j - \theta_i \right)$$

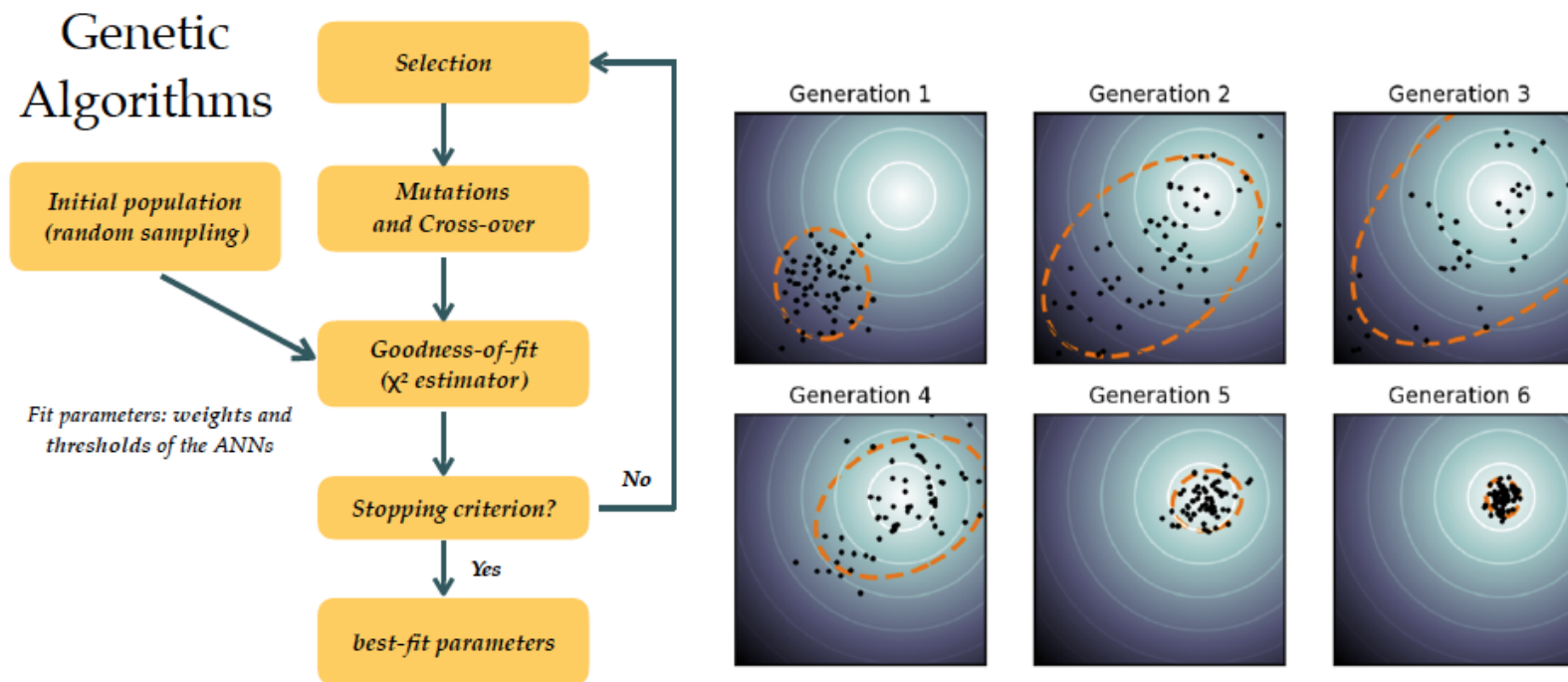
SIMPLEST EXAMPLE 1-2-1

$$f(x) = \frac{1}{1 + e^{\theta_1^{(3)} - \frac{\omega_{11}^{(2)}}{1 + e^{\theta_1^{(2)} - x\omega_{11}^{(1)}}} - \frac{\omega_{12}^{(2)}}{1 + e^{\theta_2^{(2)} - x\omega_{21}^{(1)}}}}}$$

NNPDF: 2 – 5 – 3 – 1 NN FOR EACH PDF: $37 \times 8 = 296$ PARAMETERS

SUPERVISED LEARNING GENETIC ALGORITHMS

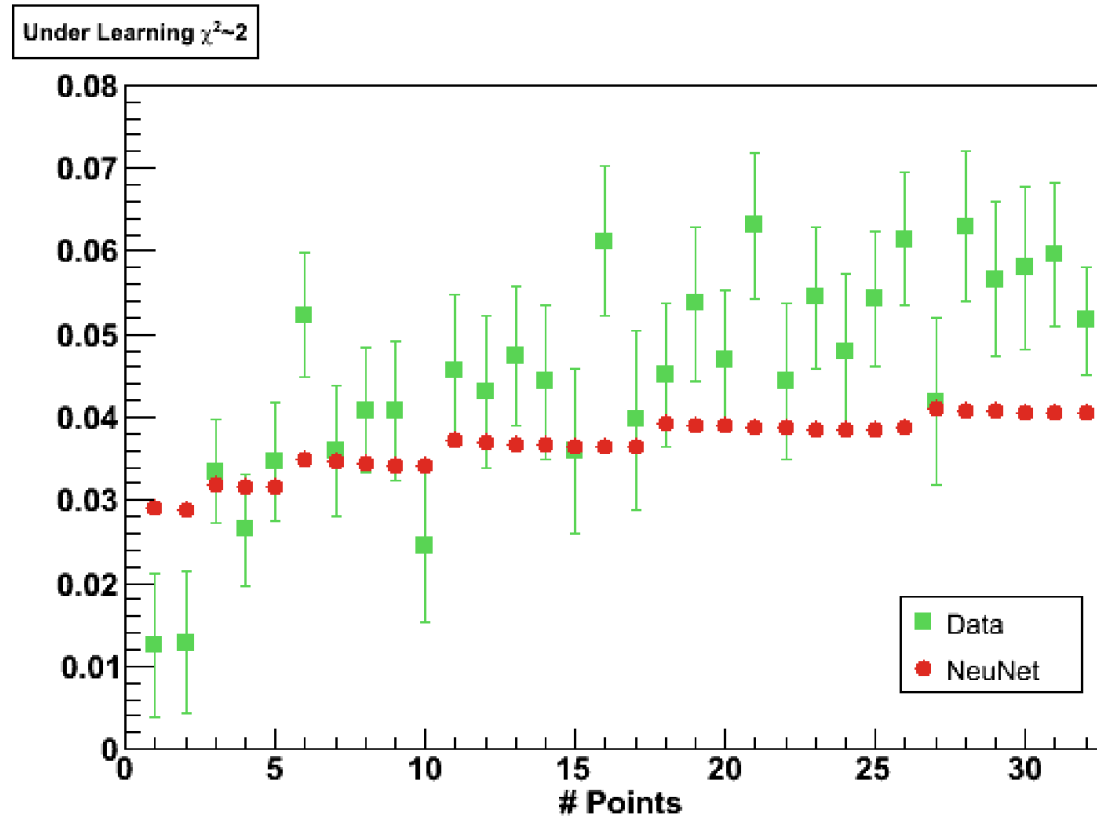
- **BASIC IDEA:** RANDOM MUTATION OF THE NN PARAMETER
- **SELECTION OF THE FITTEST**



NEURAL LEARNING

- COMPLEXITY INCREASES AS THE FITTING PROCEEDS
- UNTIL LEARNING NOISE
- WHEN SHOULD ONE STOP?

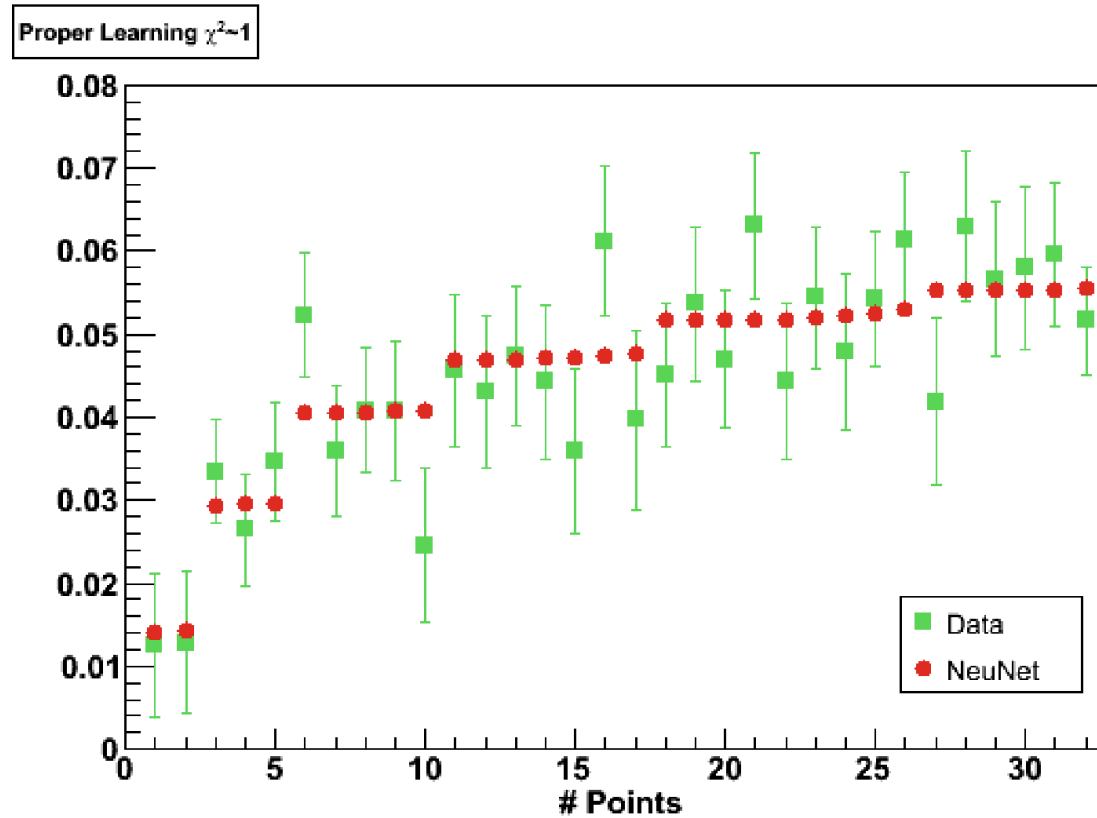
UNDERLEARNING



NEURAL LEARNING

- COMPLEXITY INCREASES AS THE FITTING PROCEEDS
- UNTIL LEARNING NOISE
- WHEN SHOULD ONE STOP?

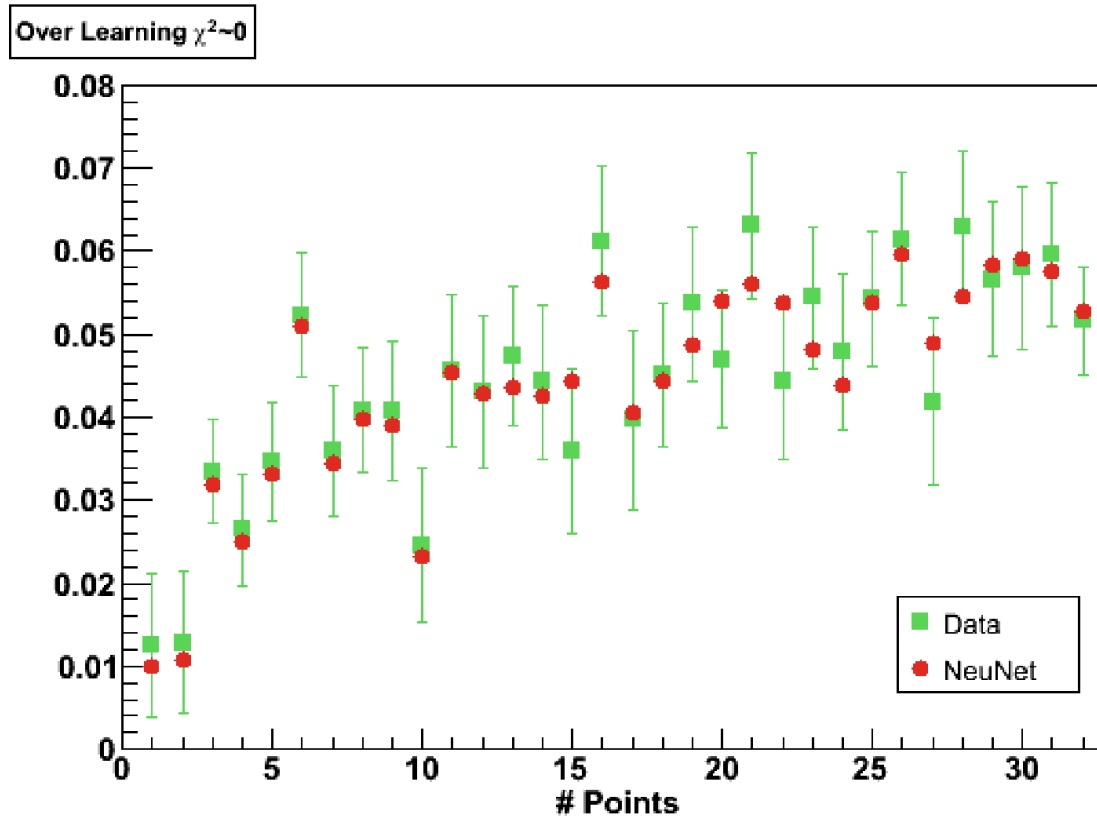
PROPER LEARNING



NEURAL LEARNING

- COMPLEXITY INCREASES AS THE FITTING PROCEEDS
- UNTIL LEARNING NOISE
- WHEN SHOULD ONE STOP?

OVERLEARNING

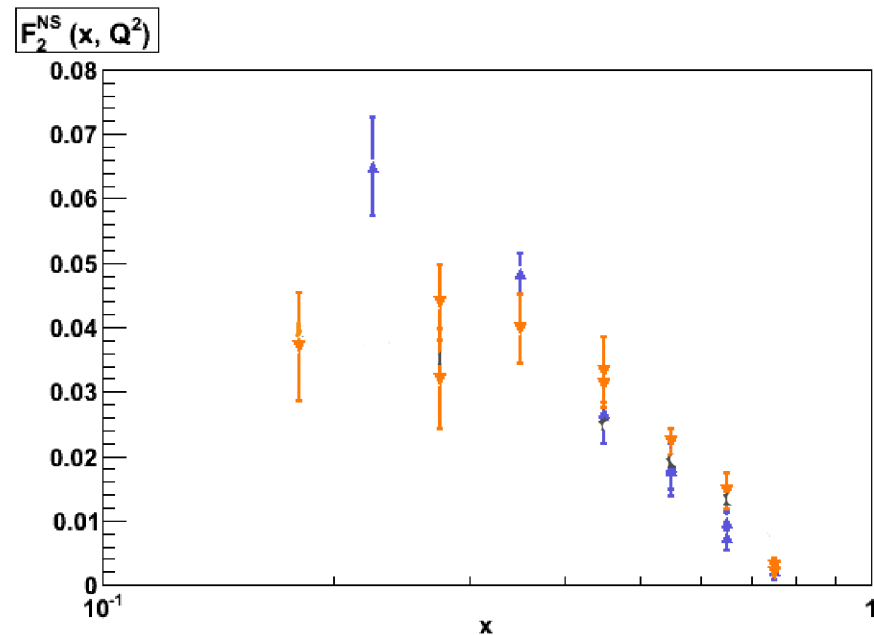


OPTIMAL FIT: CROSS-VALIDATION

GENETIC MINIMIZATION:

AT EACH GENERATION, χ^2 EITHER UNCHANGED OR DECREASING

- DIVIDE THE DATA IN TWO SETS: TRAINING AND VALIDATION
- MINIMIZE THE χ^2 OF THE DATA IN THE TRAINING SET
- AT EACH ITERATION, COMPUTE THE χ^2 FOR THE DATA IN THE VALIDATION SET (NOT USED FOR FITTING)
- WHEN THE VALIDATION χ^2 STOPS DECREASING, STOP THE FIT



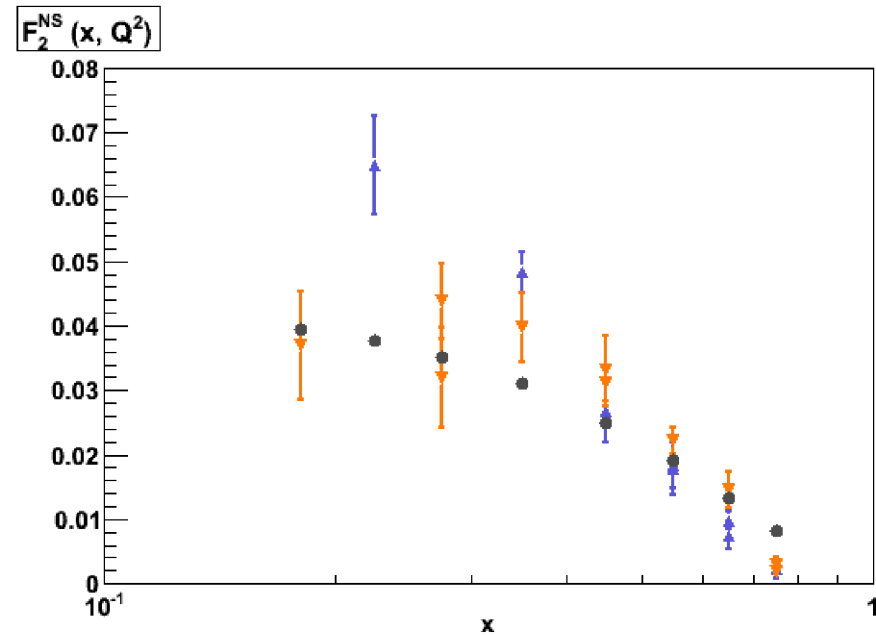
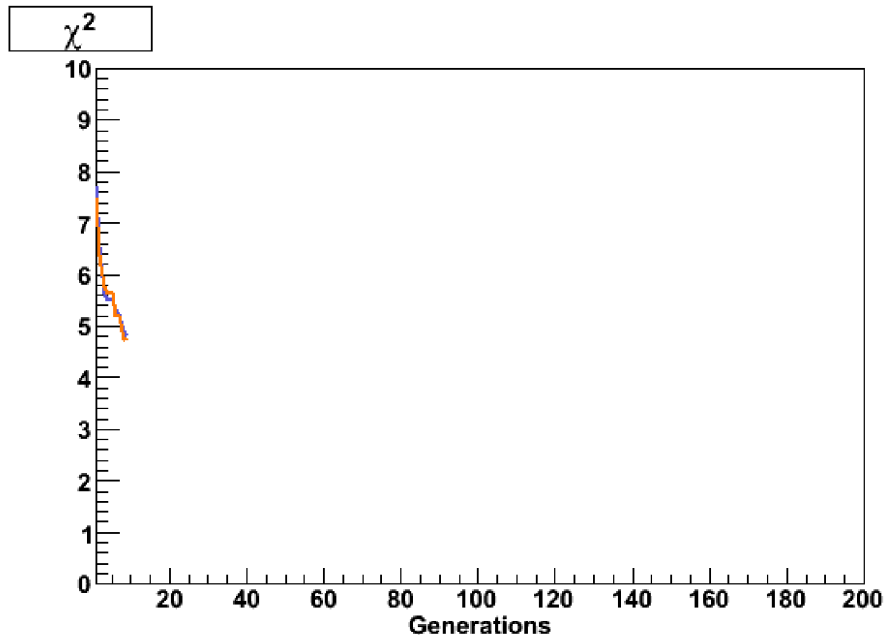
OPTIMAL FIT: CROSS-VALIDATION

GENETIC MINIMIZATION:

AT EACH GENERATION, χ^2 EITHER UNCHANGED OR DECREASING

- DIVIDE THE DATA IN TWO SETS: TRAINING AND VALIDATION
- MINIMIZE THE χ^2 OF THE DATA IN THE TRAINING SET
- AT EACH ITERATION, COMPUTE THE χ^2 FOR THE DATA IN THE VALIDATION SET (NOT USED FOR FITTING)
- WHEN THE VALIDATION χ^2 STOPS DECREASING, STOP THE FIT

GO!



