Thermodynamic quark susceptibilities in the PNJL model

PhysRevD.81.114017

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Introduction

Introduction

The sign problem

$$\det(\not{D} + \mu\gamma_0 + m) = e^{i\theta}|\det(\not{D} + \mu\gamma_0 + m)|$$

- If the average phase factor vanishes in the thermodynamic limit, Monte-Carlo simulations are not possible !

To overcome the obstacle:

- Taylor expansion: thermodynamic potential expanded in μ around 0

$$P(T, \mu_u, \mu_d) = \sum_{n_u, n_d} \frac{1}{n_u! n_d!} \chi_{n_u, n_d}(T) \mu_u^{n_u} \mu_d^{n_d}$$

Studying the radius of convergence of the expansion is possible to give an **estimate** of the critical point.

Introduction

Taylor expansion with models

$$P(T,\mu_u,\mu_d) = \sum_{n_u,n_d} \frac{1}{n_u! n_d!} \chi_{n_u,n_d}(T) \mu_u^{n_u} \mu_d^{n_d}$$

- The predictions of the expansion coefficients are sensitive to effects beyond mean field.

if we reproduce Lattice data

↓

- **The model includes the correct degrees of freedom to describe the phase transition**

MONTE CARLO PNJL MODEL

PNJL model

PNJL

NJL

Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122,345

- Gauge fields are integrated out from the QCD partition function
- Four fermion interaction: chiral symmetry is spontaneously broken
- **No confinement**

PNJL

K. Fukushima, PRD 68, 045004
C. Ratti, M. Thaler and W Weise PRD 73, 014019

- In pure gauge QCD the Polyakov loop is a good order parameter for de-confinement
- Introduce the Polyakov Loop into the NJL model via a Landau-Ginzburg effective potential for Φ, Φ*
- **Effects connected with confinement appear**

$$\mathcal{L}_{NJL} = \bar\psi(\not\partial - m_0)\psi - g(\bar\psi\gamma^\mu\lambda_a\psi)(\bar\psi\gamma_\mu\lambda_a\psi)$$

$$\mathcal{Z} = \mathcal{N}\int \mathcal{D}\phi\mathcal{D}\sigma\mathcal{D}\vec\pi \ \det[S^{-1}(\sigma,\pi,\beta)]\exp\Big[-\beta\int d^3x\Big(\frac{\sigma^2+\pi^2}{2G}\Big)\Big]$$

$$\frac{\mathcal{U}(\Phi,\Phi^*,T)}{T^4} = -\frac{1}{2}a(T)\Phi^*\Phi + b(T)\ln[1-6\Phi^*\Phi+4({\Phi^*}^3+\Phi^3)-3(\Phi^*\Phi)^2]$$

$$a(T) = \sum_{i=0,3} a_i T^i$$

$$\mathcal{Z} = \mathcal{N}\int \mathcal{D}\phi\mathcal{D}\sigma\mathcal{D}\vec\pi \ \det[S^{-1}(\phi,\sigma,\pi,\beta)]\exp\Big[-\beta\int d^3x\Big(\mathcal{U}(\phi,\beta)+\Big(\frac{\sigma^2+\pi^2}{2G}\Big)\Big)\Big]$$

PNJL

The effective potential

$$\frac{\mathcal{U}(\Phi,\Phi^*,T)}{T^4} = -\frac{1}{2}a(T)\Phi^*\Phi + b(T)\ln[1-6\Phi^*\Phi+4({\Phi^*}^3+\Phi^3)-3(\Phi^*\Phi)^2]$$

mass term

integration over non-diag gluonic degrees of freedom

- Considering:

$$A^\mu(x) = \delta^{\mu 4}(t_3 A_3^4 + t_8 A_8^4)$$

CONSTANT

- we have:

$$\phi = \frac{1}{3}\left(e^{i\left(\frac{A_3^4}{T}+\frac{A_8^4}{\sqrt{3}T}\right)} + e^{i\left(-\frac{A_3^4}{T}+\frac{A_8^4}{\sqrt{3}T}\right)} + e^{i\left(-\frac{2A_8^4}{\sqrt{3}T}\right)}\right)$$

