THE COSMIC MATTER-ANTIMATTER ASYMMETRY

a two fold problem

how to generate a net baryon number $\Delta B = n_B - n_{\bar{B}} \neq 0$ starting from a baryon-symmetric $(n_B = n_{\bar{B}})$ Universe ?

(so far, no experimental evidence for primordial (i.e. not coming from stellar evolution processes) cosmic ANTIMATTER - if primordial antimatter exists today in the Universe it must be separated from baryonic matter by enormous distances (otherwise

$\bar{B}$ B $\gamma \leftarrow$ γ's from boundary regions of B and $\bar{B}$) $> 10^{14} M_\odot$

→ difficulty to realize such cosmic separation

how to account for the smallness of the baryon-to-photon ratio:

$$\eta_B = \frac{n_{B,0}}{n_{\gamma,0}} \simeq 6 \times 10^{-10}$$

or

$$\Delta_B = \frac{n_B - n_{\bar{B}}}{s} \simeq 0.9 \times 10^{-10}$$

baryon-to-entropy ratio

if $n_B = n_{\bar{B}}$

→ nucleons in equilibrium $\Omega_B \sim 10^{-11}$ instead of $\Omega_B \sim 5\%$

through B $\bar{B}$ ← annihilation

(1967 !)

<u>Sakharov's conditions</u> to give rise to a baryon asymmetry starting from a B-symmetric ($n_B = n_{\bar{B}}$) situation (<u>necessary</u> conditions)

1) <u>Baryon number non-conservation</u> $\cancel{B}$
2) <u>C- and CP- violation</u> $\cancel{C}, \cancel{CP}$
3) <u>Thermal non-equilibrium</u>

2) if C or CP are conserved

processes with
- matter: $q\ q \;\; \ell\ \ell \;\; \dots$
- antimatter: $\bar{q}\ \bar{q} \;\; \bar{\ell}\ \bar{\ell} \;\; \dots$

$\downarrow$

occur in the same way

$\downarrow$

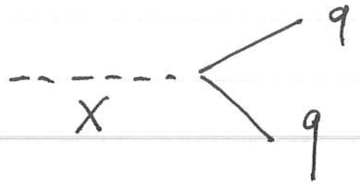
no asymmetry is generated

3) if in thermal equilibrium (w.r.t. $\cancel{B}$ interactions) any ΔB arising from $\cancel{B}$ processes tends to be washed out to reprisitinate $\Delta B = 0$.

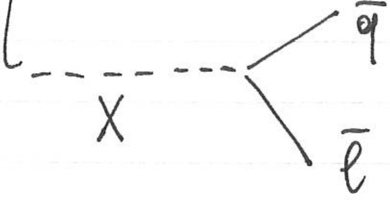
BARYOGENESIS IN GUTs

Ex.:

$\cancel{B}$



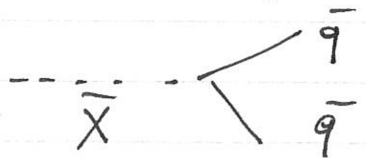
$B = 2/3$ $\qquad BR(X \to qq) = \varepsilon$



$B = -1/3$ $\qquad BR(X \to \bar{q}\bar{\ell}) = 1-\varepsilon$

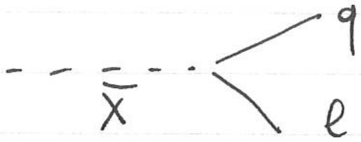
assume X has only these 2 decay channels with $B \neq 0$

$BR(\bar{X} \to \bar{q}\bar{q}) = \bar{\varepsilon}$



$B = -2/3$

$BR(\bar{X} \to q\ell) = 1-\bar{\varepsilon}$



$B = +1/3$

$$\Delta B_X \,(\text{in } X \text{ decays}) = \varepsilon \cdot \left(\frac{2}{3}\right) + (1-\varepsilon)\cdot\left(-\frac{1}{3}\right)$$

$$\Delta B_{\bar{X}} \,(\text{in } \bar{X} \text{ ''}) = \bar{\varepsilon}\cdot\left(-\frac{2}{3}\right) + (1-\bar{\varepsilon})\cdot\left(+\frac{1}{3}\right)$$

$$\Delta B_X + \Delta B_{\bar{X}} \equiv \Delta B = \varepsilon - \bar{\varepsilon}$$

if C and CP are violated $\rightarrow$ $\varepsilon \neq \bar{\varepsilon} \rightarrow \Delta B \neq 0$

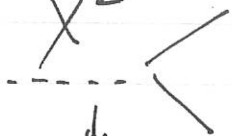
1^{st} - 2^{nd} conditions respected

$\hookrightarrow \cancel{B}$ $\quad \cancel{C}, \cancel{CP}$ $\qquad$ look for 3^{rd} condition (out-of-equilibrium condition):

$X \leftarrow$ vector

or

$X \leftarrow$ scalar



$\downarrow$

ex: X_μ
left-quark gauge boson of $SU(5)$

$SU(3)$ $SU(2)$ Q
$(3, 2, -4/3)\, X^\mu$

or $(3, 2, +1/3)\, Y$

$\downarrow$

new $SU(5)$ Higgs multiplets breaking
$SU(5)$
$\downarrow$
$SU(3) \times SU(2) \times U(1)$

$\langle 24 \rangle \to \dfrac{SU(5)}{SU(3) \times SU(2) \times U(1)}$

$\alpha \to$ coupling of X to the fermions

(for the vector gauge bosons $\alpha \to \alpha_{GUT}$)

$$\Gamma^X_D \equiv \Gamma^X_{\text{decay rate}} \simeq \alpha M_X$$

<u>X decays occur when $\Gamma^X_D \geq H$</u>, i.e. $\tau_D = (\Gamma^X_D)^{-1} <$ age of the Univ H^{-1}

$$H \to \frac{T^2}{M^*_{Pl}}$$

$$\alpha M_X \geq \frac{T^2}{M^*_{Pl}} \quad \to \quad T^2 < \alpha M_X M^*_{Pl}$$

<u>X are in equilibrium at $T = M_X$</u> if:

$$\alpha M_X \geq \frac{M_X^2}{M^*_{Pl}} \quad \Rightarrow \quad \underline{M_X \leq \alpha M^*_{Pl} \sim \alpha \cdot 10^{18}\,\text{GeV}}$$

$\Rightarrow$ if <u>$M_X < \alpha \cdot 10^{18}$ GeV</u> X still in equilibrium when T drops below X $\to$ all ΔB produced in X decays is erased if X decays (i.e. the $\cancel{B}$ processes) are in equilibrium if $M_X \gtrsim \alpha \cdot 10^{18}\,\text{GeV} \rightarrow \Gamma_D^X\Big|_{T=M_X} < H\Big|_{T=M_X}$, i.e.