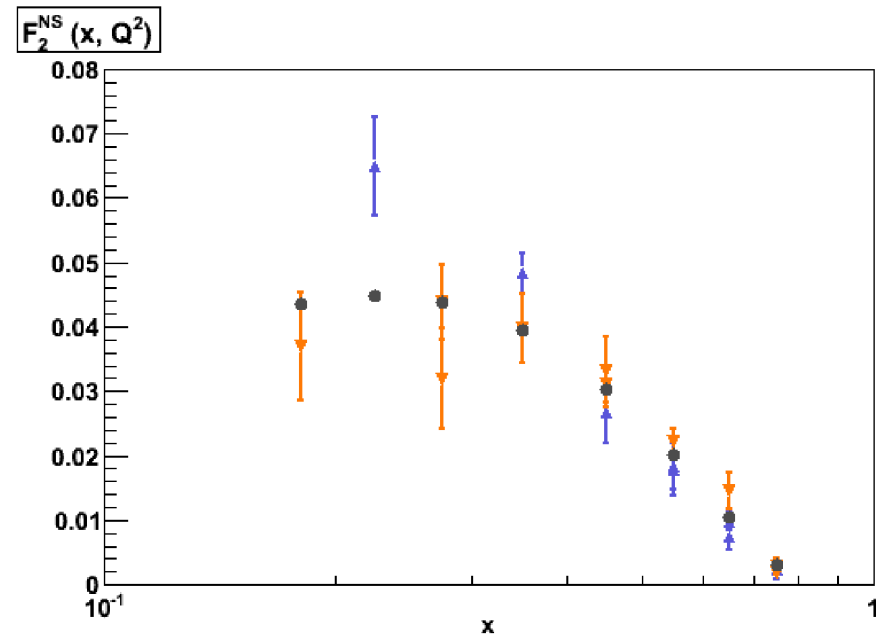
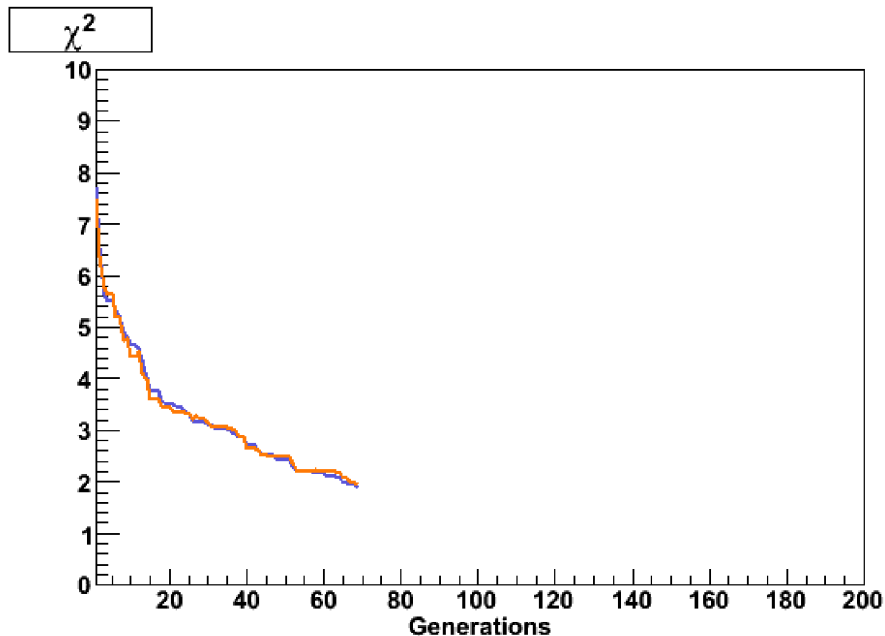
OPTIMAL FIT: CROSS-VALIDATION

GENETIC MINIMIZATION:

AT EACH GENERATION, χ^2 EITHER UNCHANGED OR DECREASING

- DIVIDE THE DATA IN TWO SETS: TRAINING AND VALIDATION
- MINIMIZE THE χ^2 OF THE DATA IN THE TRAINING SET
- AT EACH ITERATION, COMPUTE THE χ^2 FOR THE DATA IN THE VALIDATION SET (NOT USED FOR FITTING)
- WHEN THE VALIDATION χ^2 STOPS DECREASING, STOP THE FIT

STOP!



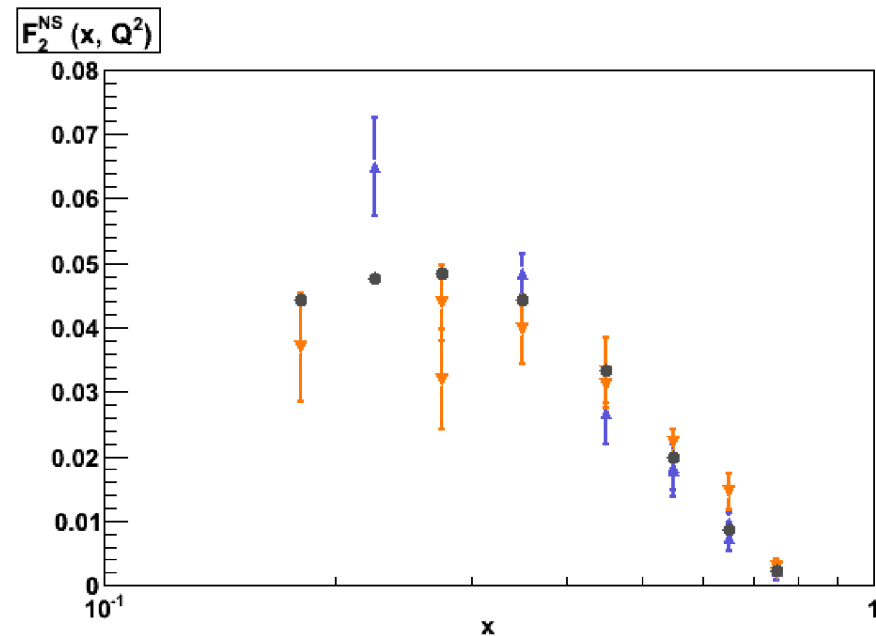
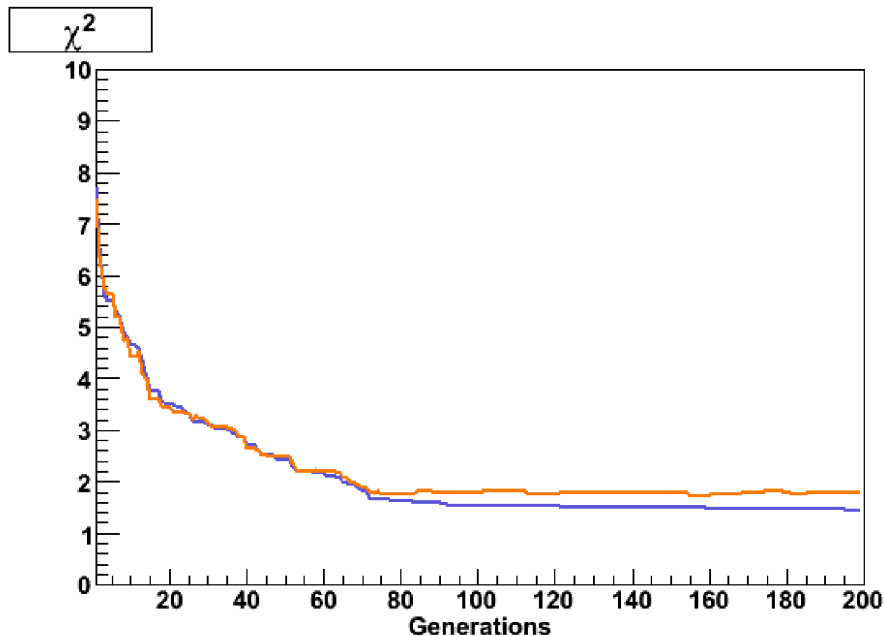
OPTIMAL FIT: CROSS-VALIDATION

GENETIC MINIMIZATION:

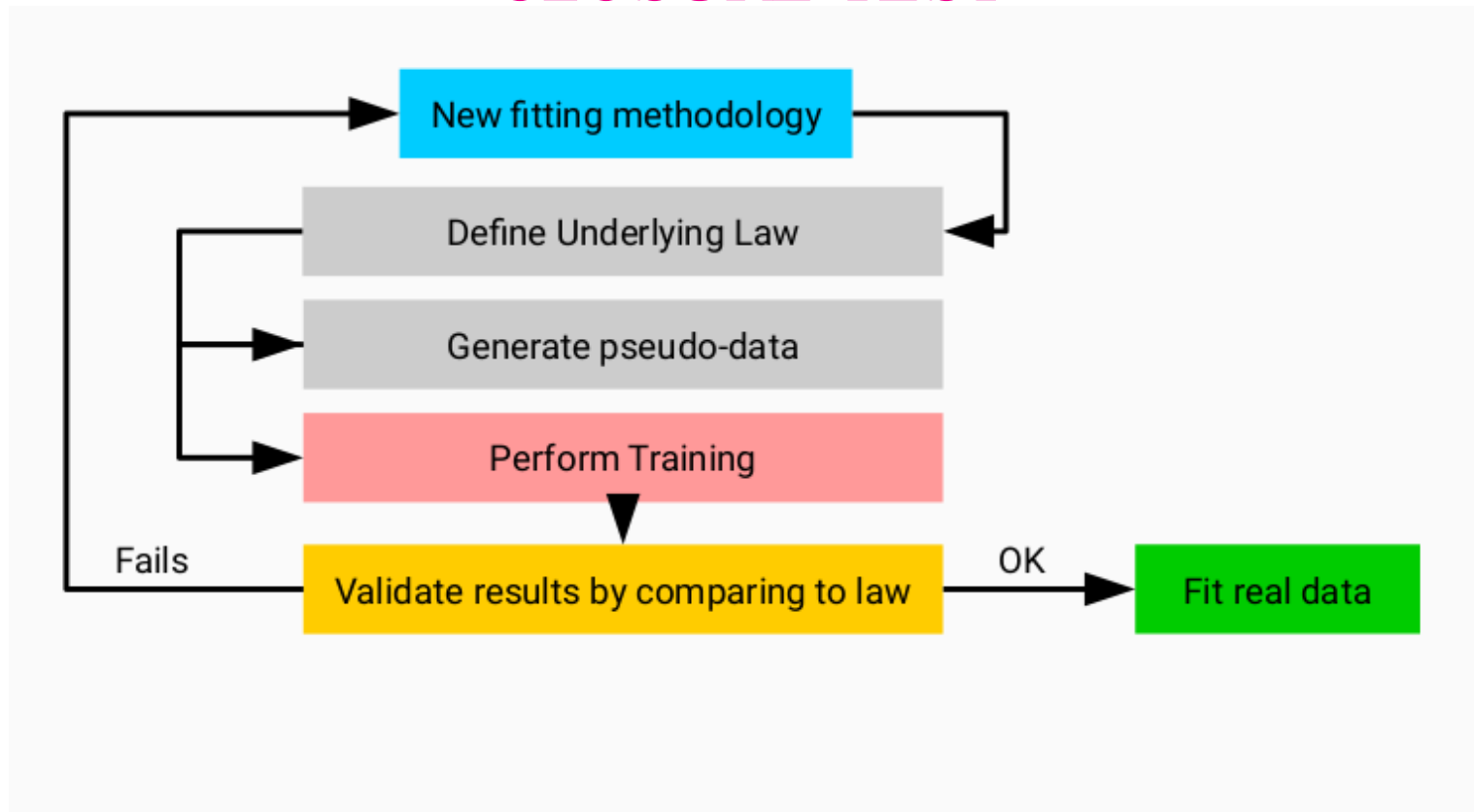
AT EACH GENERATION, χ^2 EITHER UNCHANGED OR DECREASING

- DIVIDE THE DATA IN TWO SETS: TRAINING AND VALIDATION
- MINIMIZE THE χ^2 OF THE DATA IN THE TRAINING SET
- AT EACH ITERATION, COMPUTE THE χ^2 FOR THE DATA IN THE VALIDATION SET (NOT USED FOR FITTING)
- WHEN THE VALIDATION χ^2 STOPS DECREASING, STOP THE FIT

TOO LATE!



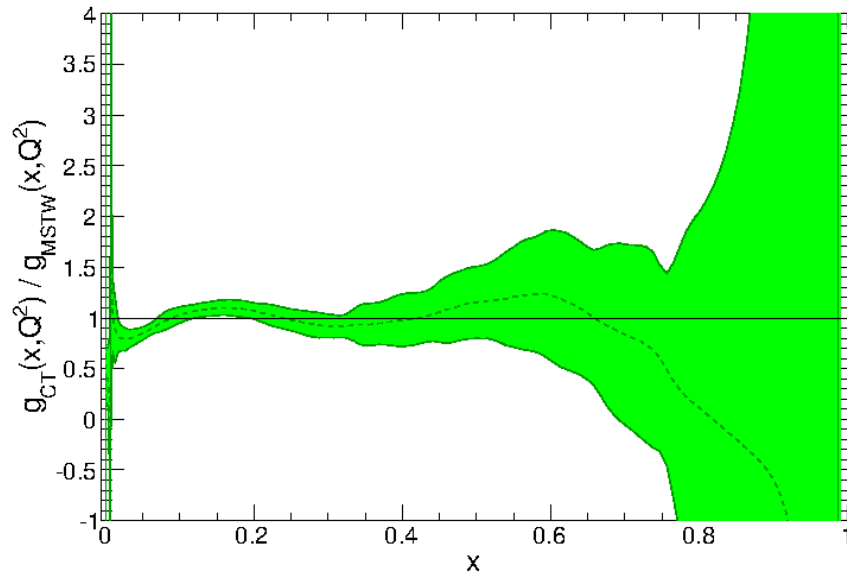
HOW DO WE KNOW THAT WE GOT THE RIGHT ANSWER? CLOSURE TEST



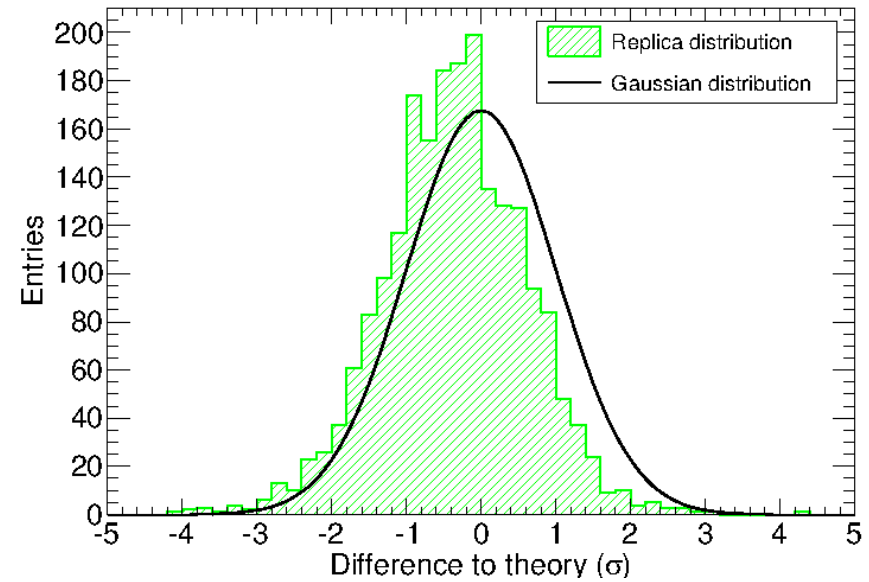
FIRST CLOSURE TEST (NNPDF3.0; 2014)

NORMALIZED DISTRIBUTION OF DEVIATIONS

THE GLUON: RESULT/"TRUTH"
Ratio of Closure Test g to MSTW2008



Distribution of single replica fits in level 2 uncertainties



1σ : 70% (should be 68%)

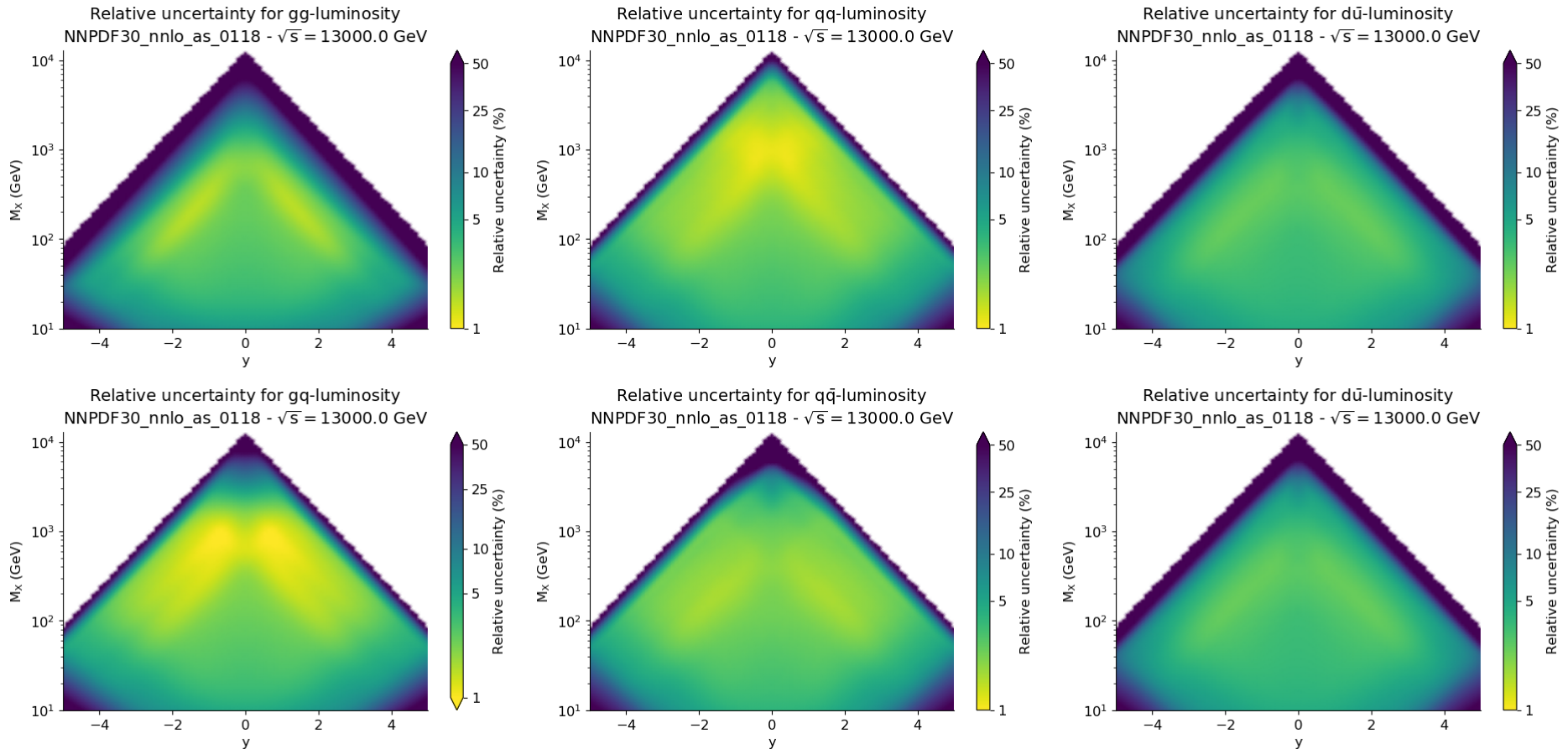
- THE METHODOLOGY IS FAITHFUL

THE STATE OF THE ART: PRECISION PDF4LHC PDFs (2014) NNPDF3.0 NNLO

GLUON

SINGLET

FLAVORS



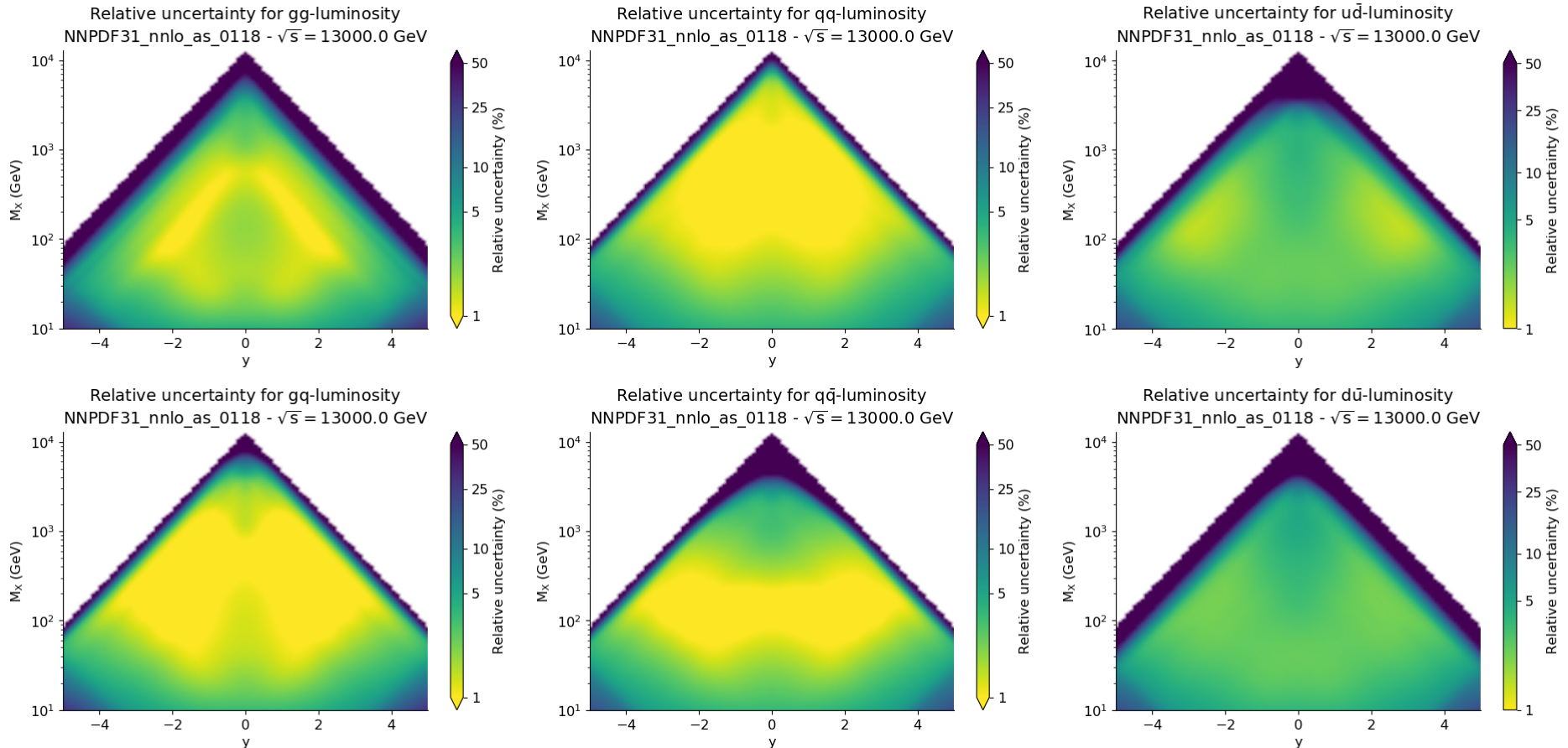
- GLUON BETTER KNOWN AT SMALL x , VALENCE QUARKS AT LARGE x , SEA QUARKS IN BETWEEN
- TYPICAL UNCERTAINTIES IN DATA REGION $\sim 3 - 5\%$
- SWEET SPOT: VALENCE Q - G; DOWN TO 1%
- UP BETTER KNOWN THAN DOWN; FLAVOR SINGLET BETTER THAN INDIVIDUAL FLAVORS