POLYAKOV LOOP

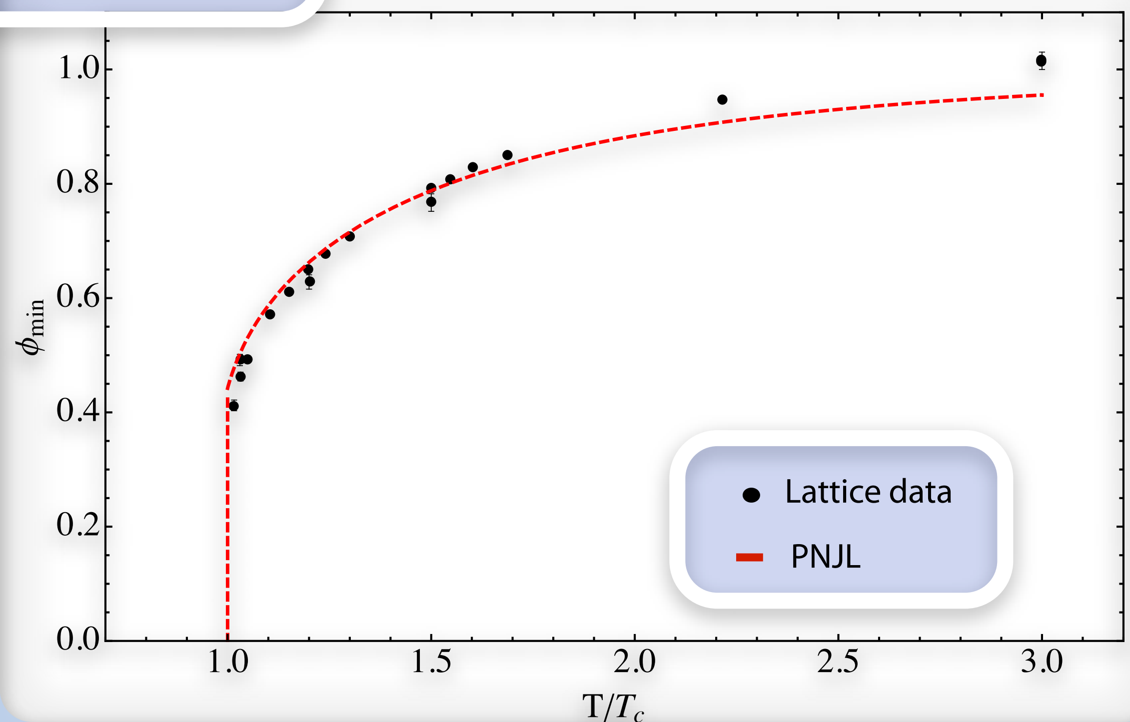
- **The effective potential is a function of A_3 and A_8**

PNJL

mean field PNJL

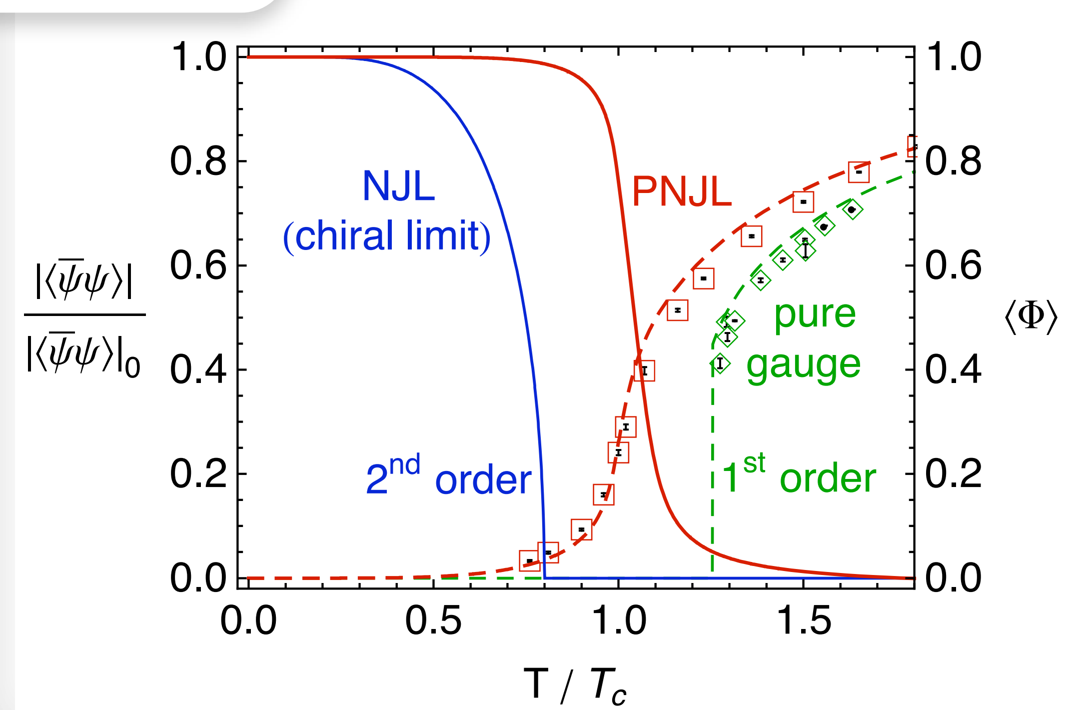
$$\mathcal{Z} = \mathcal{N} \int \mathcal{D}\phi \mathcal{D}\sigma \mathcal{D}\vec{\pi}\ \det[S^{-1}(\phi,\sigma,\pi,\beta)] \exp\Big[-\beta \int d^3x \Big(\mathcal{U}(\phi,\beta) + \Big(\frac{\sigma^2+\pi^2}{2G}\Big)\Big)\Big]$$