$\tau_X\Big|_{T\sim M_X} >$ age of the Univ.$\Big|_{T\sim M_X}$

$\rightarrow$ X starts decaying when $T < M_X$

but in this case X decays are occurring out of equilibrium

($n_X/n_\gamma \sim 1$ since X could not decay until that moment, but the equilibrium X density would be $\sim (M_X T)^{3/2} e^{-M_X/T}$ hence $\frac{n_X^{equil}}{n_\gamma} \ll 1$)

the net $\Delta B \neq 0$ $(\Delta B = \bar{r} - r)$ cannot be erased

$n_B = (\Delta B)\, n_X$ X at $T = M_X$ not yet decayed

$\rightarrow n_X = n_{\bar{X}} \sim n_\gamma$

$\rightarrow n_B \sim (\Delta B)\, n_\gamma$ assuming that no other

$\not{B}$ process is still effective after X decays

$\rightarrow$ ΔB is frozen, but n_γ is not conserved (in general)

in the subsequent evolution of the Universe

(ex. e^+e^- disappearance $\rightarrow$ increase of n_γ)

→ replace n_B/n_γ with

n_B/s → specif entropy ratio $s = \frac{2\pi^2}{45} g_s(T) T^3$

$s = \frac{\pi^4}{45\zeta(3)} g_s(T) n_\gamma$ at $T \lesssim M_X \sim 10^{15}-10^{16}$ GeV

$\neq g(T)$ entering ρ * $g(T) \sim O(100)$

→ $s \sim O(100)\, n_\gamma \big|_{T \lesssim M_X}$; $\frac{n_B}{s} \sim O(10^{-2})\, \Delta B$

$\eta = \frac{n_B}{n_\gamma}$ boundary from BBN (nucleosynthesis)

$5\times10^{-10} < \eta < 7\times10^{-10}$

See * $g(T)$ relativistic gas $\rho = \left(\sum_{bosons} g_B + \frac{7}{8}\sum_{ferm} g_F\right) \times \frac{\pi^2}{30} T^4 = \frac{\pi^2}{30} g(T) T^4$

$s = \frac{4}{3}\frac{1}{T}(\rho + p) = \frac{4}{3}\frac{\rho}{T} = \frac{2\pi^2}{45} g(T) T^3$

$p \to \rho/3$

$\rho \sim T^4$ $s \sim T^3$ (relativistic regime)

at $T \lesssim 0.5$ MeV $T_\nu < T_\gamma$ → g_ρ (i.e. the $g(T)$ from T^4 in ρ

entering ρ → $g_\rho \big|_{T<0.5\,\text{MeV}} = g_\gamma + \left(\frac{4}{11}\right)^{4/3} g_\nu$

~ 3.36 ; $g_\gamma \to 2$; $g_\nu \to \frac{7}{8}\times 2\times 3$

$g_s \leftarrow g(T)$ entering s

$g_s \big|_{T<0.5\,\text{MeV}} = g_\gamma + \left(\frac{4}{11}\right)^{3/3=1} g_\nu \sim 3.91$

$g_\gamma \to 2$

Today:

$$s = \frac{\pi^4}{45\,\zeta(3)}\, g_s(T)\Big|_{\text{Today}} n_\gamma \simeq 7 \times n_\gamma$$

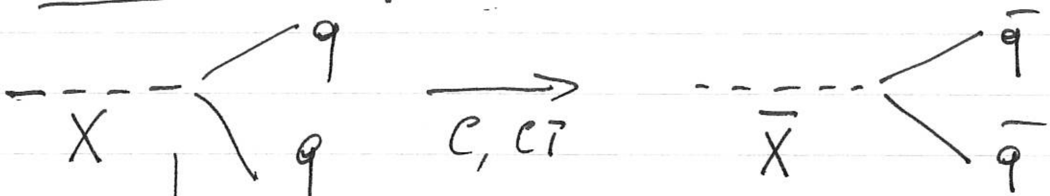
$\Rightarrow$ n_B/s $\rightsquigarrow$ (bound for BBN)

$$7 \times 10^{-11} < n_B/s < 10^{-10}$$

$n_B/s \sim 10^{-2}\, \Delta B$ $\hookrightarrow$ $\bar{\varepsilon} - \varepsilon \longrightarrow BR(\bar{X} \to \bar{q}\bar{q}) - BR(X \to qq)$

if this difference is $\sim 10^{-8}$ $\rightarrow$ GUTs would be promising candidate scenarios for baryogenesis

For $\underline{\varepsilon \text{ to be} \neq \bar{\varepsilon}}$ need $\underline{C \text{ and } CP}$ $\underline{\text{violation}}$ (if C and CP are conserved



$$\Rightarrow BR(X \to qq) = \frac{\Gamma(X \to qq)}{\Gamma^X_{TOT}} \qquad BR(\bar{X} \to \bar{q}\bar{q}) = \frac{\Gamma(\bar{X} \to \bar{q}\bar{q})}{\Gamma^{\bar{X}}_{TOT}}$$

$\Gamma^X_{TOT} = \Gamma^{\bar{X}}_{TOT} \rightarrow$ for CPT conservation

and $\Gamma(X \to qq) = \Gamma(\bar{X} \to \bar{q}\bar{q})$ if C and CP conserved but (even if CP and C are violated) $\Gamma(X \to qq)$ and $\Gamma(\bar{X} \to \bar{q}\bar{q})$ are equal at the TREE LEVEL

$\tau - \bar{\tau} = 0$ at the tree level

$\Gamma(X \to qq) \propto gg^* M_X \cdot$ kinem. factor

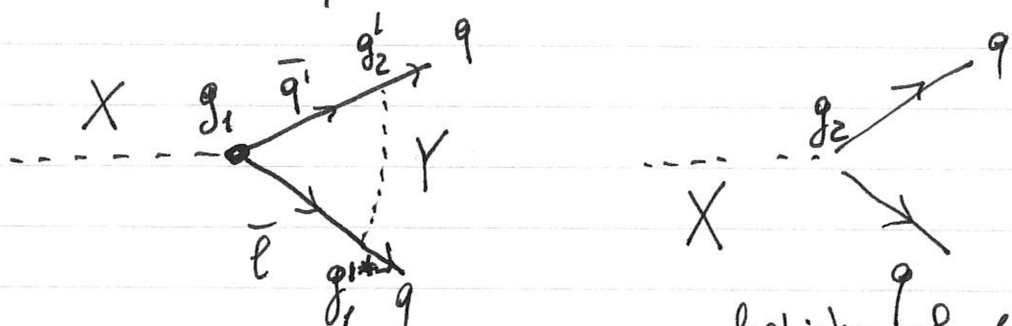
$\Gamma(\bar{X} \to \bar{q}\bar{q}) \propto g^* g\, M_{\bar{X}} \cdot$ " " $\Big\}$ same kinem. factor

