Coalescing compact binaries: the theory

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THE THEORY

Is needed to compute waveform templates for characterizing the source (GWs are detected routinely...but WHAT is detected?)

Theory is needed to study the 2-body problem in General Relativity (dynamics & gravitational wave emission)

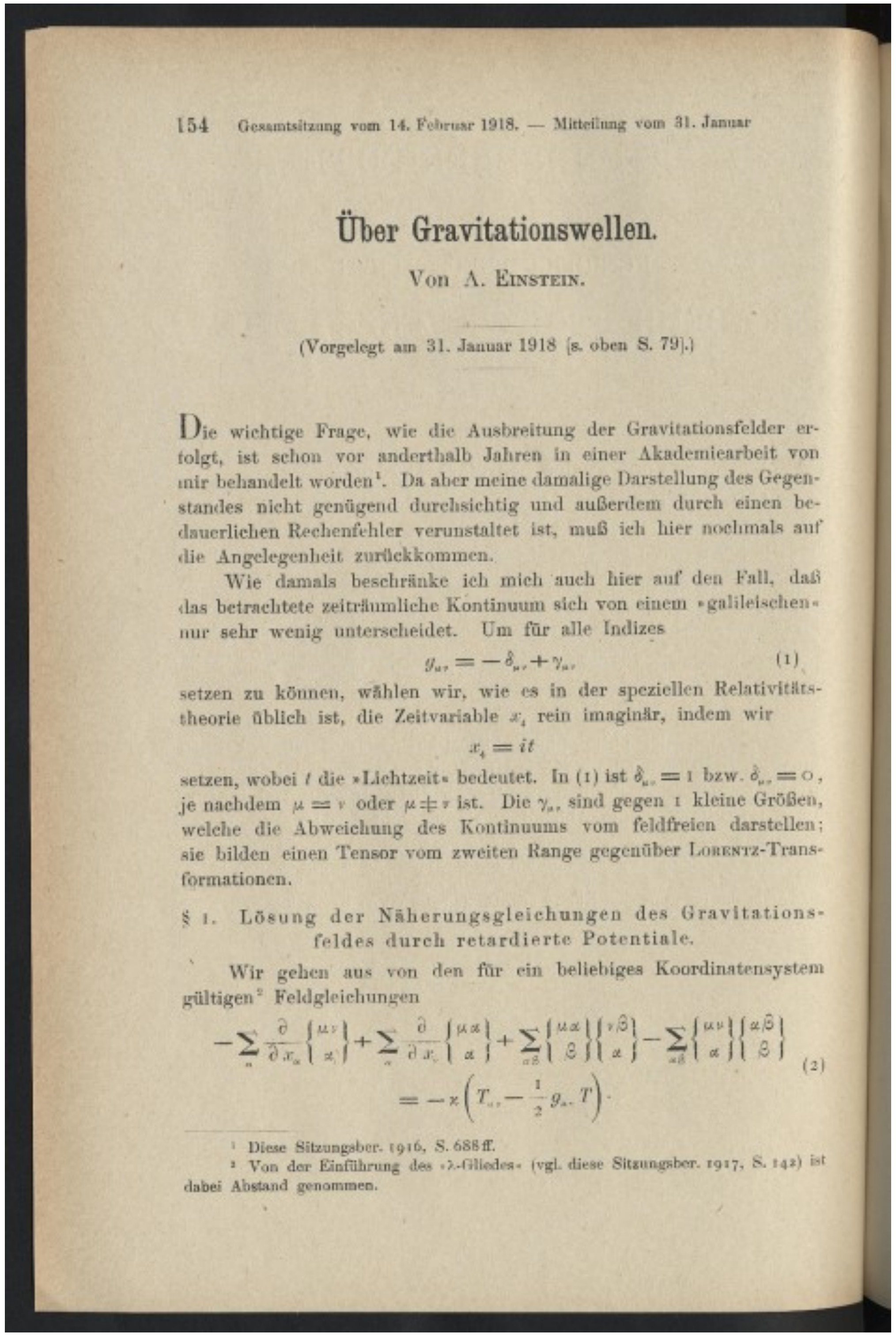
Theory: SYNERGY between

Analytical and Numerical General Relativity

(AR/NR)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Gravitational waves - 1918


154 Gesamtsitzung vom 14. Februar 1918. — Mitteilung vom 31. Januar

Über Gravitationswellen.

Von A. Einstein.

(Vorgelegt am 31. Januar 1918 [s. oben S. 79].)

Die wichtige Frage, wie die Ausbreitung der Gravitationsfelder erfolgt, ist schon vor anderthalb Jahren in einer Akademiearbeit von mir behandelt worden[1]. Da aber meine damalige Darstellung des Gegenstandes nicht genügend durchsichtig und außerdem durch einen bedauerlichen Rechenfehler verunstaltet ist, muß ich hier nochmals auf die Angelegenheit zurückkommen.

Wie damals beschränke ich mich auch hier auf den Fall, daß das betrachtete zeiträumliche Kontinuum sich von einem »galileischen« nur sehr wenig unterscheidet. Um für alle Indizes

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \qquad (1)$$

setzen zu können, wählen wir, wie es in der speziellen Relativitätstheorie üblich ist, die Zeitvariable x_4 rein imaginär, indem wir

$$x_4 = it$$

setzen, wobei t die »Lichtzeit« bedeutet. In (1) ist $\delta_{\mu\nu} = 1$ bzw. $\delta_{\mu\nu} = 0$, je nachdem $\mu = \nu$ oder $\mu \neq \nu$ ist. Die $\gamma_{\mu\nu}$ sind gegen 1 kleine Größen, welche die Abweichung des Kontinuums vom feldfreien darstellen; sie bilden einen Tensor vom zweiten Range gegenüber Lorentz-Transformationen.

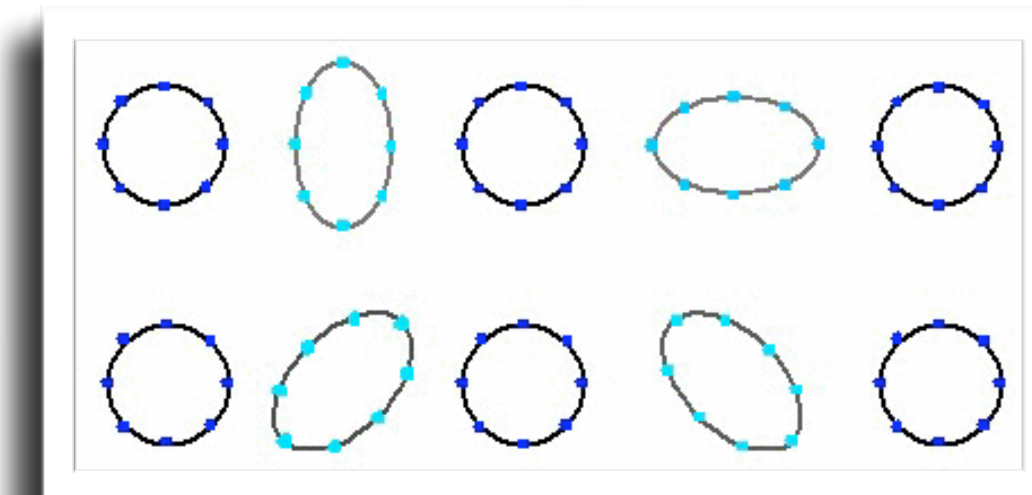
§ 1. Lösung der Näherungsgleichungen des Gravitationsfeldes durch retardierte Potentiale.

Wir gehen aus von den für ein beliebiges Koordinatensystem gültigen[2] Feldgleichungen

$$-\sum_{\alpha} \frac{\partial}{\partial x_\alpha} \begin{Bmatrix} \mu\nu \\ \alpha \end{Bmatrix} + \sum_{\alpha} \frac{\partial}{\partial x_\nu} \begin{Bmatrix} \mu\alpha \\ \alpha \end{Bmatrix} + \sum_{\alpha\beta} \begin{Bmatrix} \mu\alpha \\ \beta \end{Bmatrix} \begin{Bmatrix} \nu\beta \\ \alpha \end{Bmatrix} - \sum_{\alpha\beta} \begin{Bmatrix} \mu\nu \\ \alpha \end{Bmatrix} \begin{Bmatrix} \alpha\beta \\ \beta \end{Bmatrix} = -\varkappa \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right). \qquad (2)$$

[1] Diese Sitzungsber. 1916, S. 688 ff.
[2] Von der Einführung des »λ-Gliedes« (vgl. diese Sitzungsber. 1917, S. 142) ist dabei Abstand genommen.

Transverse and traceless
Speed of light
Two polarizations
Carry energy and momentum
“Deform” macroscopical objects



h_+

$h_\times$

$$\bar{h}_{ij}(t, r) = \frac{2G}{c^4 r} \ddot{I}_{ij}(t - r)$$

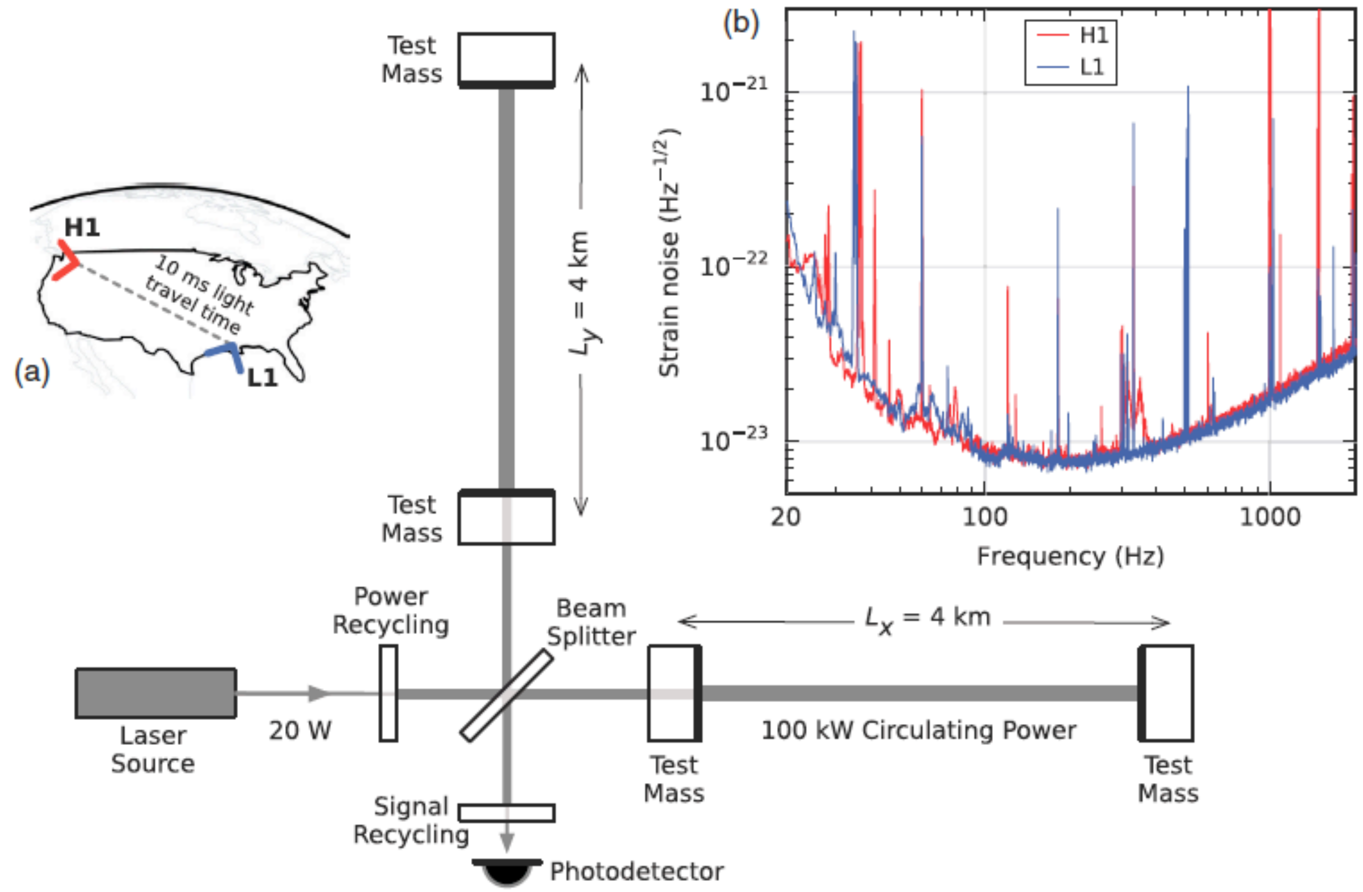
$$G = (6.67408 \pm 0.00031) \times 10^{-20}\, km^3/(kg\, s^2)$$

$$c = 299792.458\, km/s$$

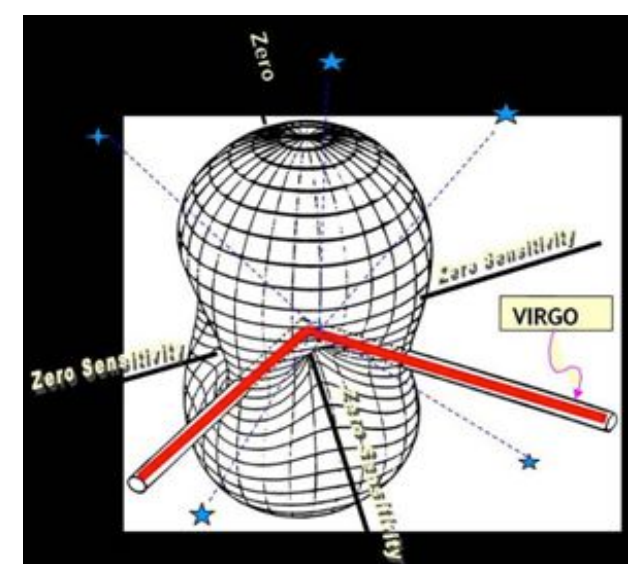
Measurable sources: extreme astrophysical events, black hole or neutron star binaries

$$\frac{2GM}{c^2 R} \approx 1$$

LIGO/Virgo interferometers

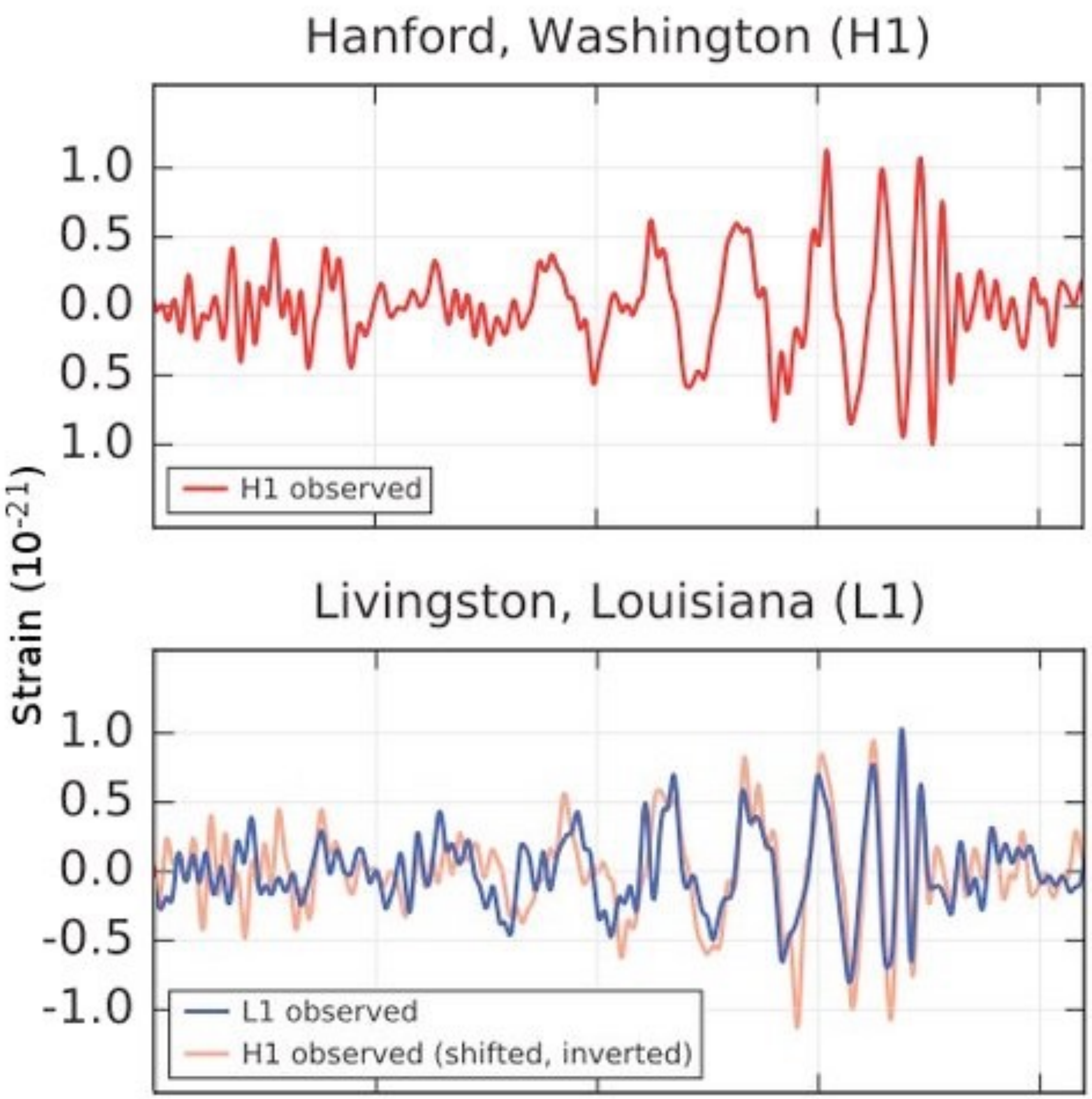


$$h = \frac{\Delta L}{L} \approx 10^{-21}$$



O3: April1st 2019-
KAGRA will join soon

GW150914



$$\text{strain} = \frac{\delta L}{L}$$

GW150914 parameters:

$$m_1 = 35.7 M_\odot$$
$$m_2 = 29.1 M_\odot$$
$$M_f = 61.8 M_\odot$$

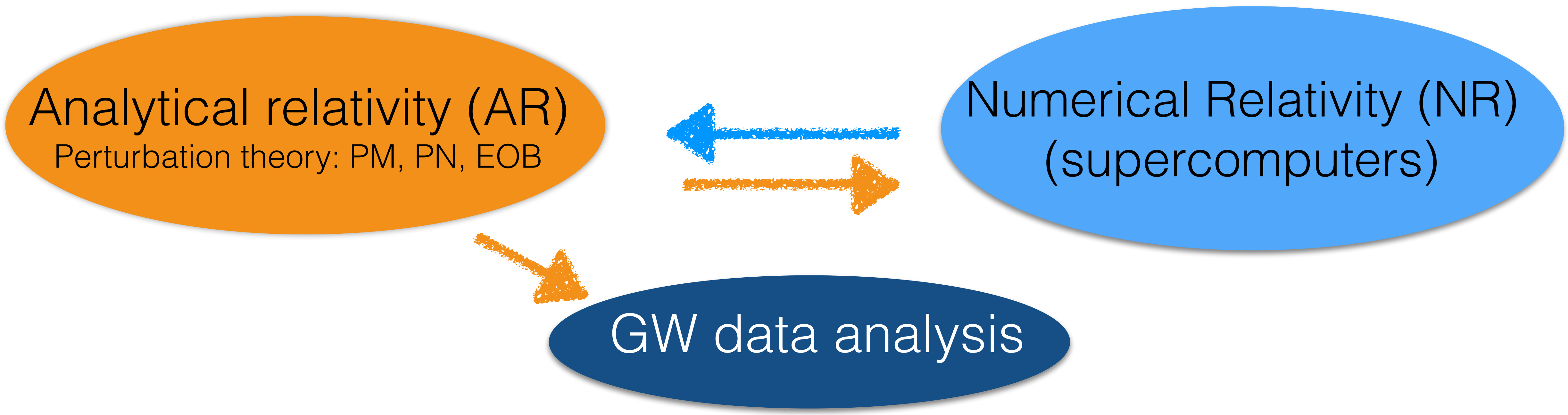
$$a_1 \equiv S_1/(m_1^2) = 0.31^{+0.48}_{-0.28}$$
$$a_2 \equiv S_2/(m_2^2) = 0.46^{+0.48}_{-0.42}$$
$$a_f \equiv \frac{J_f}{M_f^2} = 0.67$$
$$q \equiv \frac{m_1}{m_2} = 1.27$$

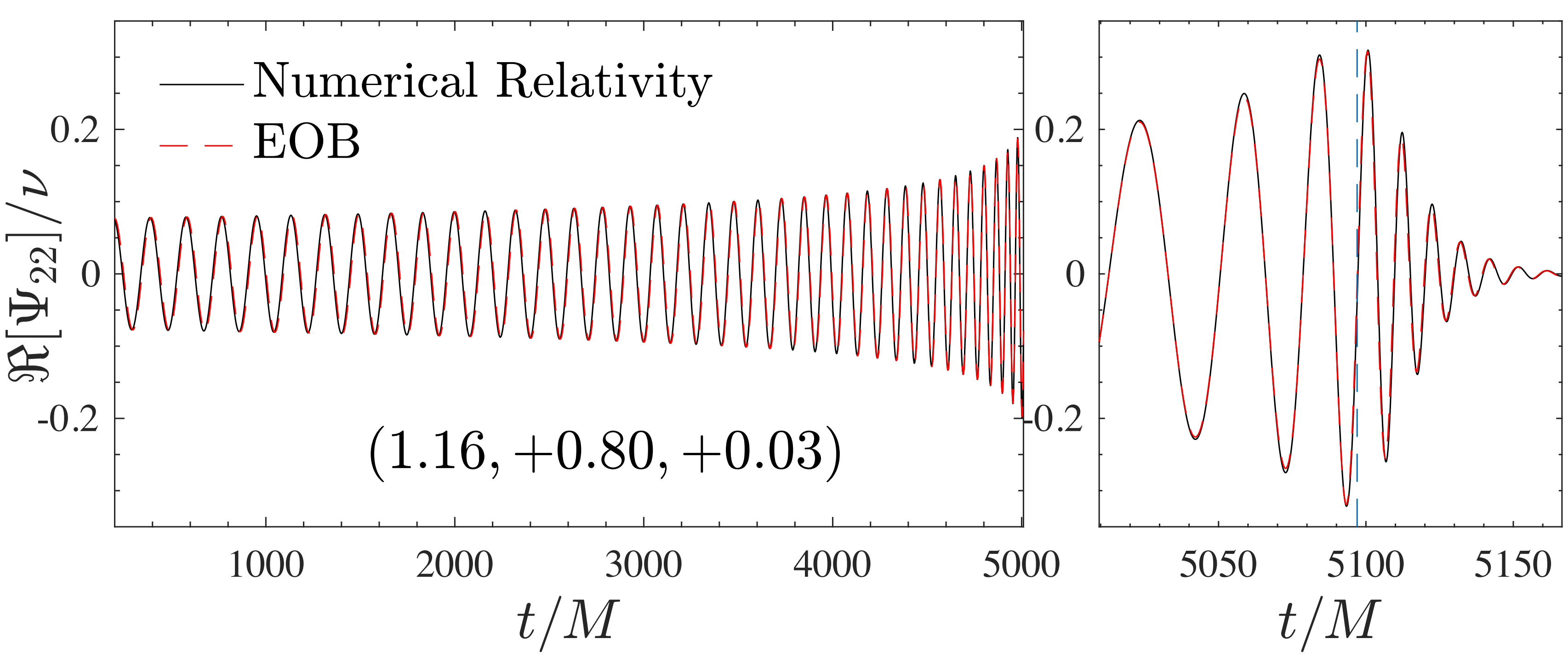
Symmetric mass ratio

$$\nu \equiv \frac{m_1 m_2}{(m_1 + m_2)^2} = 0.2466$$

THEORY for C_{ompact}B_{inary}C_{oalescence}

- Interface between Analytical & Numerical Relativity for GW data-analysis
- 2-body problem in General Relativity

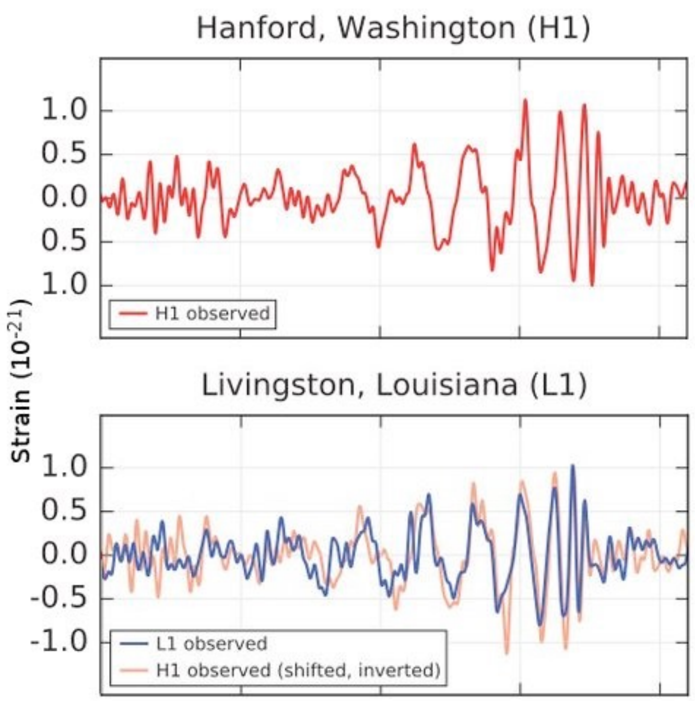




Challenges:

- physical completeness
- accuracy
- efficiency (AR vs NR)
- 10^7 templates needed for a single event

Why waveform templates?