Pure Gauge



2 flavor

S. Rößnerr, T. Hell, C. Ratti and W. Weise, arxiv:0712.3152



- Agreement with lattice data
- Critical temperature: ~200 MeV

PNJL

mean field PNJL: susceptibilities

$$\Omega(T,\mu) = \frac{1}{VT^3}\ln\mathcal{Z} = \sum_{i,j=0}^{\infty}\chi_{i,j}(T)\left(\frac{\mu_u}{T}\right)^i\left(\frac{\mu_d}{T}\right)^j$$

$$\chi_{i,j}(T) = \frac{1}{i!j!}\,\frac{\partial^{i+j}\Omega}{\partial(\mu_u/T)^i\partial(\mu_d/T)^j}\bigg|_{\mu_{u,d}=0}$$

- Partition function depends from μ through the fermionic determinant

$$\mathcal{Z} = \mathcal{N}\int\mathcal{D}\phi\mathcal{D}\sigma\mathcal{D}\vec{\pi}\,\det[S^{-1}(\mu_u,\mu_d)]\exp\Big[-\beta\int d^3x\Big(\mathcal{U}(\phi,\beta)+\Big(\frac{\sigma^2+\pi^2}{2G}\Big)\Big)\Big]$$

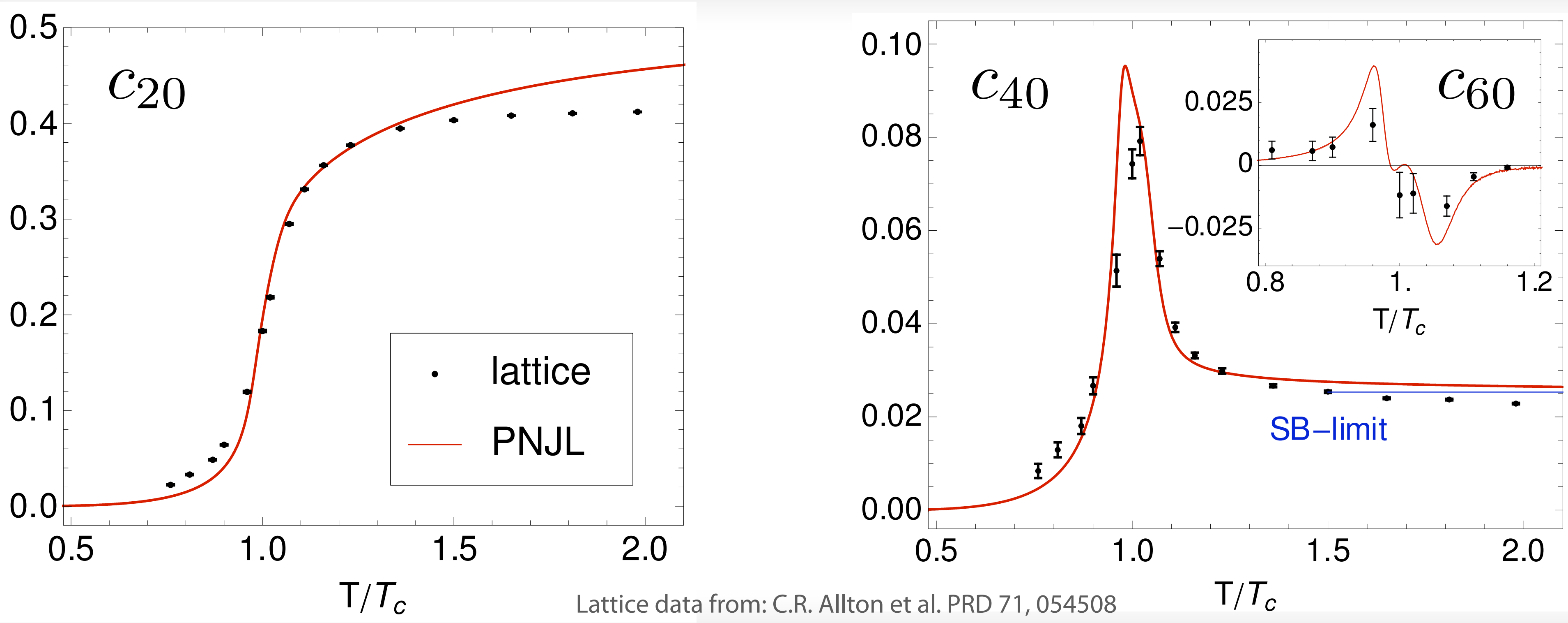
$$S^{-1} = \begin{pmatrix} -\not\partial+(\mu_u-iA_4)\gamma_0+i\gamma_5\pi^0-M & i\gamma_5\pi^+ \\ i\gamma_5\pi^- & -\not\partial+(\mu_d-iA_4)\gamma_0+i\gamma_5\pi^0-M \end{pmatrix}$$