$M_X = M_{\bar{X}}$ (CPT)

but $\Gamma(X \to qq)$ and $\Gamma(\bar{X} \to \bar{q}\bar{q})$ can $\boxed{\text{differ}}$

going to the LOOP LEVEL

ex: 1-loop level



loop integral complex number

$$\Gamma^{X \to qq}_{\text{1-loop + tree}} = \text{const.} \left| g_2 + I\, g_1 g_1'^* g_2' \right|^2$$

$$\left|\sum \text{amplitude}\right|^2 = \left|\sum \text{tree level amplit.} + \text{one-loop ampl.}\right|^2$$

$$\Gamma^{\bar{X} \to \bar{q}\bar{q}}_{\text{tree + 1loop}} = \text{const.} \left| g_2^* + I^*\, g_1^* g_1' g_2'^* \right|^2$$

$$\varepsilon - \bar{\varepsilon} \propto \frac{\Gamma(X \to qq) - \Gamma(\bar{X} \to \bar{q}\bar{q})}{\Gamma_{tot}} =$$

$$= \frac{2\,\mathrm{Im}\, I \;\; \mathrm{Im}\left(g_2 g_1^* g_1' g_2'^*\right)}{|g_1|^2 + |g_2|^2}$$

taking all g_1, g_2 of the same order ($g_1 \sim g_2 \sim g$) and assuming large uncorrelated phases of g_1 and g_2

$$\Rightarrow \quad \varepsilon - \bar{\varepsilon} \sim \frac{g^2}{4\pi} f\left(\frac{m_X}{m_Y}\right)$$

$$\left(\text{if } X = Y \text{ or } m_X = m_Y \Rightarrow \varepsilon - \bar{\varepsilon} = 0\right)$$

$f\left(\frac{m_X}{m_Y}\right) \sim O(1)$ for $m_X \gtrsim m_Y$

and $f(m_X/m_Y)$ decreases for $\frac{m_X}{m_Y}$ decreasing

$$\Rightarrow \varepsilon - \bar{\varepsilon} < \frac{g^2}{4\pi} \quad \to \frac{g^2}{4\pi} \gtrsim 10^{-8}$$

out-of-equilibrium condition : $M_X \gtrsim \overset{g^2/4\pi}{\alpha}\, \underset{\to\, 10^{18}\,\mathrm{GeV}}{M_{Pl}^*}$

if α has to be $> 10^{-8}$ to ensure ΔB large enough

$\Rightarrow M_X > 10^{10}$ GeV, i.e. X has to be a superheavy boson → to implement this baryogenesis scheme ⇒ need a reheating temp. of the Universe quite high ($T_{RH} > 10^{10}$ GeV) to have around the X bosons

Is ΔB preserved after it's generation from X decays at high T ($T \gg M_W$)?
→ one could argue that since
B is conserved by $SU(3)_c \times SU(2)_L \times U(1)_Y$ interactions
and since $G_{GUT} \xrightarrow{M_{GUT}} SU(3)_c \times SU(2) \times U(1)$
↳ where $\cancel{B}$ ↳ where B is conserved (??) see later
then there is no problem to preserve a ΔB generated $\sim M_{GUT}$ by superheavy X-boson decays occurring when we enter in a phase of the Universe where B is conserved
but is B really conserved in the SM?

ELECTROWEAK BARYOGENESIS

$\mathcal{L}_{class}$ → invariant under $q \to e^{i\alpha} q$ for any q

→ $U(1)_B$ $U(1)$ global Baryon-number symmetry

→ invariant also under $l_i \to e^{i\beta_i} l_i$ for $m_{\nu_i} = 0$

$i = 1,2,3$ ← generation number

$l_1 \to \nu_e \quad e$
$l_2 \to \nu_\mu \quad \mu$
$l_3 \to \nu_\tau \quad \tau$

→ $U(1)_{L_e}$, $U(1)_{L_\mu}$, $U(1)_{L_\tau}$ global Flavour ($i = 1,2,3$) Lepton-numbers symmetries

$\mathcal{L}^{SM}$ invariant under $U(1)_B$ and $\underbrace{U(1)_{L_e}, U(1)_{L_\mu}, U(1)_{L_\tau}}_{\text{if } m_\nu = 0}$

or just $U(1)_L$ if $m_\nu \neq 0$ with $L = L_e + L_\mu + L_\tau$

but j_μ^B and $j_\mu^{L_i}$ are anomalous at the quantum level:

$$\partial_\mu j^{\mu B} = 3 \frac{g_2^2}{32\pi^2} W^{\mu\nu a} \tilde{W}^a_{\mu\nu}$$

$$\partial_\mu j^{\mu L_i} = \frac{g_2^2}{32\pi^2} W^{\mu\nu a} \tilde{W}^a_{\mu\nu} \qquad i = 1,2,3$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\, \varepsilon^{abc} W^b_\mu W^c_\nu$$

↳ FIELD STRENGTH of the $SU(2)_L$ gauge field

$$\tilde{W}^a_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\lambda\rho} W^{\lambda\rho a}$$ dual tensor

[in general $\quad \partial_\mu j^\mu_A = \partial_\mu \left(j^\mu_R - j^\mu_L \right) = \frac{g^2}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$

↳ axial current

$\bar{\psi} \gamma^\mu \gamma^5 \psi$

for $SU(2)_L \rightarrow$ no j^μ_R $\quad q_L$ interacts with W_i

q_R does NOT interact with W_i

" $SU(3)_c \rightarrow q_L, q_R$

gluons interact equally with q_L and q_R

$\rightarrow$ B conserved in STRONG interactions

on the contrary for $SU(2)_L$:

$$\Delta B = B(t_f) - B(t_i) = \int_{t_i}^{t_f} dt \int d^3\vec{x}\, \partial^\mu j^B_\mu =$$

$$= 3 \int_{t_i}^{t_f} \frac{g_2^2}{32\pi^2} W^{\mu\nu a} \tilde{W}^a_{\mu\nu} \, d^4x$$

$t_i \rightarrow$ initial time; $\quad t_f \rightarrow$ final time

$$\Delta L_i = L_i(t_f) - L(t_i) = \int_{t_i}^{t_f} \frac{g_2^2}{32\pi^2} W^{\mu\nu a} \tilde{W}^a_{\mu\nu} \, d^4x$$