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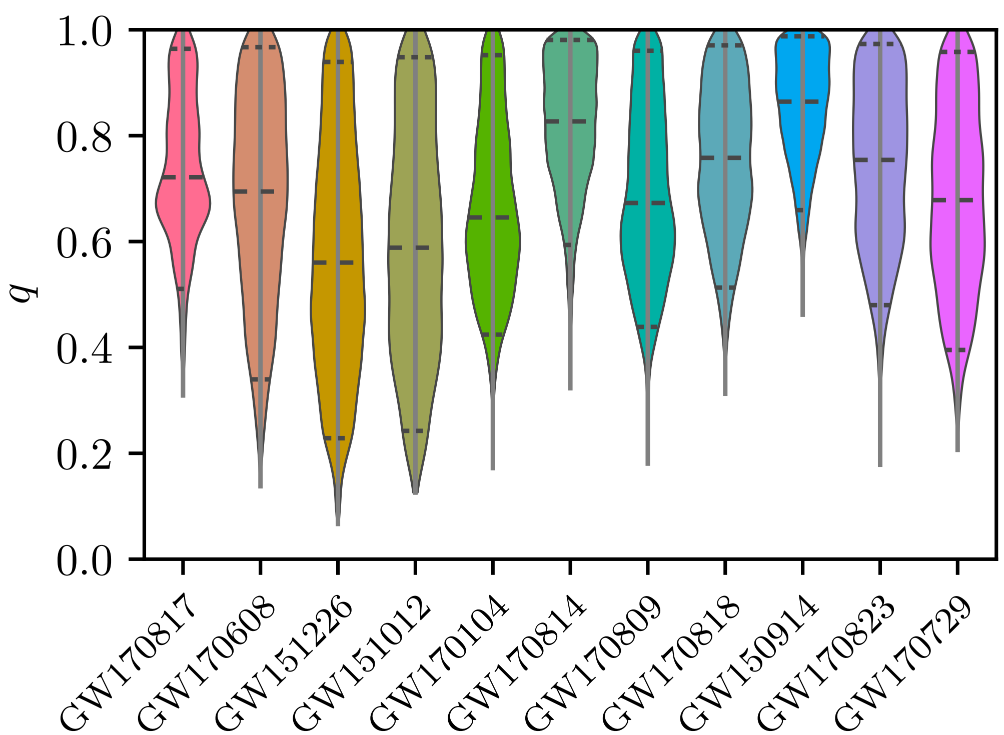
O2 events: GWTC-1: arXiv:1811.12907



Matched filtering: detection and parameter estimation

$$\langle output | h_{\rm template} \rangle = \int \frac{df}{S_n(f)} o(f) h^*_{\rm template}(f)$$

Analytical formalism:

theoretical understanding of the coalescence process

BINARY SYSTEMS: NEWTONIAN PRELIMINARIES

GWS FROM COMPACT BINARIES: BASICS

Newtonian binary systems in circular orbits: Kepler's law

$$GM = \Omega^2 R^3$$

$$\frac{v^2}{c^2} = \frac{GM}{c^2 R} = \left(\frac{GM\Omega}{c^3}\right)^{2/3}$$

$$M = m_1 + m_2$$

Einstein (1918) quadrupole formula: GW luminosity (energy flux)

$$P_{\rm gw} = \frac{dE_{\rm gw}}{dt} = \frac{32}{5}\frac{c^5}{G}\nu^2 x^5$$

$$x = \left(\frac{v}{c}\right)^2$$

$$\nu = \frac{\mu}{M} = \frac{m_1 m_2}{M^2}$$

GWS FROM COMPACT BINARIES: BASICS

$$E^{\text{orbital}} = E^{\text{kinetic}} + E^{\text{potential}} = -\frac{1}{2}\frac{m_1 m_2}{R} = -\frac{1}{2}\mu x$$

Balance argument

$$\frac{dE^{\text{orbital}}}{dt} = P_{GW} = \frac{dE_{\text{GW}}}{dt}$$

$$\omega_{22}^{\text{GW}} = 2\pi f_{22}^{\text{GW}} = 2\Omega^{\text{orbital}}$$

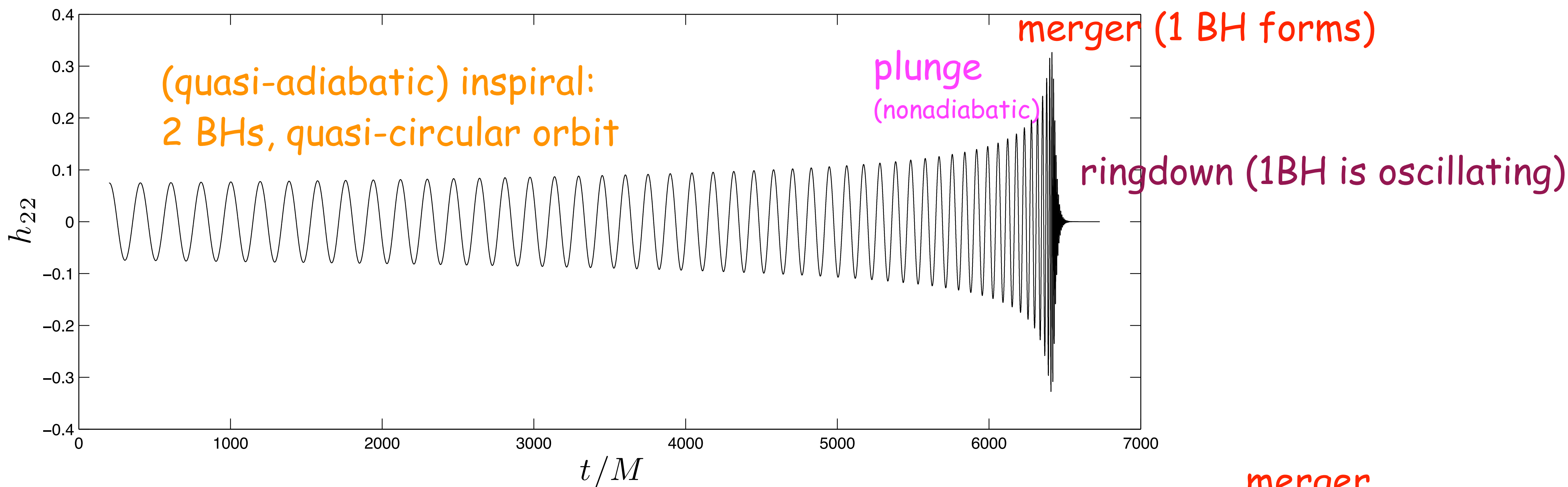
$$f_{GW}^{22} = \frac{1}{\pi}\left(\frac{5}{256\nu}\right)^{3/8}\left(\frac{1}{t - t_{\text{coalescence}}}\right)^{3/8}$$

MONOTONICALLY GROWING FREQUENCY: CHIRP!

BBHS: WAVEFORM OVERWIEV

$$h_+ - ih_\times = \frac{1}{r}\sum_{\ell m} h_{\ell m}\ {}_{-2}Y_{\ell m}(\theta,\phi)$$

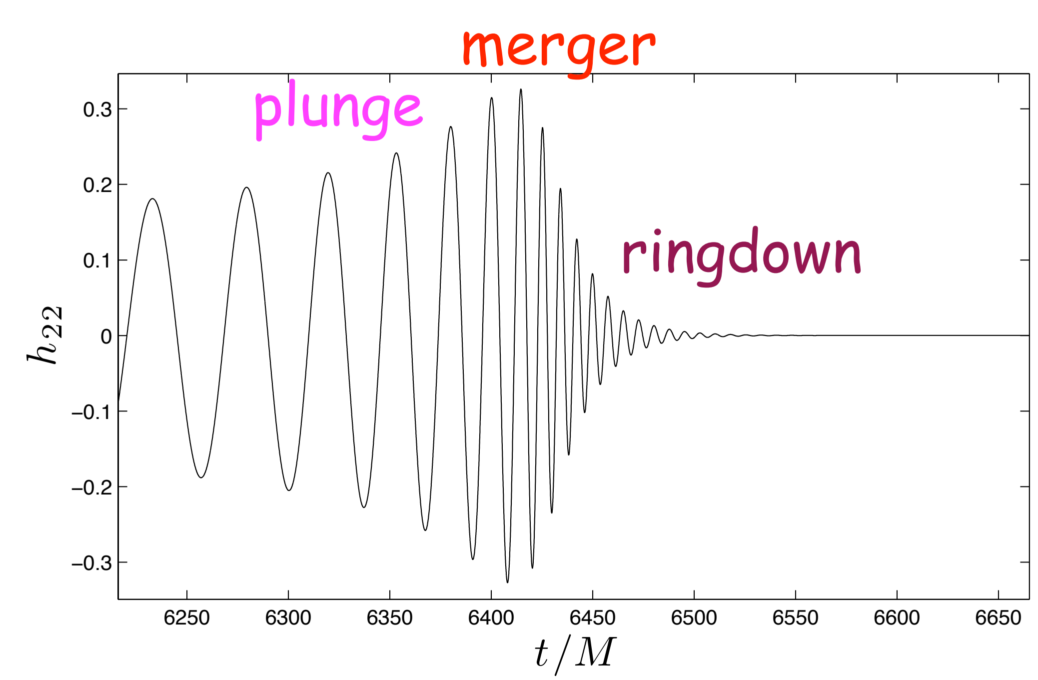
$$h\left(m_1, m_2, \vec{S}_1, \vec{S}_2\right)$$



e.g: equal-mass BBH, aligned-spins

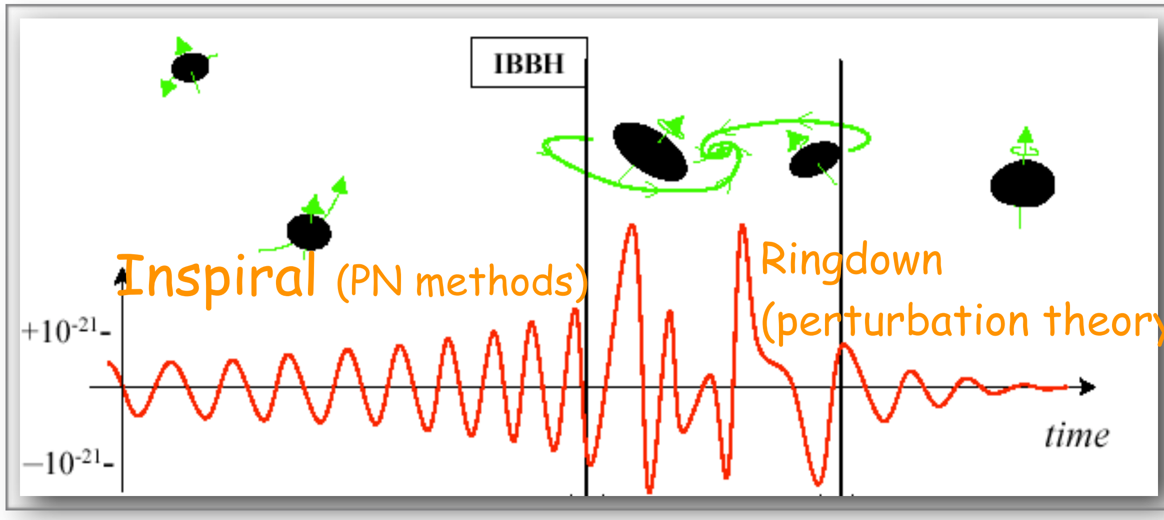
$$\chi_1 = \chi_2 = +0.98$$

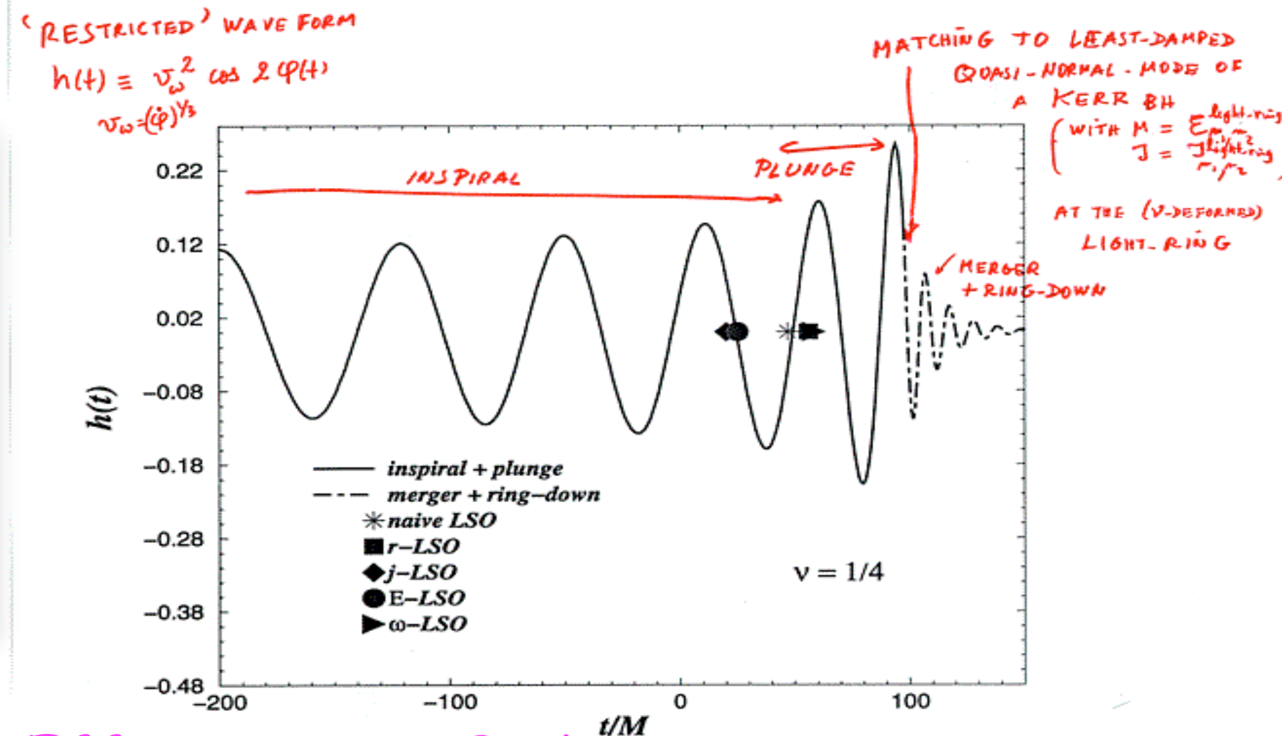
- SXS (Simulating eXtreme Spacetimes) collaboration
- www.blackholes.org
- Free catalog of waveforms (downloadable)



TEMPLATES FOR GWS FROM BBH COALESCENCE

Brady, Craighton & Thorne, 1998





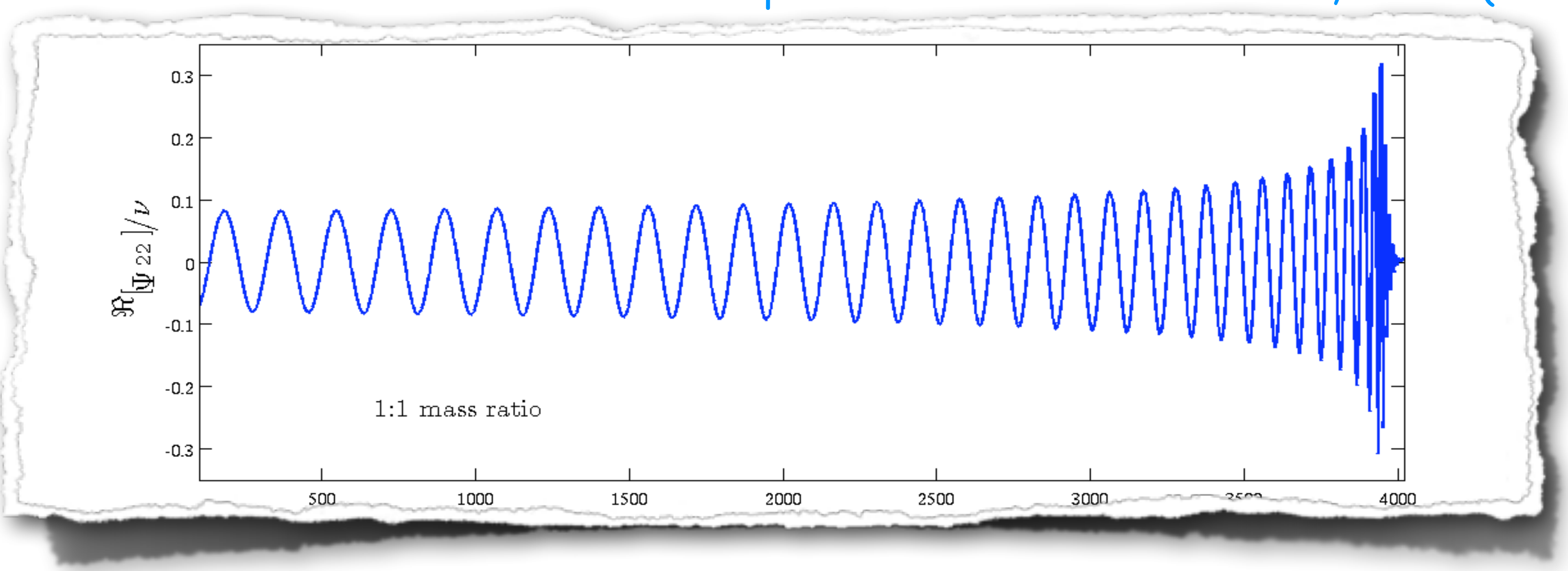
Merger:
Numerical Relativity

Effective-One-Body (Buonanno & Damour (2000)

PN-resummation (Damour,Iyer, Sathyaprakash (1998)

Numerical Relativity: >= 2005 (F. Pretorius, Campanelli et al., Baker et al.)