PNJL

mean field PNJL: susceptibilities

$$\chi_{i,j}(T) = \frac{1}{i!j!}\frac{\partial^{i+j}\Omega}{\partial(\mu_u/T)^i\partial(\mu_d/T)^j}\bigg|_{\mu_{u,d}=0}$$

c_{n0}

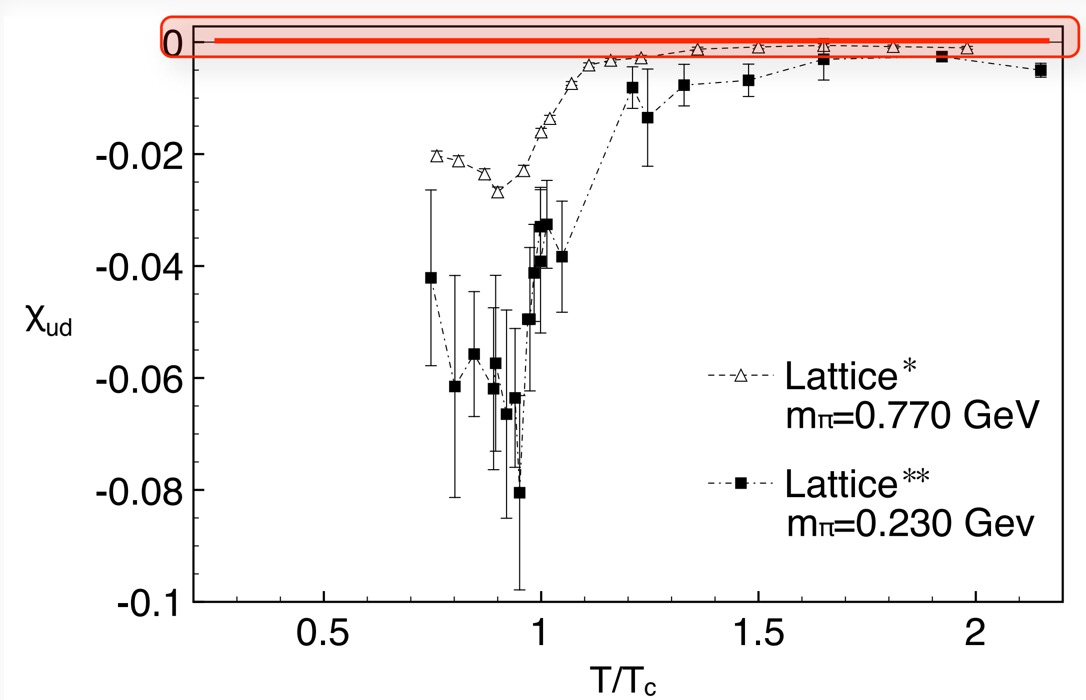


Lattice data from: C.R. Allton et al. PRD 71, 054508

mean field PNJL: susceptibilities

$$\chi_{i,j}(T) = \frac{1}{i!j!}\,\frac{\partial^{i+j}\Omega}{\partial(\mu_u/T)^i\partial(\mu_d/T)^j}\bigg|_{\mu_{u,d}=0}$$

c_{11}



* C.R. Allton et al. PRD 71, 054508 ** R.V. Gavai and S. Gupta PRD 78, 114503

At the **mean field** level the second non-diagonal coefficient is **zero**:

$$S^{-1} = \begin{pmatrix} s_{11}(\mu,\mu_I,\phi,\sigma) & i\gamma_5\pi \\ i\gamma_5\pi & s_{22}(\mu,\mu_I,\phi,\sigma) \end{pmatrix}$$

The expectation value of **π** and **A8** is **zero** for $\mu=\mu_I=0$

Monte-Carlo PNJL

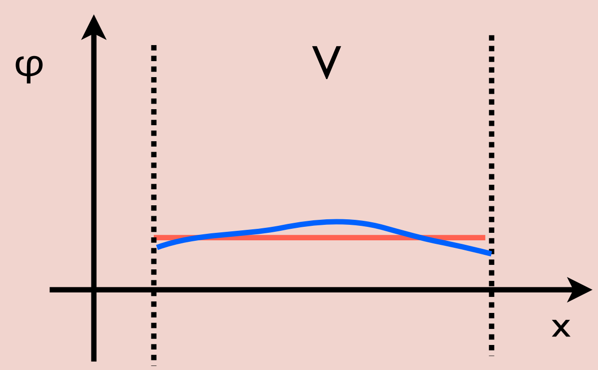
Monte-Carlo PNJL

MC-PNJL

PNJL in a box

$$\mathcal{Z} = \int \mathcal{D}\phi\mathcal{D}\sigma\mathcal{D}\pi \exp\left[\beta V \frac{1}{2}\sum_n \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \mathrm{Tr}\ln[S^{-1}(i\omega_n, \vec{p})] - \beta V \left(\mathcal{U}(\phi,\beta) + \left(\frac{\sigma^2+\pi^2}{2G}\right)\right)\right]$$