B, L_i not conserved

if the surface term in $\int_\Omega d^4x \; F\tilde{F} =$

$= \int dS_\mu K_\mu$ does not vanish

$\downarrow K_\mu : \quad F_{\mu\nu} \tilde{F}^{\mu\nu} = \partial^\mu K_\mu$

for abelian symm: $\int dS_\mu K_\mu \longrightarrow 0$

$\Rightarrow F\tilde{F}$ does not influence physical quantities

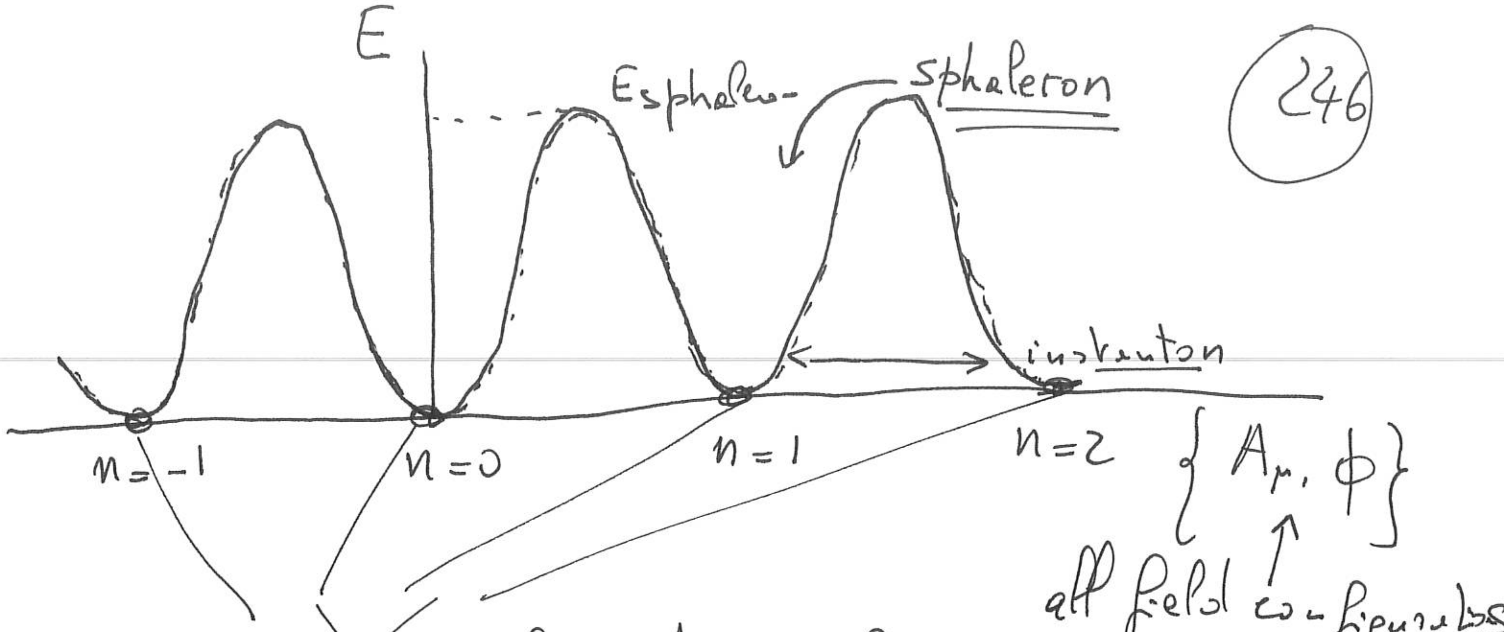
for non-abelian symm.: $\int dS_\mu K_\mu \not\rightarrow 0$

$$\frac{g_2^2}{16\pi^2} \int d^4x \; \mathrm{tr}\left(F_{\mu\nu} \tilde{F}^{\mu\nu}\right) = \frac{g_2^2}{32\pi^2} \int dS_\mu K^\mu = \nu$$

$$\nu = - \frac{i g_2^3}{24\pi^2} \int dS_\mu \; \varepsilon_{\mu\nu\sigma\tau} \; \mathrm{tr}\left[A_\nu A_\sigma A_\tau\right]$$

$\downarrow$

winding number or Chern-Simon number



pure gauge configurations of $E=0$
in which the fields A_μ, ϕ have different
topologically distinct vacua

from t_i to t_f if $\Delta\nu = 1$ $\rightarrow$ fields evolve with topological winding number n to the vacuum
with " " " $n+1$

$\Delta\nu = 1 \Rightarrow$

$$\Delta L_e = \Delta L_\mu = \Delta L_\tau = \frac{1}{3}\Delta L = \frac{1}{3}\Delta B$$

$$\Rightarrow \underline{\Delta B = \Delta L} \Rightarrow \boxed{B-L} \text{ IS CONSERVED!}$$

adding electric and colour charges conservation

$$\Delta\nu = 1 \rightarrow |0\rangle \rightarrow |u_L u_L d_L e_L^- + c_L c_L s_L \mu_L^- + t_L t_L b_L \tau_L^-|$$

p-decay possible!

Height of the energy barrier:

$$E_{sph} \sim \frac{M_W}{g_2^2}$$

→ exact computation

$$E_{sph} = \frac{2 M_W}{\alpha_W} f\left(\frac{M_H}{M_W}\right)$$

↳ $f\left(\frac{m_H}{m_W}\right) = 2.4$ for $m_H = 125\,\text{GeV}$

At $T=0$ ⇒ transmission through energy barrier only possible via QUANTUM TUNNELING:

$\Gamma \propto e^{-4\pi/\alpha_W}$ → $\Gamma \sim 10^{-165}$

→ tunneling probability

⇒ $\cancel{B}$ practically 0 at $T=0$

but at $T \neq 0$ ← not only quantum but also THERMAL fluctuations

$$\Gamma_{sphal} \sim \cancel{M_W(T)}\; T^4 \exp - \left(\frac{M_W(T)}{T\,\alpha_W}\right)$$

→ prob. per unit time per " volume

but at $T > \langle\phi\rangle = v \sim \mathcal{O}(100\,\text{GeV})$

→ $M_W(T) = 0$ (unbroken phase $v=0$)

at high T $(T \gg M_W)$:

$$\underline{\Gamma_{sphal} \sim (\alpha_W T)^4}$$

no more exponential suppression

electroweak $\cancel{B}$ processes are in thermal equilibrium

$\frac{\Gamma_{sphal}}{T^3}$ (rate per particle $n \sim T^3$)

$\hookrightarrow > H(T) = \frac{T^2}{M_{Pl}^*}$

$\rightarrow T \lesssim \underline{10^{12}\ GeV}$

$\Rightarrow$ $T \sim 100\,GeV$ < $\underline{\text{sphaleron processes in equilibrium}}$ < $\underline{T \sim 10^{12}\,GeV}$

$\underbrace{B-L, \quad L_i - L_j}$ $\quad i,j = e, \mu, \tau$

$\hookrightarrow$ conserved numbers in the range $10^2 - 10^{12}\ GeV$

but $\cancel{B}, \cancel{L_i}$ violated

$\rightarrow$ possible to create $\Delta B \neq 0$ but are all the Sackharov cond. respected?

Problems for a successful Electroweak Baryogenesis from the 2 Sakharov conditions

- CP violation
- out-of-equilibrium condition

The CP violation in the SM

→ CKM mixing matrix arising from the mismatch in the "rotations" performed to diagonalize the up-quark and down-quark mass matrices.

CKM 3×3 unitary matrix → 3 angles + 1 phase → CP violation

if we had 2 fermionic generations instead of 3 → $V_{CKM} \xrightarrow[\text{2 generations}]{} \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$

$\theta_c \equiv$ Cabibbo angle → no phase is present

→ no CP violation. 3 generations is the MINIMAL number of generations to have CP$\neq$!

