Optical Resolution at the Quantum Fisher Information Limit

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Abstract

There are many problems where optics and quantum theory overlap, one of them being the fundamental limit upon the resolution. The spatial resolution of any imaging device is restricted by diffraction, which causes a sharp point on the object to blur into a finite-sized spot in the image. This intrinsic blurring is encoded in the point-spread function, which hinders to distinguish two neighbourhood points- an effect known as "Rayleigh curse". The same problem can be reconsidered from the perspective of quantum estimation theory, as done by Tsang and coworkers, see [Tsang:2016]. Here the constraints on resolution are more fundamental and correspond to the so-called Fisher information and Cramér-Rao lower bound (CRLB) for parameter estimation.

When only light intensity at the image plane is measured on the basis of all the traditional techniques such as CCD detection, the Fisher information falls to zero as the separation between two sources decreases in accordance with Rayleigh curse. On the other hand, when the Fisher information is calculated for optimal measurement saturating the quantum Cramer-Rao Lower bound (qCRLB), it remains constant implying that the Rayleigh limit is subsidiary to the problem and super-resolution is in principle achievable see [Tsang:2016, Rehacek: 2017a].

The optimal measurements, on the contrary to the direct imaging, depends crucially on the number of parameters, which have to be estimated.

Multi-parameter quantum Cramer-Rao bound for simultaneously estimating the centroid, the separation, and the relative intensities of two incoherent optical point sources using a linear imaging system were addressed in [Rehacek: 2017b]. For equally bright sources, CRLB is independent of their separation as before. However, for the general case of unequally bright sources, the amount of information one can gain about the separation falls to zero, but we show that there is always a quadratic improvement in an optimal detection in comparison with the intensity measurements. This advantage can be of utmost importance in realistic scenarios, such as observational astronomy. Attainability of the qCRLB and feasibility of the multi-parameter estimation scheme was addressed in [Rehacek: 2018]. Estimation of 3 parameters requires at least 4 detected channels constructed from the projection into the superposition of the modes spanned by derivatives of the PSF. Quantum-inspired imaging techniques can be nontrivially extended in several ways, for example to the time-frequency domain using mode-selective sum-frequency generation with shaped ultrafast pulses demonstrating the resolution up to ten-fold improvement in precision over the intensity detection [Donohue: 2018], for axial resolution reaching the limit of quantum Fisher information even for intensity detection [Rehacek:2019] or clarifying the role of coherence in estimation problems [Hradil:2019, 2021].

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