Most accurate data: Caltech-Cornell spectral code: M. Scheel et al., 2008 (SXS collaboration)



Spectral code

Extrapolation (radius & resolution)

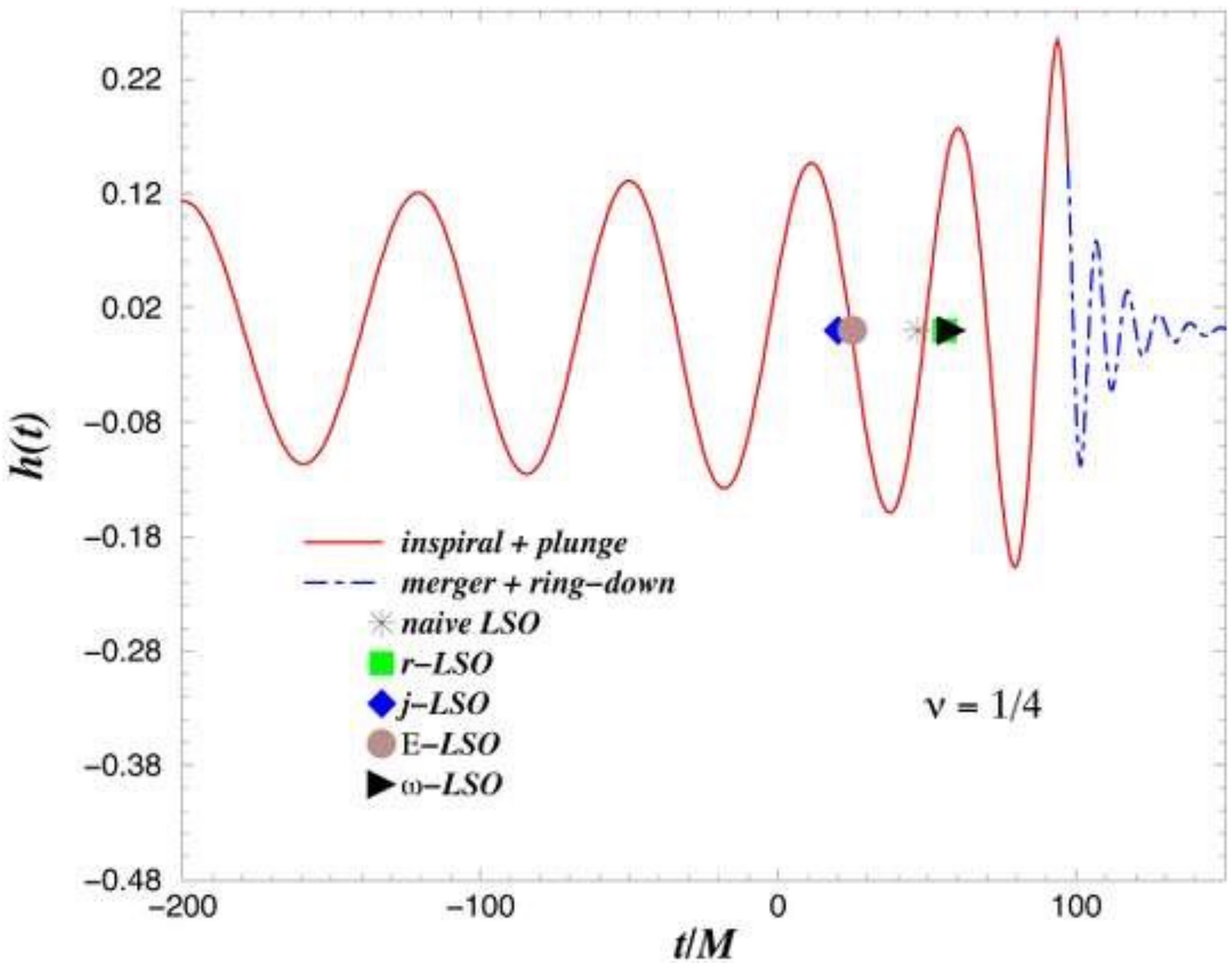
Phase error:

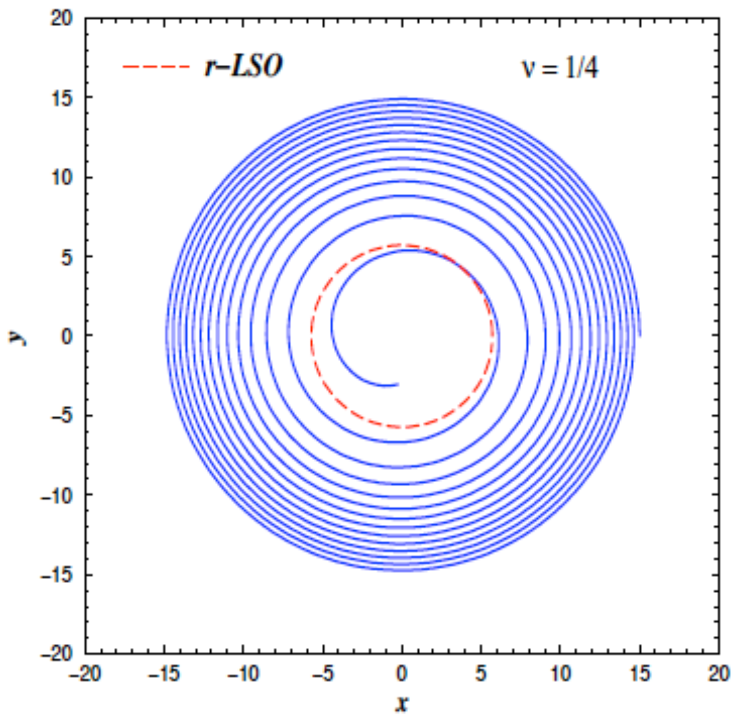
< 0.02 rad (inspiral)

<0.1 rad (ringdown)

EFFECTIVE ONE BODY (EOB): 2000

Numerical Relativity was not working (yet...)
EOB formalism was predictive, qualitatively and semi-quantitatively correct (10%)





A. Buonanno & T. Damour, PRD 59 (1999) 084006
A. Buonanno & T. Damour, PRD 62 (2000) 064015

- Blurred transition from inspiral to plunge
- Final black-hole mass
- Final black hole spin
- Complete waveform

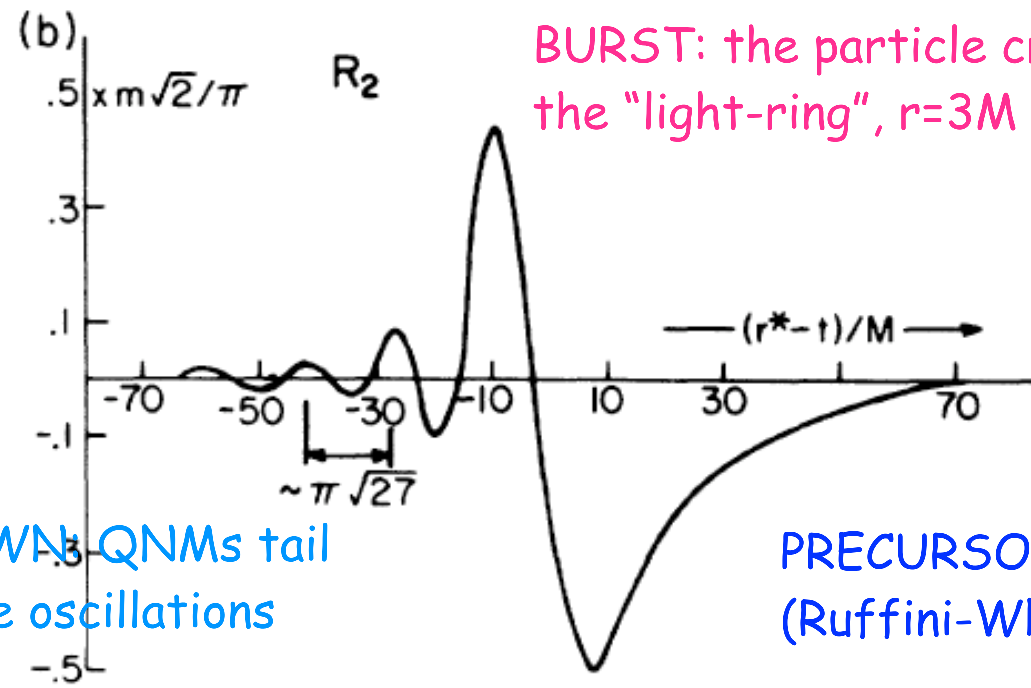
$$\nu = \frac{m_1 m_2}{(m_1+m_2)^2} = \frac{\mu}{M}$$

> 2005: Developing EOB & interfacing with NR
2 groups did (and are doing) it

- A.Buonanno et al. (AEI)
- T.Damour & AN + (>2005)

PRECURSOR-BURST-RINGDOWN STRUCTURE :1972

Davis, Ruffini & Tiomno: radial plunge of a test-particle onto a Schwarzschild black hole (Regge-Wheeler-Zerilli BH perturbation theory)



BURST: the particle crosses the "light-ring", r=3M

RINGDOWN: QNMs tail
Spacetime oscillations

PRECURSOR: Quadrupole formula
(Ruffini-Wheeler approximation)

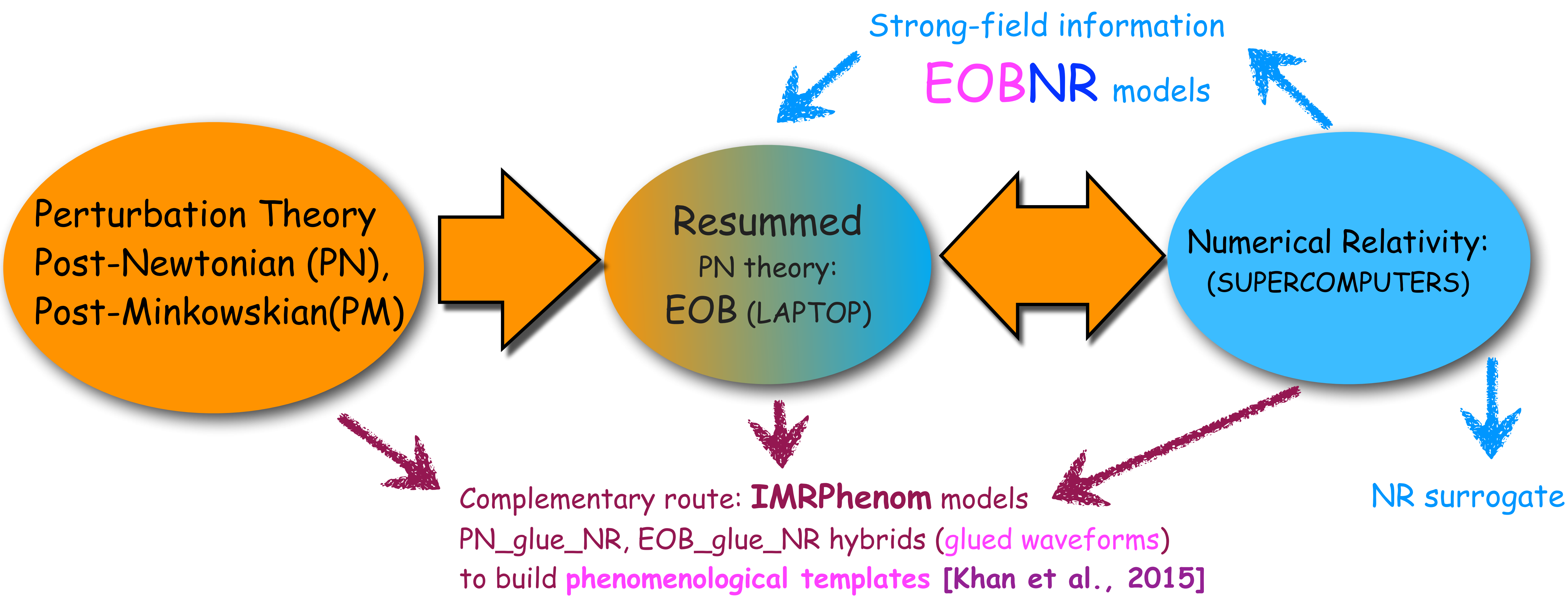
2-body problem in GR

Hamiltonian: conservative part of the dynamics

Radiation reaction: mechanical energy/angular momentum goes away in GWs and backreacts on the system.

The (closed) orbit CIRCULARIZES and SHRiNks with time

Waveform



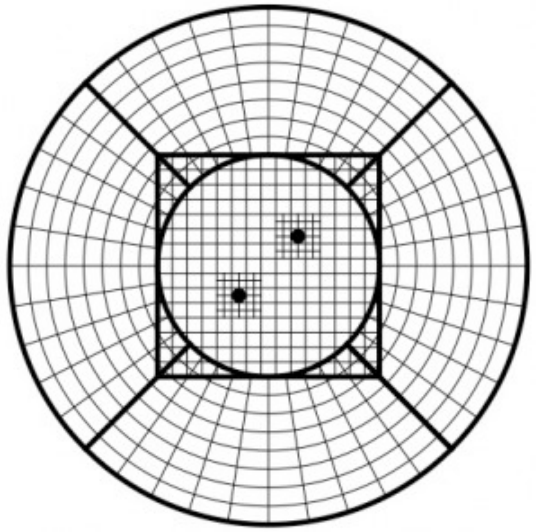
BBH & BNS COALESCENCE: NUMERICAL RELATIVITY

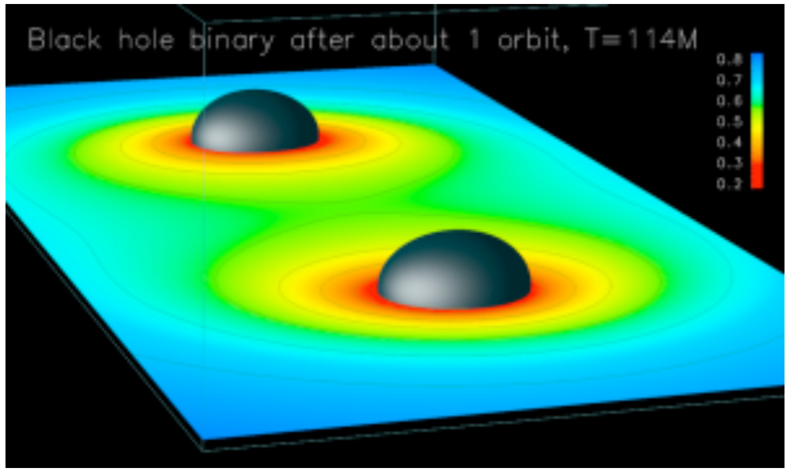
Numerical relativity is complicated & computationally expensive:

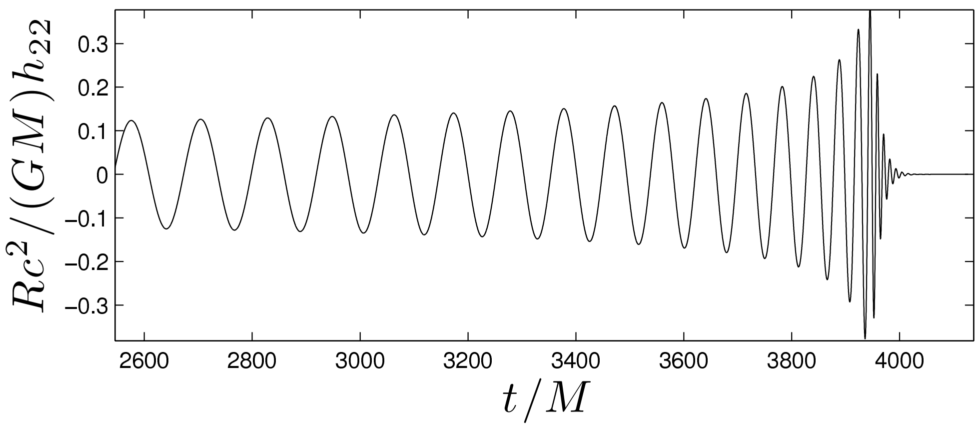
- Formulation of Einstein equations (BSSN, harmonic, Z4c,...)
- Setting up initial data (solution of the constraints)
- Gauge choice
- Numerical approach (finite-differencing (FD, e.g. Llama) vs spectral (SpEC,SXS))
- High-order FD operators
- Treatment of BH singularity (excision vs punctures)
- Wave extraction problem on finite-size grids (Cauchy-Characteristic vs extrapolation)
- Huge computational resources (mass-ratios 1:10; spin)
- Adaptive-mesh-refinement
- Error budget (convergence rates are far from clean...)
- For BNS: further complications due to GR-Hydrodynamics for matter
- Months of running/analysis to get one accurate waveform....

Multi-patch grid structure

(Llama FD code, Pollney & Reisswig)







A catalog of 171 high-quality binary black-hole simulations for gravitational-wave astronomy

[PRL 111 (2013) 241104]

Abdul H. Mroué,[1] Mark A. Scheel,[2] Béla Szilágyi,[2] Harald P. Pfeiffer,[1] Michael Boyle,[3] Daniel A. Hemberger,[3] Lawrence E. Kidder,[3] Geoffrey Lovelace,[4,2] Sergei Ossokine,[1,5] Nicholas W. Taylor,[2] Anıl Zenginoğlu,[2] Luisa T. Buchman,[2] Tony Chu,[1] Evan Foley,[4] Matthew Giesler,[4] Robert Owen,[6] and Saul A. Teukolsky[3]

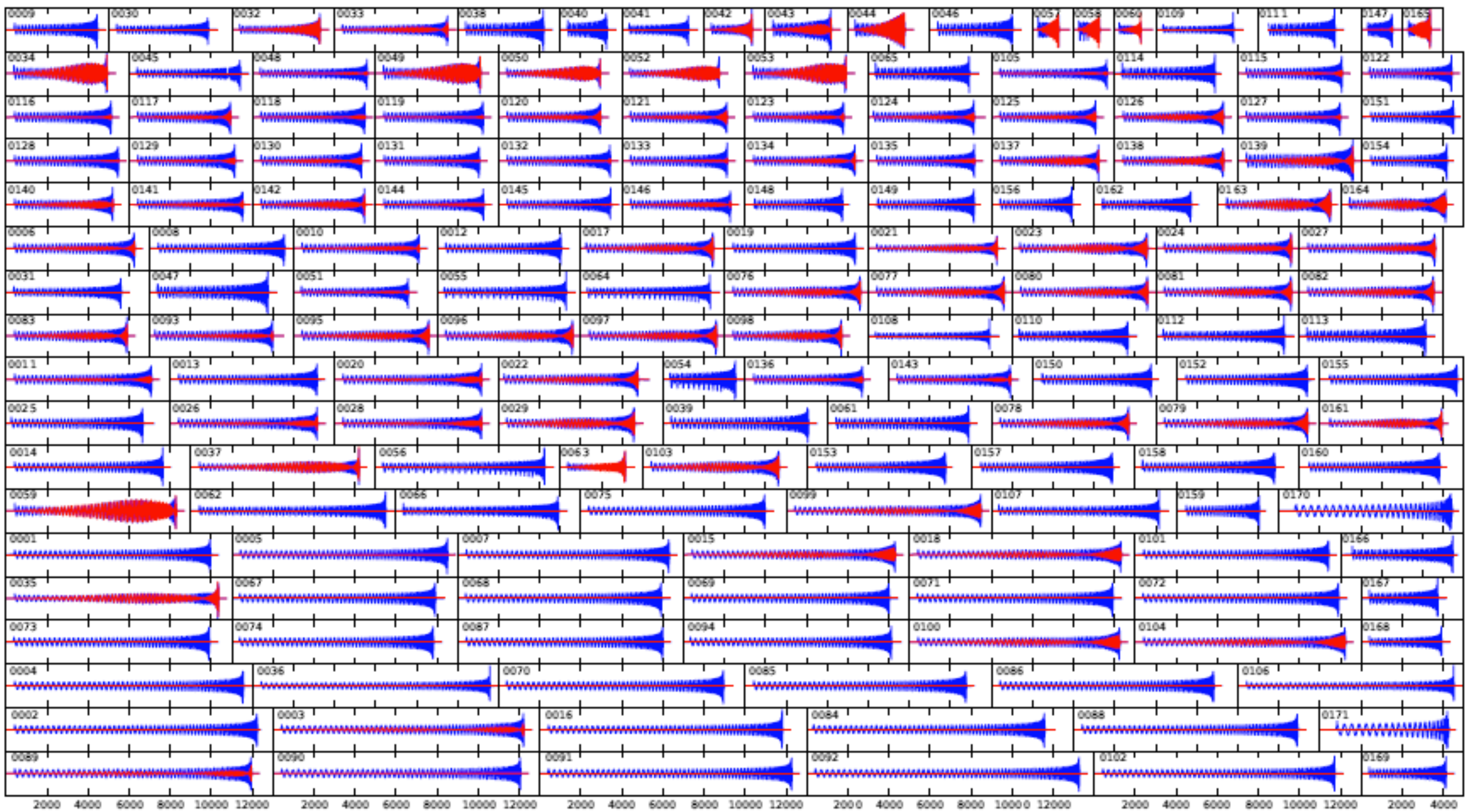


FIG. 3: Waveforms from all simulations in the catalog. Shown here are h_+ (blue) and $h_\times$ (red) in a sky direction parallel to the initial orbital plane of each simulation. All plots have the same horizontal scale, with each tick representing a time interval of $2000M$, where M is the total mass.

- www.blackholes.org

But (at least) 250.000 templates were used...

ANALYTICALLY: MOTION AND GW IN GR

Hamiltonian: conservative part of the dynamics

Radiation reaction: mechanical energy/angular momentum goes away in GWs and backreacts on the system.

The (closed) orbit CIRCULARIZES and SHRiNks with time

Waveform

General Relativity is NONLINEAR!

Post-Newtonian (PN) approximation: expansion in $\frac{v^2}{c^2}$

PROBLEM OF MOTION IN GENERAL RELATIVITY

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)\ ,\ h_{\mu\nu} \ll 1$$

$$h_{00} \sim h_{ij} \sim \frac{v^2}{c^2}\ ,\ h_{0i} \sim \frac{v^3}{c^3}\ ,\ \partial_0 h \sim \frac{v}{c}\partial_i h$$

Approximation methods

- post-Minkowskian (Einstein 1916)
- post-Newtonian (Droste 1916)
- Matching of asymptotic expansions: body zone/near zone/wave zone
- Numerical Relativity

One-chart versus Multi-chart approaches

Coupling between Einstein field equations and equations of motion

Strongly self-gravitating bodies: neutron stars or black holes

$$h_{\mu\nu}(x) \sim 1$$

Skeletonized: $T_{\mu\nu}$ point-masses ? delta-functions in GR

Multipolar Expansion

Need to go to very high-orders of approximation

QFT-like calculations

Use a "cocktail": PM, PN, MPM, MAE, EFT,an. reg., dim. reg.,...