The CP violation in the SM is TOO SMALL to generate the observed B asymmetry in the Universe

$$\Rightarrow A_{CP} = (m_t^2 - m_c^2)\cdot(m_t^2 - m_u^2)\cdot(m_c^2 - m_u^2)\cdot(m_b^2 - m_s^2)\cdot(m_b^2 - m_d^2)\cdot J$$

Jarlskog determinant

J = Jarlskog ~~determinant~~ invariant =

$$= \mathrm{Im}\left(V_{us} V_{cb} V_{ub}^* V_{cs}^*\right) \simeq \sin\theta_{12} \sin\theta_{23} \sin\theta_{13} \sin\delta_{CKM} \simeq 3\times 10^{-5}$$

The relevant CP violating quantity at the moment of the electroweak phase transition is:

$$\delta_{CP} \sim \frac{A_c}{T_c^{12}} \sim \underline{\underline{10^{-20}}}$$

← too small to account for $n_B/n_\gamma \sim 10^{-10}$

$T_c \sim O(100\ \mathrm{GeV})$

to have <u>enough</u> <u>CP violation</u> for electroweak baryogenesis → go <u>beyond the SM</u>, for instance introducing <u>two</u> Higgs doublets instead of one

→ the new mechanisms of CP violation required by electroweak baryogenesis likely to show up experimentally in the <u>Electric Dipole Moments</u>

present exp. bounds on EDMs

$$\left\{ \begin{array}{l} \underline{d_e} < 10^{-27}\, e\,cm \sim 10^{-12}\, e\, GeV^{-1} \\ \text{electron} \\ \underline{d_n} < 10^{-26}\, e\,cm \sim 10^{-11}\, e\, GeV^{-1} \\ \text{neutron} \end{array} \right.$$

e = electric charge

In the SM : the electroweak phase transition is NOT FIRST ORDER

(in fact it is likely that in the SM we don't have a "phase transition" but rather a <u>smooth</u> <u>crossover</u>

(at $T \sim 100$ GeV $\rightarrow$ Hubble time

$$t_U \sim H^{-1}\Big|_{T=100\,\text{GeV}} = \frac{M_{Pl}^*}{T^2}\Big|_{T=100\,\text{GeV}}$$

$$\sim 10^{-10}\,s$$

time scale for elw. interactions between particles in the plasma:

$$t_{\text{interaction}} \sim \frac{1}{\alpha_W T} \sim 1\,\text{GeV}^{-1} \sim 10^{-24}\,s.$$

$\rightarrow$ if there is no strong phase transition (i.e. 1st order phase transition) $\Longrightarrow$ no <u>strong deviation from thermal equilibrium</u>

LEPTOGENESIS

Baryogenesis ($\Delta B \to n_B/n_\gamma \sim 10^{-10}$) through

Leptogenesis $\Delta L \neq 0$

$\Rightarrow$ correlation between $\begin{cases} - \text{smallness of } m_\nu \\ - B \text{ asymmetry} \end{cases}$

see-saw mechanism to obtain small m_ν

$\downarrow$

heavy ν_R Majorana mass $\to$ decays of

" " $\Rightarrow \Delta L \neq 0 \Rightarrow$ sphaleron processes

conserve $B-L$, i.e. $\Delta B = \Delta L$, hence

LEPTON ASYMMETRY converted BARYON ASYMM.

m_ν

no ν_R introduced

$h\, \nu_L^i \Delta^{ij} \nu_L^j$ i,j $SU(2)_L$ indices

new Higgs $SU(2)_L$ triplet

$\downarrow$

$\langle \Delta \rangle = v' \ll v \sim O(100\,\text{GeV})$

$\downarrow$

$\to \Delta L = 2 \to$ only ν_L m_ν Majorana mass

introduction of ν_R

$h\, \bar{\nu}_L \phi \nu_R$

usual Higgs doublet

$\downarrow$

$M \nu_R \nu_R$ enforce global $U(1)_{Lept.}$ to avoid it

$\to \nu$ Dirac particle

$m_\nu = h \langle \phi \rangle$ $h \ll 1$

if global $U(1)_{\text{Left}}$ is imposed (i.e. if ν_R is present $U(1)_{\text{Lepton}}$ is no longer an SM

ACCIDENTAL SYMMETRY $\rightarrow$ $M \nu_R \nu_R$ allowed by space-time and GAUGE symmetries of the SM

$\rightarrow$ possible problems of GLOBAL symm. to be preserved by gravity)

if global $U(1)_L$ is not imposed $\Rightarrow M \nu_R \nu_R$ is allowed M not related to M_W, possibly $M \gg M_W$ M corresponding to the energy scale where some new (gauge) symmetry is present ~~with~~ and under which ν_R is no longer a singlet

ex. $\rightarrow$ $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$ or $SO(10)$

At the SM $\mathcal{L}_{SM}$ $\xrightarrow[\text{new additional terms}]{}$ $\Delta\mathcal{L}_{\nu_R} = M_\alpha N_{\alpha R} N_{\alpha R} + y_{\alpha\beta} L_\alpha \phi N_{\beta R}$

$\alpha, \beta = 1,2,3$ generation indices

if $N_R = \nu_R$ more precisely the Majorana mass term is $M \bar{N}_R^c N_R$

N^c charge conjugate of N

$$\nu = \begin{pmatrix} \nu_L \\ N_R \end{pmatrix} \begin{matrix} \leftarrow \text{2-compon. spinor} \\ \leftarrow \text{2-compon. spinor} \end{matrix}$$

$\rightarrow \mathcal{L}^{\text{Major}}_{N_R} \Rightarrow N_R^T \, i\sigma_2 \, N_R \leftarrow N^a N^b \varepsilon_{ab}$

a, b ~~SU~~ Lorentz indices
Lor. group ~ ~~SO(2)$_L$×SO~~ $SU(2)_L \times SU(2)_R$

$\mathcal{L}^{\text{Dirac}}_{\nu-N} \Rightarrow \nu_L^+ \, N_R$

$(\nu_{1L} \; \nu_{2L})_R \quad \begin{pmatrix} N_{1R} \\ N_{2R} \end{pmatrix}$

3 R-H ν $\Rightarrow$ N_1 lightest RH ν

$N_1 \rightarrow l_L + \phi$ $\qquad$ $N_1 \rightarrow l_L^c + \phi$

N does not have a definite L number

$MNN \rightarrow \Delta L = 2 \rightarrow L$-number symmetry is <u>not</u> a symmetry of $\mathcal{L}$ $\rightarrow$ N coincides with N^c $\rightarrow$ hence $\underset{\uparrow L=+1}{N \rightarrow l}$ but also

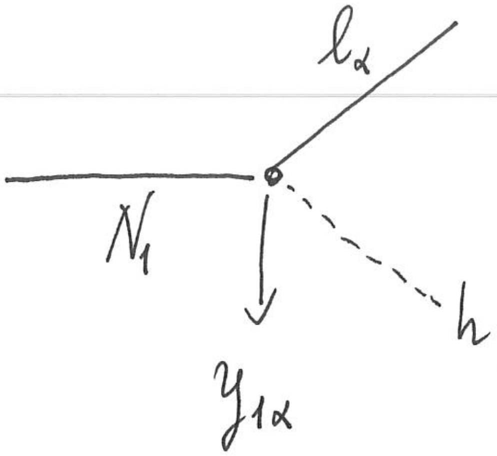
$\underset{\nwarrow L=-1}{N \rightarrow l^c}$