New parameter V



zero mode fluctuations

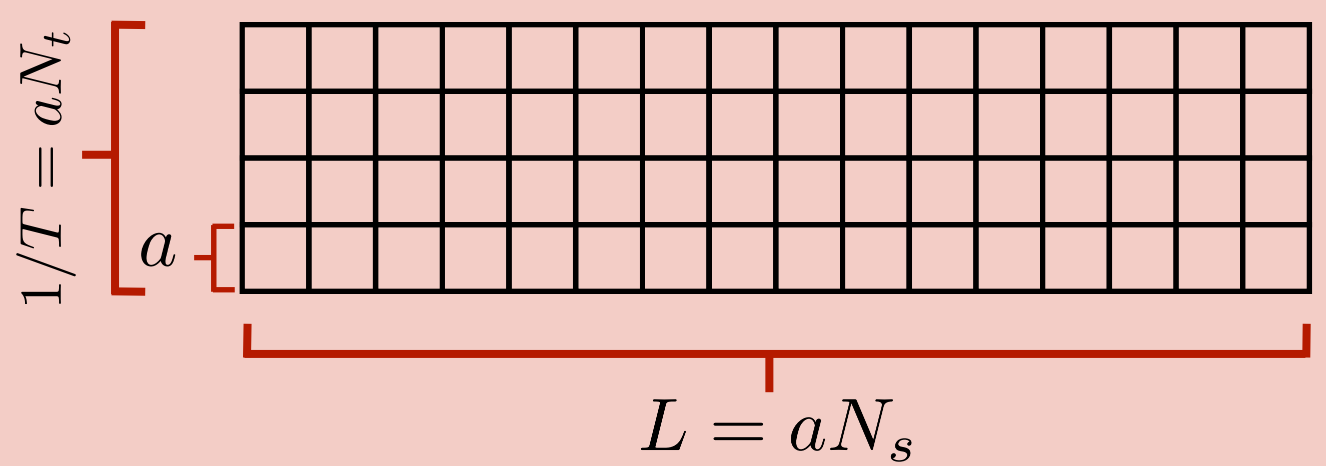
Monte-Carlo method to compute the partition function

$$\sigma^j = \sigma^i + \delta_\sigma \cdot r()$$
$$\pi^j = \pi^i + \delta_\pi \cdot r()$$
$$\phi_3^j = \phi_3^i + \delta_{\phi_3} \cdot r()$$
$$\phi_8^j = \phi_8^i + \delta_{\phi_8} \cdot r()$$