POST-NEWTONIAN HAMILTONIAN (C.O.M)

$$\widehat{H}^{\rm NR}_{\rm real}(\mathbf{q},\mathbf{p}) = \widehat{H}_{\rm N}(\mathbf{q},\mathbf{p}) + \widehat{H}_{\rm 1PN}(\mathbf{q},\mathbf{p}) + \widehat{H}_{\rm 2PN}(\mathbf{q},\mathbf{p}) + \widehat{H}_{\rm 3PN}(\mathbf{q},\mathbf{p}), \qquad (4.27)$$

where

$$\widehat{H}_{\rm N}(\mathbf{q},\mathbf{p}) = \frac{\mathbf{p}^2}{2} - \frac{1}{q}, \qquad (4.28\text{a})$$

Newton (0PN)

$$\widehat{H}_{\rm 1PN}(\mathbf{q},\mathbf{p}) = \frac{1}{8}(3\nu-1)(\mathbf{p}^2)^2 - \frac{1}{2}\left[(3+\nu)\mathbf{p}^2 + \nu(\mathbf{n}\cdot\mathbf{p})^2\right]\frac{1}{q} + \frac{1}{2q^2}, \qquad (4.28\text{b})$$

(1PN, 1938) - [Einstein-Infeld-Hoffman]

$$\widehat{H}_{\rm 2PN}(\mathbf{q},\mathbf{p}) = \frac{1}{16}\left(1-5\nu+5\nu^2\right)(\mathbf{p}^2)^3 + \frac{1}{8}\left[\left(5-20\nu-3\nu^2\right)(\mathbf{p}^2)^2 - 2\nu^2(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 - 3\nu^2(\mathbf{n}\cdot\mathbf{p})^4\right]\frac{1}{q}$$
$$+ \frac{1}{2}\left[(5+8\nu)\mathbf{p}^2 + 3\nu(\mathbf{n}\cdot\mathbf{p})^2\right]\frac{1}{q^2} - \frac{1}{4}(1+3\nu)\frac{1}{q^3}, \qquad (4.28\text{c})$$

(2PN, 1982/83) - [Damour-Deruelle]

$$\widehat{H}_{\rm 3PN}(\mathbf{q},\mathbf{p}) = \frac{1}{128}\left(-5+35\nu-70\nu^2+35\nu^3\right)(\mathbf{p}^2)^4$$
$$+ \frac{1}{16}\left[\left(-7+42\nu-53\nu^2-5\nu^3\right)(\mathbf{p}^2)^3 + (2-3\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^2(\mathbf{p}^2)^2 + 3(1-\nu)\nu^2(\mathbf{n}\cdot\mathbf{p})^4\mathbf{p}^2 - 5\nu^3(\mathbf{n}\cdot\mathbf{p})^6\right]\frac{1}{q}$$
$$+ \left[\frac{1}{16}\left(-27+136\nu+109\nu^2\right)(\mathbf{p}^2)^2 + \frac{1}{16}(17+30\nu)\nu(\mathbf{n}\cdot\mathbf{p})^2\mathbf{p}^2 + \frac{1}{12}(5+43\nu)\nu(\mathbf{n}\cdot\mathbf{p})^4\right]\frac{1}{q^2}$$
$$+ \left\{\left[-\frac{25}{8} + \left(\frac{1}{64}\pi^2 - \frac{335}{48}\right)\nu - \frac{23}{8}\nu^2\right]\mathbf{p}^2 + \left(-\frac{85}{16} - \frac{3}{64}\pi^2 - \frac{7}{4}\nu\right)\nu(\mathbf{n}\cdot\mathbf{p})^2\right\}\frac{1}{q^3}$$
$$+ \left[\frac{1}{8} + \left(\frac{109}{12} - \frac{21}{32}\pi^2 + \omega_{\rm static}\right)\nu\right]\frac{1}{q^4}. \qquad (4.28\text{d})$$

(3PN, 2000) - [Damour, Jaranowski,Schaefer]

...and 4PN too, [Damour, Jaranowski&Schaefer 2014/2015] - 4 loop calculation

$$\mathbf{q} = \mathbf{q}_1 - \mathbf{q}_2$$

$$\mathbf{p} = \mathbf{p}_1 = -\mathbf{p}_2$$

PN-EXPANDED (CIRCULAR) ENERGY FLUX (3.5PN)

$$\frac{dE}{dt} = -\mathcal{L}$$

balance equation

Mechanical loss GW luminosity

$$\mathcal{L} = \frac{32c^5}{5G}\nu^2 x^5 \Big\{ 1 + \left(-\frac{1247}{336} - \frac{35}{12}\nu\right)x + 4\pi x^{3/2} + \left(-\frac{44711}{9072} + \frac{9271}{504}\nu + \frac{65}{18}\nu^2\right)x^2$$
$$+ \left(-\frac{8191}{672} - \frac{583}{24}\nu\right)\pi x^{5/2}$$
$$+ \left[\frac{6643739519}{69854400} + \frac{16}{3}\pi^2 - \frac{1712}{105}C - \frac{856}{105}\ln(16\,x)\right.$$
$$\left. + \left(-\frac{134543}{7776} + \frac{41}{48}\pi^2\right)\nu - \frac{94403}{3024}\nu^2 - \frac{775}{324}\nu^3\right]x^3$$
$$+ \left(-\frac{16285}{504} + \frac{214745}{1728}\nu + \frac{193385}{3024}\nu^2\right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right)\Big\}.$$

Newtonian quadrupole formula

$$C = \gamma_E = 0.5772156649...$$

TAYLOR-EXPANDED (CIRCULAR) 3PN WAVEFORM

Blanchet, Iyer&Joguet, 02; Blanchet, Damour, Iyer&Esposito-Farese, 04; Kidder07; Blanchet et al.,08

$$h^{22} = -8\sqrt{\frac{\pi}{5}}\frac{G\nu m}{c^2 R}e^{-2i\phi}x\Big\{1 - x\Big(\frac{107}{42} - \frac{55}{42}\nu\Big) + x^{3/2}\Big[2\pi + 6i\ln\Big(\frac{x}{x_0}\Big)\Big] - x^2\Big(\frac{2173}{1512} + \frac{1069}{216}\nu - \frac{2047}{1512}\nu^2\Big)$$
$$- x^{5/2}\Big[\Big(\frac{107}{21} - \frac{34}{21}\nu\Big)\pi + 24i\nu + \Big(\frac{107i}{7} - \frac{34i}{7}\nu\Big)\ln\Big(\frac{x}{x_0}\Big)\Big]$$
$$+ x^3\Big[\frac{27\,027\,409}{646\,800} - \frac{856}{105}\gamma_E + \frac{2}{3}\pi^2 - \frac{1712}{105}\ln 2 - \frac{428}{105}\ln x$$
$$- 18\Big[\ln\Big(\frac{x}{x_0}\Big)\Big]^2 - \Big(\frac{278\,185}{33\,264} - \frac{41}{96}\pi^2\Big)\nu - \frac{20\,261}{2772}\nu^2 + \frac{114\,635}{99\,792}\nu^3 + \frac{428i}{105}\pi + 12i\pi\ln\Big(\frac{x}{x_0}\Big)\Big] + O(\epsilon^{7/2})\Big\},$$

$$x = (M\Omega)^{2/3} \sim v^2/c^2$$

$$M = m_1 + m_2$$

$$\nu = \frac{m_1 m_2}{M^2}$$

EFFECTIVE-ONE-BODY (EOB)

approach to the general relativistic two-body problem

(Buonanno-Damour 99, 00, Damour-Jaranowski-Schäfer 00, Damour 01, Damour-Nagar 07, Damour-Iyer-Nagar 08)

key ideas:

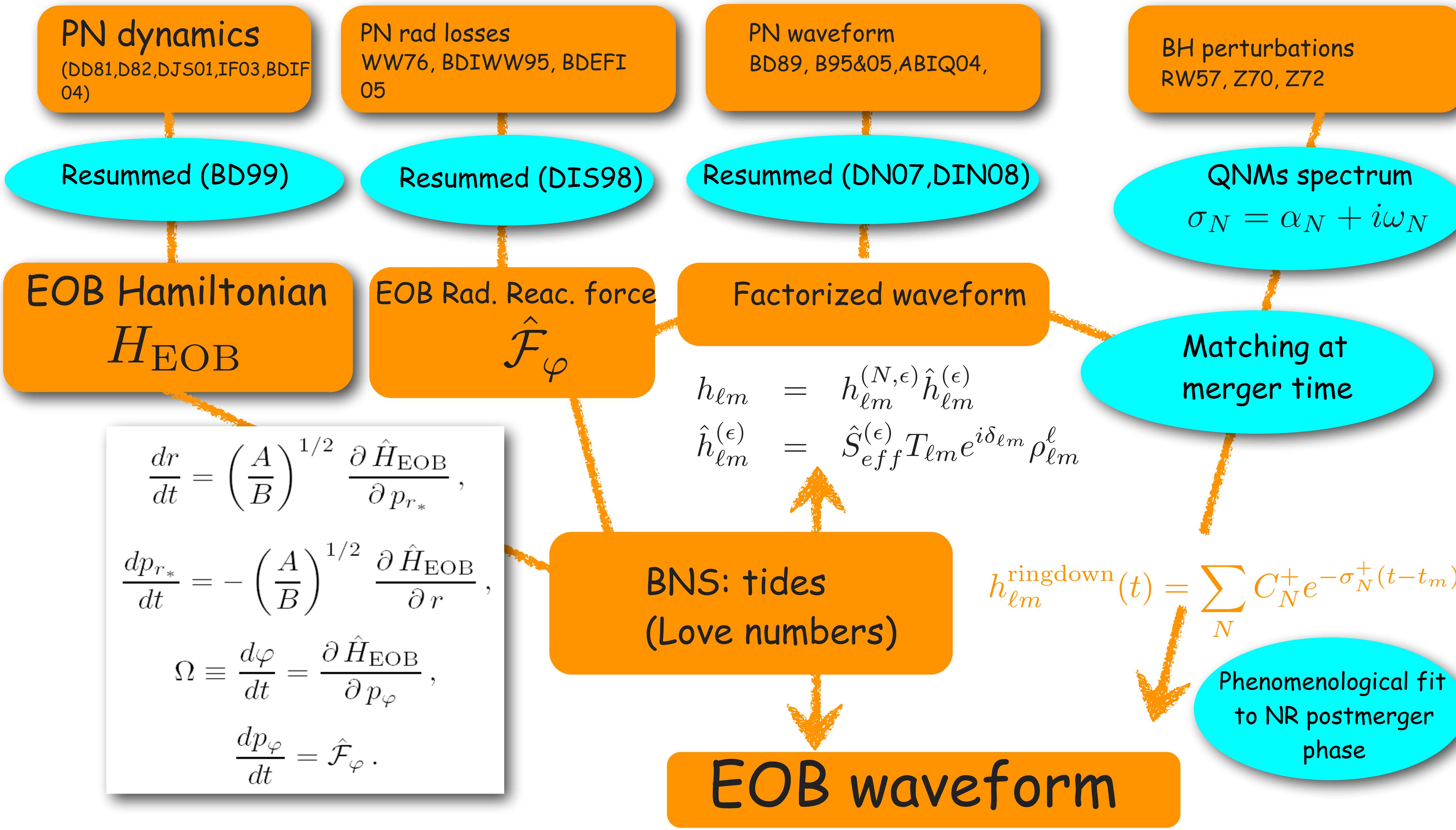
(1) Replace two-body dynamics (m_1, m_2) by dynamics of a particle $(\mu \equiv m_1 m_2/(m_1 + m_2))$ in an effective metric $g_{\mu\nu}^{\rm eff}(u)$, with

$$u \equiv GM/c^2 R\,, \quad M \equiv m_1 + m_2$$

(2) Systematically use RESUMMATION of PN expressions (both $g_{\mu\nu}^{\rm eff}$ and $\mathcal{F}_{RR}$) based on various physical requirements

(3) Require continuous deformation w.r.t. $\nu \equiv \mu/M \equiv m_1 m_2/(m_1 + m_2)^2$ in the interval $0 \leq \nu \leq \frac{1}{4}$

STRUCTURE OF THE EOB FORMALISM

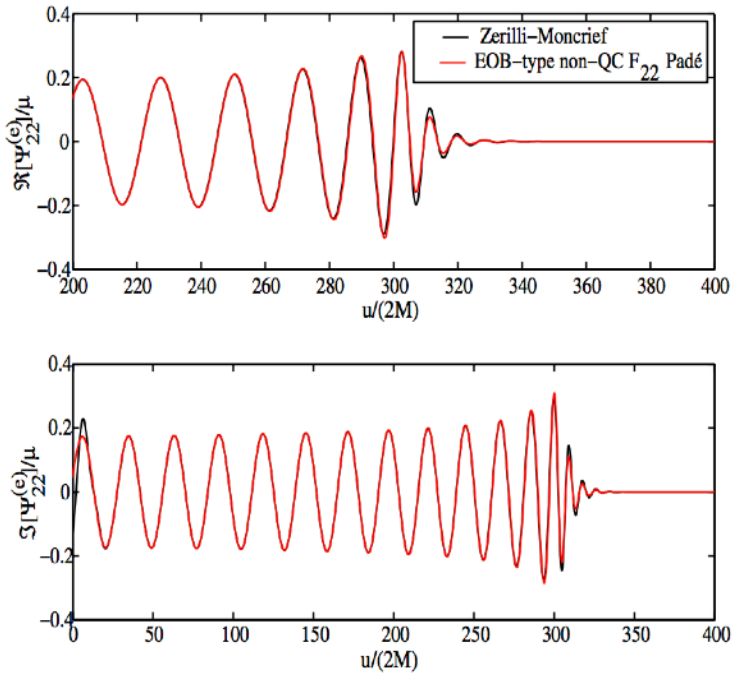


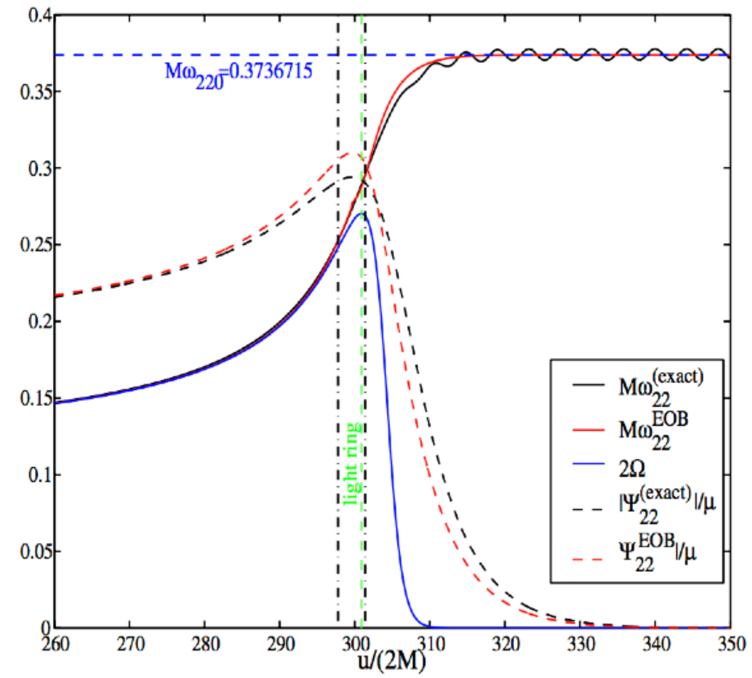
$$h^{\rm EOB}_{\ell m} = \theta(t_m - t) h^{\rm insplunge}_{\ell m}(t) + \theta(t - t_m) h^{\rm ringdown}_{\ell m}(t)$$

+ GSF + EOB based on Post-Minkowskian approximation

Extreme-mass-ratio limit (2007)

- Laboratory to learn each physical element entering the coalescence
- Accurate waveform computation using Regge-Wheeler-Zerilli (Schwarzschild) or Teukolsky (Kerr) perturbation equations



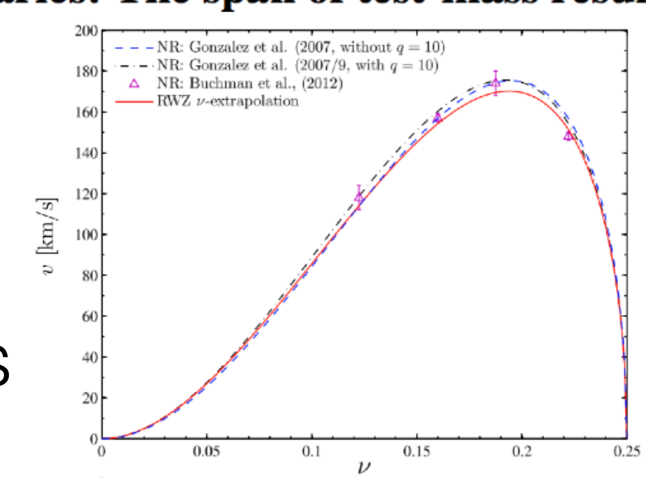


- Several aspects of the phenomenon explored in detail
- Several papers with Bernuzzi+ (multipoles, GW-recoil, spin etc.): Teukode

PHYSICAL REVIEW D **88,** 121501(R) (2013)

Gravitational recoil in nonspinning black-hole binaries: The span of test-mass results

Alessandro Nagar



RWZ-extrapolated final recoil velocity and "full" NR calculations

EOBNR Models

AEI (Ligo): LAL
SEOBNRv4 (spin-aligned)
SEOBNRv4_HM (spin-aligned, 22,21,33,44,55 modes)
SEOBNRv4P_HM (precessing spins, 22,21,33,44,55 modes)
SEOBNRv4T (tides)

(**Virgo**): standalone C & LAL implementation
TEOBResumS (spin-aligned,tides, BBH, BNS,BHNS)
TEOBiResumS(higher modes, in progress)

Differences:

- Hamiltonian (gauge + spin sector. Spin-spin)
- Resummation of the interaction potential
- Radiation reaction
- Effective representation of merger and post merger
- ESSENTIALLY: different deformation of the test-mass limit

EOB Hamiltonian

EOB Hamiltonian

$$H_{\rm EOB} = M\sqrt{1 + 2\nu\left(\hat{H}_{\rm eff} - 1\right)}$$

All functions are a ν-dependent deformation of the Schwarzschild ones

$$A(r) = 1 - 2u + 2\nu u^3 + a_4 \nu u^4$$

$$a_4 = \frac{94}{3} - \frac{41}{32}\pi^2 \simeq 18.6879027$$

$$A(r)B(r) = 1 - 6\nu u^2 + 2(3\nu - 26)\nu u^3$$

$$u = GM/(c^2 R)$$

Simple effective Hamiltonian:

$$\hat{H}_{\rm eff} \equiv \sqrt{p_{r_*}^2 + A(r)\left(1 + \frac{p_\varphi^2}{r^2} + z_3 \frac{p_{r_*}^4}{r^2}\right)}$$

$$p_{r_*} = \left(\frac{A}{B}\right)^{1/2} p_r$$

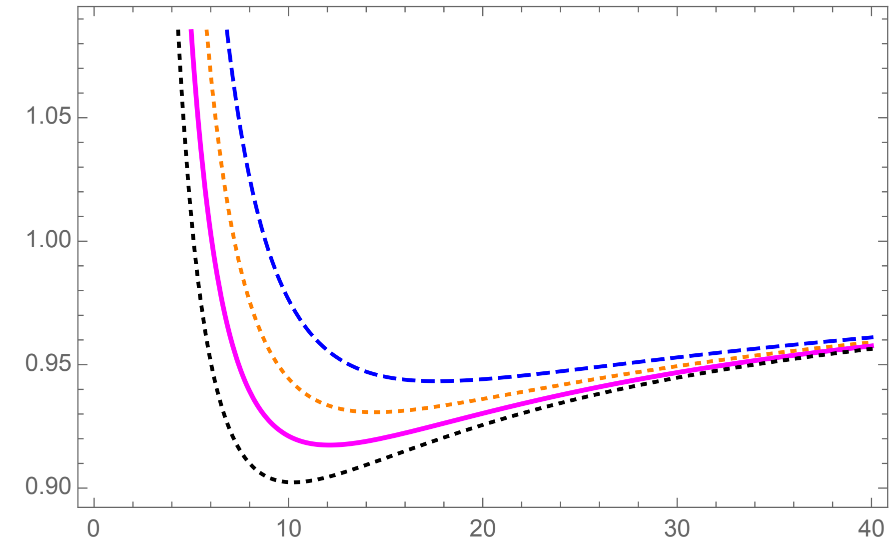
Crucial EOB radial potential

Contribution at 3PN

EFFECTIVE POTENTIALS

Newtonian gravity (any mass ratio):
circular orbits are always stable. No plunge.

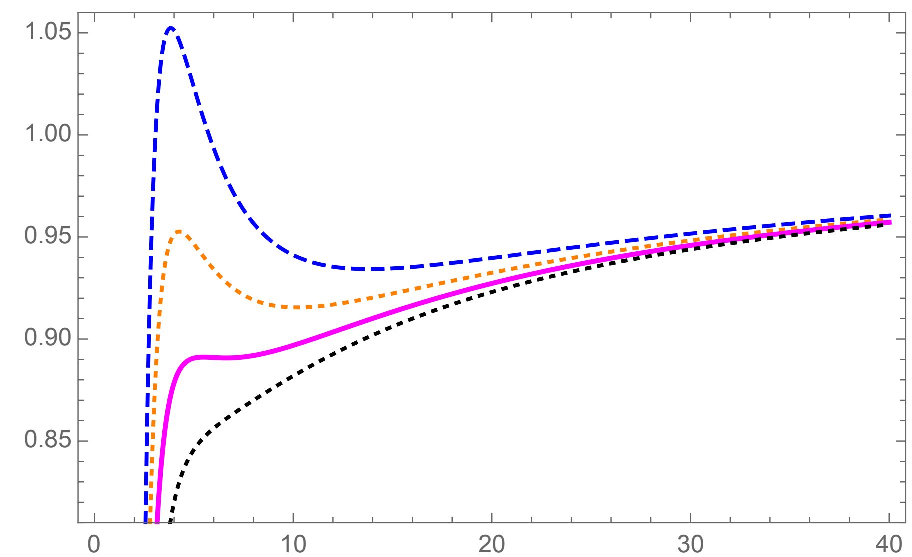
$$W^{\rm eff}_{\rm Newt} = 1 - \frac{2}{r} + \frac{p_\varphi^2}{r^2}$$



Test-body on Schwarzschild black hole:

last stable orbit (LSO) at **r=6M**; plunge

$$W^{\rm eff}_{\rm Schwarzschild} = \left(1 - \frac{2}{r}\right)\left(1 + \frac{p_\varphi^2}{r^2}\right)$$



EOB, Black-hole binary, any mass ratio:

last stable orbit (LSO) at **r<6M** plunge

$$W^{\rm eff}_{\rm EOB} = A(r;\,\nu)\left(1 + \frac{p_\varphi^2}{r^2}\right)$$

ν -deformation of the Schwarzschild case!