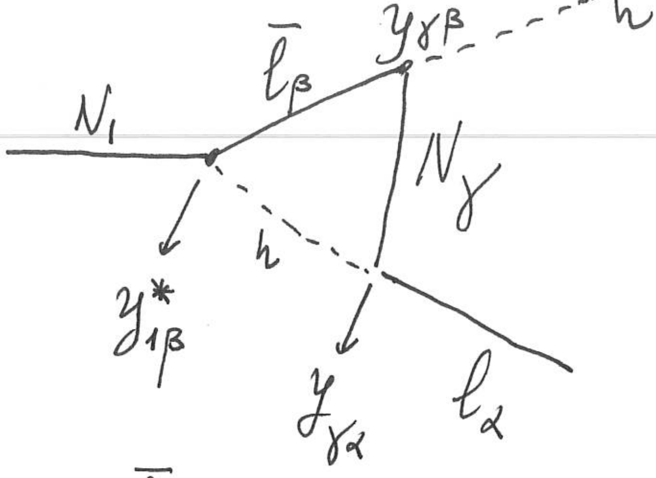
ΔL generated if $BR(N \rightarrow l\phi) \neq BR(N \rightarrow \bar{l}\phi)$

and $BR(N \rightarrow l\phi) \neq$ ~~BR~~ $BR(N \rightarrow \bar{l}\phi)$ can occur at 1-loop level if CP is violated)

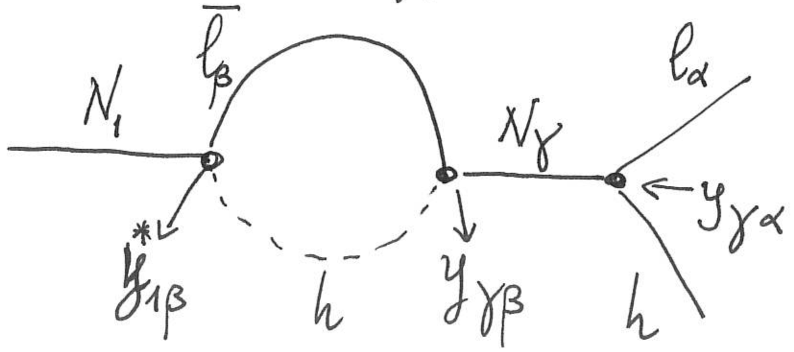
INTERFERENCE OF

TREE LEVEL with ONE LOOP





Yukawa couplings
α, β, γ generation indices



$$\Gamma(N_1 \to \ell h) = \text{const} \sum_\alpha \left| y_{1\alpha} + \sum_{\beta,\gamma} f\left(\frac{M_1}{M_\gamma}\right) y^*_{1\beta}\, y_{\gamma\alpha}\, y_{\gamma\beta} \right|^2$$

one-loop partial width $N_1 \to \ell h$

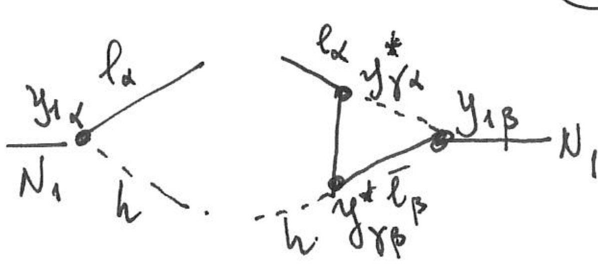
tree-level contrib.

one-loop contrib.

$M_1 \to m_{N_1}$ $\quad M_\gamma \to m_{N_\gamma}$ (masses of SM particles ~ 0)

$M_\alpha \gg O(100\,\text{GeV})$ see later

$f\left(\frac{M_1}{M_\gamma}\right)$ comes from the sum of the loop integrals



$y_{1\alpha}\, y^*_{\gamma\alpha}\, y^*_{\gamma\beta}\, y_{1\beta}$

if $\gamma = 1 \to$ $\underline{y_{1\alpha} y^*_{1\alpha}}$ $\underline{y^*_{1\beta} y_{1\beta}}$ real

$$\Gamma(N_1 \to \bar{\ell} h) = \text{const} \sum_\alpha \left| y^*_{1\alpha} + \sum_{\beta,\gamma} f\left(\frac{M_1}{M_\gamma}\right) y_{1\beta}\, y^*_{\gamma\alpha}\, y^*_{\gamma\beta} \right|^2$$

$$\Delta L \equiv \frac{\Gamma(N_1 \to \ell h) - \Gamma(N_1 \to \bar{\ell} h)}{\Gamma_{tot}}$$

$$= - \sum_{\gamma=2,3} \mathrm{Im} f\left(\frac{M_1}{M_\gamma}\right) \frac{\mathrm{Im}\left(\sum_\alpha y_{1\alpha} y^*_{\gamma\alpha}\right)^2}{\sum_\alpha |y_{1\alpha}|^2}$$

interference term

tree-level, one-loop level

$$\mathrm{Im} f\left(\frac{M_1}{M_\gamma}\right) = \frac{1}{8\pi}\, g\left(\frac{M_1}{M_\gamma}\right)$$

$$\Delta L = -\frac{1}{8\pi} \sum_{\gamma=2,3} g\left(\frac{M_1}{M_\gamma}\right) \cdot \frac{\mathrm{Im}\left(\sum_\alpha y_{1\alpha} y^*_{\gamma\alpha}\right)^2}{\sum_\alpha |y_{1\alpha}|^2}$$

if $M_1 << M_{2,3} \to g\left(\frac{M_1}{M_\gamma}\right) \sim -\frac{3}{2} \frac{M_1}{M_\gamma}$

$$\Delta L \simeq \frac{3}{16\pi} M_1 \frac{1}{\sum_\alpha |y_{1\alpha}|^2} \sum_{\alpha,\beta,\gamma} \mathrm{Im}\left[y_{1\alpha} y_{1\beta} \left(y^*_{\gamma\alpha} \frac{1}{M_\gamma} y^*_{\gamma\beta}\right)\right]$$

In the see saw mechanism the

NEUTRINO MASS MATRIX is:

$$\begin{array}{c|c} & \begin{array}{cc} \nu_L & \nu_R \end{array} \\ \hline \begin{array}{c} \nu_L \\ \nu_R \end{array} & \left(\begin{array}{c|c} 0 & m_D \\ \hline m_D & M \end{array}\right) \end{array}$$