THE STATE OF THE ART: PRECISION CURRENT PDFs (2017) NNPDF3.1 NNLO

GLUON

SINGLET

FLAVORS



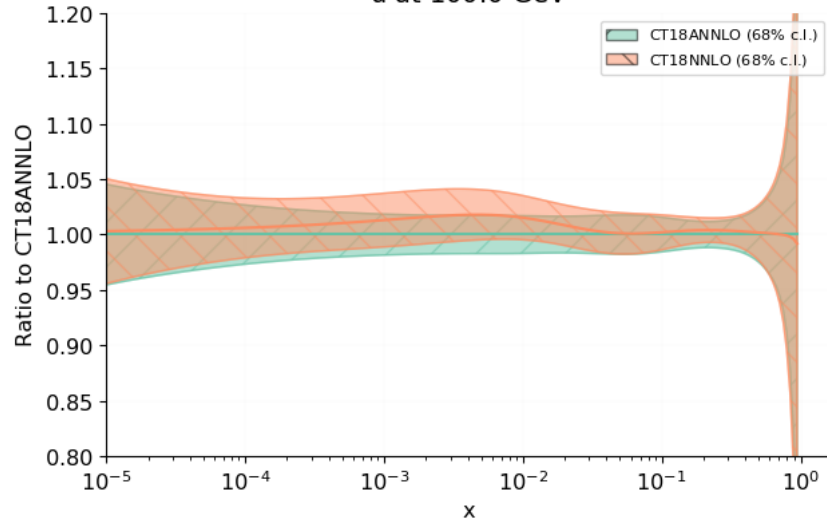
- **GLUON** BETTER KNOWN AT SMALL x , **VALENCE** QUARKS AT LARGE x , SEA QUARKS IN BETWEEN
- **TYPICAL** UNCERTAINTIES IN DATA REGION $\sim 1 - 3\%$
- **SWEET SPOT**: VALENCE Q - G; 1% OR BELOW
- UP BETTER KNOWN THAN DOWN; FLAVOR SINGLET BETTER THAN INDIVIDUAL FLAVORS

THE STATE OF THE ART: CONSISTENCY

IMPACT OF ATLAS W/Z 7TeV DATA

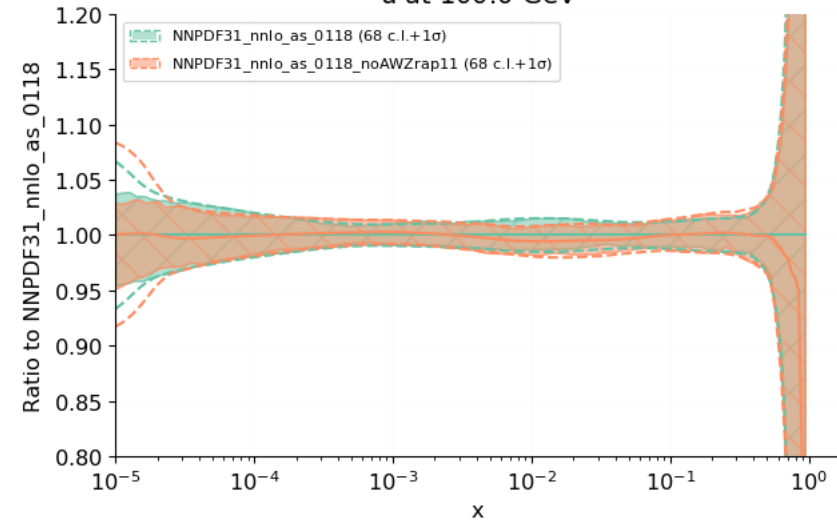
CT18

u at 100.0 GeV

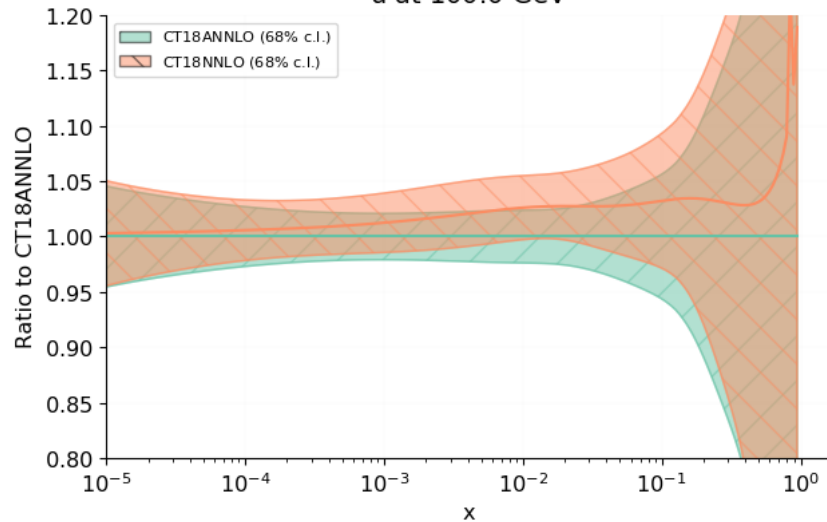


NNPDF3.1

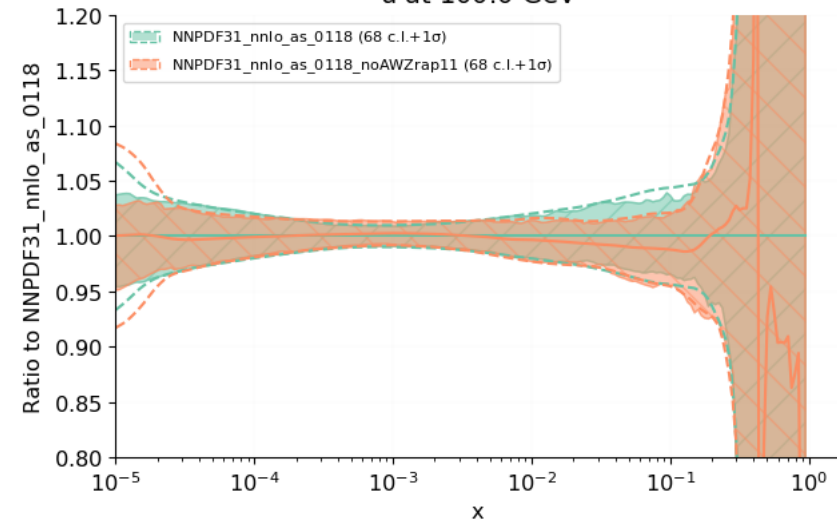
u at 100.0 GeV



\bar{u} at 100.0 GeV

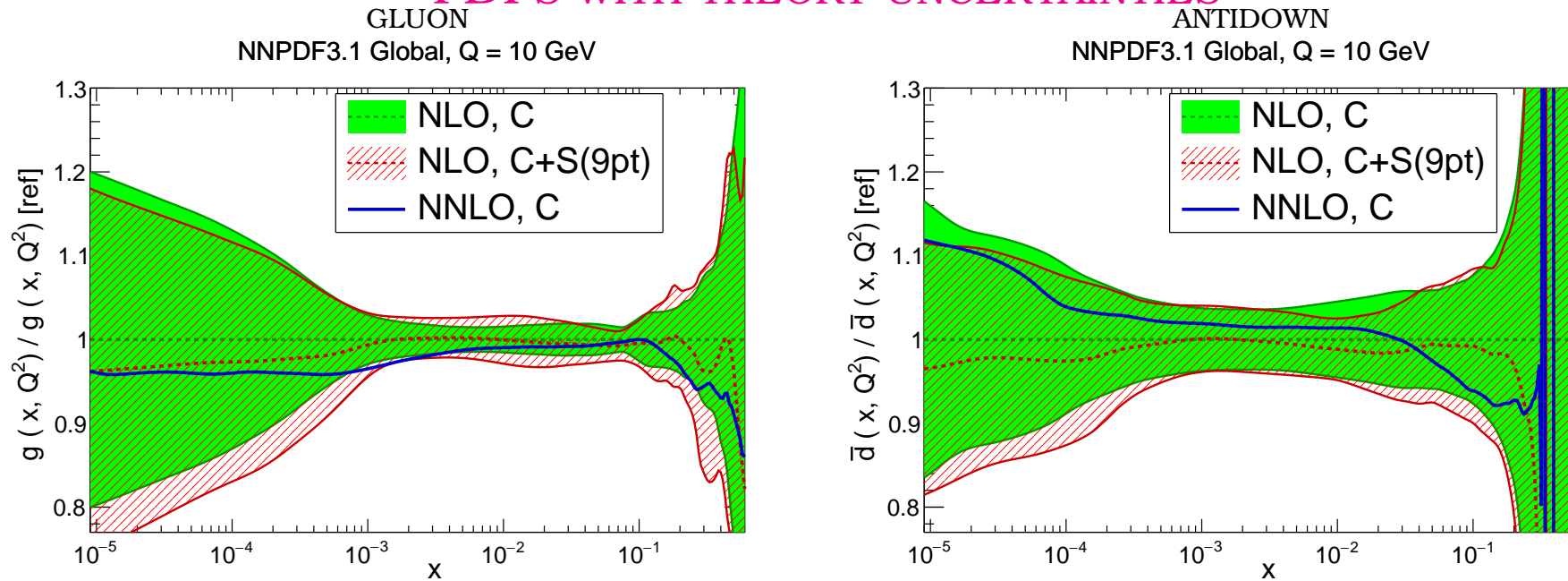


\bar{u} at 100.0 GeV



- CT18: PDF SETS RELEASED WITH/WITHOUT ATLAS W/Z DATA INCLUDED
- NNPDF3.1: CONSISTENCY OF ALL DATASETS INCLUDED

THE STATE OF THE ART: ACCURACY PDFs WITH THEORY UNCERTAINTIES



	C	$C + S^{(9\text{pt})}$
χ^2	1.139	1.109
ϕ	0.314	0.415

- FIT QUALITY χ^2 IMPROVES
- RELATIVE ERROR ϕ ON PREDICTION MILDLY INCREASED
- CENTRAL VALUE MOVES TOWARDS KNOWN NNLO

EQUALLY PRECISE BUT MORE ACCURATE RESULT!

THE STATE OF THE ART:

QUESTIONS

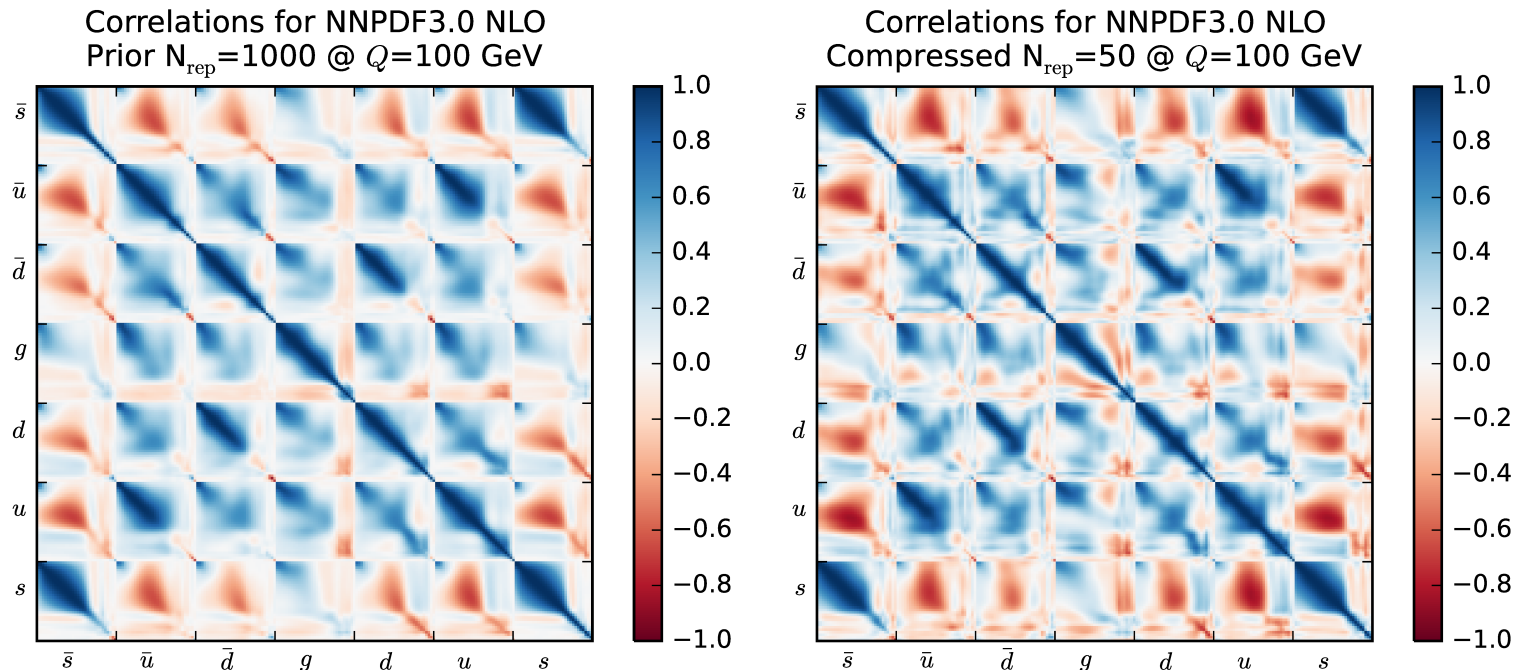
- DO WE **REALLY NEED** 1000 **REPLICAS**? OR 100? \Rightarrow **EFFICIENCY**
- ARE 1000 **REPLICAS ENOUGH**? OR 10000? \Rightarrow **ACCURACY**
- PDF UNCERTAINTIES ARE FAITHFUL, BUT **ARE THEY OPTIMAL**?
 \Rightarrow **PRECISION**

PDFS FROM AI TO ML

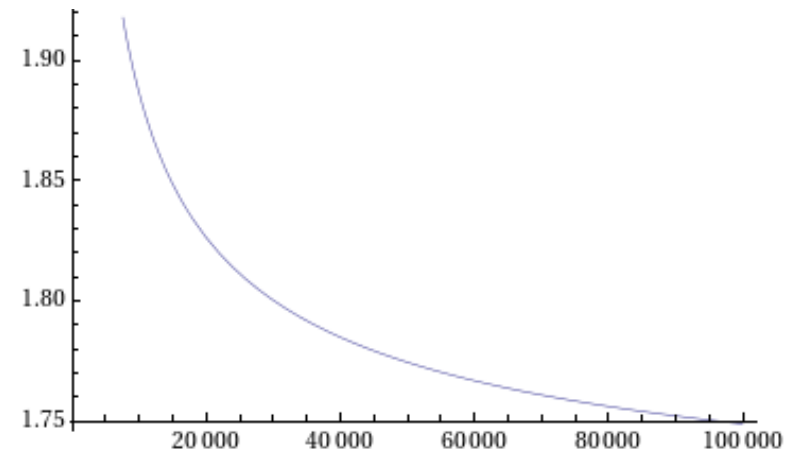
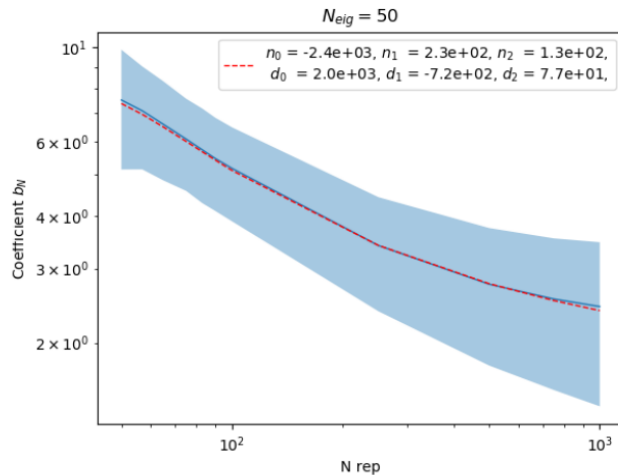
ML: UNSUPERVISED LEARNING OPTIMIZATION I

- HOW TO **MAXIMIZE ACCURACY?**
- LARGE (PRIOR) REPLICAS SET
- GENETIC SELECTION \Rightarrow **OPTIMIZATION** OF STATISTICAL INDICATORS (KULLBACK-LEIBLER DIVERGENCE)
- 50 **OPTIMIZES REPLICAS** \Leftrightarrow 1000 STARTING REPLICAS

CORRELATION MATRIX



ML: SUPERVISED LEARNING
OPTIMIZATION II
HOW MANY PDF REPLICAS DO WE NEED?
FINITE-SIZE EFFECTS
 ONE- σ $\Delta\chi^2$ VS NUMBER OF REPLICAS

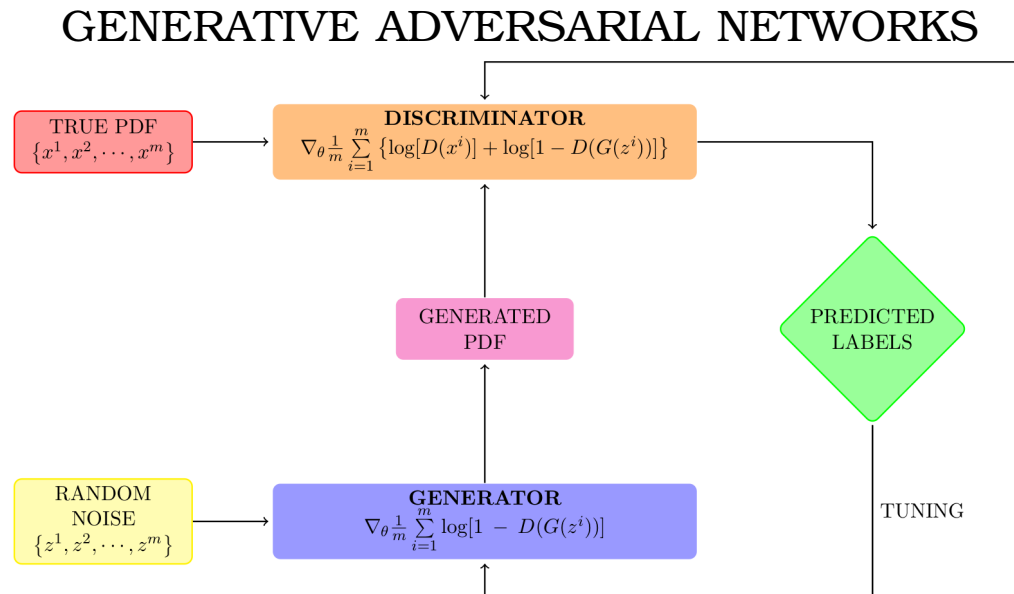


- SIGNIFICANT **DEPENDENCE ON NUMBER** OF REPLICAS
- ASYMPTOTIC “TOLERANCE” $T = 1.3 \pm 0.3$; $\Delta\chi^2 = 1.7 \pm 0.7$
- FOR $N_{rep} = 100$, $T = 2.3$, EVEN FOR $N_{rep} = 1000$, $T = 1.6$