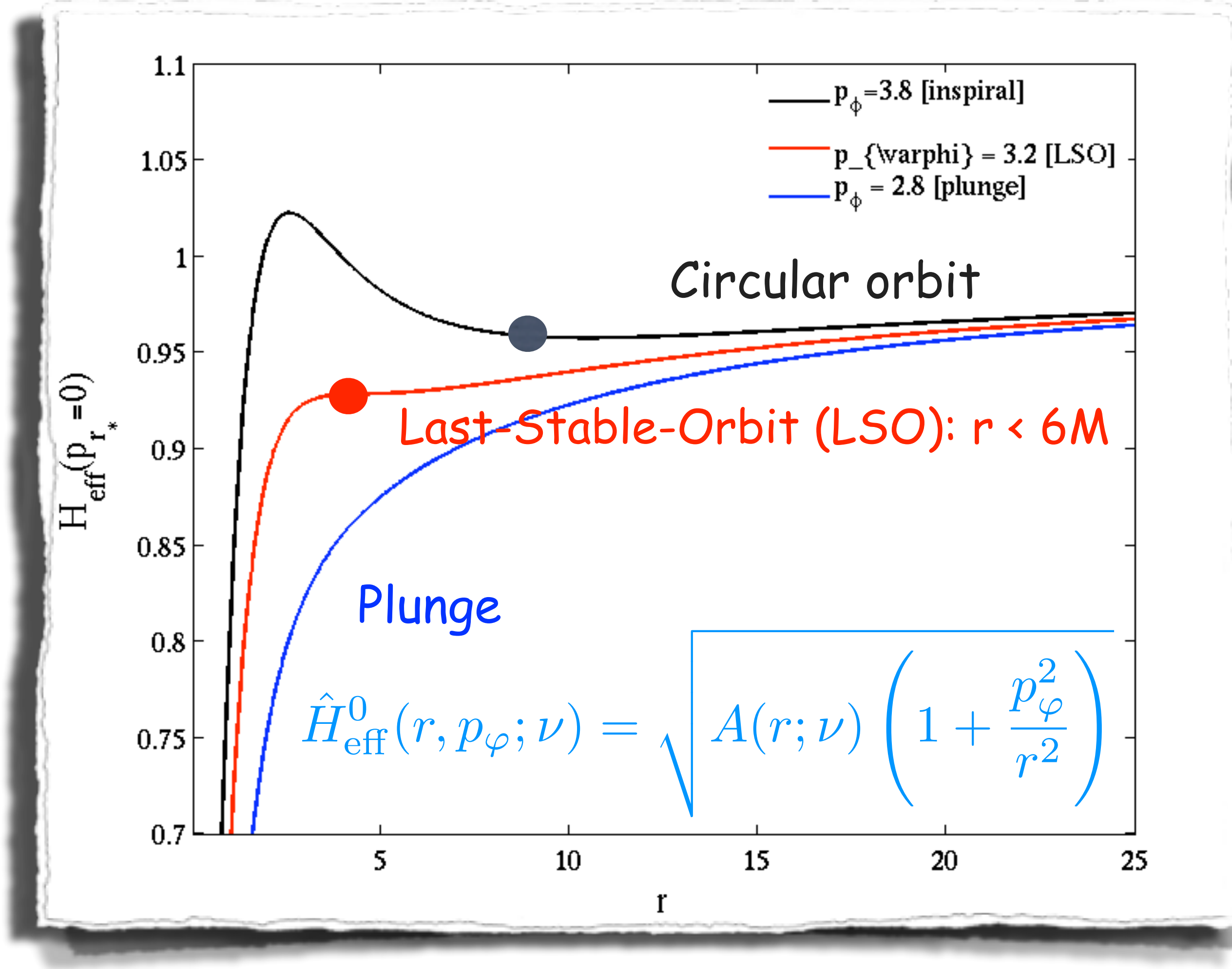
HAMILTON'S EQUATIONS & RADIATION REACTION

$$\dot{r} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}}$$

$$\dot{\varphi} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{\varphi}} \equiv \Omega$$

$$\dot{p}_{r_*} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r} + \hat{\mathcal{F}}_{r_*}$$

$$\dot{p}_{\varphi} = \hat{\mathcal{F}}_{\varphi}$$



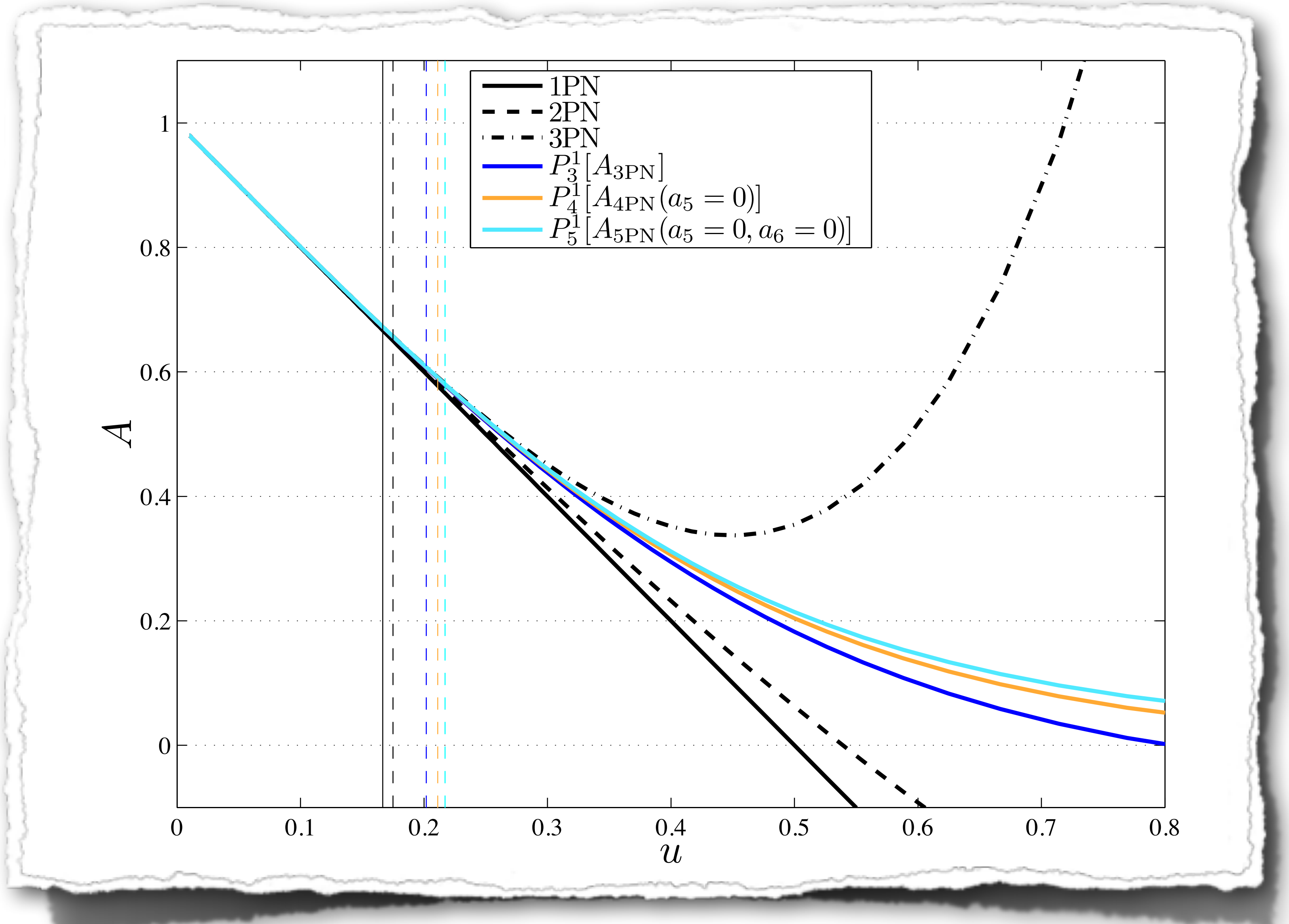
- The system must radiate angular momentum
- How?Use PN-based (Taylor-expanded) radiation reaction force (ang-mom flux)
- Need flux resummation

$$\hat{\mathcal{F}}_{\varphi}^{\rm Taylor} = -\frac{32}{5}\nu\Omega^5 r_\Omega^4 \hat{F}^{\rm Taylor}(v_\varphi)$$

Plus horizon contribution [AN&Akcay2012]

Resummation multipole by multipole (Damour&Nagar 2007, Damour, Iyer & Nagar 2008, Damour & Nagar, 2009)

USE OF PADE APPROXIMANTS



- Continuity with Schwarzschild metric: A(r) needs to have a zero
- Simple (possible) prescription: use a Padé representation of the potential

$$A(r) = P^1_3[A^{3\mathrm{PN}}(r)] = \frac{1 + n_1 u}{1 + d_1 u + d_2 u^2 + d_3 u^3}$$

TEOBResumS - I

4PN analytically complete + 5PN logarithmic term in the A(u) function:

[Damour 2009, Blanchet et al. 2010, Barack, Damour & Sago 2010, Le Tiec et al. 2011, Barausse et al. 2011, Akcay et al. 2012, Bini& Damour2013, DamourJaranowski&Schaefer 2014].

$$A_{5\mathrm{PN}}^{\mathrm{Taylor}} = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\nu u^4 + \nu[a_5^c(\nu) + a_5^{\ln}\ln u]u^5 + \nu[a_6^c(\nu) + a_6^{\ln}\ln u]u^6$$

1PN 2PN 3PN 4PN 5PN

$$a_5^{\log} = \frac{64}{5}$$

$$a_5^c = a_{5_0}^c + \nu a_{5_1}^c$$

$$a_{5_0}^c = -\frac{4237}{60} + \frac{2275}{512}\pi^2 + \frac{256}{5}\log(2) + \frac{128}{5}\gamma$$

$$a_{5_1}^c = -\frac{221}{6} + \frac{41}{32}\pi^2$$

4PN fully known ANALYTICALLY!

$$a_6^{\log} = -\frac{7004}{105} - \frac{144}{5}\nu$$

5PN logarithmic term (analytically known)

NEED ONE "effective" 5PN parameter from NR waveform data: $a_6^c(\nu)$

State-of-the-art EOB potential (5PN-resummed):

$$A(u;\nu, a_6^c) = P_5^1[A_{5\mathrm{PN}}^{\mathrm{Taylor}}(u;\nu, a_6^c)]$$

TEOBResumS - II

Resummation of the waveform (and flux) multipole by multipole (CRUCIAL!)

[Damour&Nagar 2007, Damour, Iyer, Nagar 2008]

Next-to-quasi-circular correction

$$h_{\ell m} \equiv h^{(N,\epsilon)}_{\ell m} \hat{h}^{(\epsilon)}_{\ell m} \hat{h}^{\rm NQC}_{\ell m}$$

Newtonian × PN × NQC

PN-correction

$$\hat{h}^{(\epsilon)}_{\ell m} = \hat{S}^{(\epsilon)}_{\rm eff} T_{\ell m} e^{i\delta_{\ell m}} \rho^{\ell}_{\ell m}$$

Remnant phase and modulus corrections: "improved" PN series

The "Tail factor"

$$T_{\ell m} = \frac{\Gamma(\ell + 1 - 2i\hat{\hat{k}})}{\Gamma(\ell+1)} e^{\pi \hat{\hat{k}}} e^{2i\hat{\hat{k}} \ln(2kr_0)}$$

Resums an infinite number of leading logarithms in tail effects (hereditary contributions)

Effective source:

EOB (effective) energy (even-parity modes)

EOB angular momentum (odd-parity modes)

TEOBResumS - III

Damour&AN 2014: NR-based phenomenological description of postmerger phase

Factorize the fundamental QNM, fit what remains

$$h(\tau) = e^{\sigma_1 \tau - i\phi_0} \bar{h}(\tau)$$

$$\bar{h}(\tau) \equiv A_{\bar{h}} e^{i\phi_{\bar{h}}(\tau)}.$$

$$A_{\bar{h}}(\tau) = c_1^A \tanh(c_2^A \tau + c_3^A) + c_4^A,$$

$$\phi_{\bar{h}}(\tau) = -c_1^\phi \ln\left(\frac{1 + c_3^\phi e^{-c_2^\phi \tau} + c_4^\phi e^{-2c_2^\phi \tau}}{1 + c_3^\phi + c_4^\phi}\right)$$

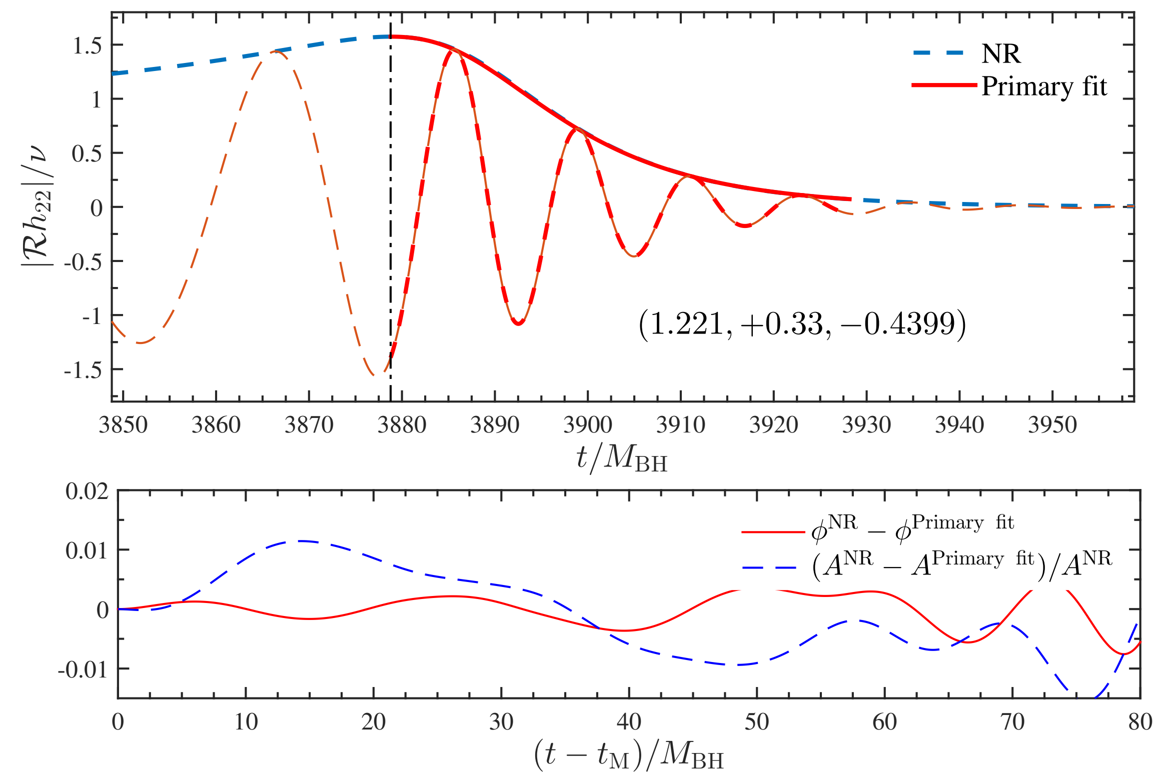
$$c_2^A = \frac{1}{2}\alpha_{21}, \qquad \alpha_{21} = \alpha_2 - \alpha_1$$

$$c_4^A = \hat{A}_{22}^{\rm mrg} - c_1^A \tanh(c_3^A),$$

$$c_1^A = \hat{A}_{22}^{\rm mrg}\, \alpha_1 \frac{\cosh^2(c_3^A)}{c_2^A},$$

$$c_1^\phi = \Delta\omega \frac{1 + c_3^\phi + c_4^\phi}{c_2^\phi (c_3^\phi + 2c_4^\phi)}, \qquad \Delta\omega \equiv \omega_1 - M_{\rm BH}\omega_{22}^{\rm mrg}$$

$$c_2^\phi = \alpha_{21},$$



Good performance of primary fits (modulo details...)

Do this for various SXS dataset and then build up a (simple-minded) interpolating fit

Black-list:

(1) the structure due to m<0 modes is not included (yet)

(2) large-mass ratios/high spin: amplitude problems

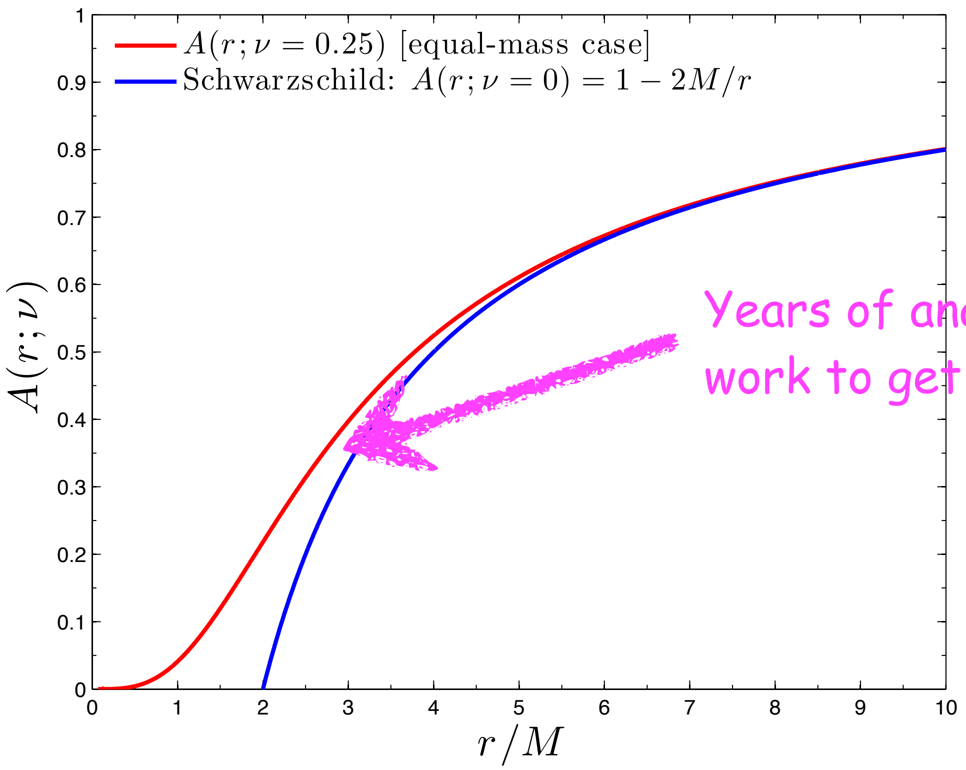
(3) problems are extreme for high-spin EMRL waves

(4) more flexible fit-template needed

(5) improve/check over all datasets (SXS & BAM for large mass-ratios & consistency with EMRL)

Del Pozzo & AN, PRD 95 (2017) 124034

TEOBResumS point-mass potential

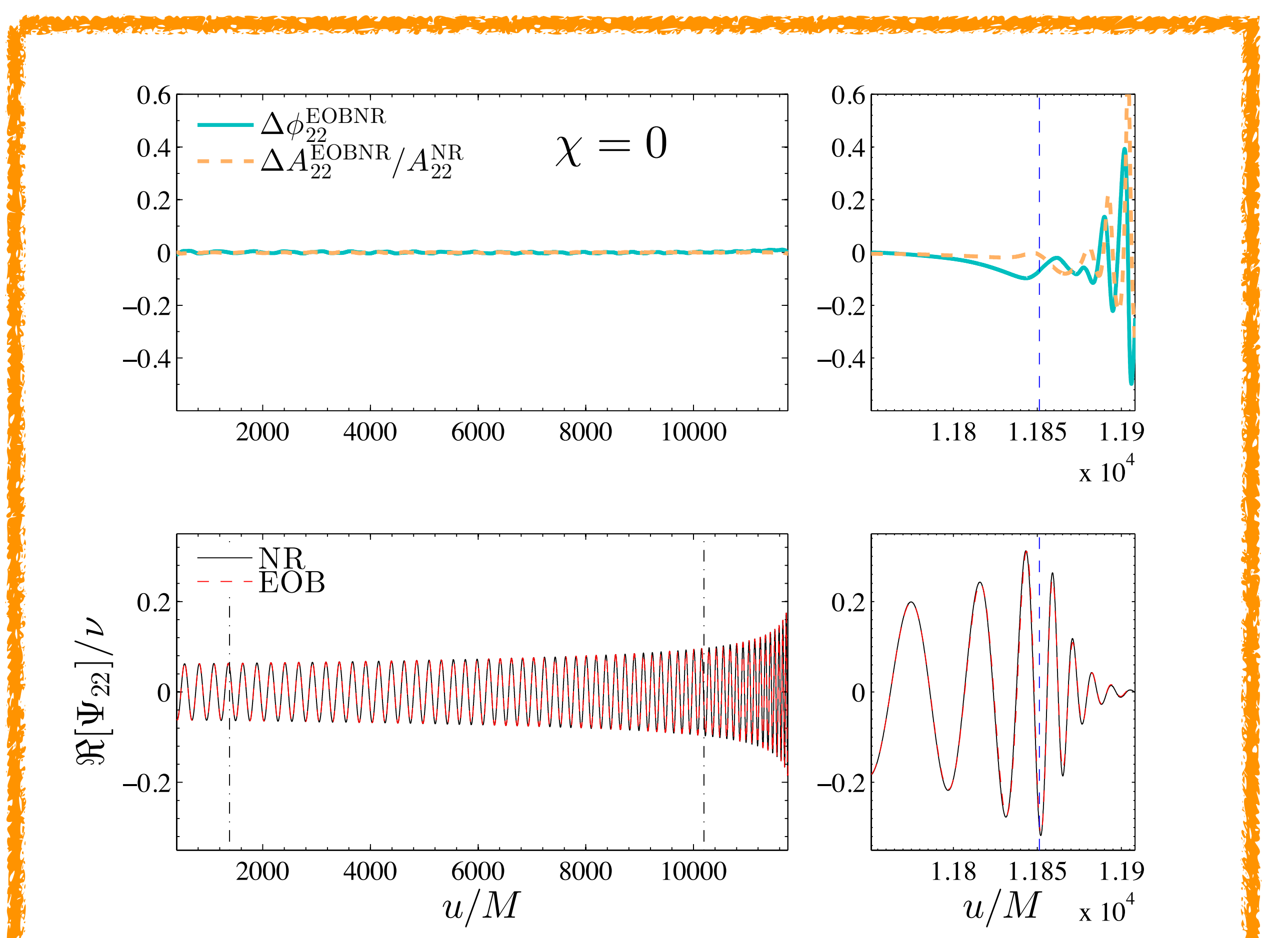


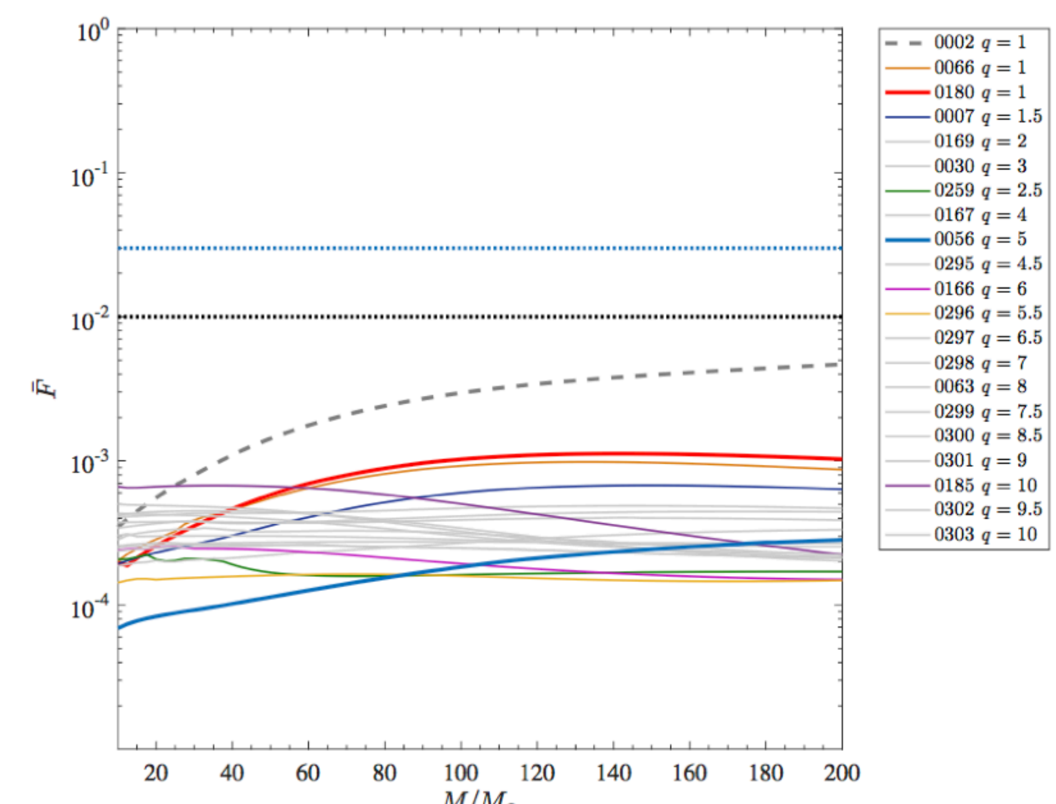
From EOB/NR-fitting:

$$a_6^c(\nu) = 3097.3\nu^2 - 1330.6\nu + 81.3804$$

Years of analytical and numerical work to get this strong-field difference!