← 3x3 block matrix

$$\Rightarrow m = - m_D M^{-1} m_D^T \; ; \quad M = \begin{pmatrix} M_1 & & 0 \\ & M_2 & \\ 0 & & M_3 \end{pmatrix}$$

$$m_{D_{\alpha\beta}} = \frac{y_{\alpha\beta} v}{\sqrt{2}}$$

$\alpha, \beta = 1,2,3$ generation indices

$$m_{\alpha\beta} = -\frac{v^2}{2} \sum_\gamma y_{\gamma\alpha} \frac{1}{M_\gamma} y_{\gamma\beta}$$

combinations of the Yukawa couplings y different from the combination entering ΔL:

$$\sum_{\alpha,\beta,\gamma} \mathrm{Im} \left[y_{1\alpha} y_{1\beta} \left(y^*_{\gamma\alpha} \frac{1}{M_\gamma} y^*_{\gamma\beta} \right) \right.$$

if we change the basis for ℓ_α:

$$y \to y U \qquad (U \text{ unitary transformation})$$

m (neutrino mass matrix) changes

but $(y y^\dagger)_{1\gamma} (y y^\dagger)_{1\gamma} \longrightarrow$ does not

$$\underset{U U^\dagger}{(y y^\dagger)_{1\gamma}} \; \underset{U U^\dagger}{(y y^\dagger)_{1\gamma}}$$

change, i.e. ΔL insensitive to the leptonic change of basis

$\rightarrow$ no DIRECT connection between the PMNS (Pontecorvo - Maki - Nakagawa - Sakata) matrix describing neutrino (light ν) mixings (PMNS is in a sense the analogue of the V_{CKM} mixing matrix in the quark sector) with the combination of couplings entering leptogenesis, but still information on ΔL from measurements of ν OSCILLATIONS and, in the future, CP violation in the ν sector (LONG BASELINE ν oscill. exps.)

$$\frac{n_L}{s} \sim 10^{-2}\, \Delta L$$

to obtain an effection lepton-antilepton asymmetry $\left(n_L - n_{\bar{L}} \neq 0 \right)$ we must verify the <u>OUT-OF-EQUILIBRIUM</u> Sacharov condition:

$$\boxed{\Gamma_{tot}(M_1) < H(T = M_1)}$$

$$\downarrow$$

$$\frac{M_1}{8\pi} \sum_\alpha |y_{1\alpha}|^2 < \frac{T^2}{M_{Pe}^*} \Bigg|_{T=M_1}$$

$$\sum_\alpha |y_{1\alpha}|^2 \frac{1}{2M_1} < \frac{4\pi}{M_{Pe}^*}$$

cfr. with $m_{\alpha\beta} = -\frac{v^2}{2} \sum_\gamma y_{\gamma\alpha} \frac{1}{M_\gamma} y_{\gamma\beta}$

$$\rightarrow \sum_\alpha |y_{1\alpha}|^2 \frac{1}{2M_1} v^2 < \frac{4\pi}{M_{Pe}^*} v^2$$

Σ of the N_1 contribution to the ν mass matrix but N_1 is assumed to be the lightest of the 3 N_i particles $\rightarrow$ if all $y_{\alpha\beta}$ Yukawa couplings are of the same order then:

$$m_\nu < \frac{4\pi}{M_{PL}^*} v^2 \sim \frac{10}{10^{18}\,GeV} 10^4\,GeV^2 \sim 10^{-13}\,GeV \sim 10^{-4}\,eV$$

all the 3 ν masses

m_ν would be too small to account ~~of~~ for $\Delta m^2_{\nu_{atm}}$ and $\Delta m^2_{\nu_{sol}}$ $\Rightarrow$ there must be a hierarchy of values of the y ν Yukawa couplings (not surprising: for instance, for the Yukawa couplings of the charged leptons $m_e \sim 0.5\,MeV$ $m_\mu \sim 100\,MeV$ $m_\tau \sim 1.7\,GeV$ $y_e/y_\mu \sim \frac{1}{200}$ $y_e/y_\tau \sim \frac{0.5}{1700}$)

Hence, assuming that the $y_{\alpha\beta}$ of ν mass matrix are getting values compatible with the experimentally observed ν oscillation, let's

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come back to the ~~a~~ implementation of the out-of-equilibrium condition

$\Gamma_{tot}(M_1) < H(T=M_1)$:

$$\left(\sum_\alpha |y_{1\alpha}|^2\right)\frac{M_1}{8\pi} < \frac{M_1^2}{M_{Pe}^*}$$

$$\rightarrow \quad M_1 > \frac{M_{Pe}^*}{8\pi}\sum_\alpha |y_{1\alpha}|^2$$

express $\sum_\alpha |y_{1\alpha}|^2$ in terms of the ν mass matrix (to exploit our knowledge of $\Delta m_{\nu_{atm}}$, $\Delta m_{\nu_{sol}}$ ← only direct experimental information on $m_\nu \neq 0$)

going back to (page 257):

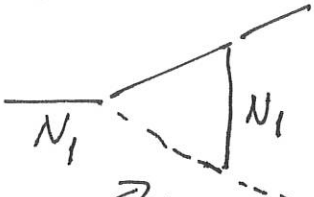
$$\Delta L = \frac{3}{16\pi} M_1 \frac{1}{\sum_\alpha |y_{1\alpha}|^2}\sum_{\alpha,\beta,\gamma}\mathrm{Im}\left[y_{1\alpha}y_{1\beta}\left(y^*_{\gamma\alpha}\frac{1}{M_\gamma}y^*_{\gamma\beta}\right)\right]$$

$$= \frac{3M_1}{8\pi}\frac{1}{\sum_\alpha |y_{1\alpha}|^2}\sum_{\alpha\beta}\mathrm{Im}\left[y_{1\alpha}y_{1\beta}\sum_{\gamma=2,3}\frac{m^\gamma_{\alpha\beta}}{v^2}\right]$$

$$m^{\gamma}_{\alpha\beta} = y_{\alpha\gamma}\,\frac{v^2}{2M_\gamma}\,y_{\gamma\beta}$$

contribution of N_γ to the ν mass matrix

($\sum_{\gamma=2,3}$ because if $\gamma = 1$ i.e.



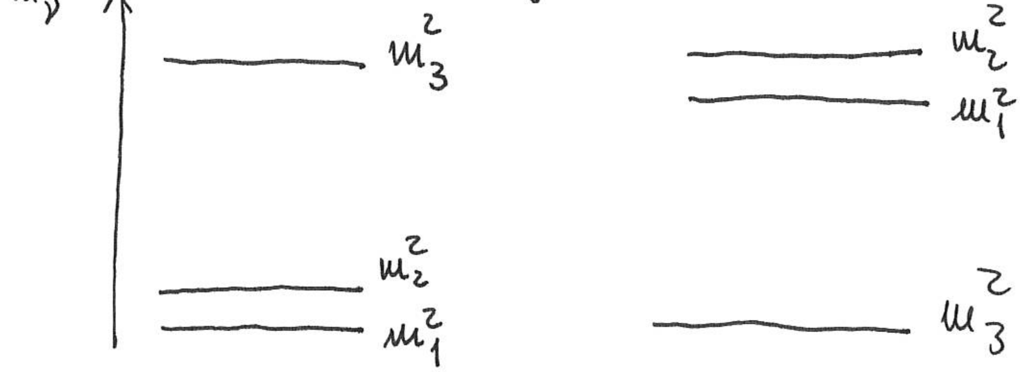
there is no $\mathrm{Im} \neq 0$)

experimentally :

$$\Delta m^2_{21} \equiv \Delta m^2_{solar} = m_2^2 - m_1^2 \simeq (7\text{-}8) \times 10^{-5}\, eV^2$$

$$\Delta m^2_{32} \equiv \Delta m^2_{atm} = m_3^2 - m_2^2 \simeq (2\text{-}3) \times 10^{-3}\, eV^2$$