DO WE HAVE TO **FIT 10000 REPLICAS?** DO WE HAVE TO **USE 10000 REPLICAS?**

ML: SUPERVISED LEARNING OPTIMIZATION II

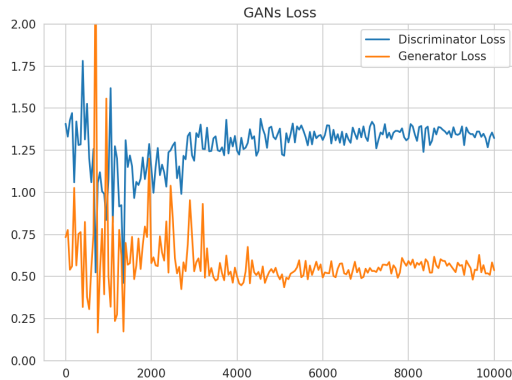
- CAN WE **REDUCE THE NUMBER OF COMPRESSED REPLICAS WITHOUT LOSS OF INFORMATION?** **SOLUTION FOR USER**
- CAN WE **INCREASE THE NUMBER OF REPLICAS WITHOUT REFITTING?** **SOLUTION FOR PDF FITTER**



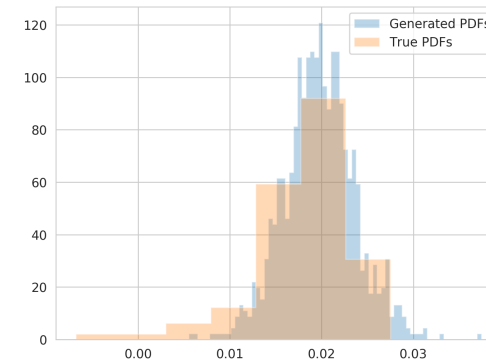
- TRAIN A NETWORK TO **SIMULATE** THE TRUE DISTRIBUTION (**GENERATOR**)
- TRAIN A NETWORK TO **DISCRIMINATE** TRUTH FROM SIMULATION (**DISCRIMINATOR**)
- TRAIN THE **GENERATOR TO TRICK THE DISCRIMINATOR**

SOLVING THE PROBLEM.... GAN REPLICA GENERATION

GAN TRAINING



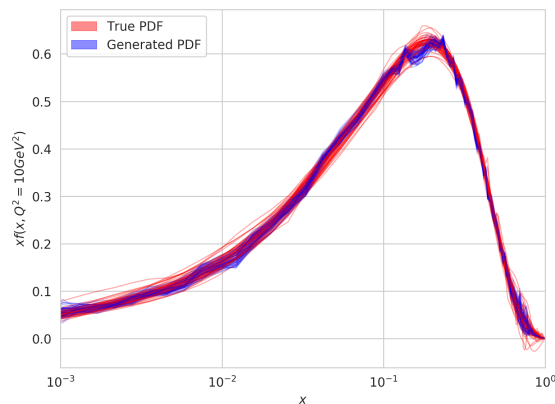
UP VALENCE AT FIXED x



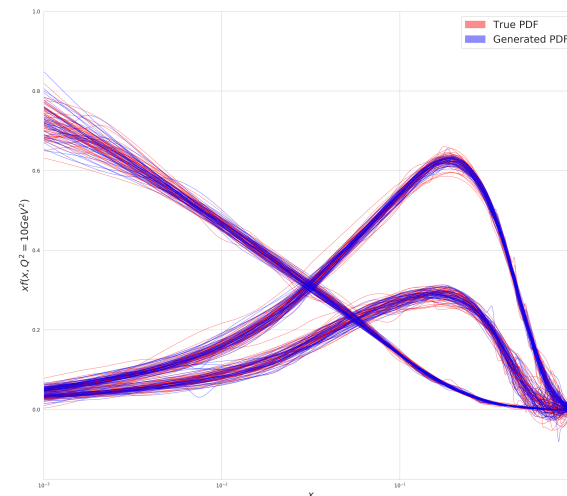
- **1D GAN:** REPRODUCE THE INFORMATION IN THE UNDERLYING REPLICA SET, BUT **NO GAIN** (WIGGLY REPLICAS)
 \Rightarrow REDUCE THE NUMBER OF COMPRESSED REPLICA WITH **FIXED NUMBER** OF FITTED REPLICAS W/O INFORMATION LOSS
- **2D GAN:** COMBINE CORRELATED INFORMATION FROM UNDERLYING REPLICA SET **INFERRING** THE **TRUE** UNDERLYING DISTRIBUTION
 \Rightarrow REDUCE THE NUMBER OF INPUT REPLICAS W/O INFORMATION LOSS



ONE-DIMENSIONAL



TWO-DIMENSIONAL

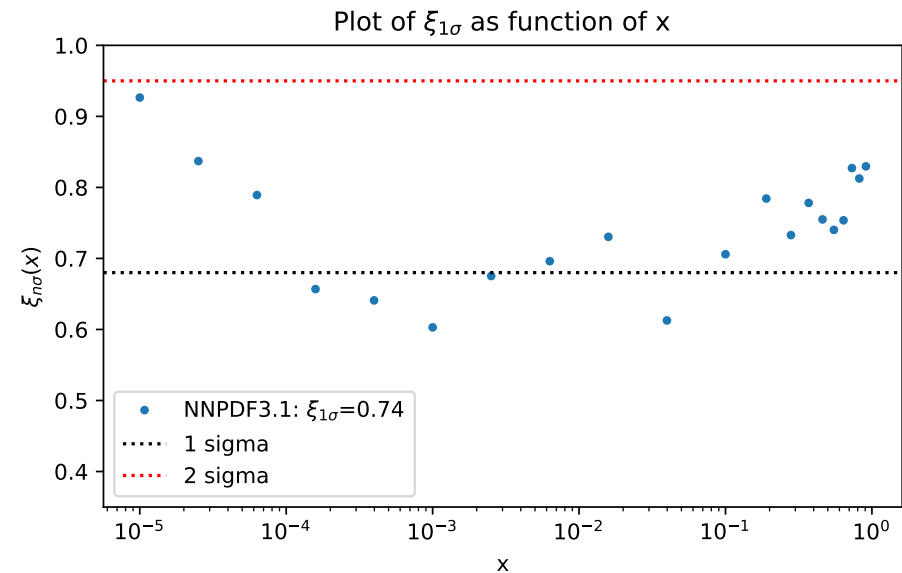


CLOSURE TEST: A CLOSER LOOK (NNPDF3.1)

ONE σ : ACTUAL/PREDICTED
FOR DATA, BY EXPERIMENT

experiment	NNPDF3.1 ratio
NMC	0.882828
SLAC	0.767063
BCDMS	0.730569
CHORUS	0.698907
NTVDMN	0.991090
HERACOMB	0.847359
HERAF2CHARM	1.867597
F2BOTTOM	1.124157
DYE886	0.655955
DYE605	0.585725
CDF	0.961652
D0	0.881199
ATLAS	0.904127
CMS	1.090241
LHCb	1.092194
Total	0.842168

ONE σ VALUE
FOR PDFS, VS x



- **UNCERTAINTIES OVERESTIMATED**
- $1\sigma > 68\%$ AT VERY SMALL AND VERY LARGE x ;
 $1\sigma < 68\%$ AT INTERMEDIATE x

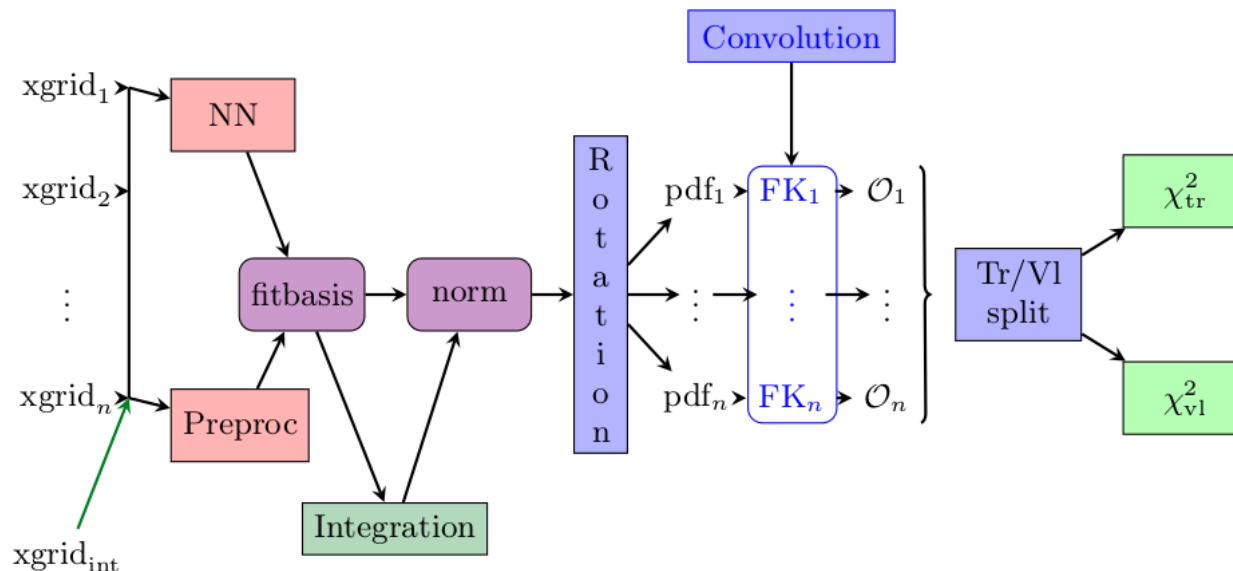
CAN WE DO BETTER?

FITTING THE METHODOLOGY

THE N3FIT PROJECT

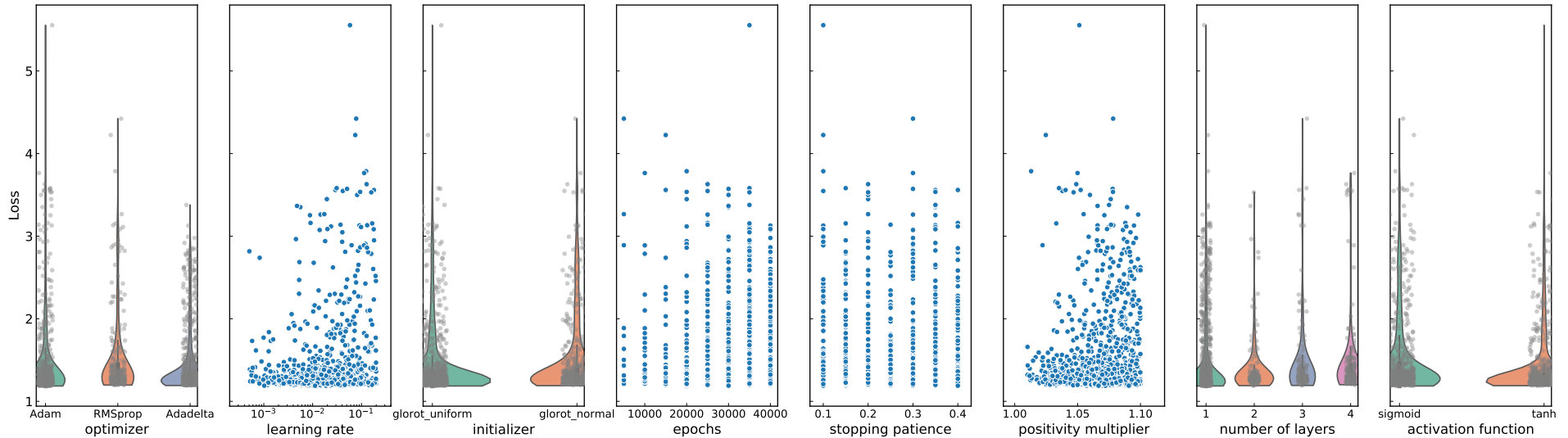
HOW DO WE KNOW THAT THE METHODOLOGY IS THE BEST?
“ACCUMULATED WISDOM” INEFFICIENT AND SLOW

CHANGE OF PHILOSOPHY \Rightarrow DETERMINISTIC MINIMIZATION (GRADIENT DESCENT)
GO FOR THE ABSOLUTE MINIMUM, AND (HYPER)OPTIMIZE



- PYTHON-BASED KERAS + TENSORFLOW FRAMEWORK
- EACH BLOCK INDEPENDENT LAYER
- CAN VARY ALL ASPECT OF METHODOLOGY

FITTING THE METHODOLOGY HYPEROPTIMIZATION SCANS



HYPEROPT PARAMETERS

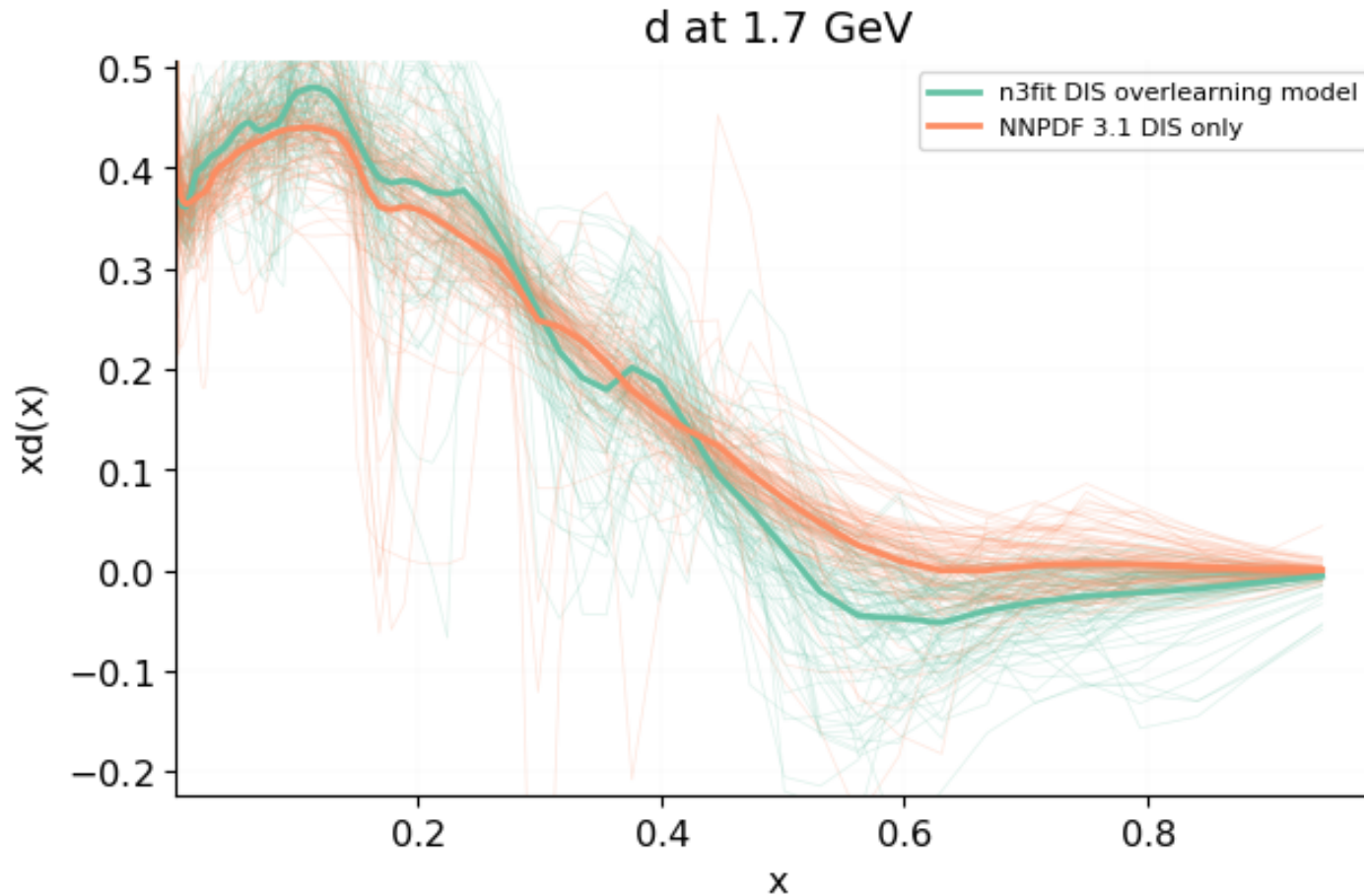
NEURAL NETWORK	FIT OPTIONS
NUMBER OF LAYERS (*)	OPTIMIZER (*)
SIZE OF EACH LAYER	INITIAL LEARNING RATE (*)
DROPOUT	MAXIMUM NUMBER OF EPOCHS (*)
ACTIVATION FUNCTIONS (*)	STOPPING PATIENCE (*)
INITIALIZATION FUNCTIONS (*)	POSITIVITY MULTIPLIER (*)

- **SCAN** PARAMETER SPACE
- **OPTIMIZE** FIGURE OF MERIT: **VALIDATION** χ^2
- **BAYESIAN** UPDATING

FITTING THE METHODOLOGY

THE OVERFITTING PROBLEM

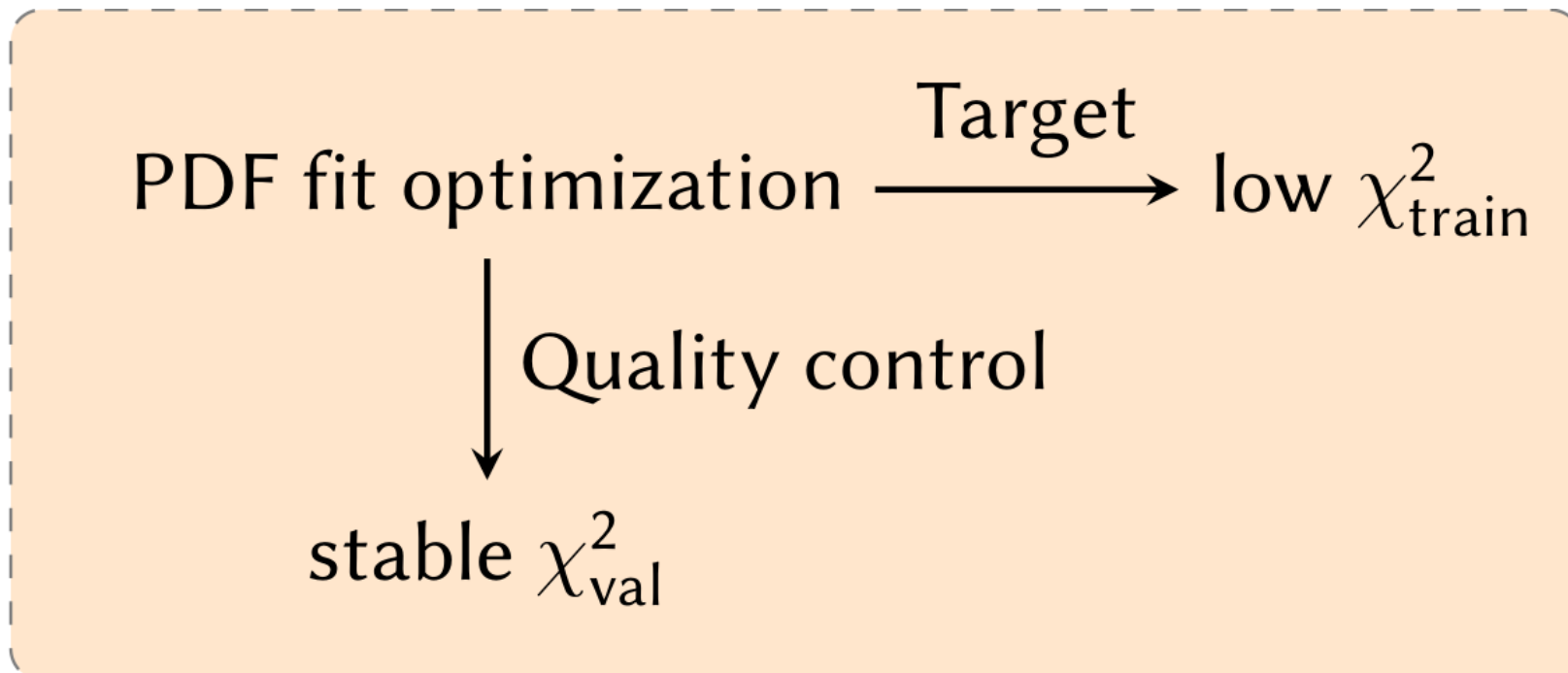
DOWN QUARK: HYPEROPTIMIZED VS. STANDARD



- **NNPDF3.1**: **WIGGLES**: **FINITE SIZE** \Rightarrow WILL GO AWAY AS N_{rep} GROWS
- **N3FIT**: **WIGGLY** PDFS \Leftrightarrow **OVERFITTING** \Rightarrow WILL **NOT** GO AWAY ($\chi^2_{\text{train}} \ll \chi^2_{\text{valid}}$!!)

WHAT HAPPENED?