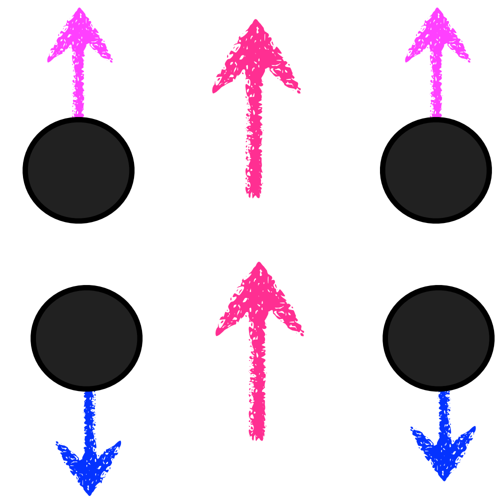
$$\bar{F}(M) \equiv 1 - F = 1 - \max_{t_0,\phi_0} \frac{\langle h_{22}^{\rm EOB}, h_{22}^{\rm NR}\rangle}{||h_{22}^{\rm EOB}||\,||h_{22}^{\rm NR}||},$$





Nagar, Riemenschneider, Pratten 2017

Spinning BBHs



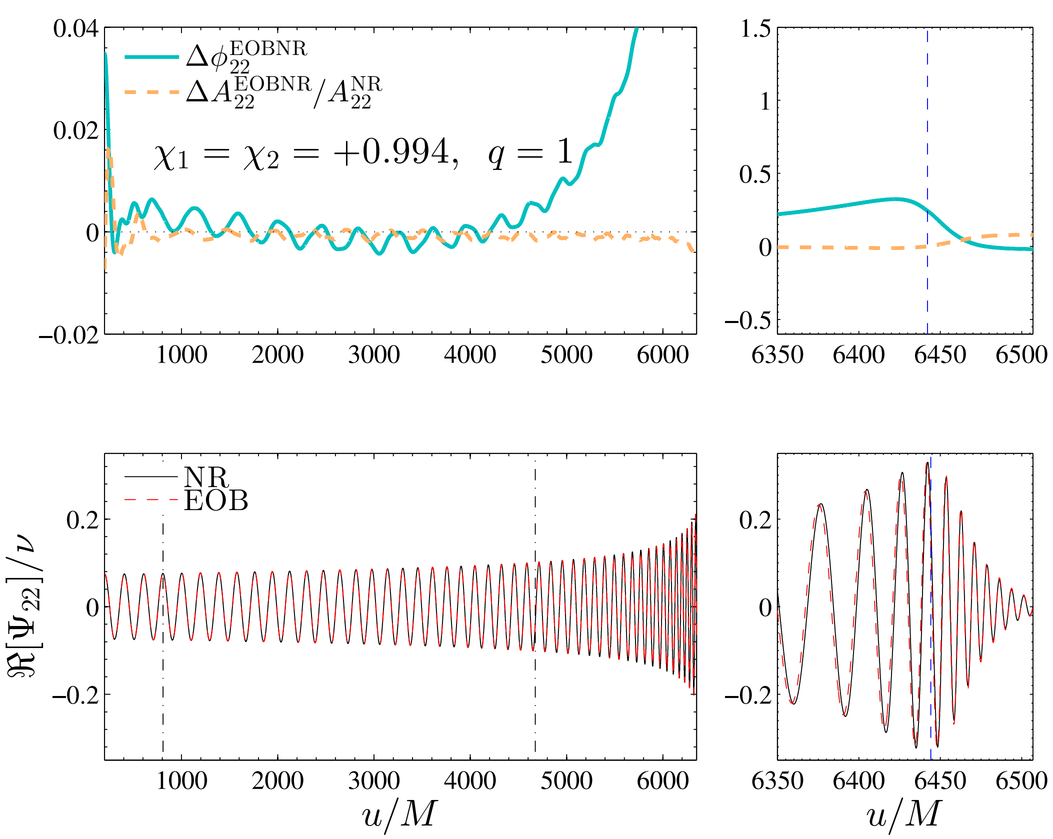
Spin-orbit & spin-spin couplings

(i) Spins aligned with L: repulsive (slower) L-o-n-g-e-r INSPIRAL

(ii) Spins anti-aligned with L: attractive (faster) Shorter INSPIRAL

(iii) Misaligned spins: precession of the orbital plane (waveform modulation)

$$\chi_{1,2} = \frac{c\,\mathbf{S}_{1,2}}{G m_{1,2}^2}$$



EOB/NR agreement: sophisticated (though rather simple) model for spin-aligned binaries

Damour&Nagar, PRD90 (2014), 024054 (Hamiltonian)
Damour&Nagar, PRD90 (2014), 044018 (Ringdown)
Nagar,Damour, Reisswig & Pollney, PRD 93 (2016), 044046

AEI model, SEOBNRv4, Bohe et al., arXiv:1611.03703v1 (PRD in press)

Spin-Spin in Kerr Hamiltonian

Particle: (μ, S_*) Kerr black-hole: (M, S)

$$H_{\rm Kerr} = H^{\rm Kerr}_{\rm orb} + H^{S}_{\rm SO}(\mathbf{S}) + H^{S_*}_{\rm SO}(\mathbf{S}_*)$$

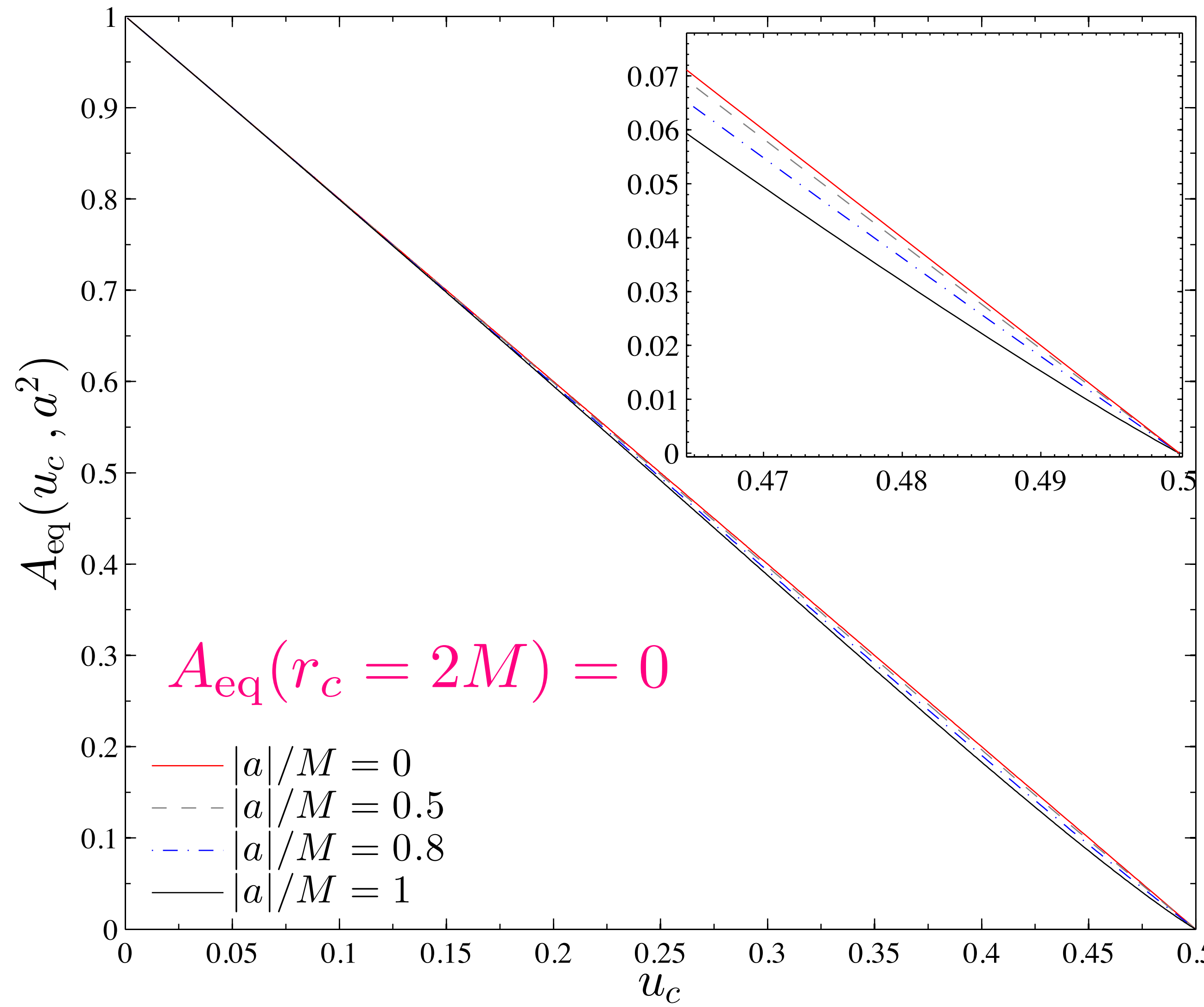
$$H^{\rm Kerr}_{\rm orb,eq}(r, p_r, p_\varphi) = \sqrt{A^{\rm eq}(r)\left(\mu^2 + \frac{p_\varphi^2}{r_c^2} + \frac{p_r^2}{B^{\rm eq}(r)}\right)}.$$

$$A_{\rm eq}(r) \equiv \frac{\Delta(r)}{r_c^2} = \left(1 - \frac{2M}{r_c}\right)\frac{1 + \frac{2M}{r_c}}{1 + \frac{2M}{r}}$$

centrifugal radius

$$r_c^2 = r^2 + a^2 + \frac{2Ma^2}{r}$$

EOB: Identify a similar centrifugal radius in the comparable mass case and devise a similar deformation of A



Similar, though different, in SEOB

The effective Hamiltonian

$$\hat{H}_{\rm eff} = \frac{g_S^{\rm eff}}{r^3}\mathbf{L}\cdot\mathbf{S} + \frac{g_{S^*}^{\rm eff}}{r^3}\mathbf{L}\cdot\mathbf{S}^* + \sqrt{A(1+\gamma^{ij}p_i p_j + Q_4(p))}$$

with the structure

$$g_S^{\rm eff} = 2 + \nu(\text{PN corrections}) + (\text{spin})^2\text{corrections}$$

$$g_{S^*}^{\rm eff} = \left(\frac{3}{2} + \text{test mass coupling}\right) + \nu(\text{PN corrections}) + (\text{spin})^2\text{corrections}$$

$$A = 1 - \frac{2}{r} + \nu(\text{PN corrections}) + (\text{spin})^2\text{corrections}$$

$$\gamma^{ij} = \gamma^{ij}_{\rm Kerr} + \nu(\text{PN corrections}) + \dots$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = M^2(X_1^2\chi_1 + X_2^2\chi_2)$$

$$X_i = m_i/M$$

$$\mathbf{S}^* = \frac{m_2}{m1}\mathbf{S}_1 + \frac{m_1}{m_2}\mathbf{S}_2 = M^2\nu(\chi_1+\chi_2)$$

$$-1 \le \chi_i \le 1$$

THE TWO TYPES OF SPIN-ORBIT COUPLINGS

$$\hat{H}_{\rm SO}^{\rm eff} = G_S \mathbf{L}\cdot\mathbf{S} + G_{S^*}\mathbf{L}\cdot\mathbf{S}^* \qquad G_S = \frac{1}{r^3} g_S^{\rm eff}, \quad G_{S^*} = \frac{1}{r^3} g_{S^*}^{\rm eff}$$

In the Kerr limit, only S-type gyro-gravitomagnetic ratio enters:

$$g_S^{\rm eff} = 2\frac{r^2}{r^2 + a^2\left[(1-\cos^2\theta)\left(1+\frac{2}{r}\right) + 2\cos^2\theta\right] + \frac{a^4}{r^2}\cos^2\theta} = 2 + \mathcal{O}[({\rm spin})^2]$$

PN calculations yield (in some spin gauge)[DJS08, Hartung&Steinhoff11,Nagar11,Barausse&Buonanno11]

"Effective" NNNLO SO-coupling

$$g_S^{\rm eff} = 2 + \frac{1}{c^2}\left\{-\frac{1}{r}\frac{5}{8}\nu - \frac{33}{8}(\mathbf{n}\cdot\mathbf{p})^2\right\} + \frac{1}{c^4}\left\{-\frac{1}{r^2}\left(\frac{51}{4}\nu + \frac{\nu^2}{8}\right) + \frac{1}{r}\left(-\frac{21}{2}\nu + \frac{23}{8}\nu^2\right)(\mathbf{n}\cdot\mathbf{p})^2 + \frac{5}{8}\nu(1+7\nu)(\mathbf{n}\cdot\mathbf{p})^4\right\}, \quad + \frac{1}{c^6}\frac{\nu c_3}{r^3}$$

$$g_{S^*}^{\rm eff} = \frac{3}{2} + \frac{1}{c^2}\left\{-\frac{1}{r}\left(\frac{9}{8} + \frac{3}{4}\nu\right) - \left(\frac{9}{4}\nu + \frac{15}{8}\right)(\mathbf{n}\cdot\mathbf{p})^2\right\} + \frac{1}{c^4}\left\{-\frac{1}{r^2}\left(\frac{27}{16} + \frac{39}{4}\nu + \frac{3}{16}\nu^2\right) + \frac{1}{r}\left(\frac{69}{16} - \frac{9}{4}\nu + \frac{57}{16}\nu^2\right)(\mathbf{n}\cdot\mathbf{p})^2 + \left(\frac{35}{16} + \frac{5}{2}\nu + \frac{45}{16}\nu^2\right)(\mathbf{n}\cdot\mathbf{p})^4\right\} + \frac{1}{c^6}\frac{\nu c_3}{r^3}$$

This functions are resummed taking their Taylor-inverse

The NR-informed effective parameter makes the spin-orbit coupling stronger or weaker with respect to the straight analytical prediction

Spin-Spin within TEOBResumS

Define a "centrifugal radius": at LO reads

$$r_c^2 = r^2 + \hat{a}_0^2 \left(1 + \frac{2}{r}\right)$$

$$A_{\rm eq}(\nu, \chi_1, \chi_2) = A_{\rm orb}^{\rm EOB}(\nu, \kappa, r) \frac{1 + \frac{2}{r_c}}{1 + \frac{2}{r}}$$

where:

$$\hat{a}_0^2 = \tilde{a}_1^2 + 2\tilde{a}_1\tilde{a}_2 + \tilde{a}_2^2$$

BBH case

or

$$\hat{a}_0^2 = C_{Q1}(\tilde{a}_1)^2 + 2\tilde{a}_1\tilde{a}_2 + C_{Q2}(\tilde{a}_2)^2$$

BNS case

$C_Q = 1$ is the BH case. In general, from I-Love-Q [Yagi-Yunes]

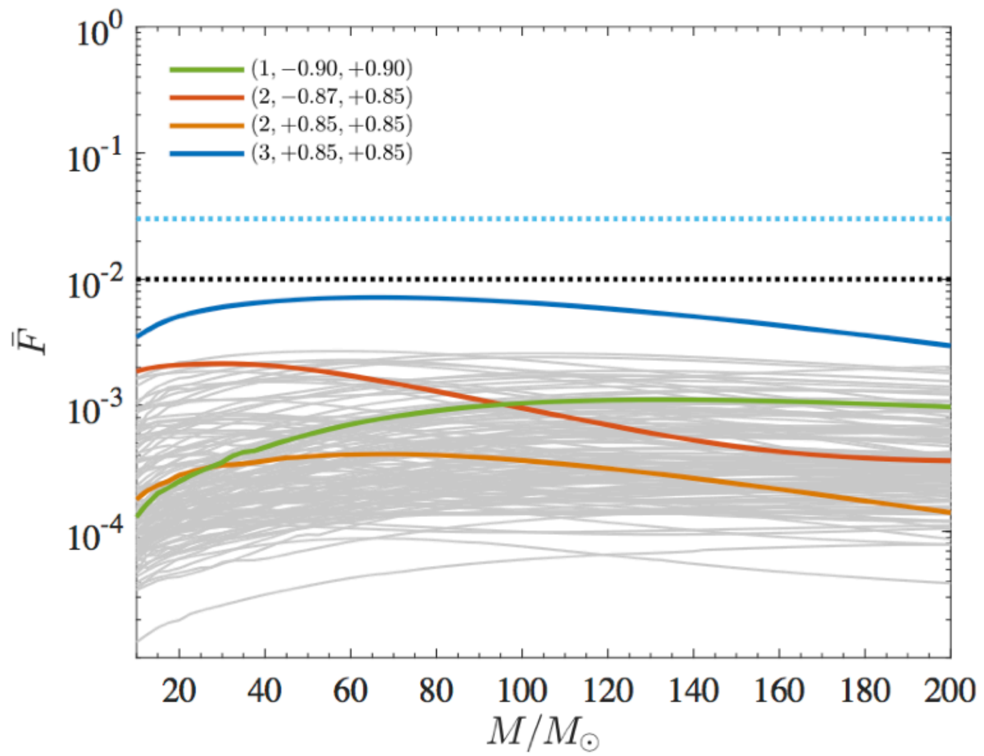
One verifies that once plugged in the EOB Hamiltonian and re-expanded one obtains the standard PN Hamiltonian @LO [e.g., cf. Levi-Steinhoff]

Similarly one can act on LO SS terms in the waveform & flux

$$r_c^2 = r^2 + \hat{a}_0^2 \left(1 + \frac{2}{r}\right) + \delta\hat{a}^2,$$

NLO contribution

TEOBResumS: spin-aligned + tides



- spin-orbit parameter informed by 30 BBH NR simulations
- BEST faithfulness with all NR available (200 simulations)
- Robust and simple
- Tides and spin-induced moment included (BNS)
- ONLY publicly available stand-alone EOB code

$$\bar{F}(M) \equiv 1 - F = 1 - \max_{t_0,\phi_0} \frac{\langle h_{22}^{\rm EOB}, h_{22}^{\rm NR}\rangle}{||h_{22}^{\rm EOB}||\,||h_{22}^{\rm NR}||},$$

Nagar, Bernuzzi, Del Pozzo et al., PRD98.104052

effective NNNLO spin-orbit "function"

$$c_3(\tilde{a}_A, \tilde{a}_B, \nu) = p_0 \frac{1 + n_1 \hat{a}_0 + n_2 \hat{a}_0^2}{1 + d_1 \hat{a}_0} + \left(p_1 \nu + p_2 \nu^2 + p_3 \nu^3\right) \hat{a}_0 \sqrt{1 - 4\nu} + p_4 \left(\tilde{a}_A - \tilde{a}_B\right) \nu^2, \qquad (17)$$

$$\tilde{a}_{1,2} = X_{1,2} \chi_{1,2}$$

$$X_{1,2} \equiv \frac{m_{1,2}}{M}$$

$$\hat{a}_0 \equiv \frac{S + S_*}{M^2} = X_A \chi_A + X_B \chi_B = \tilde{a}_A + \tilde{a}_B$$

ONLY 2 EOBNR models
TEOBResumS
SEOBNRv4 (AEI)

See Rettegno, Martinetti, Nagar+2019, arXiv:1911.10818

TEOBResumS + Post Adiabatic Approx

ODEs are slow: 1-2s for BNS waveforms (10Hz) not good for DA

Shared solution: ROMs (surrogate models. Fast but not flexible)

Are ROMs really needed?

EOB equations of motion

$$\frac{d\varphi}{dt} = \frac{1}{\nu \hat{H}_{\rm EOB} \hat{H}_{\rm eff}^{\rm orb}} \left[A \frac{p_\varphi}{r_c^2} + \hat{H}_{\rm eff}^{\rm orb} \tilde{G} \right], \tag{1}$$

$$\frac{dr}{dt} = \left(\frac{A}{B}\right)^{1/2} \frac{1}{\nu \hat{H}_{\rm EOB} \hat{H}_{\rm eff}^{\rm orb}} \times \times \left[p_{r_*} \left(1 + 2 z_3 \frac{A}{r_c^2} p_{r_*}^2\right) + \hat{H}_{\rm eff}^{\rm orb} p_\varphi \frac{\partial \tilde{G}}{\partial p_{r_*}} \right], \tag{2}$$

$$\frac{dp_\varphi}{dt} = \hat{\mathcal{F}}_\varphi, \tag{3}$$

$$\frac{dp_{r_*}}{dt} = -\left(\frac{A}{B}\right)^{1/2} \frac{1}{2\nu \hat{H}_{\rm EOB} \hat{H}_{\rm eff}^{\rm orb}} \left[A' + p_\varphi^2 \left(\frac{A}{r_c^2}\right)' + + z_3\, p_{r_*}^4 \left(\frac{A}{r_c^2}\right)' + 2 \hat{H}_{\rm eff}^{\rm orb} p_\varphi \tilde{G}' \right], \tag{4}$$

$$\tilde{G} \equiv G_S S + G_{S_*} S_*$$

TEOBResumS + Post Adiabatic Approx

Post-adiabatic approximation (Damour & AN, 2007)

2PA: used to have eccentricity free ID for the EOB EoM

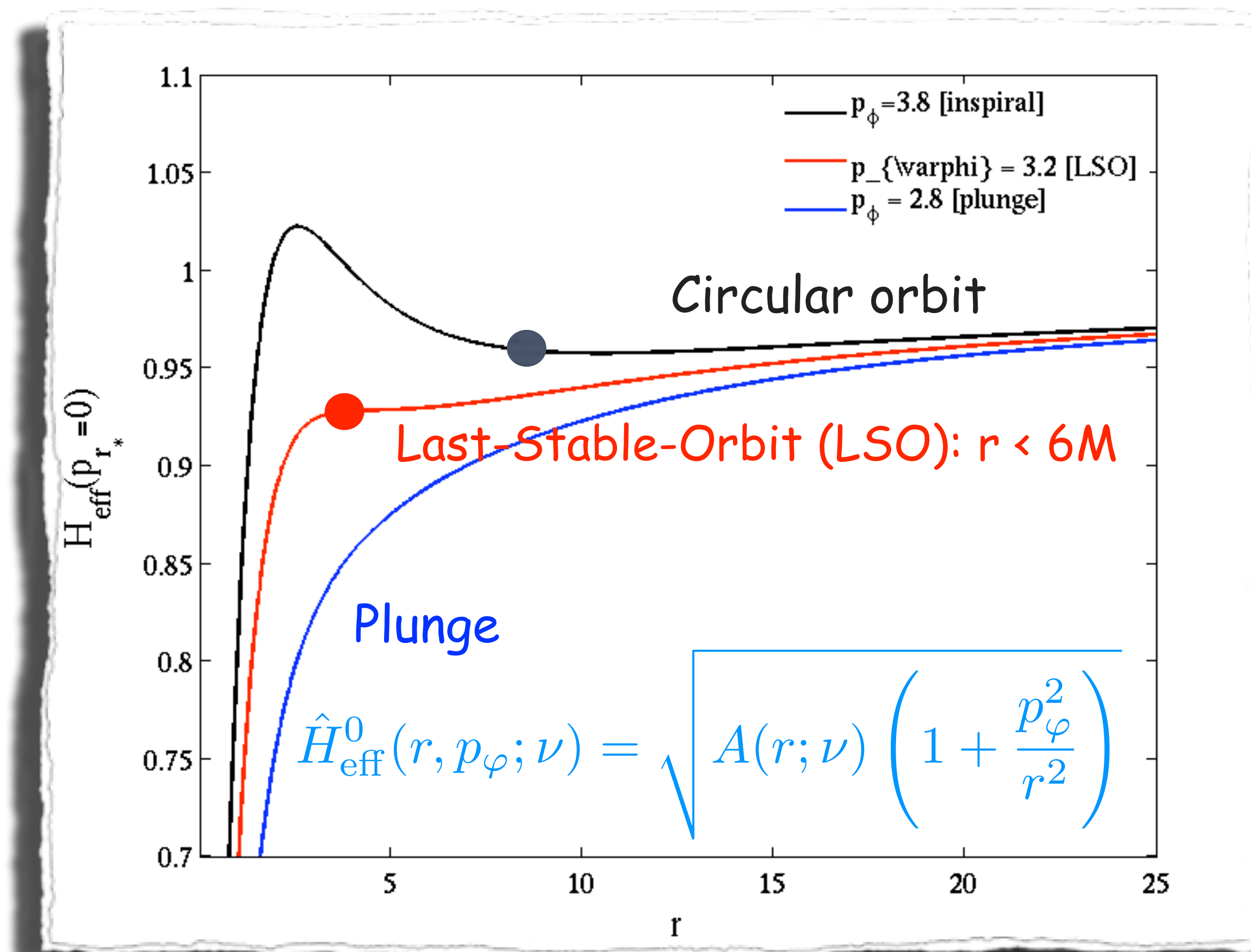
$$\hat{\mathcal{F}}_\varphi(r) = \sum_{n=0}^{\infty} \mathcal{F}_{2n+1}(r)\ \varepsilon^{2n+1}$$

$$p_\varphi^2(r) = j_0^2(r)\left(1 + \sum_{n=1}^{\infty} k_{2n}(r)\ \varepsilon^{2n}\right)$$

$$p_{r_*}(r) = \sum_{n=0}^{\infty} \pi_{2n+1}(r)\ \varepsilon^{2n+1}$$

Iterate up to nth order at a given radius to obtain the momenta with high accuracy