MC-PNJL

Fixing the volume

- Calculations performed at different volume size V

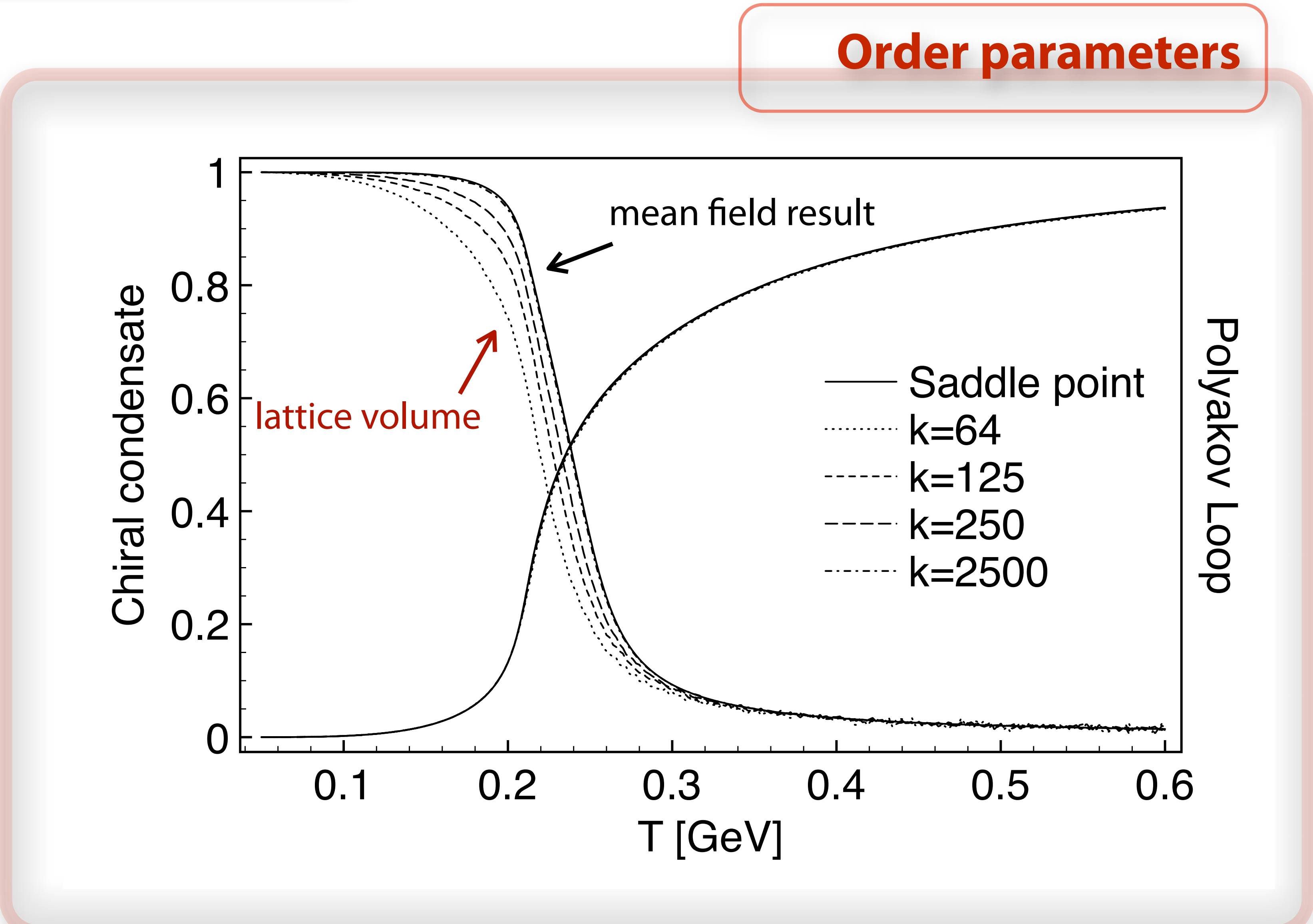


Available Lattice data

$$a = \frac{1}{N_t T} \quad \rightarrow \quad V = N_s^3 a^3 = \frac{N_s^3}{N_t^3 T^3} \qquad N_s = 4N_t \rightarrow V = \frac{64}{T^3}$$

MC-PNJL

Order parameters

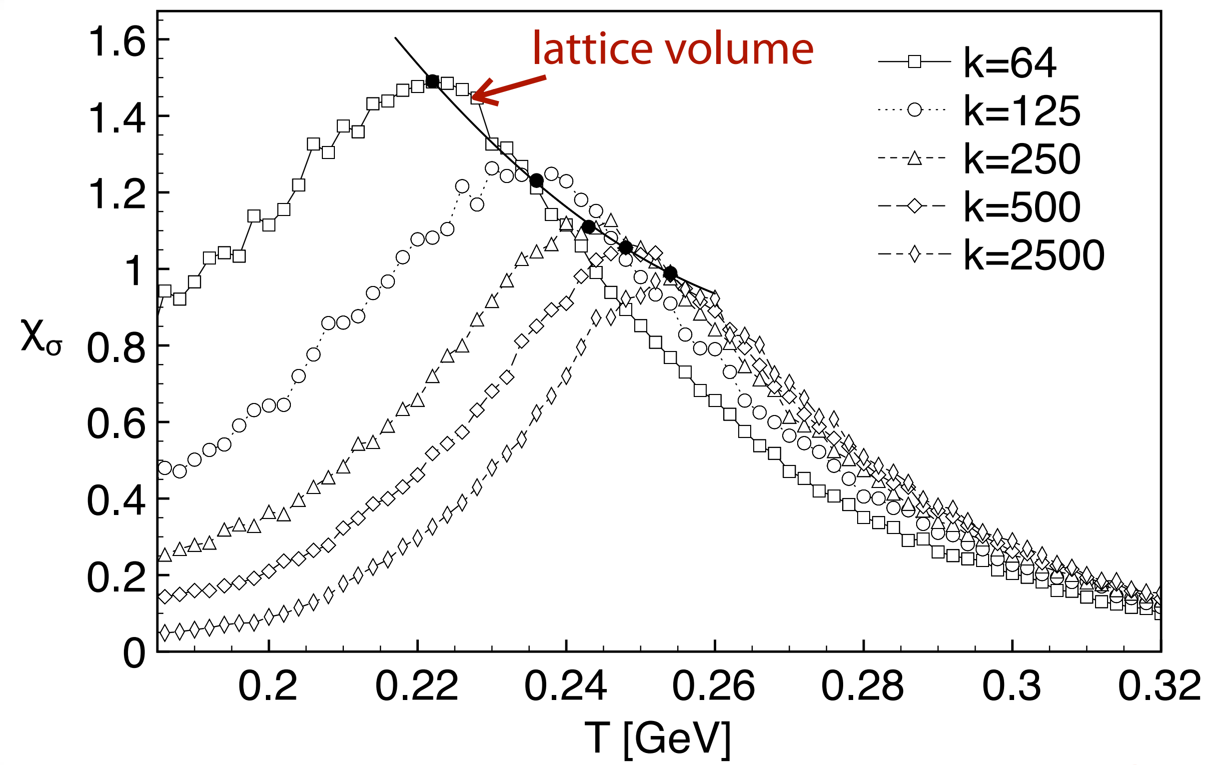


- The effect of fluctuations is negligible for the Polyakov loop
- Only small corrections for the chiral condensate