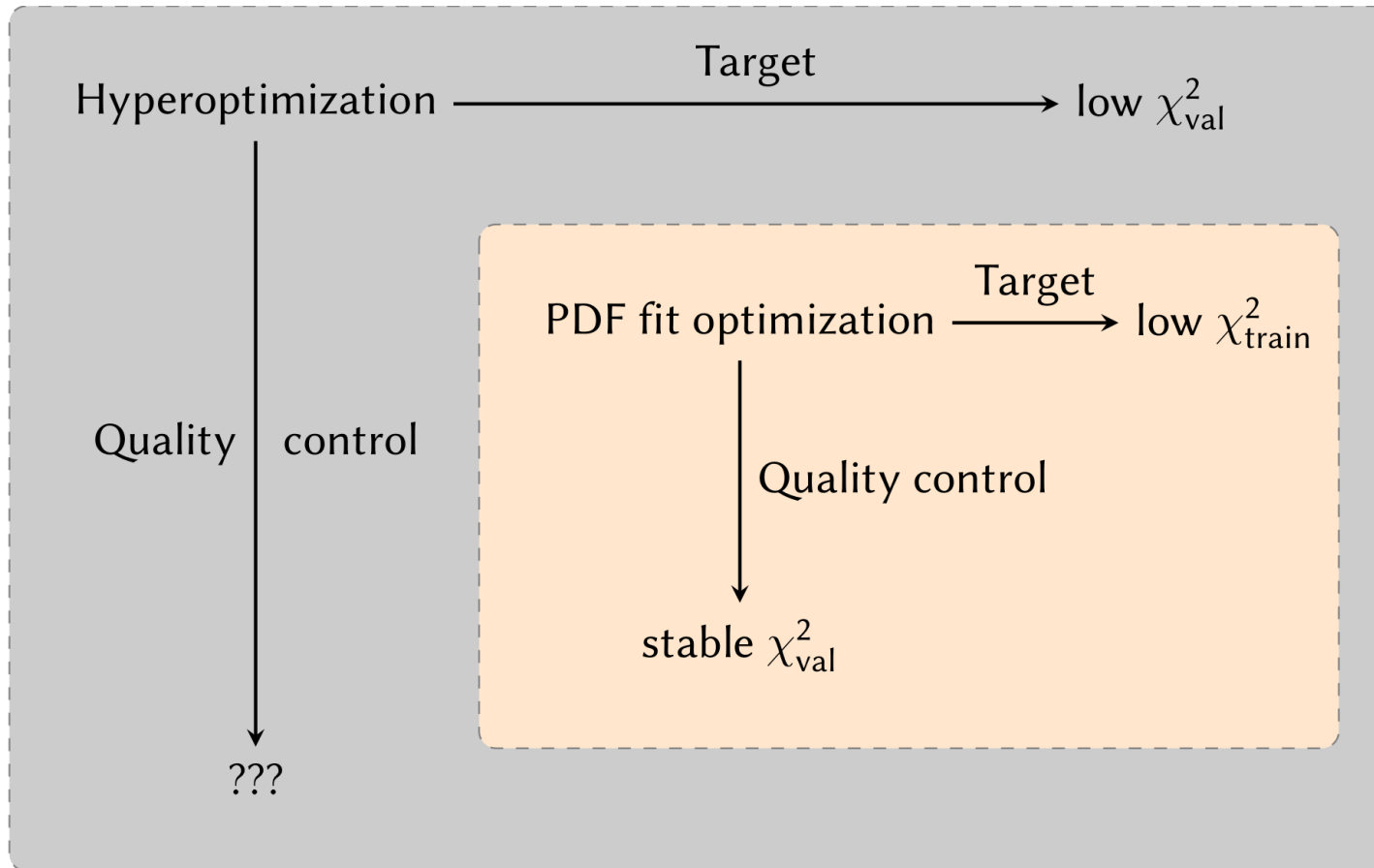
OPTIMIZATION



CROSS-VALIDATION SELECTS THE OPTIMAL MINIMUM

WHAT HAPPENED?

HYPEROPTIMIZATION

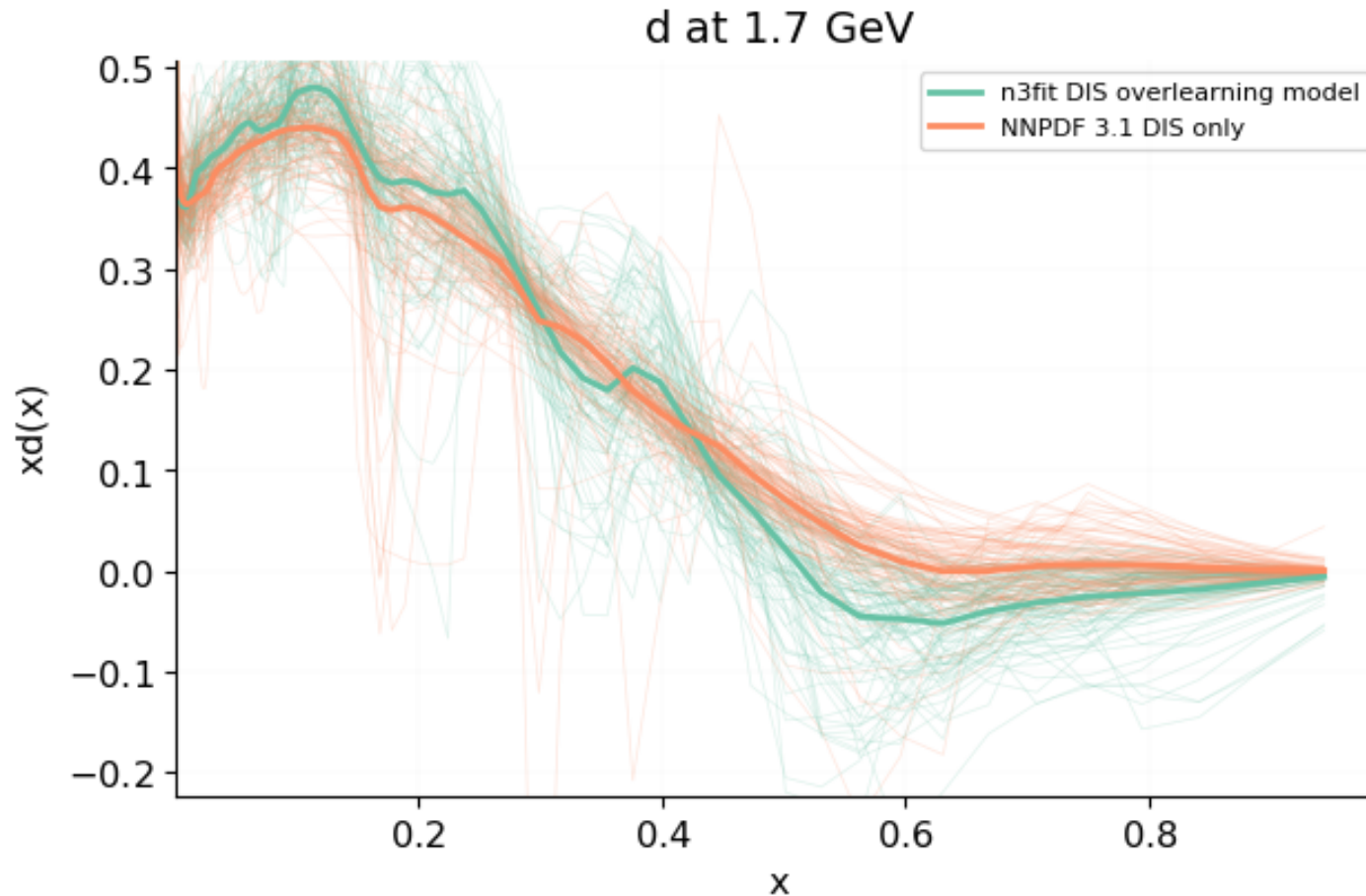


WE ARE MISSING A SELECTION CRITERION

FITTING THE METHODOLOGY

THE OVERFITTING PROBLEM

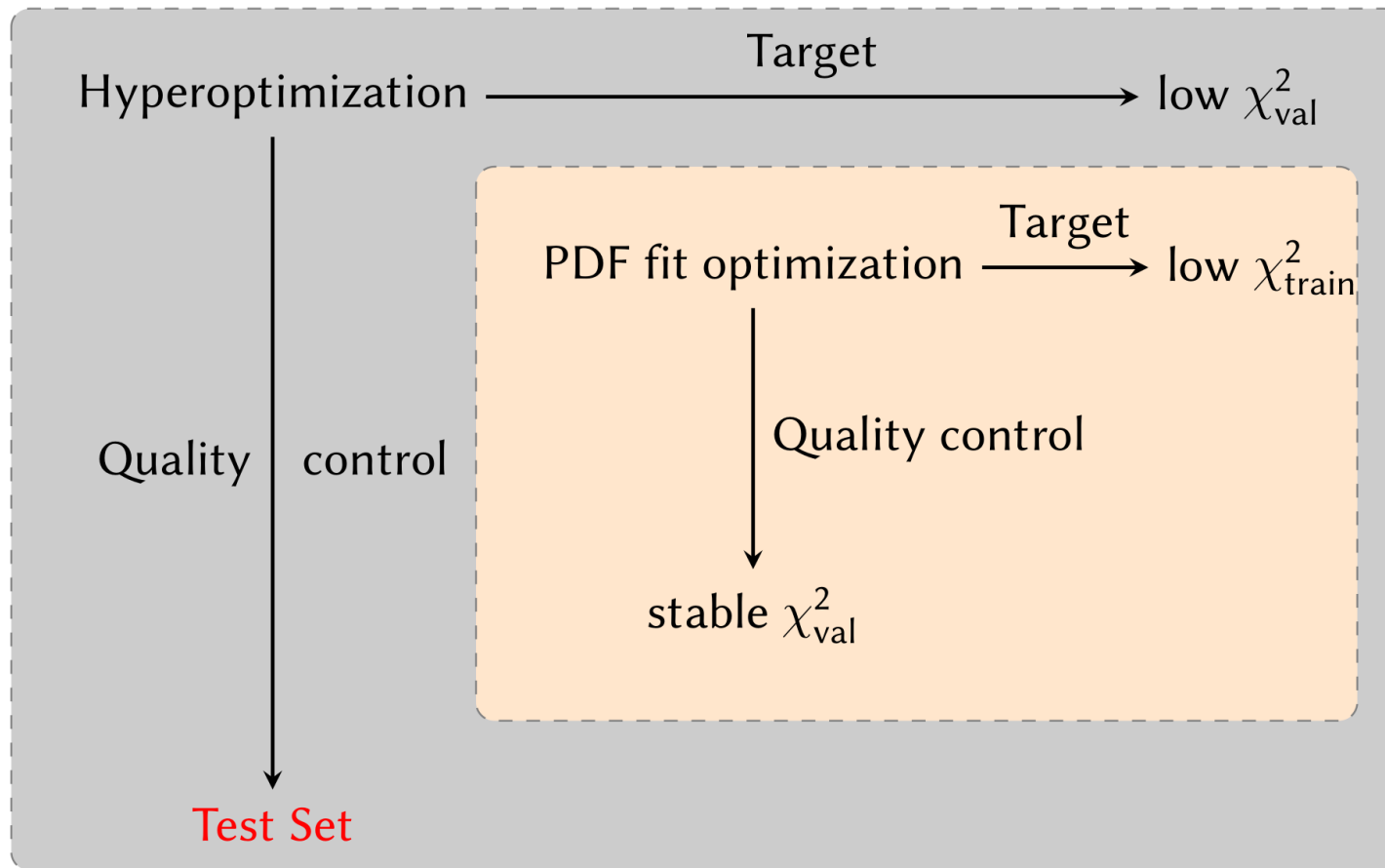
DOWN QUARK: HYPEROPTIMIZED VS. STANDARD



- **NNPDF3.1: WIGGLES: FINITE SIZE** \Rightarrow WILL GO AWAY AS N_{rep} GROWS
- **N3FIT: WIGGLY PDFS** \Leftrightarrow **OVERFITTING** \Rightarrow WILL **NOT** GO AWAY ($\chi_{\text{train}}^2 \ll \chi_{\text{valid}}^2$!!)
- **CORRELATIONS** BETWEEN TRAINING AND VALIDATION DATA

MACHINE LEARNING THE SOLUTION

TUNED HYPEROPTIMIZATION



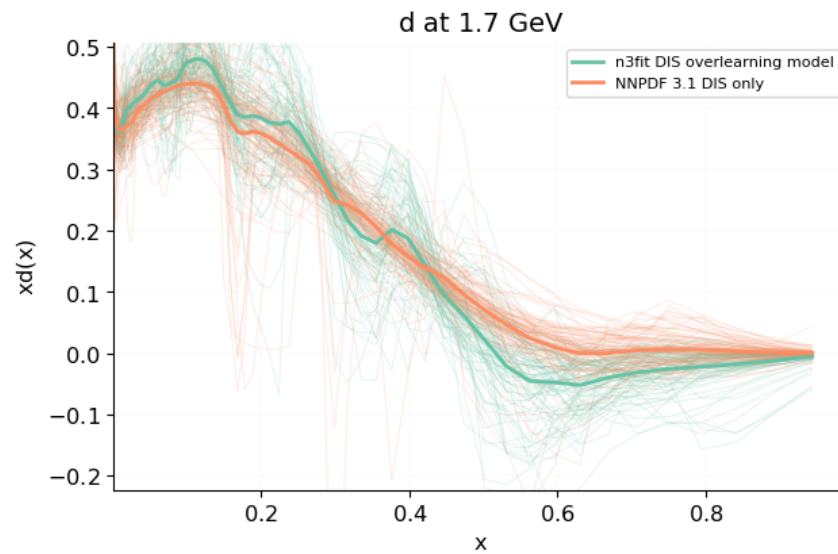
COMPARE TO A **A TEST SET** (NEW SET OF DATA PREVIOUSLY NOT USED AT ALL)
TESTS **GENERALIZATION POWER**

THE TEST SET METHOD

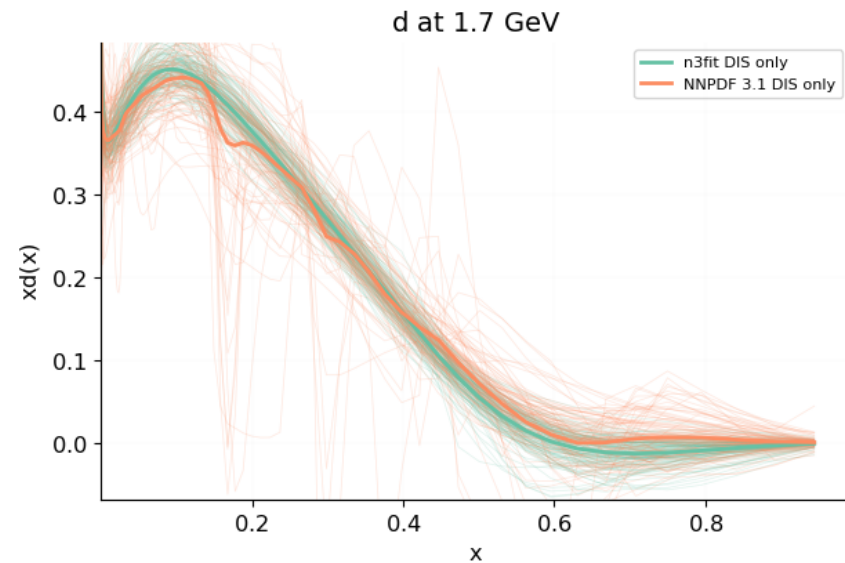
- **COMPLETELY UNCORRELATED** TEST SET
- OPTIMIZE ON WEIGHTED **AVERAGE** OF **VALIDATION AND TEST**
⇒ **NO OVERLEARNING**

OPTIMIZED PDFs DOWN QUARK

N3 OVERFIT vs NNPDF3.1

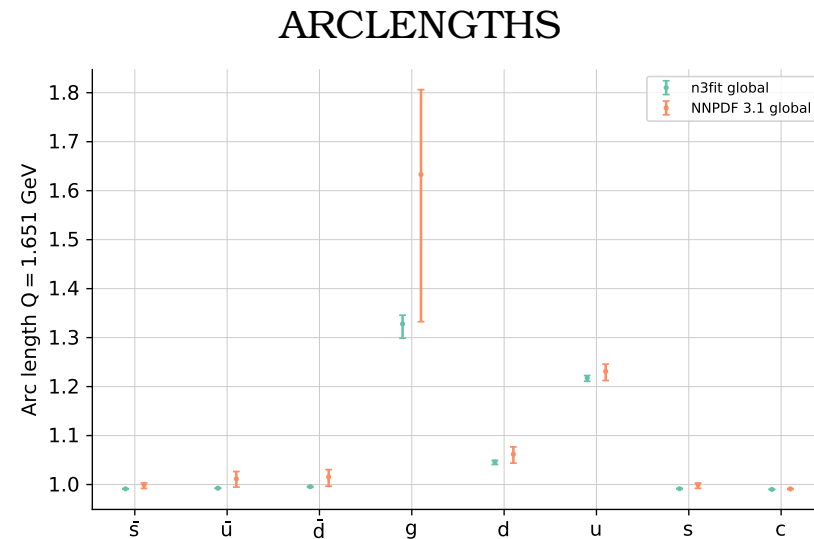
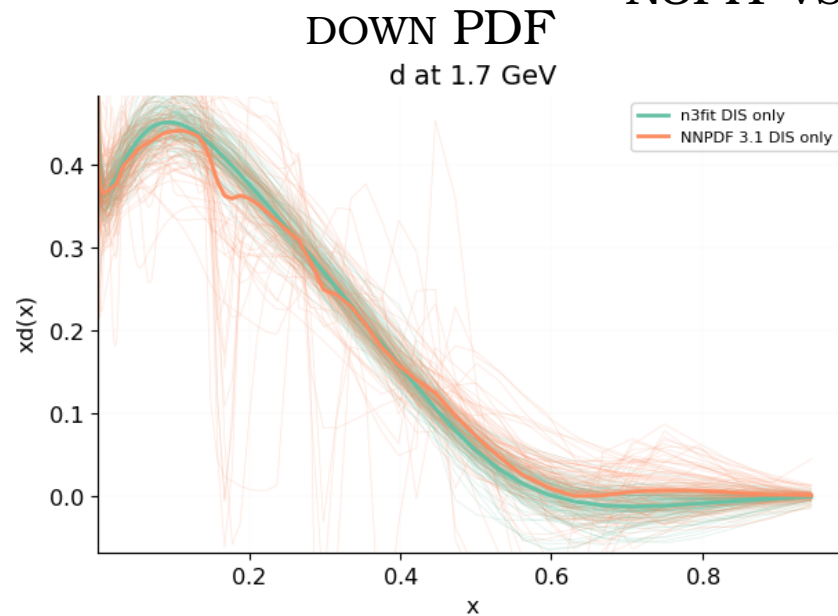


N3FIT vs NNPDF3.1



THE TEST SET METHOD

N3FIT vs NNPDF3.1



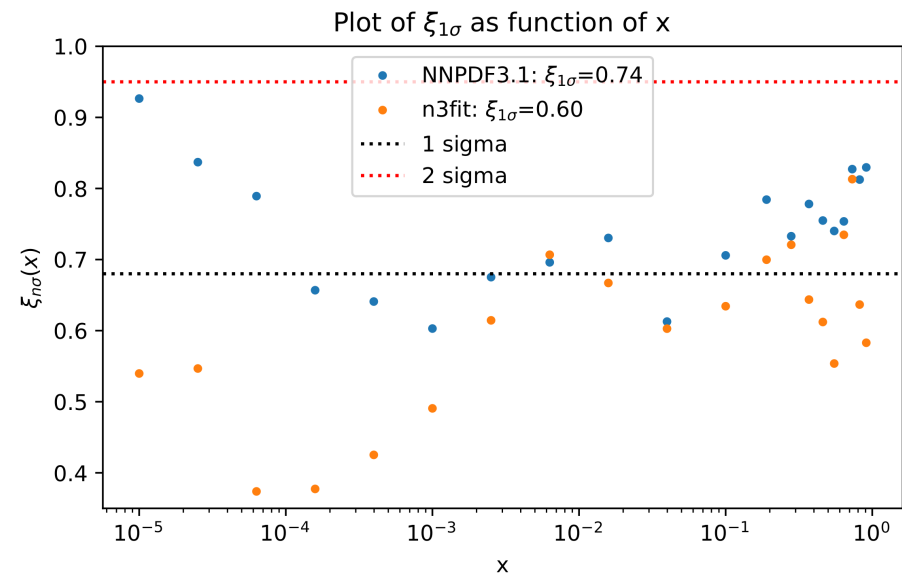
- **NO OVERFITTING**
- **COMPARED TO NNPDF3.1**
 - **MUCH** GREATER **STABILITY** \Rightarrow **FEWER REPLICAS** FOR EQUAL ACCURACY
 - **UNCERTAINTIES** SOMEWHAT **REDUCED**

CLOSURE TESTS AGAIN

ONE σ : ACTUAL/PREDICTED
FOR DATA, BY EXPERIMENT

experiment	NNPDF3.1 ratio	n3fit ratio
NMC	0.882828	0.843427
SLAC	0.767063	0.690118
BCDMS	0.730569	0.770704
CHORUS	0.698907	0.734656
NTVDMN	0.991090	0.797017
HERACOMB	0.847359	1.326333
HERAF2CHARM	1.867597	3.566076
F2BOTTOM	1.124157	1.532634
DYE886	0.655955	0.857915
DYE605	0.585725	0.870151
CDF	0.961652	0.779424
D0	0.881199	1.015202
ATLAS	0.904127	1.132229
CMS	1.090241	1.017136
LHCb	1.092194	0.993525
Total	0.842168	0.940737

ONE σ VALUE
FOR PDFS, VS x



- UNCERTAINTIES WELL ESTIMATED ON AVERAGE;
BUT **SIZABLE FLUCTUATIONS**
- ONE σ PERFECT IN DATA REGION;
BUT **UNDERESTIMATED IN EXTRAPOLATION**

BEYOND THE STATE OF THE ART:

DREAMS

- WHAT IS THE **UNCERTAINTY** WHERE THERE IS **NO DATA**?
- WHAT IS THE **UNCERTAINTY** WHERE THERE IS **NO THEORY**?