[Nagar&Rettegno, 2018]



TEOBResumS_rush

$$t = \int_{r_{\max}}^{r} dr (\partial_{p_r} \hat{H})^{-1}$$

$$\varphi = \int_0^t dt \partial_{p_\varphi} \hat{H} = \int_{r_{\max}}^{r} dr \partial_{p_\varphi} \hat{H} (\partial_{p_r} \hat{H})^{-1}$$

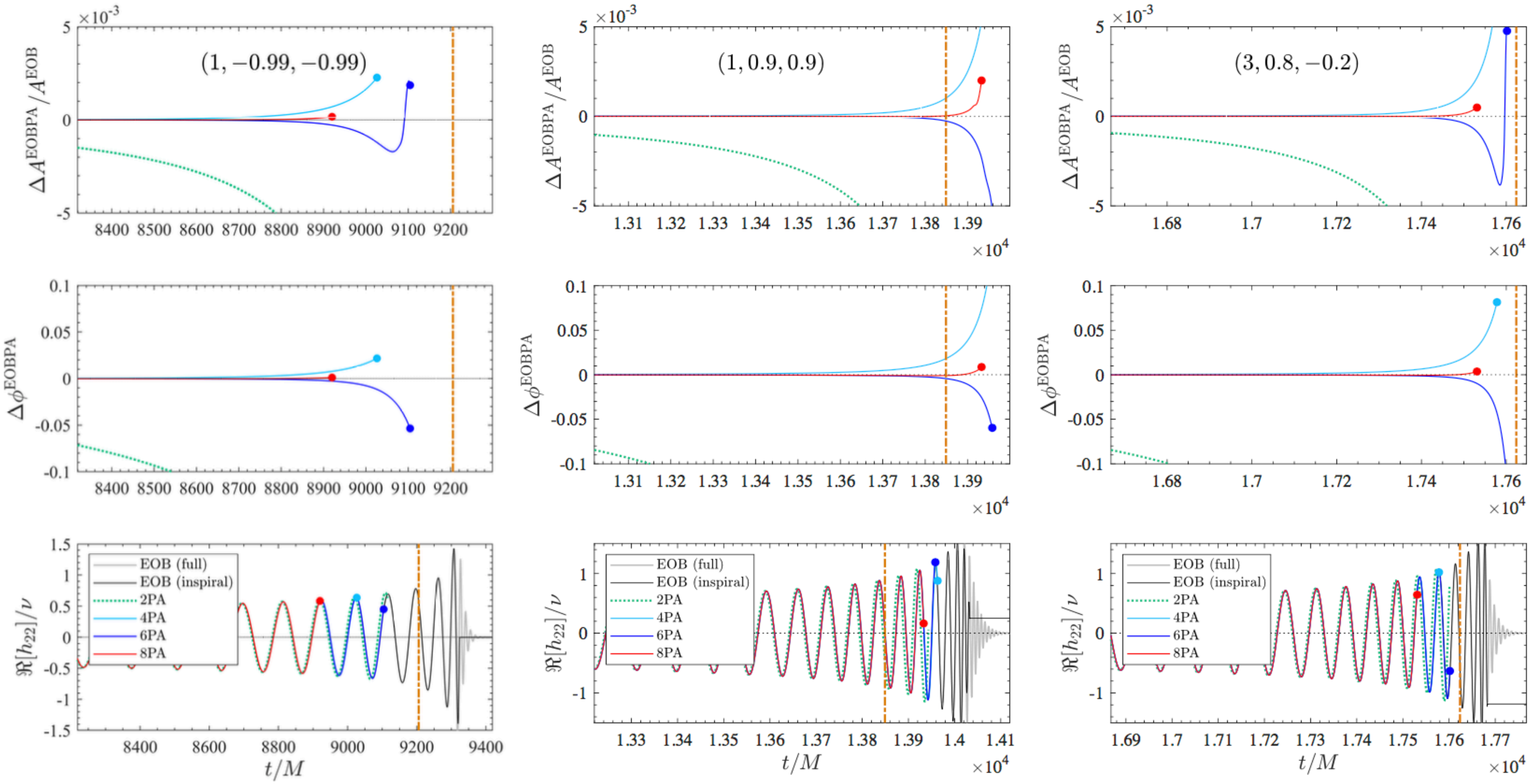


FIG. 1. Waveform comparison, $\ell = m = 2$ strain mode: EOB_{PA} inspiral (colors) versus EOB inspiral obtained solving the ODEs (black). The orange vertical line marks the EOB LSO crossing for $(1, -0.99, -0.99)$ and $(3, +0.80, -0.20)$, while it corresponds to $r = 6$-crossing for $(1, +0.90, +0.90)$. The 4PA approximation already delivers an acceptable $\text{EOB}/\text{EOB}_{\text{PA}}$ agreement for both phase, ϕ, and amplitude, A. This is improved further by the successive approximations. At 8PA, the GW phase difference is $\lesssim 10^{-3}$ rad up to ~ 3 orbits before merger. The light-gray curve also incorporates the EOB-merger and ringdown.

TEOBResumS and GW150914

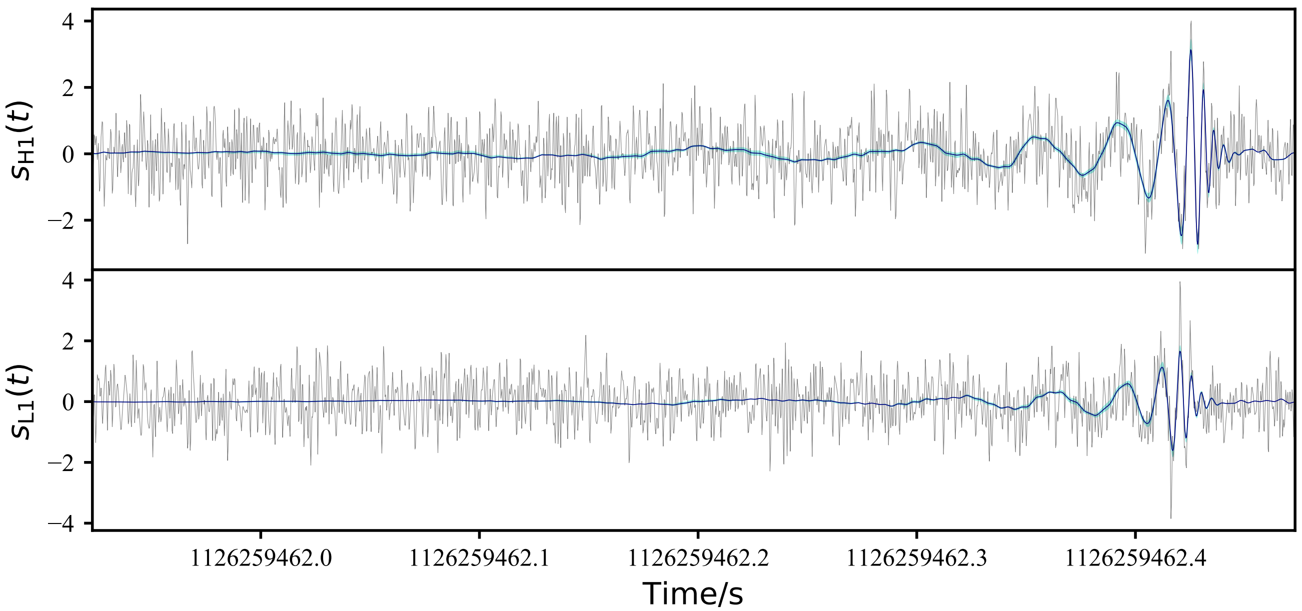
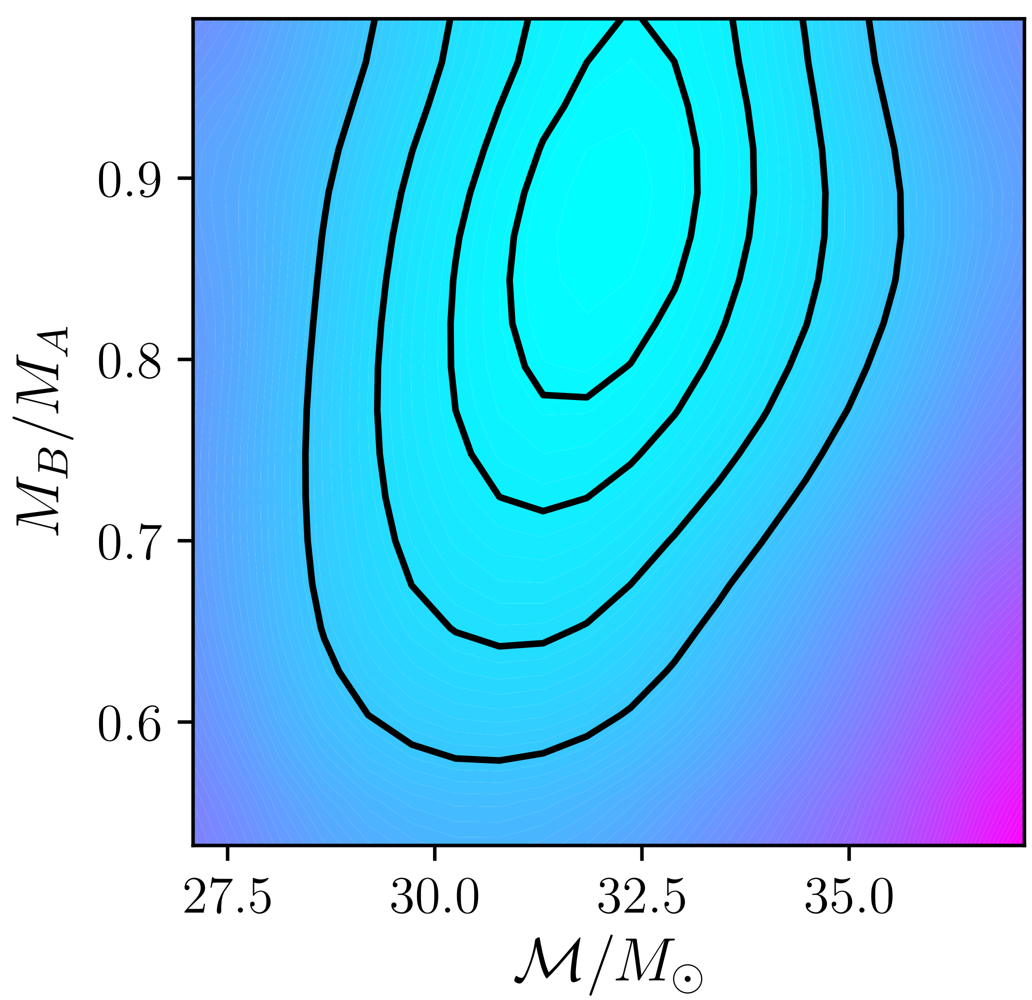


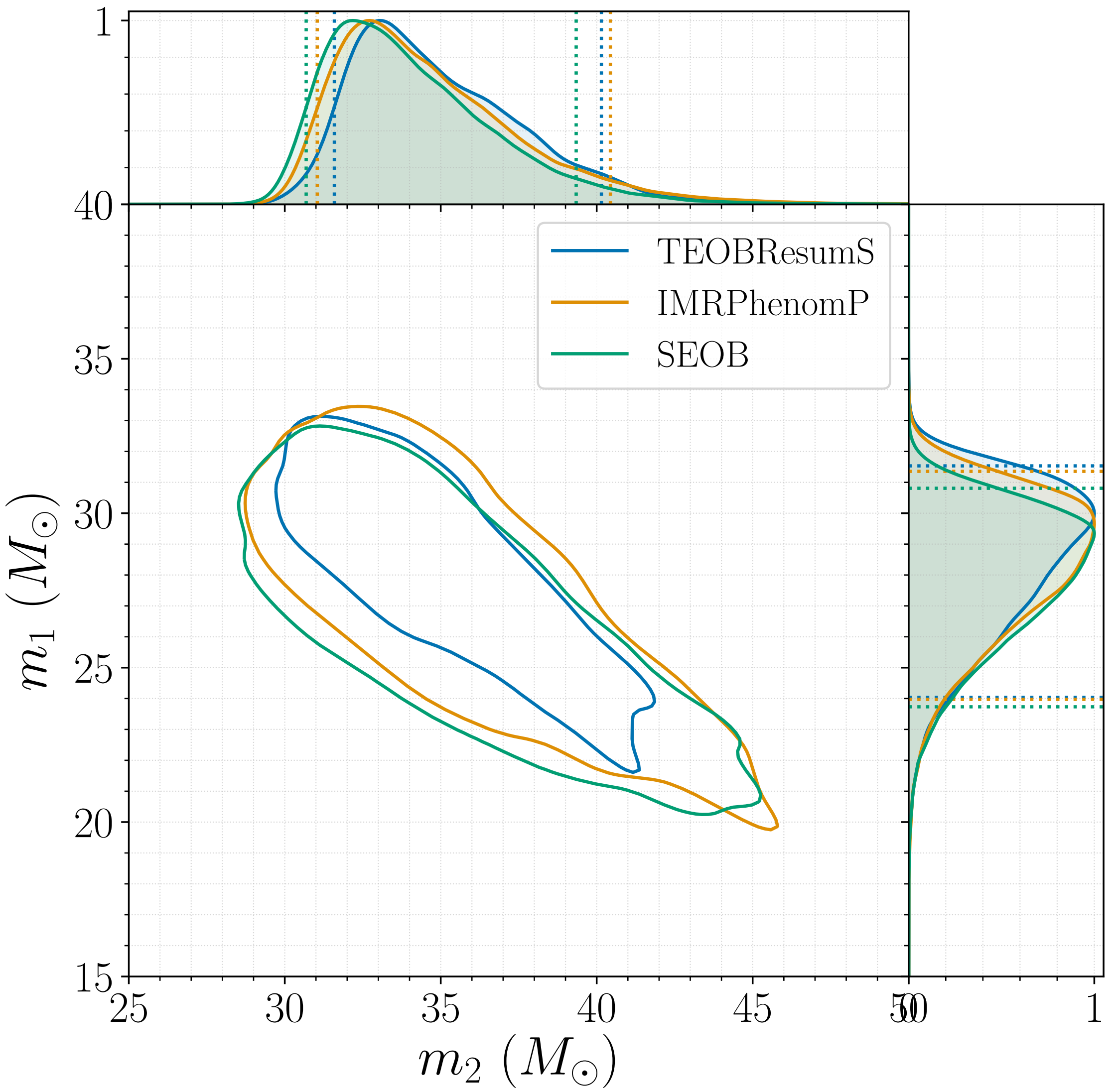
TABLE IV. Summary of the parameters that characterize GW150914 as found by `cpnest` and using `TEOBResumS` as template waveform, compared with the values found by the LVC collaboration [135]. We report the median value as well as the 90% credible interval. For the magnitude of the dimensionless spins $|\chi_A|$ and $|\chi_B|$ we also report the 90% upper bound. Note that we use the notation $\chi_{\rm eff} \equiv \hat{a}_0$ for the effective spin, as introduced in Eq. (8).

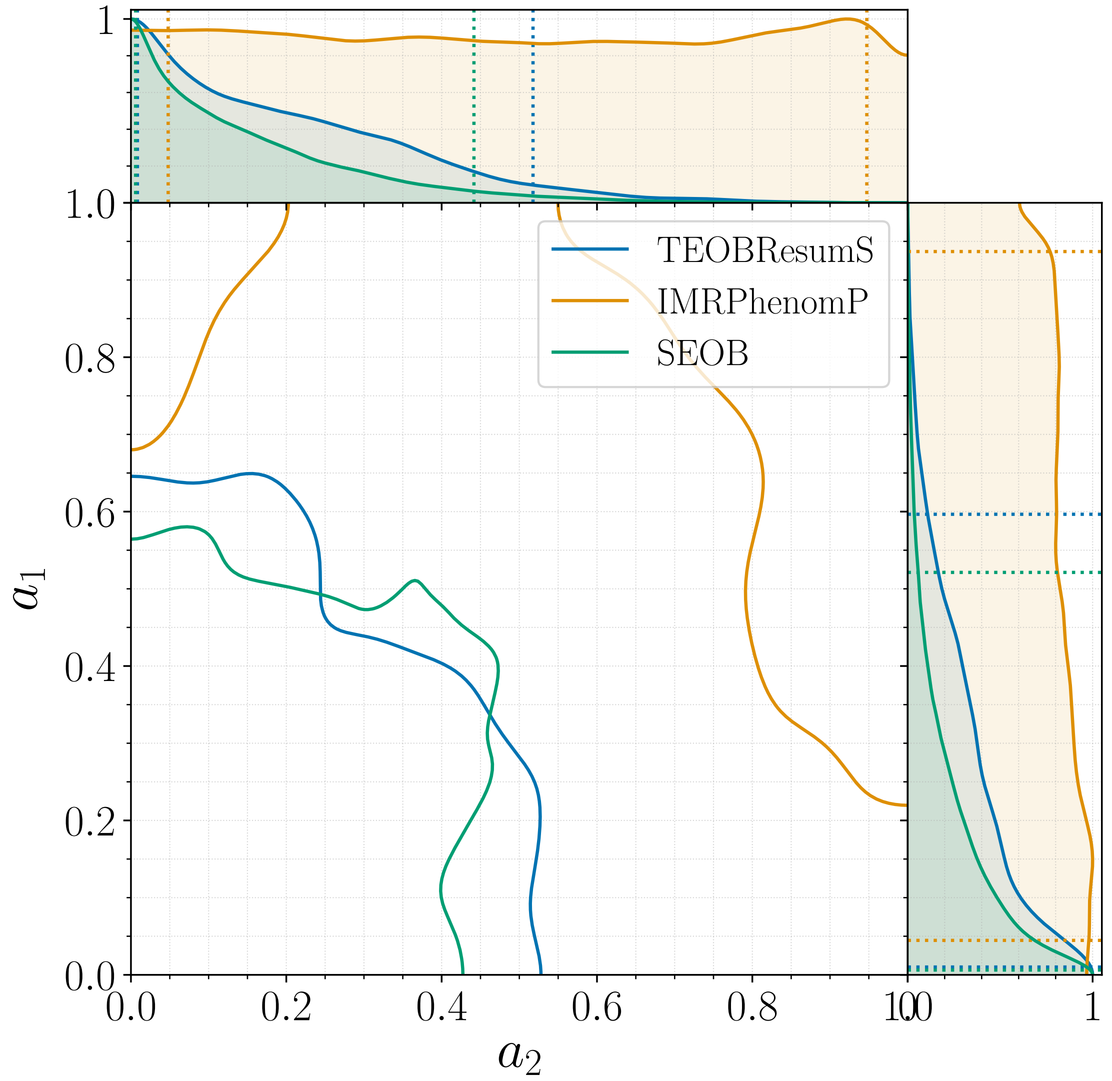
	`TEOBResumS`	LVC
Detector-frame total mass $M/M_\odot$	$73.6^{+5.7}_{-5.2}$	$70.6^{+4.6}_{-4.5}$
Detector-frame chirp mass $\mathcal{M}/M_\odot$	$31.8^{+2.6}_{-2.4}$	$30.4^{+2.1}_{-1.9}$
Detector-frame remnant mass $M_f/M_\odot$	$70.0^{+5.0}_{-4.6}$	$67.4^{+4.1}_{-4.0}$
Magnitude of remnant spin $\hat{a}_f$	$0.71^{+0.05}_{-0.07}$	$0.67^{+0.05}_{-0.07}$
Detector-frame primary mass $M_A/M_\odot$	$40.2^{+5.1}_{-3.7}$	$38.9^{+5.6}_{-4.3}$
Detector-frame secondary mass $M_B/M_\odot$	$33.5^{+4.0}_{-5.5}$	$31.6^{+4.2}_{-4.7}$
Mass ratio M_B/M_A	$0.8^{+0.1}_{-0.2}$	$0.82^{+0.20}_{-0.17}$
Orbital component of primary spin χ_A	$0.2^{+0.6}_{-0.8}$	$0.32^{+0.49}_{-0.29}$
Orbital component of secondary spin χ_B	$0.0^{+0.9}_{-0.8}$	$0.44^{+0.50}_{-0.40}$
Effective aligned spin $\chi_{\rm eff}$	$0.1^{+0.1}_{-0.2}$	$-0.07^{+0.16}_{-0.17}$
Magnitude of primary spin $|\chi_A|$	≤ 0.7	≤ 0.69
Magnitude of secondary spin $|\chi_B|$	≤ 0.9	≤ 0.89
Luminosity distance $d_{\rm L}/{\rm Mpc}$	479^{+188}_{-235}	410^{+160}_{-180}



Nagar, Bernuzzi, Del Pozzo et al., PRD, arXiv:1806.01772

TEOBResumS on GW150914





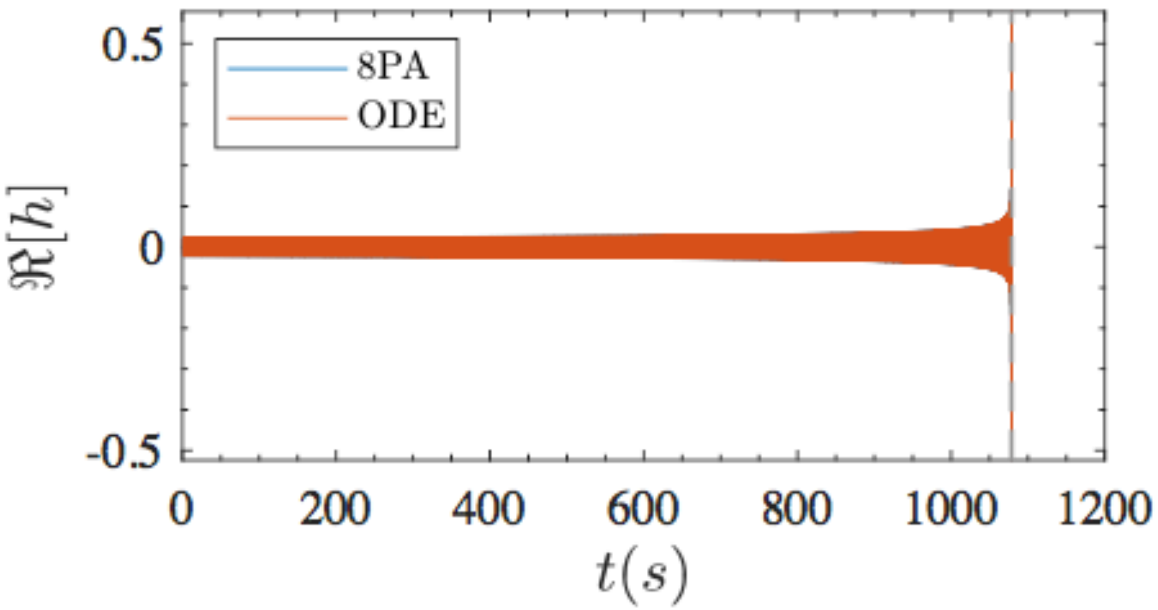
State of the art:

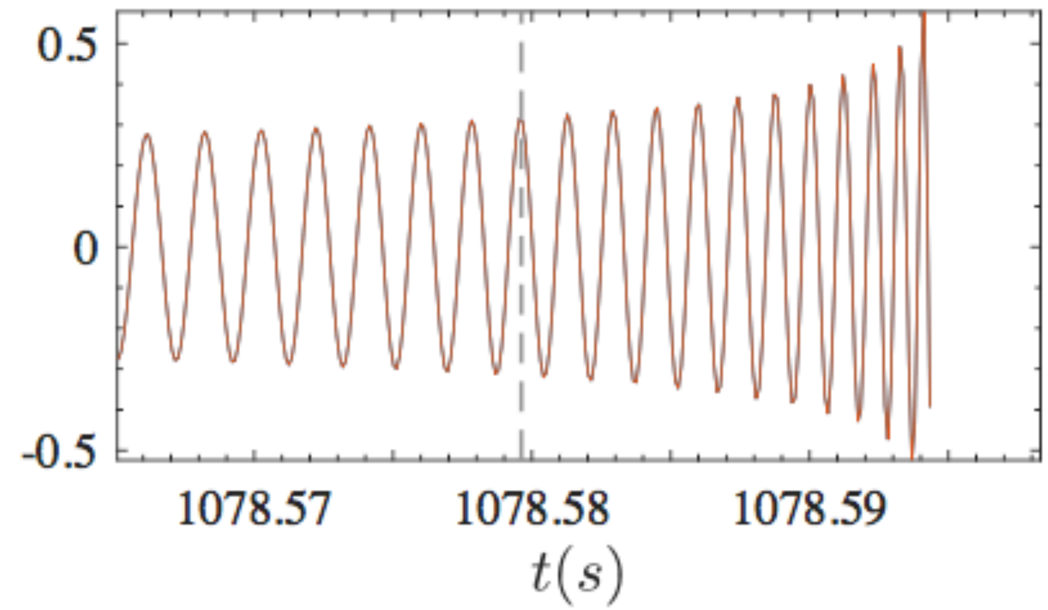
TEOBResumS: PA approx + ODE+LAL implementation

Neutron stars: tides & spin

TEOBResumS today [AN+, PRD98, 2018,104052]

- tidal effects + nonlinear-in-spin-effects *(S^2, S^3,S^4,…)* [AN+, PRD99, 2019,044007]
- analytically very complete model (almost final)
- l=3 GSF-informed + gravitomagnetic tides [Akcay+, PRD, 2019, in press]
- checked with (state-of-the-art but short) NR simulations up to merger
- EFFICIENT due to the post-adiabatic approximation [AN & Rettegno PRD99, 2019 021501]
- no precession (yet!)