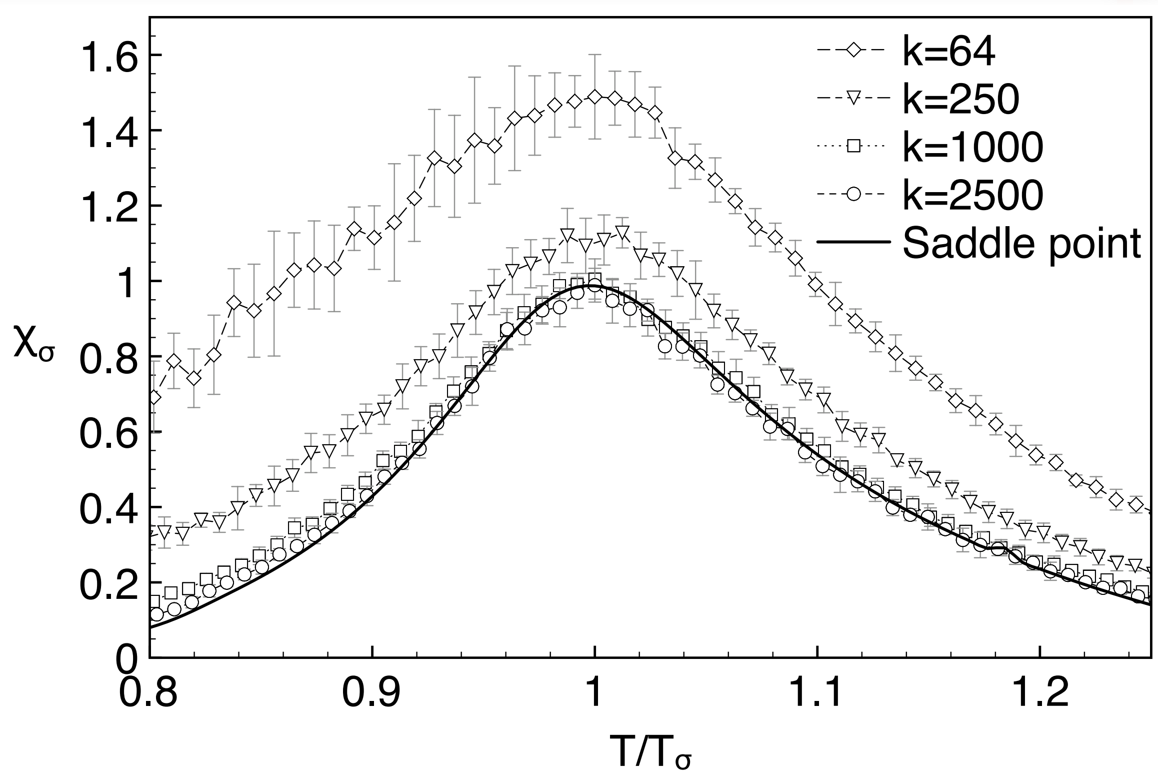
MC-PNJL

Chiral susceptibility

$$
\begin{aligned}
\chi_{\overline{\psi}\psi} &= \frac{T}{V}\frac{\partial^2}{\partial m^2}\ln\mathcal{Z} \\
&= \frac{V}{T}\Big[\Big\langle\frac{\partial \ln\det M[m,T,f,A]}{\partial m}^2\Big\rangle - \\
&\quad \Big(\Big\langle\frac{\partial \ln\det M[m,T,f,A]}{\partial m}\Big\rangle\Big)^2\Big] + \\
&\quad \Big\langle\frac{\partial^2 \ln\det M[m,T,f,A]}{\partial m^2}\Big\rangle
\end{aligned}
$$



Chiral susceptibility



Saddle point limit

Off-diagonal
2nd moment of the pressure

2nd moment of the pressure
Off-diagonal

Off-diagonal 2nd moment of the pressure

Definitions

$$\chi_{ud} = \frac{T^2}{VT^3} \frac{\partial^2}{\partial\mu_u \partial\mu_d} \ln \mathcal{Z}$$

μ dependence only in this term

$$\mathcal{Z} = \int \mathcal{D}\phi_3 \mathcal{D}\phi_8 \mathcal{D}\sigma \mathcal{D}\vec{\pi} \Bigg(\exp\left[\beta V \sum_n \int \frac{d^3p}{(2\pi)^3} \ln\det[M(\phi_3, \phi_8, \sigma, \vec{\pi}; T, \mu_u, \mu_d; i\omega_n, \vec{p})] \right] - \exp\left[\beta V \left(\mathcal{U}(\phi_3, \phi_8; \beta) + \left(\frac{\sigma^2 + \vec{\pi}^2}{2G} \right) \right) \right] \Bigg)$$

$$\begin{aligned}
\frac{\partial^2}{\partial\mu_u \partial\mu_d} \ln \mathcal{Z} = & \frac{V}{T} \left\langle \frac{\partial^2}{\partial\mu_u \partial\mu_d} \ln\det[M(f; T, \mu_u, \mu_d)] \right\rangle \\
& + \left(\frac{V}{T}\right)^2 \left\langle \left(\frac{\partial}{\partial\mu_u} \ln\det[M(f; T, \mu_u, \mu_d)] \right)^2 \right\rangle \\
& - \left(\frac{V}{T}\right)^2 \left(\left\langle \frac{\partial}{\partial\mu_u} \ln\det[M(f; T, \mu_u, \mu_d)] \right\rangle \right)^2
\end{aligned}$$