ML THE UNKNOWN

WHAT IS “PROPER LEARNING”?

FORECASTING AN UNKNOWN TRUTH \Rightarrow WHAT IS “OPTIMAL”?

MENU ▼ nature
International Journal of Science

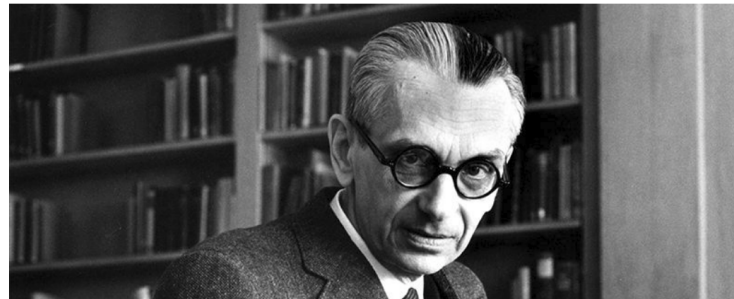
NEWS • 08 JANUARY 2019

Machine learning leads mathematicians to unsolvable problem

Simple artificial-intelligence problem puts researchers up against a logical paradox by famed mathematician Kurt Gödel.

Daive Castelvechi

[Twitter](#) [Facebook](#) [Email](#)



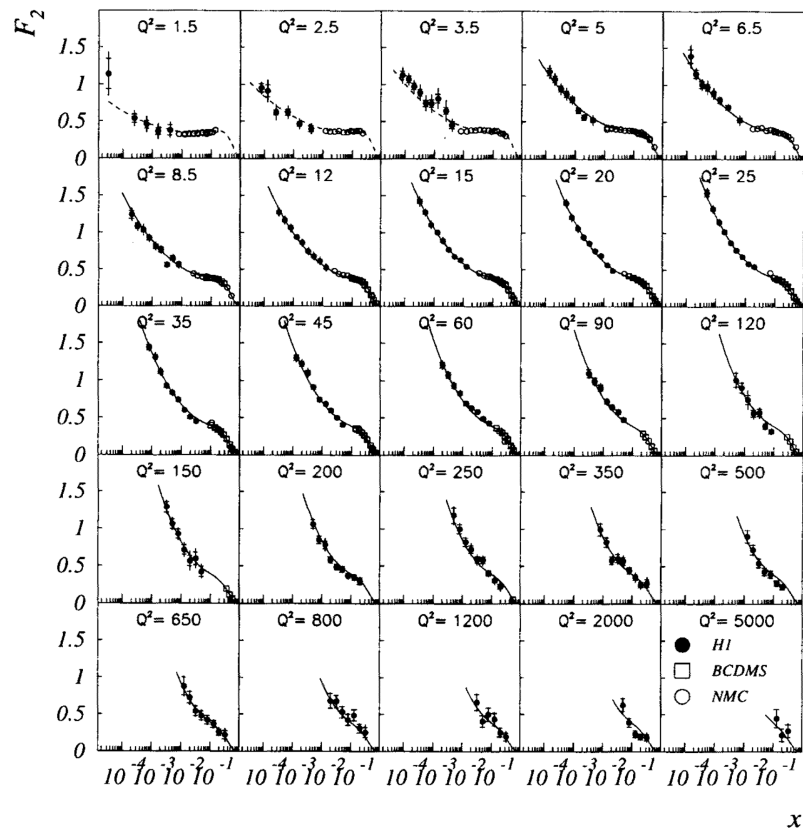
SOME POSSIBLE ANSWERS/CRITERIA

- PASS A CLOSURE TEST
- REPRODUCE THE EXPECTED STATISTICAL PROPERTIES:
ONE $\sigma \Leftrightarrow \Delta\chi^2 = 1$
- SATISFY THEORETICAL PREJUDICE?

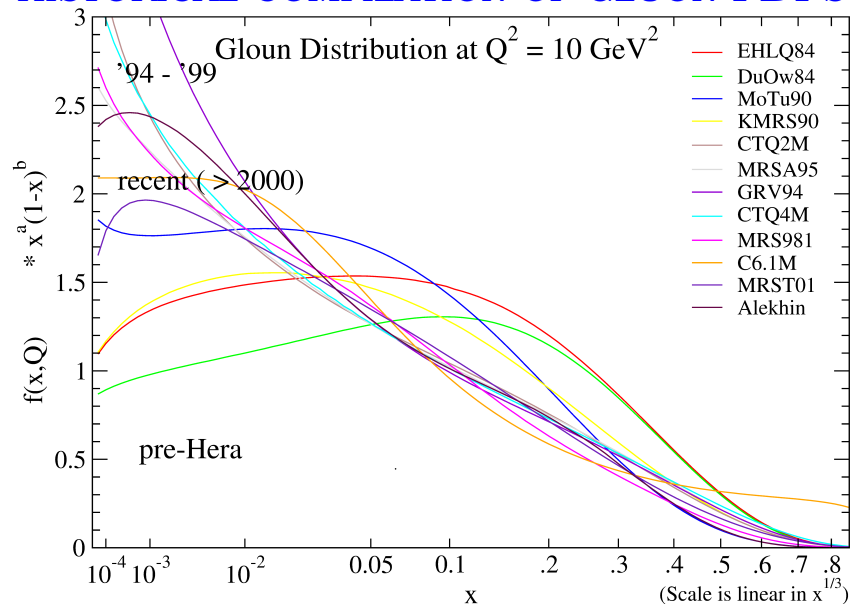
PASS A “FUTURE TEST”:
GENERALIZE TO CURRENT DATA BASED ON PAST DATA

THE "FUTURE TEST"

1995: THE RISE OF STRUCTURE FUNCTIONS AT HERA FIRST HERA DATA VS OLDER DATA



HISTORICAL COMPILATION OF GLUON PDFs



W.K.Tung, DIS 2004

A. de Roeck, Cracow epiphany conf. 1996

- **RISE** OF F_2 AT HERA CAME \Rightarrow **SURPRIZE**
- **HINTED** BY PRE-HERA **DATA**; **VETOED** BY **PREJUDICE**

COULD WE HAVE **PREDICTED** IT?

THE N3FIT FUTURE TEST

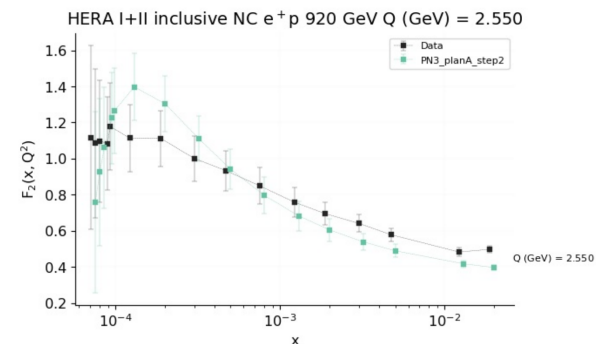
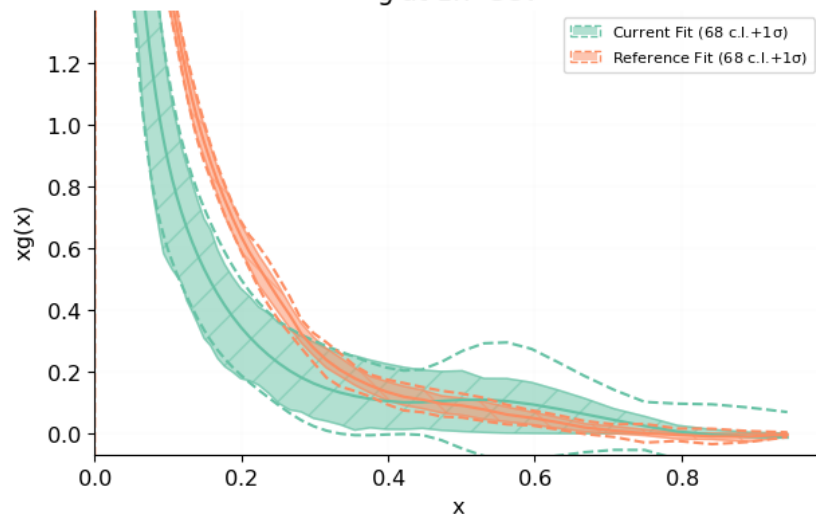
ONLY PRE-HERA DATA USED

PREDICTION COMPARED TO DATA

HERA F_2

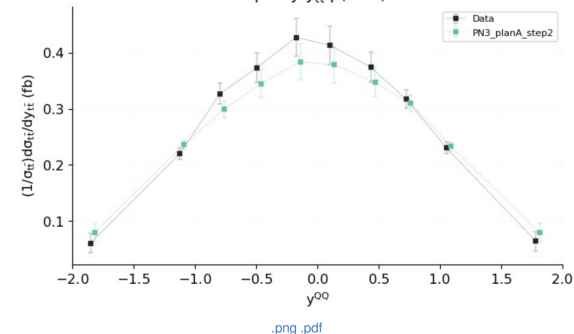
PREDICTED VS TRUE GLUON

g at 1.7 GeV



CMS TOP

CMS $t\bar{t}$ rapidity $y_{t\bar{t}}$ $\mu(\text{GeV}) = 173.3$



- N3FIT METHODOLOGY APPLIED AND HYPEROPTIMIZED TO PRE-HERA DATASET
- RESULTS WITH PDF UNCERTAINTY COMPARED TO FUTURE DATA
- $\chi^2/\text{dat}=1.1$ ON FULL PREDICTED CURRENT DATASET (ABOUT 200 DATAPOINTS)

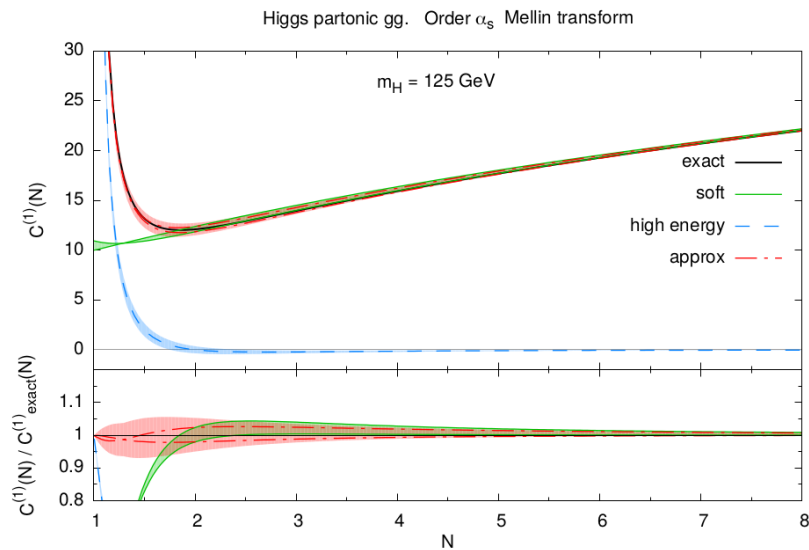
SUCCESS!

HOWEVER.... PREPROCESSING \Rightarrow TUNED METHODOLOGY

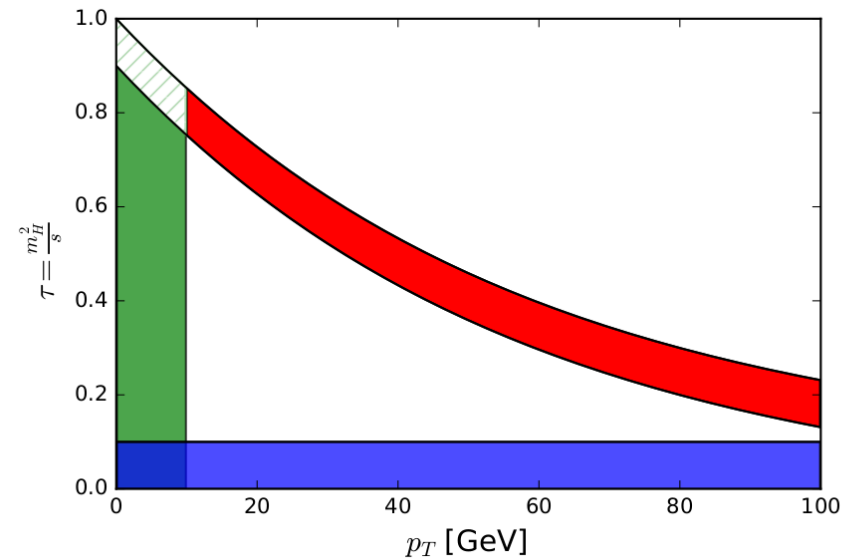
- GAUSSIAN PROCESSES? (KERNEL METHODS)
- REINFORCEMENT LEARNING

THEORY UNCERTAINTIES MISSING HIGHER ORDERS FROM RESUMMATION

N -SPACE GGHIGGS: APPROX VS. EXACT



(τ, p_T) RESUMMATION REGIONS



- THEORY **UNCERTAINTIES** \Leftrightarrow **APPROXIMATE** NEXT ORDER
- **RESUMMATION** \Rightarrow **SINGULARITIES**
- **MATCHING** THROUGH **LSTM?** (RECURRENT NN)

THE WORK OF MANY PEOPLE



NNPDF collaboration and N³PDF team meeting,
Varenna, Italy, September 2019

“Io stimo più il trovare un vero, benché di cosa leggiera, che il disputar lungamente delle massime questioni senza verità nissuna”

“I am more interested in uncovering a fact, however trifling, than to dispute at length about profound questions devoid of any truth”

Galileo Galilei, letter to Tommaso Campanella

EXTRAS

CONTEMPORARY PDF TIMELINE (ONLY PUBLISHED GLOBAL)

	2008		2009		2010		2011	2012		2013		2014		2015	2017		2019
SET	CTEG6.6	NNPDF1.0(08)	MSTW	ABKM09	NNPDF2.0(02)	CT10 (NLO)	NNPDF2.1 (NNLO)	ABM11	NNPDF2.3(07)	CT10 (NNLO)	ABM12	NNPDF3.0(10)	MMHT	CT14	ABMP16	NNPDF3.1(06)	CT18
MONTH	(02)	(08)	(01)	(08)	(02)	(07)	(07)	(02)	(07)	(02)	(10)	(10)	(12)	(06)	(01)	(06)	(12)
F. T. DIS	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
ZEUS+H1-HI	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
COMB. HI	✗	✗	✗	✗	✓	✗	some	✗	✓	✗	some	✓	✗	✗	✓	✓	✓
ZEUS+H1-HII	✗	✗	✗	✗	✗	✗	some	✗	✗	some	✗	✓	✗	✗	✓	✓	✓
HERA JETS	✗	✗	✓	✗	✗	✗	✗	✗	✗	✗	✗	✗	✓	✗	✗	✗	✗
F. T. DY	✓	✗	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
TEV W+Z	✓	✗	✓	✗	✓	✓	✓	✗	✓	✓	✗	✓	✓	✓	✗	✓	✓
LHC W+Z	✗	✗	✗	✗	✗	✗	✗	✗	✓	✗	some	✓	✓	✓	some	✓	✓
TEV JETS	✓	✗	✓	✗	✓	✓	✗	✓	✓	✓	✗	✓	✓	✓	✗	✓	✓
LHC JETS	✗	✗	✗	✗	✗	✗	✗	✗	✓	✗	✗	✓	✓	✓	✗	✓	✓
TOP TOTAL	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✓	✓	✗	✗	✓	✓	✓
SINGLE TOP TOTAL	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✓	✗	✗
TOP DIFFERENTIAL	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✓	✓
$W p_T$	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✓	✗	✗	✗	✗	✗
W+C	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✓	✗	✗	✗	✗	✗
$Z p_T$	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✗	✓	✓

THEORY PROGRESS:

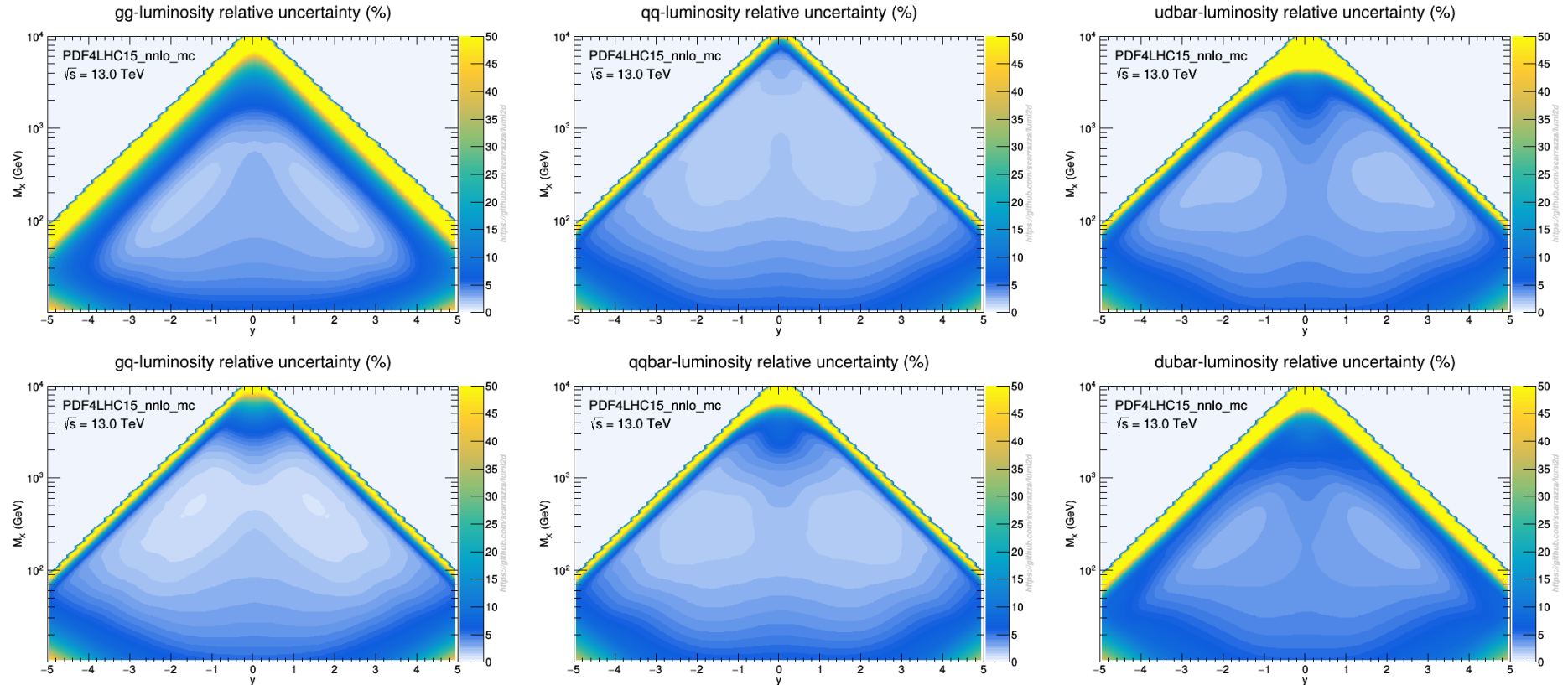
- **MSTW**, **ABKM**: all NNLO; **NNPDF** NNLO since 07/11 (2.1), **CT** since 02/13 (**CT10**); **NNPDF** THRESHOLD RESUMMATION (3.0RESUM, 07/15), SMALL x RESUMMATION (3.1SX, 10/17)
- **MSTW**, **CT**, **NNPDF** all GM-VFN; **NNPDF** since 01/11 (2.1); **ABM** FFN+ZM-VFN since 01/17 (**ABMP16**)
- **NNPDF** FITTED CHARM since 05/16 (**NNPDF3IC**)
- PHOTON PDF: (**mrst2004qed**), **NNPDF2.3QED** (08/13), **NNPDF3.0QED** (06/16), **NNPDF3.1LUXQED** (12/17)

PDF4LHC15: PDF UNCERTAINTIES (NNLO)