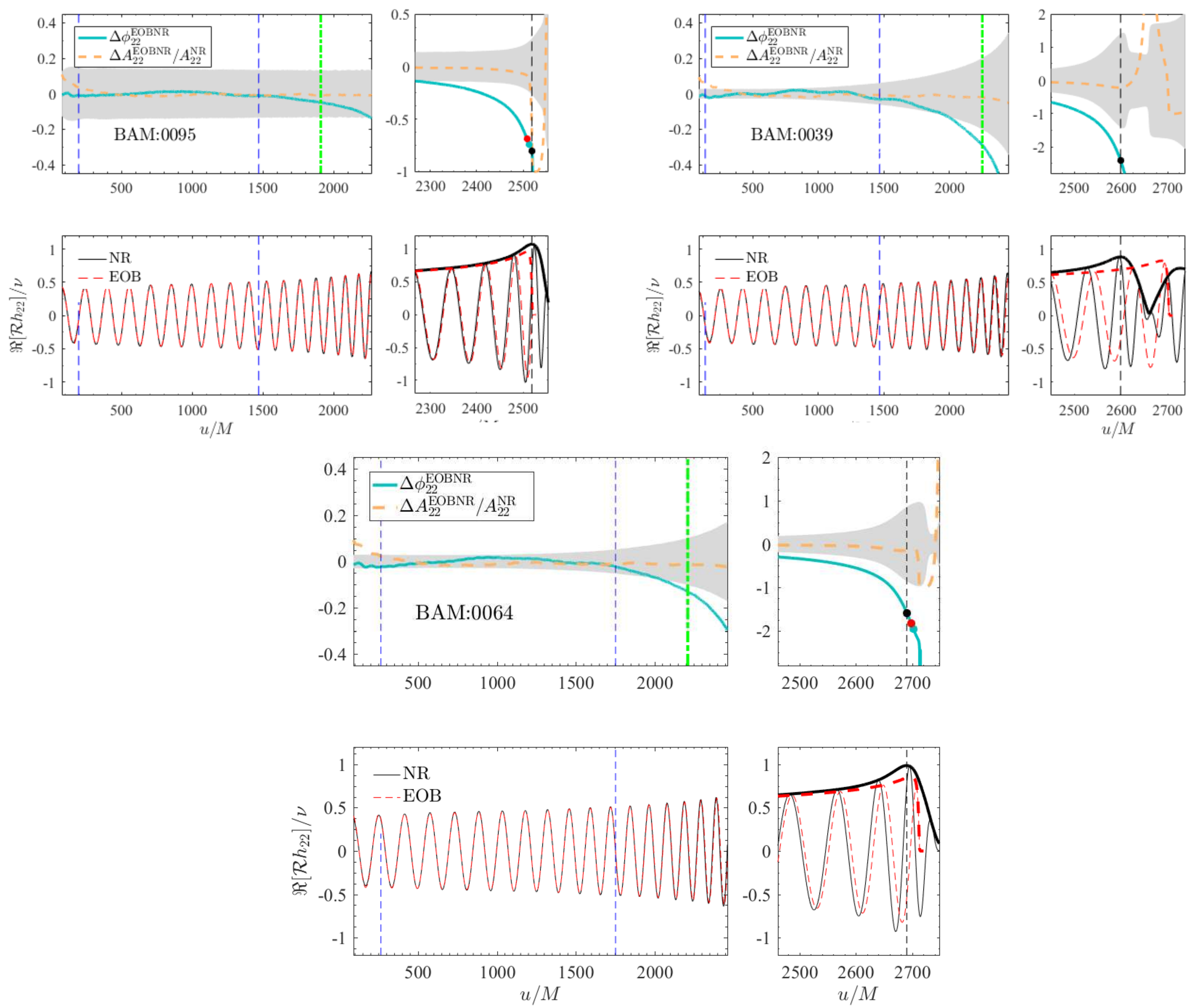


f_0 [Hz]	r_0	r_{min}	N_r	Δr	τ_{8PA} [sec]	τ_{ODE} [sec]
20	112.81	12	500	0.20	0.03	0.53
10	179.02	12	830	0.20	0.05	1.1

No real need of EOB-surrogate!

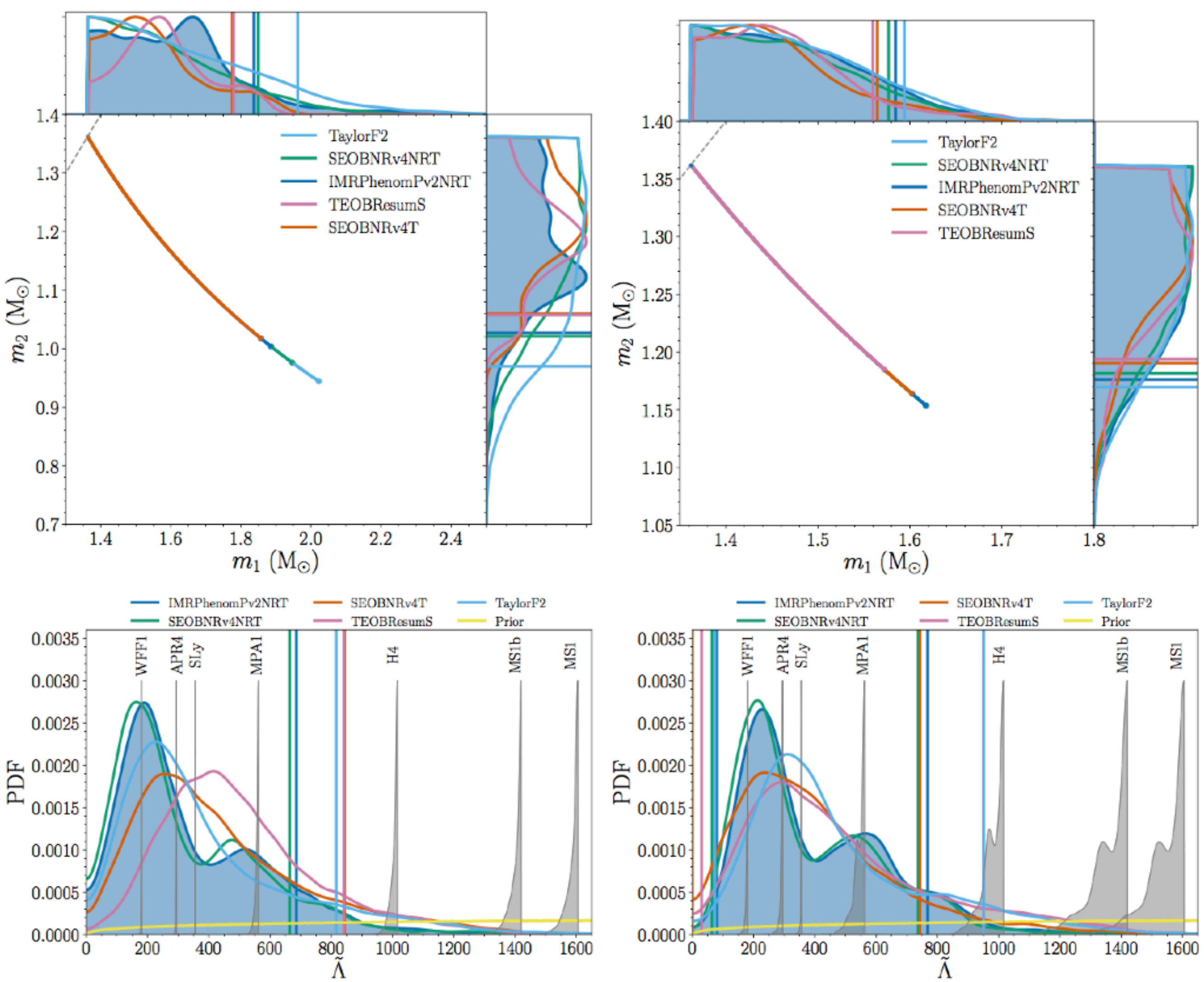
TEOBResumS vs NR: BNS

name	EOS	$M_{A,B}[M_\odot]$	$C_{A,B}$	$k_2^{A,B}$	κ_2^T	$\Lambda_2^{A,B}$	$\chi_{A,B}$	$C_{QA,QB}$
BAM:0095	SLy	1.35	0.17	0.093	73.51	392	0.0	5.491
BAM:0039	H4	1.37	0.149	0.114	191.34	1020.5	0.141	7.396
BAM:0064	MS1b	1.35	0.142	0.134	289.67	1545	0.0	8.396



GW170817- Parameter Estimation (LVC)

- Only existing EOB model independent from existing waveform models in LIGO/Virgo
- PE of the binary neutron star GW170817: arXiv:1811.12907 (GWTC-1)



$$\tilde{\Lambda} = \frac{16}{13}\frac{(m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{M^5}.$$

Masses

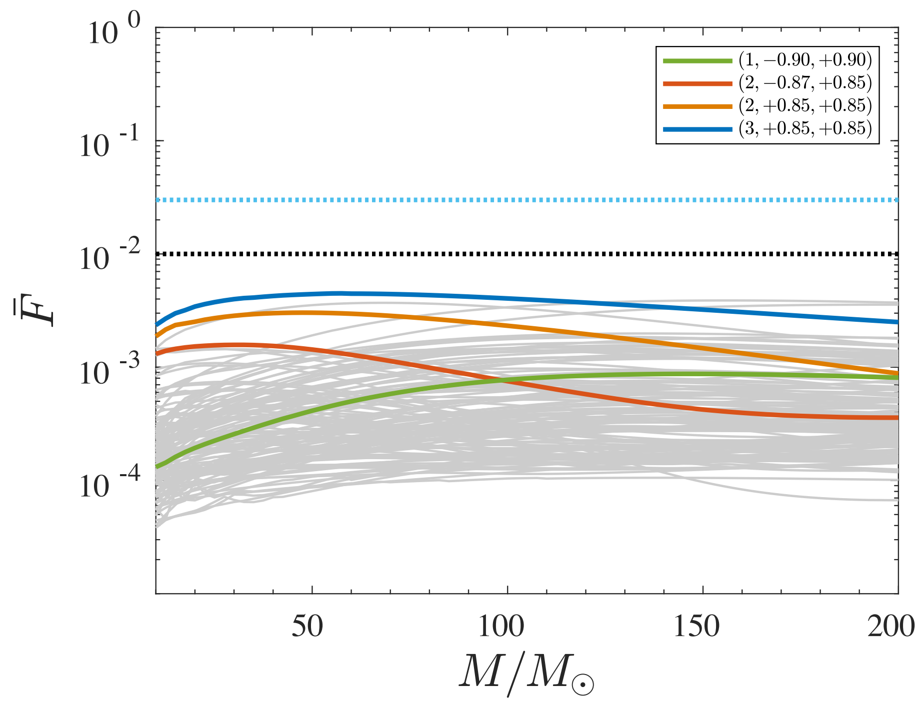
Tidal polarizability (EOS)

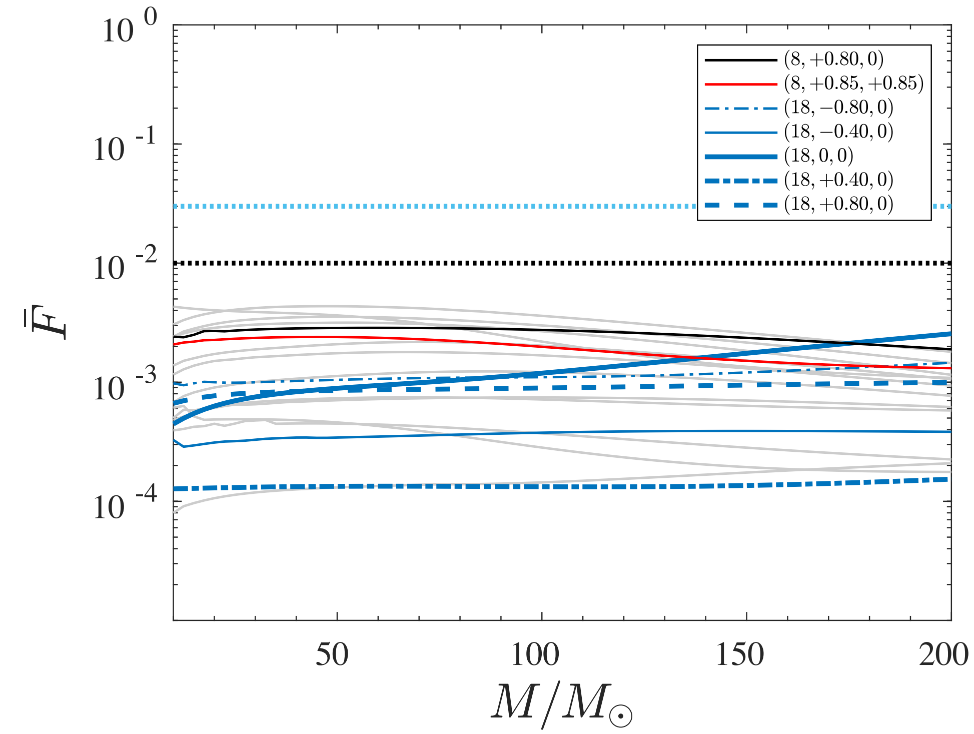
BIASES ARE POSSIBLE USING BAD TIDAL MODELS!!!!

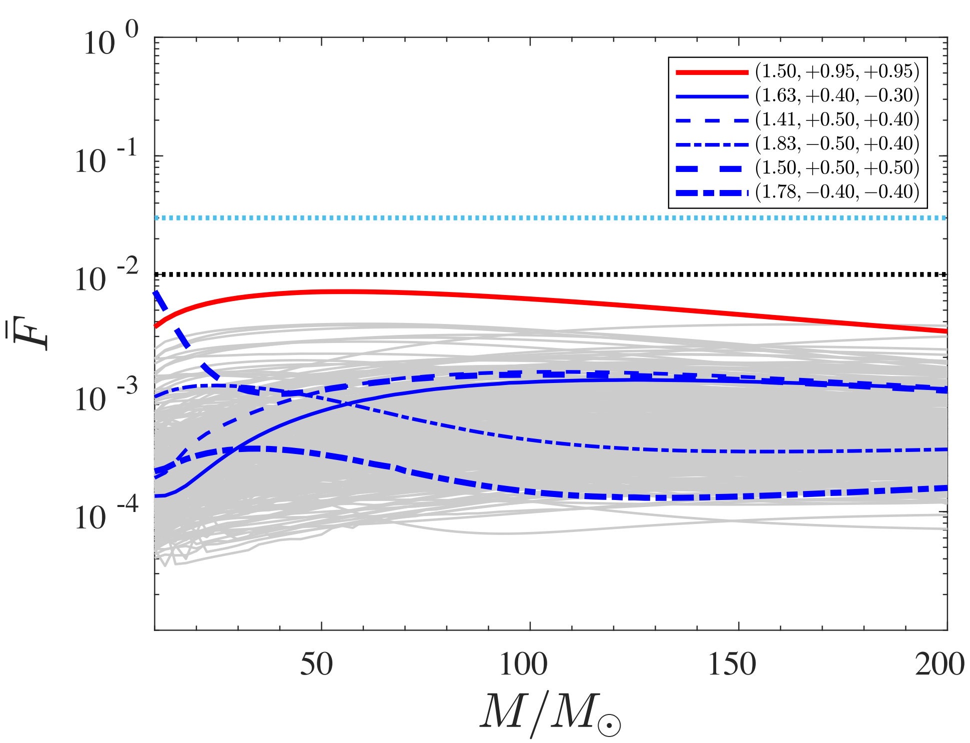
Recent development

Improved spin content in fluxes

More robust resummation of waveform amplitudes



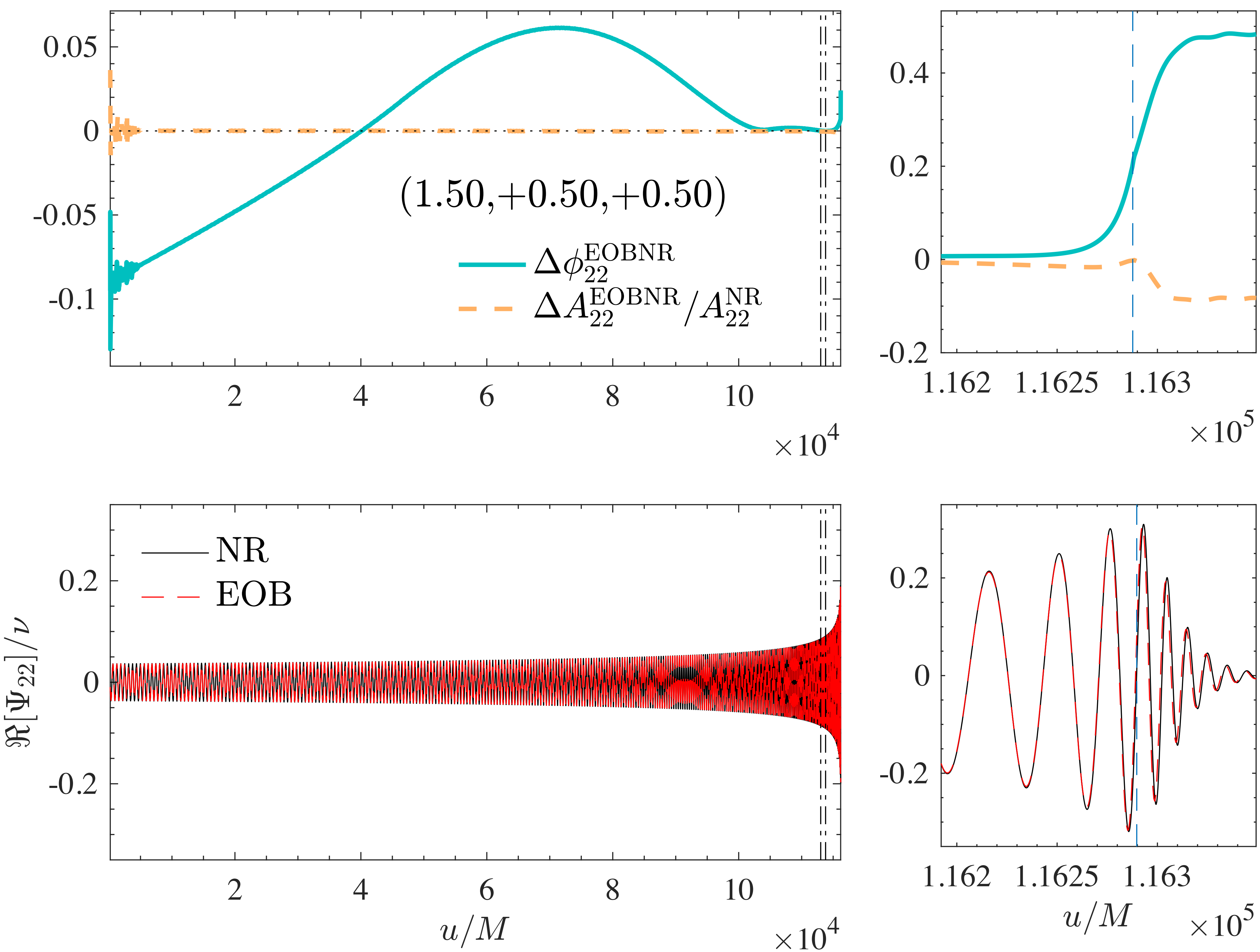




32 simulations to determine c_3

tested over132+338 waveforms

Do we trust NR?



Conclusions

SEOB vs TEOB: consistent BUT different.
Analytic differences are spelled out explicitly (see arXiv:1911.10818)
Spin sector very different!

TEOB is more efficient due to PA approx. Long inspirals.
No need of surrogate (e.g., is being used on BNS GW190426)

Good analytic modeling needed for reducing systematics. All current GW signal are going to be re-analyzed with TEOB

BBH+higher modes (no spin): arXiv:1904.09550, in press
Higher modes with spin: in progress

Next challenge: eccentricity (in progress)