Two different ν mass orderings (or ν mass hierarchies) are possible (experimentally we don't know yet which hierarchy choice is made by Nature):



NORMAL INVERSE

ORDERING (HIERARCHY) of ν MASSES

in any case at least one of the ν masses must be $m \geq m_{atm} \equiv \sqrt{\Delta m^2_{atm}} \simeq 5 \times 10^{-2}\, eV$

and another ν mass must be
$$m \geq m_{sol} \equiv \sqrt{\Delta m^2_{sol}} \simeq 9 \times 10^{-3}\, eV$$

(minimal possibility with normal hierarchy
$m_1 \ll m_{sol}$, $m_2 \simeq m_{sol}$, $m_3 \simeq m_{atm}$)

unless the 3 ν are strongly mass degenerate

i.e. 

$\leftarrow$ with average $m_\nu \sim O(eV)$ but conflict with cosmological bound on $\sum m_\nu$

if one assumes direct ν mass hierarchy: $m_{\nu_1} < m_{atm} \sim 5 \times 10^{-2}\, eV$

$$\Delta L = \frac{3 M_1}{8 \pi v^2} \frac{1}{\sum_\alpha |y_{1\alpha}|^2} \sum_{\alpha\beta} y_{1\alpha} y_{1\beta} \sum_{\gamma = 2,3} m^\gamma_{\alpha\beta}$$

$$\lesssim \frac{3 M_1}{8 \pi v^2}\, m_{atm}$$

we require to have $\frac{n_L}{s} \sim \frac{n_B}{s} \sim 10^{-10}$

$$\Delta L \sim 10^2 \frac{n_L}{s} \sim 10^{-8} \implies M_1 \gtrsim \frac{8\pi}{3} \frac{v^2}{m_{atm}} \cdot 10^{-8} \simeq$$

$$M_1 \gtrsim \frac{8\pi}{3} \frac{6\cdot 10^4\,\text{GeV}^2}{5\times 10^{-11}\,\text{GeV}} \cdot 10^{-8} \gtrsim 10^8\,\text{GeV}$$

→ the temperature of the Universe (important for reheating condition) must be $> 10^8$–10^9 GeV

COMPATIBILITY OF ν MASSES WITH LEPTOGENESIS

$$\Delta L \sim \frac{3}{16\pi} M_1 \frac{1}{\sum_\alpha |y_{1\alpha}|^2} \sum_{\alpha\beta\gamma} \mathrm{Im}\left[y_{1\alpha} y_{1\beta} \left(y^*_{\gamma\alpha} \frac{1}{M_\gamma} y^*_{\gamma\beta} \right) \right]$$

$\Delta L \to \sim 10^{-8}$ to have an efficient leptogenesis

$M_1 \to$ mass of the lightest ν_R

$$m_\nu \;\xi \;\Rightarrow\; m_D M^{-1} m_D^T = \frac{v^2}{2} \sum_\gamma y_{\gamma\alpha} \frac{1}{M_\gamma} y_{\gamma\beta}$$

$m_\nu \to$ mass matrix of the ordinary (light) $\nu_{i=1,2,3}$

$$m_{D\alpha\beta} = \frac{y_{\alpha\beta} v}{\sqrt{2}}$$

Sum of N_1 contributions to m_ν:

$$\tilde{m}_1 = \sum_\alpha \frac{|y_{1\alpha}|^2}{2M_1} \cdot v^2$$

(from out-of-equilibrium condition $\tilde{m}_1 < \frac{4\pi}{M^*_{Pl}} \cdot v^2 \sim 10^{-4}\,\text{eV}$)

$$\Delta L = \frac{3}{16\pi} M_1 \frac{1}{\sum_\alpha |y_{1\alpha}|^2} \sum_{\alpha\beta} \mathrm{Im}\left(y_{1\alpha} y_{1\beta} \sum_{\gamma=2,3} y^*_{\alpha\gamma} \frac{1}{M_\gamma} y^*_{\gamma\beta} \right)$$

$$= \frac{3}{8\pi} \frac{M_1}{v^2} \frac{1}{\sum_\alpha |y_{1\alpha}|^2} \sum_{\alpha\beta} \mathrm{Im}\left(y_{1\alpha} y_{1\beta} \; m^{\gamma\,*}_{\alpha\beta} \right)$$

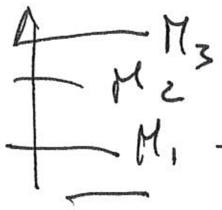
$$m^\gamma_{\alpha\beta} = y_{\alpha\gamma} \frac{v^2}{2M_\gamma} y_{\gamma\beta}$$

if one takes the minimal possibility

$m_1 < m_{sol}$ $\quad m_2 = m_{sol}$ $\quad m_3 = m_{atm}$

$$\rightarrow \Delta L < \frac{3 M_1}{8\pi v^2} m_{atm} \qquad M_1 > 10^8 \,\mathrm{GeV}$$

if $\quad m_3 \propto \frac{1}{M_1} \quad m_2 \propto \frac{1}{M_2} \quad m_1 \sim \frac{1}{M_3}$



$\longrightarrow \quad m_3 > m_2 > m_1$

$$\tilde{m}_1 \sim m_{atm} \sim 5 \times 10^{-2}\,\mathrm{eV}$$

but from the out-of-equilibrium condition

$$\tilde{m}_1 < \frac{4\pi}{M^*_{PL}} v^2 \sim 10^{-4}\,\mathrm{eV} \quad \Longrightarrow \quad \text{suppression of lepton asymmetry}$$

to get the correct $\frac{n_L}{s} \sim 10^{-10}$ we need to start with

$\Delta L > 10^{-8}$

ex. $\Delta L \gtrsim 10^{-5}$ push $M_1 > 10^{11}-10^{12}$ GeV

$\rightarrow$ if the Universe reheats up to $T > 10^{11}-10^{12}$ GeV

OK with Leptogenesis

Notice that large M_1 implies that to have an efficiently leptogenesis ν masses need to be sufficiently small:

$$m_{\nu_3} \sim y^2 \frac{v^2}{M_1} \sim y^2 \frac{(10^4-10^5 \text{ GeV})^2}{10^{12} \text{ GeV}}$$

$\sim y^2\, 10^{-8}$ GeV $\longrightarrow$ m_ν must be $\lesssim O(1\text{eV})$

and indeed for $m_\nu < O(1\text{eV}) \rightarrow H\big|_{T\sim M_1}$ is not far from $\Gamma_{tot}(N_1 \rightarrow \ell h)$

$\boxed{\text{the exp. value for } m_\nu \text{ from } \nu \text{ oscillations is COMPATIBLE with a viable LEPTOGENESIS}}$