GLUON

SINGLET

FLAVORS

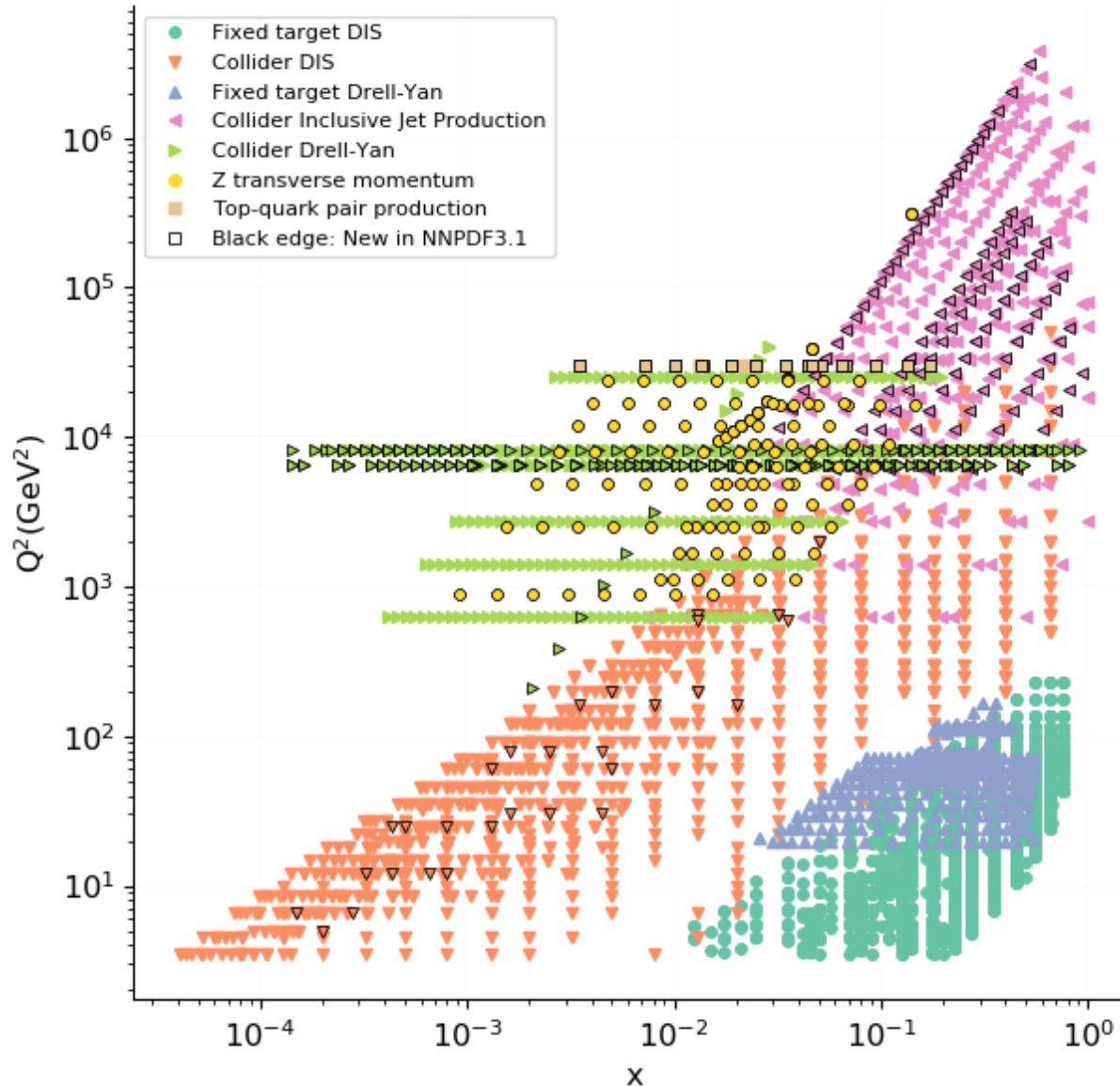


- **GLUON** BETTER KNOWN AT SMALL x , **VALENCE** QUARKS AT LARGE x , SEA QUARKS IN BETWEEN
- **TYPICAL** UNCERTAINTIES IN DATA REGION $\sim 3 - 5\%$
- **SWEET SPOT**: VALENCE Q - G; DOWN TO 1%
- UP BETTER KNOWN THAN DOWN; FLAVOR SINGLET BETTER THAN INDIVIDUAL FLAVORS
- NO QUALITATIVE DIFFERENCE BETWEEN NLO AND NNLO

DATASET WIDENING

NNPDF3.0 vs NNPDF3.1

Kinematic coverage



NEW DATA: (BLACK EDGE)

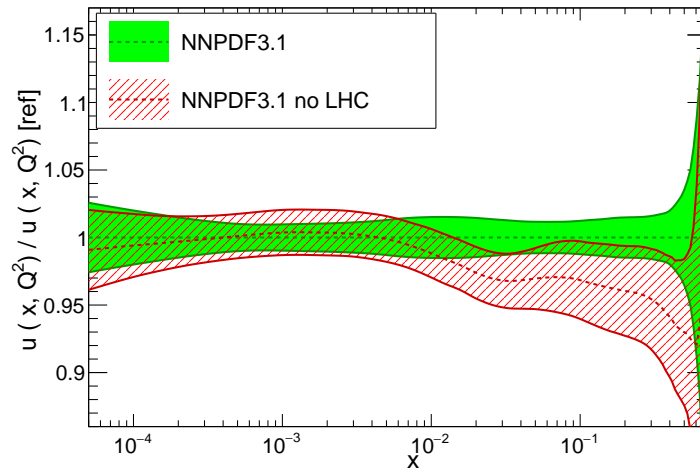
- HERA COMBINED F_2^b
- DO W LEPTON ASYMMETRY
- ATLAS W, Z 2011, HIGH & LOW MASS DY 2011;
LHCb W, Z 7TEV & 8TEV
- ATLAS 7TEV JETS 2011, CMS 2.76TEV JETS
- ATLAS & CMS TOP DIFFERENTIAL RAPIDITY
- ATLAS Z p_T DIFFERENTIAL RAPIDITY & INVARIANT MASS 8TEV,
 CMS Z p_T DIFFERENTIAL RAPIDITY 8TEV

THE IMPACT OF LHC DATA

NEXT-GENERATION PDFs **LARGELY DETERMINED BY LHC DATA: A FIRST!**

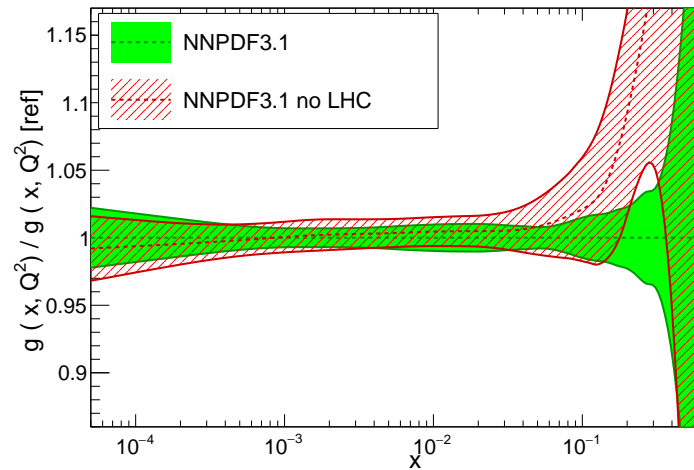
NNPDF3.1 up

NNPDF3.1 NNLO, $Q = 100$ GeV

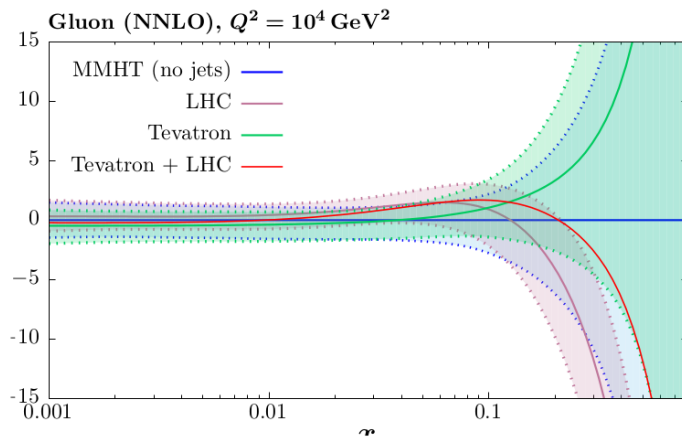


NNPDF3.1 glue

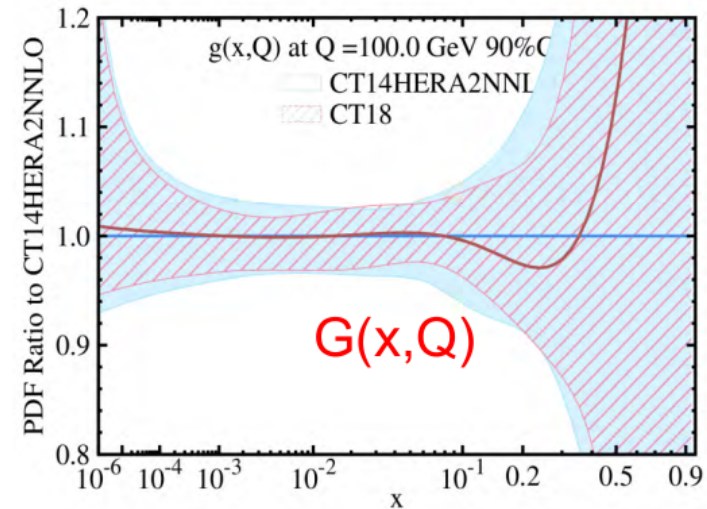
NNPDF3.1 NNLO, $Q = 100$ GeV



'MMHT' 19 glue (prelim., unpublished)



CT18 glue (preliminary, unpublished)



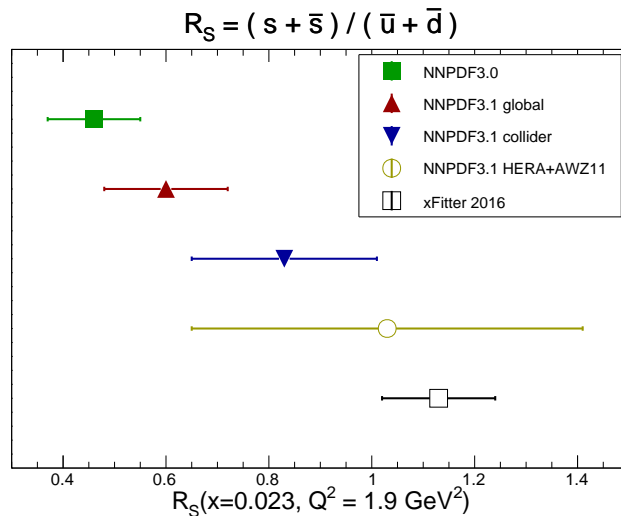
- SIGNIFICANT UNCERTAINTY REDUCTION
- MANY PDFs CHANGE BY MORE THAN ONE SIGMA
- BOTH FLAVOR SEPARATION & GLUON SIGNIFICANTLY AFFECTED

DATA VS. THEORY/METHODOLOGY

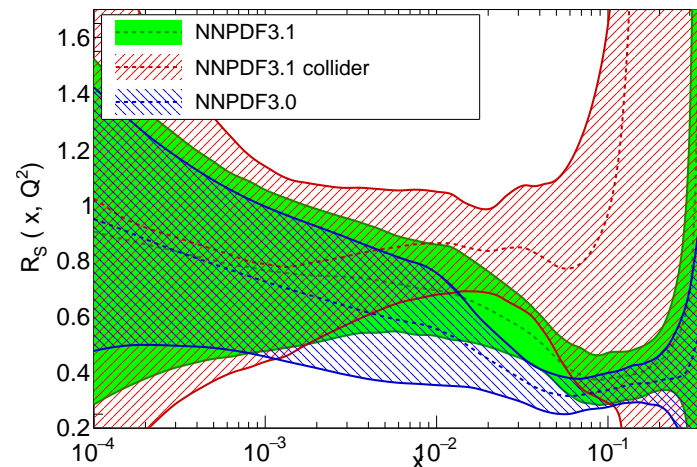
THE STRANGE PDF: DIS VS. W PRODUCTION

- **STRANGE PDF** CONTROLLED BY **NEUTRINO DIS** **CHARM** PRODUCTION + **W PRODUCTION**
- **DIS** DATA FAVOR “**SUPPRESSED STRANGE**” \Rightarrow SMALL $R_s \equiv \frac{s+\bar{s}}{\bar{u}+\bar{d}}$
- **ATLAS** FAVORS **ENHANCED** STRANGENESS
- **ATLAS** IMPACT **EXAGGERATED** IN **XFITTER** ANALYSIS
- **EVERYTHING** **CONSISTENT** WITHIN **UNCERTAINTIES** IN **GLOBAL FIT**

THE STRANGENESS SUPPRESSION
 XFITTER VS HERA+ATLAS VS. DIS ONLY VS ATLAS
 ONLY VS ALL



DIS ONLY VS ATLAS ONLY VS ALL
 NNLO, $Q=1.38 \text{ GeV}$

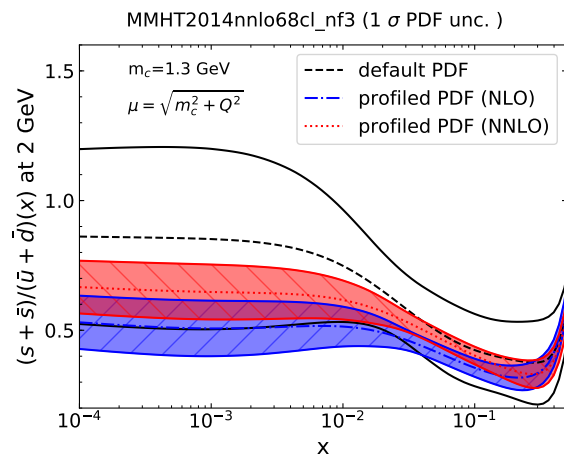


DATA VS. THEORY/METHODOLOGY

THE STRANGE PDF: DIS VS. W PRODUCTION

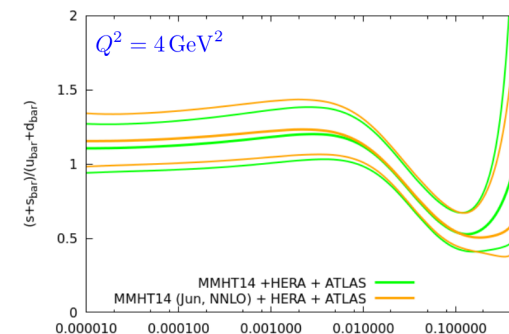
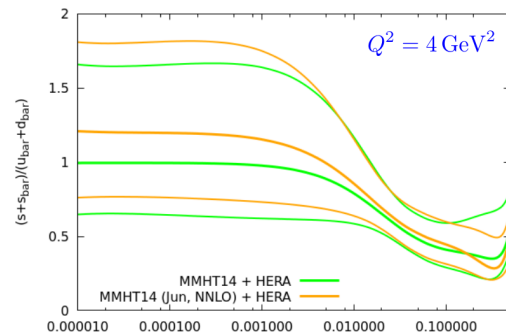
- **MASSIVE CORRECTIONS** TO CHARGED CURRENT DIS HITERTO **INCLUDED TO NLO** MASSLESS TO NNLO
- Gao, 2018 \Rightarrow **NNLO COMPUTED**
- **STRANGENESS ENHANCED BY NNLO** CORRECTIONS

HERAPDF +NLO CC DIS VS NNLO
CC DIS



(Gao, 2108)

MMHT WITH NLO VS NNLO CC DIS



Preliminary

(Harland-Lang, Thorne, prelim.)

LESSONS:

- **BEWARE** OF XFITTER **HERA+X** FITS
- IN A **GLOBAL FIT** DIFFERENT **DATA** ALWAYS **PULL IN DIFFERENT DIRECTIONS!**
- **TENSIONS** CAN BE **RESOLVED BY BETTER THEORY**

DATA VS. THEORY/METHODOLOGY

THE CHARM MASS AND TREATMENT

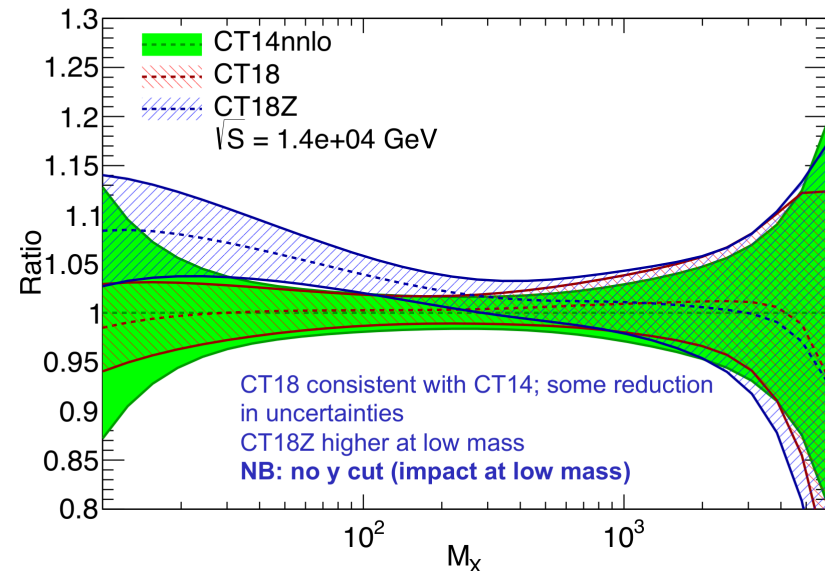
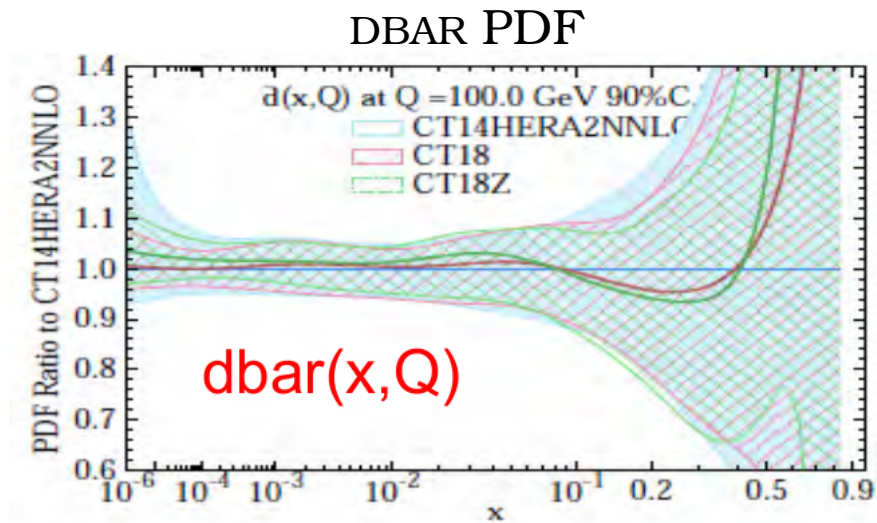
CT18 → CT18Z

- ATLAS W AND Z 7TeV RAPIDITY INCLUDED
- CHARM MASS INCREASED
- x -DEPENDENT FACTORIZATION SCALE

CT18 vs. CT18Z (preliminary, unpublished)

QQBAR LUMI

Quark - Antiquark Luminosity

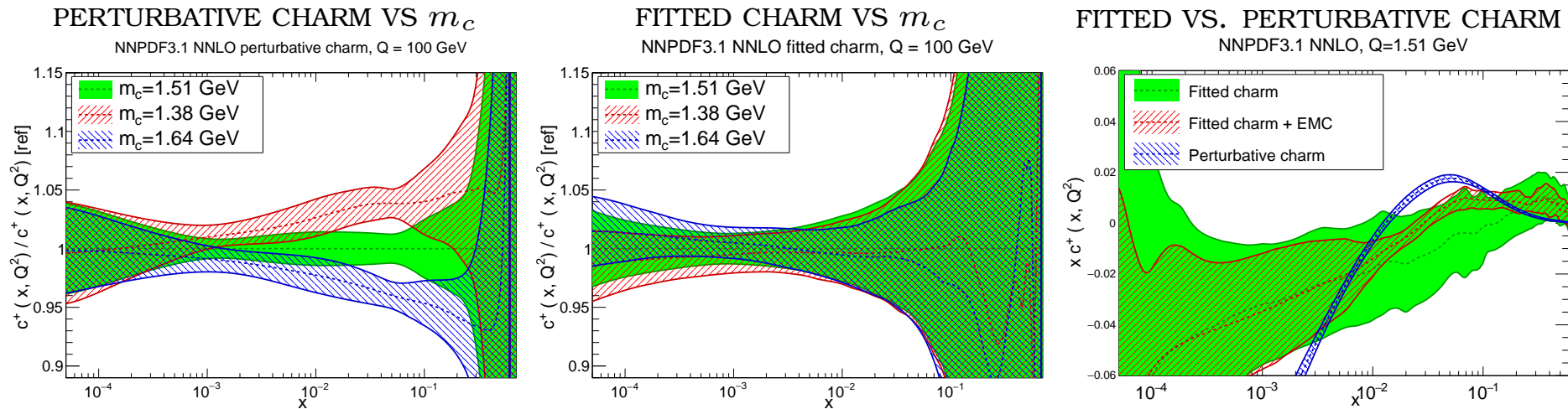


DATA VS. THEORY/METHODOLOGY

THE CHARM MASS AND TREATMENT

CHARM FROM DATA

- CHARM **SHOULD NOT DEPEND** STRONGLY ON **CHARM MASS**



- ITS **SHAPE SHOULD NOT BE DETERMINED** BY **FIRST-ORDER MATCHING** (NO HIGHER NONTRIVIAL ORDERS KNOWN)
- MIGHT EVEN HAVE A NONPERTURBATIVE COMPONENT

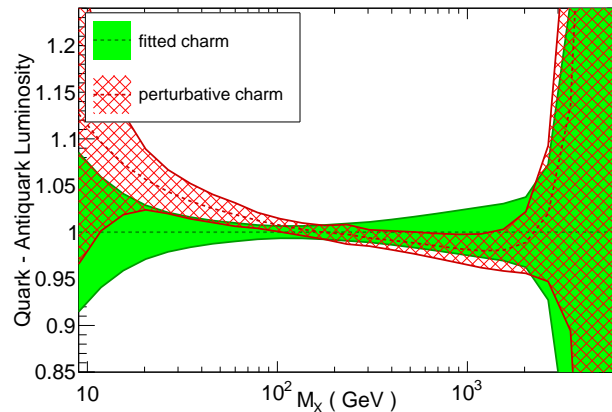
FITTED VS. PERTURBATIVE:
SUPPRESSED AT MEDIUM-SMALL x ,
ENHANCED AT VERY SMALL, VERY LARGE x

DATA VS. THEORY/METHODOLOGY

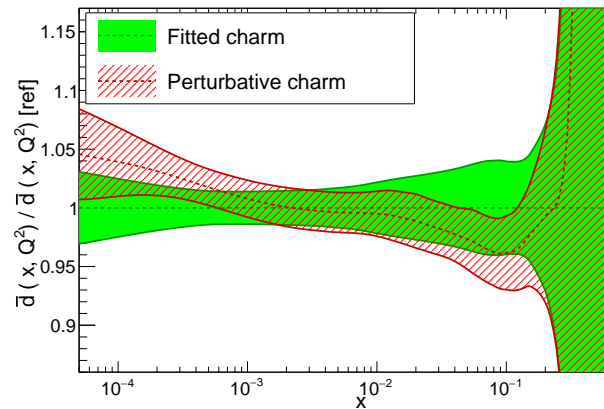
THE CHARM MASS AND TREATMENT

CHARM FROM DATA IMPACT ON LIGHT QUARK PDFS

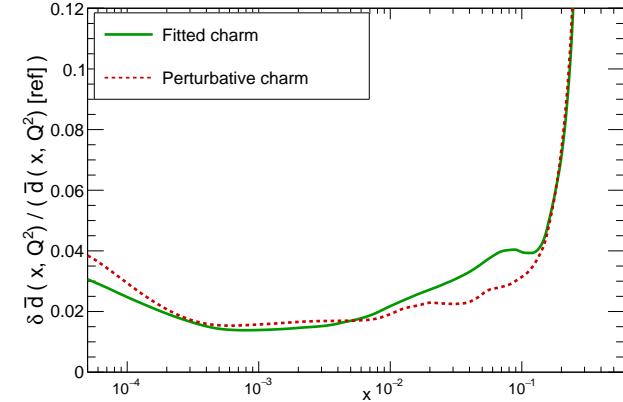
QQBAR LUMI
LHC 13 TeV, NNLO



FITTED VS. PERTURBATIVE CHARM
ANTIDOWN PDF
NNPDF3.1 NNLO, Q = 100 GeV



ANTIDOWN PDF UNCERTAINTY
NNPDF3.1 NNLO, Q = 100 GeV



- QUARK LUMI AFFECTED BECAUSE OF CHARM SUPPRESSION AT MEDIUM- x
- FLAVOR DECOMPOSITION ALTERED
- UNCERTAINTIES ON LIGHT QUARKS NOT SIGNIFICANTLY INCREASED
- AGREEMENT OF 13TeV W,Z PREDICTED CROSS-SECTIONS IMPROVES!

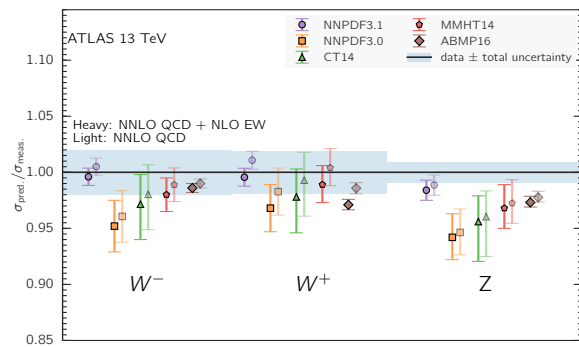
DATA VS. THEORY/METHODOLOGY

THE CHARM MASS AND TREATMENT

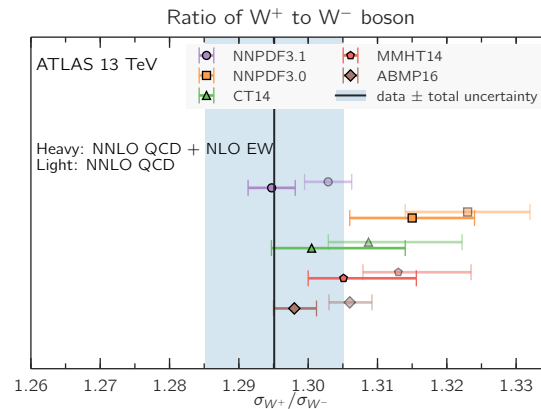
CHARM FROM DATA

IMPACT ON PHENOMENOLOGY

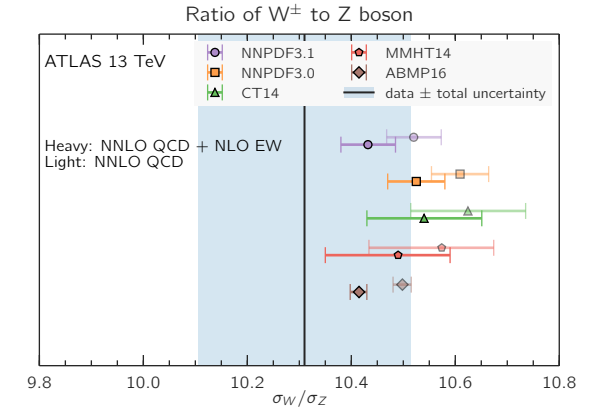
DRELL-YAN XSECTS



W^+ / W^- XSECT RATIO



W/Z XSECT RATIO



- W , Z CROSS-SECTIONS AT 13 TEV IN PERFECT AGREEMENT WITH DATA
THANKS TO FITTED CHARM!

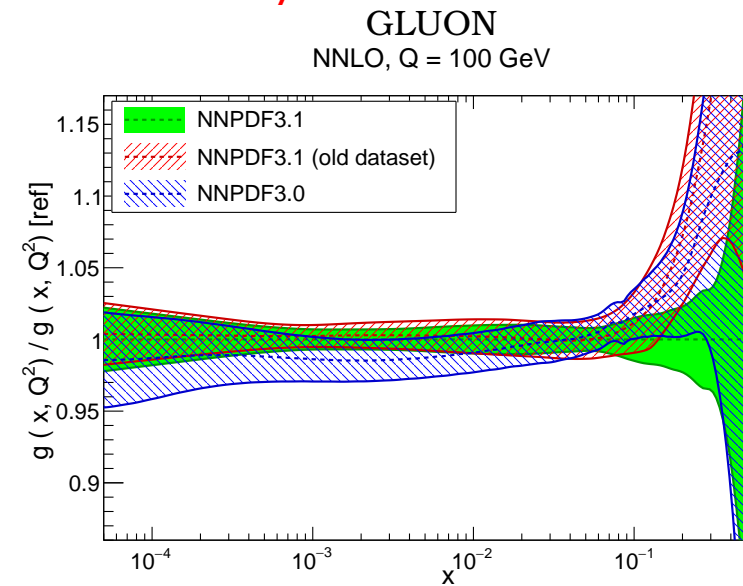
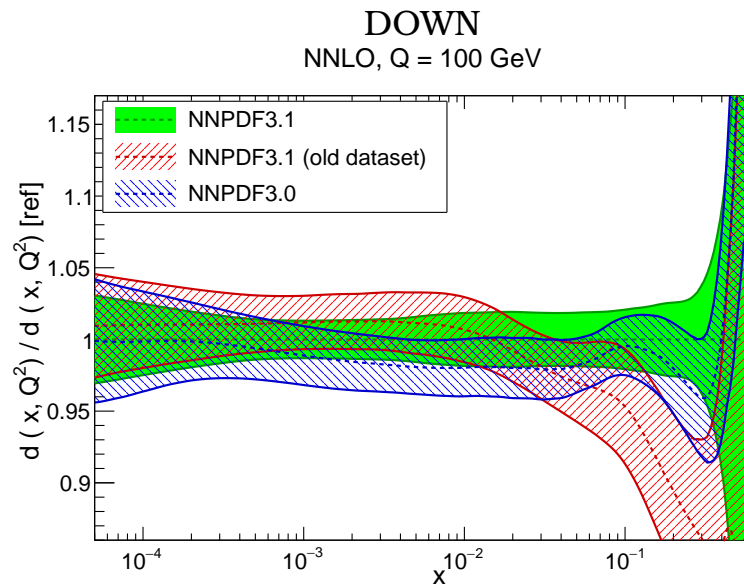
LESSONS:

- TENSIONS CAN REVEAL METHODOLOGICAL ISSUES
- MORE LIKELY AS DATASET INCREASES, EXPERIMENTAL UNCERTAINTIES DECREASE
- RESOLVED BY MORE COMPLEX METHODOLOGY

DATA vs. METHODOLOGY

- NEW DATA \Rightarrow MAJOR METHODOLOGICAL CHOICES \Rightarrow SIGNIFICANT IMPACT
- NNPDF3.1 vs NNPDF3.0: DATA AND METHODOLOGY HAVE SIMILAR IMPACT

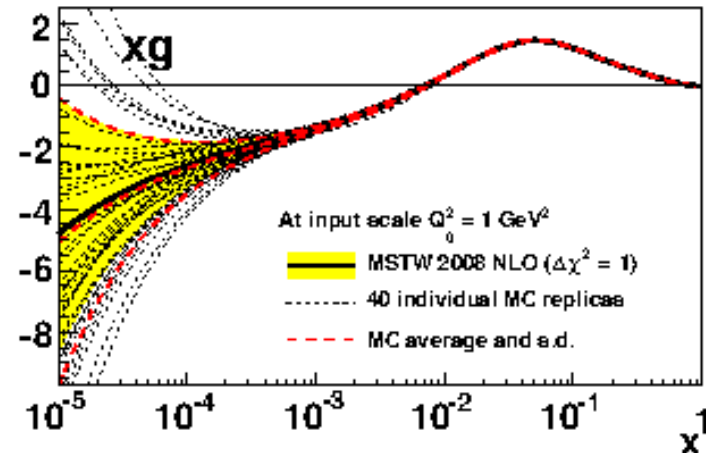
NNPDF3.0 vs. NNPDF3.1 vs. NNPDF3.1 w/ NNPDF3.0 DATASET



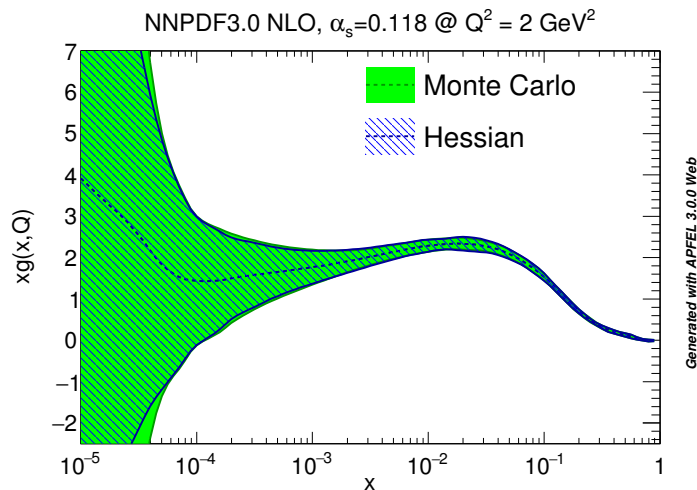
TOOLS I

MC \Leftrightarrow HESSIAN

- TO CONVERT HESSIAN INTO MONTECARLO
GENERATE MULTIGAUSSIAN REPLICAS
IN PARAMETER SPACE
- ACCURATE WHEN NUMBER OF REPLICAS
SIMILAR TO THAT WHICH REPRODUCES DATA



(Thorne, Watt, 2012)



(Carrazza, SF, Kassabov, Rojo, 2015)

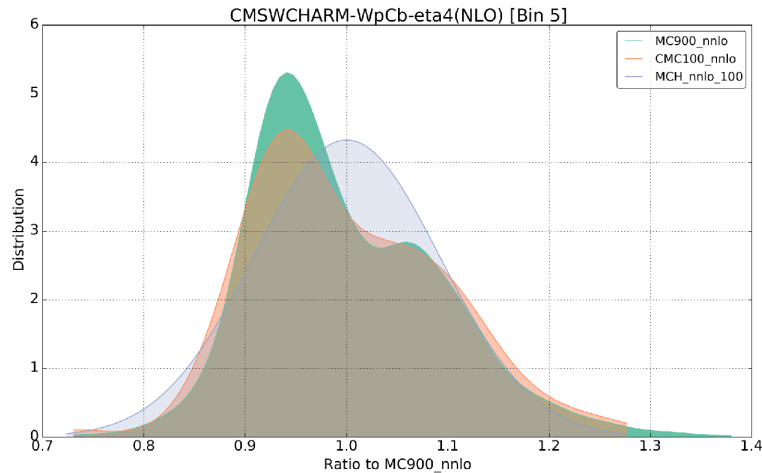
- TO CONVERT MONTE CARLO INTO HESSIAN, SAMPLE
THE REPLICAS $f_i(x)$ AT A DISCRETE SET OF POINTS &
CONSTRUCT THE ENSUING COVARIANCE MATRIX
- EIGENVECTORS OF THE COVARIANCE MATRIX AS A
BASIS IN THE VECTOR SPACE SPANNED BY THE REPLICAS
BY SINGULAR-VALUE DECOMPOSITION
- NUMBER OF DOMINANT EIGENVECTORS SIMILAR TO
NUMBER OF REPLICAS \Rightarrow ACCURATE REPRESENTATION

TOOLS II

NONGAUSSIAN BEHAVIOUR

MONTE CARLO COMPARED TO HESSIAN

CMS $W + c$ production



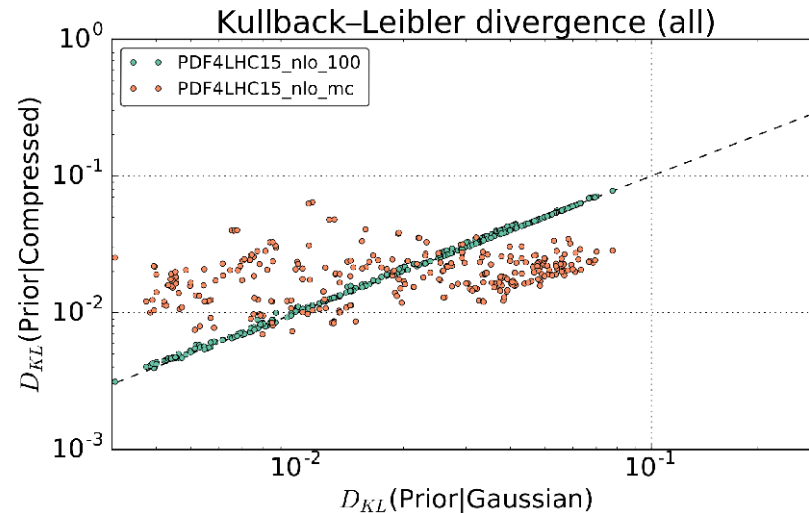
- DEFINE **KULLBACK-LEIBLER DIVERGENCE**

$$D_{KL} = \int_{-\infty}^{\infty} P(x) \frac{\ln P(x)}{\ln Q(x)} dx$$

BETWEEN A PRIOR P AND ITS REPRESENTATION Q

- D_{KL} BETWEEN PRIOR AND HESSIAN DEPENDS ON DEGREE OF GAUSSIANTY
- D_{KL} BETWEEN PRIOR AND COMPRESSED MC DOES NOT

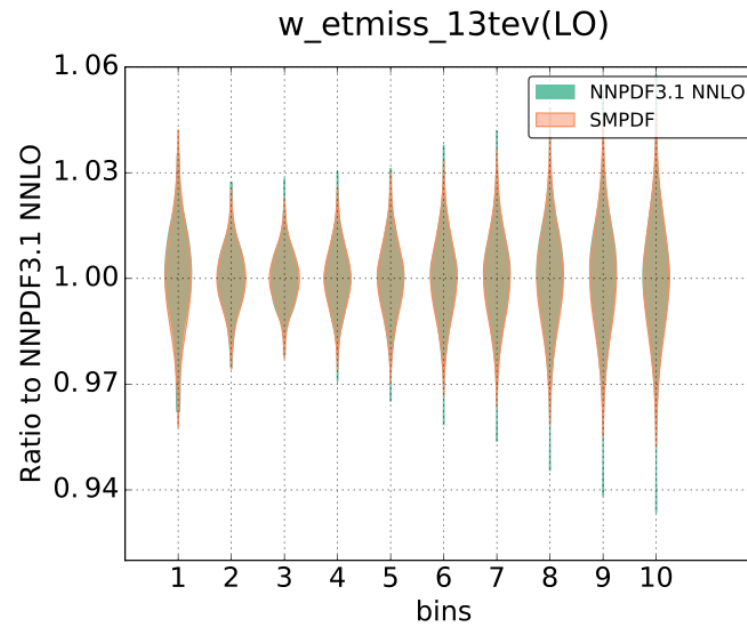
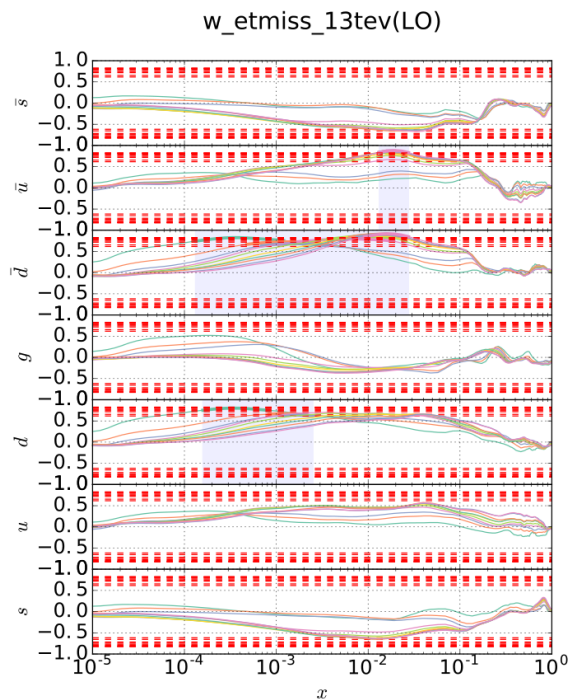
- DEVIATION FROM GAUSSIANTY E.G. AT LARGE x DUE TO LARGE UNCERTAINTY + POSITIVITY BOUNDS
⇒ RELEVANT FOR SEARCHES
- CANNOT BE REPRODUCED IN HESSIAN FRAMEWORK
- WELL REPRODUCED BY COMPRESSED MC



CAN (A) GAUGE WHEN MC IS MORE ADVANTAGEOUS THAN HESSIAN;
(B) ASSESS THE ACCURACY OF COMPRESSION

TOOLS III OPTIMIZED PDFS: SMPDF

- OLD ASPIRATION: PDFS OPTIMIZED TO PROCESSES (Pumplin 2009)
- SELECT **SUBSET OF THE COVARIANCE MATRIX CORRELATED** TO A GIVEN SET OF PROCESSES
- PERFORM **SVD ON THE REDUCED COVARIANCE MATRIX**, SELECT DOMINANT EIGENVECTOR, **PROJECT OUT** ORTHOGONAL SUBSPACE
- ITERATE UNTIL DESIRED ACCURACY REACHED
- **CAN ADD PROCESSES TO GIVEN SET; CAN COMBINE DIFFERENT OPTIMIZED SETS**
- **WEB INTERFACE AVAILABLE**



(Carrazza, SF, Kassabov, Rojo, 2016)

- EG $ggH, Hb\bar{b}, W E_T^{\text{miss}} \Rightarrow 11$ EIGENVECTORS
- STUDY **CORRELATIONS OF PDFS** TO DATA AND AMONG THEMSELVES!