Off-diagonal 2nd moment of the pressure

Definitions

$$\lambda_1 = \frac{A_3}{T} + \frac{A_8}{\sqrt{3}T}, \quad \lambda_2 = -\frac{A_3}{T} + \frac{A_8}{\sqrt{3}T}, \quad \lambda_3 = -\frac{2A_8}{\sqrt{3}T}$$

$$\begin{aligned}\frac{\partial^2}{\partial\mu_u\partial\mu_d}\ln\mathcal{Z} = & \frac{V}{T}\left\langle \frac{\partial^2}{\partial\mu_u\partial\mu_d}\ln\det[M(f;T,\mu_u,\mu_d)]\right\rangle \\ & + \left(\frac{V}{T}\right)^2\left\langle\left(\frac{\partial}{\partial\mu_u}\ln\det[M(f;T,\mu_u,\mu_d)]\right)^2\right\rangle \\ & - \left(\frac{V}{T}\right)^2\left(\left\langle\frac{\partial}{\partial\mu_u}\ln\det[M(f;T,\mu_u,\mu_d)]\right\rangle\right)^2 = 0\end{aligned}$$

$$\varepsilon(p) = \sqrt{p^2 + (m_0-\sigma)^2 + \vec{\pi}^2}$$

$$\begin{aligned}&\frac{\partial^2}{\partial\mu_u\partial\mu_d}\ln\det[M,T,\mu_u,\mu_d,f,A] = \\ &\frac{1}{\pi^2}\int_0^\infty dp\, p^2 \pi^+\pi^-\left(\sum_{i=1}^{3}\left(\frac{1+\cos(\lambda_i)(\cosh(\varepsilon(p)/T)}{T\varepsilon(p)^2(\cos(\lambda_i)+\cosh(\varepsilon(p)/T))^2} + \frac{\cos(\lambda_i)}{\varepsilon(p)^3(\cos(\lambda_i)+\cosh(\varepsilon(p)/T))}\right.\right. \\ &\left.\left. + \frac{\cosh(\varepsilon(p)/T)-\sinh(\varepsilon(p)/T)}{\varepsilon(p)^3(\cos(\lambda_i)+\cosh(\varepsilon(p)/T))}\right)\right) - \frac{1}{\pi^2}\int_0^\Lambda dp \frac{3p^2\pi^+\pi^-}{\varepsilon(p)^3}\end{aligned}$$

0 if π=0

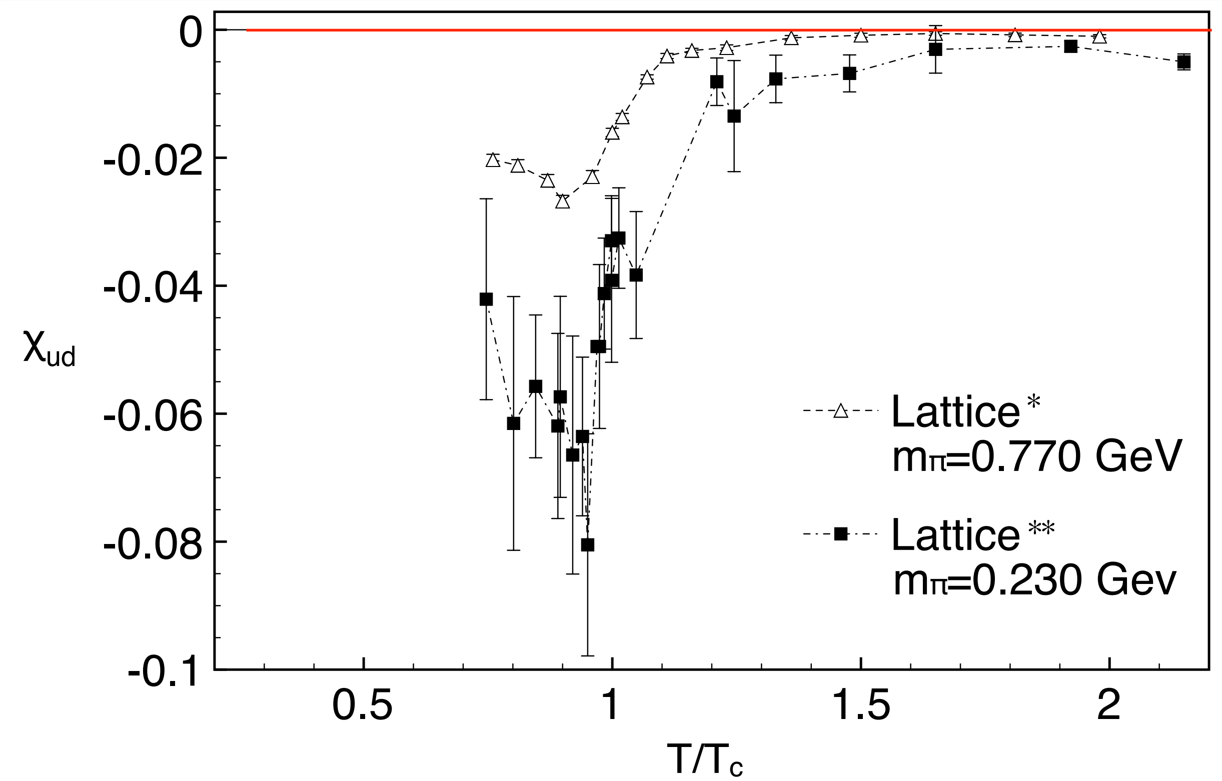
$$\begin{aligned}&\frac{\partial}{\partial\mu_u}\ln\det[M,T,\mu_u,\mu_d,f,A] = \\ &i\frac{1}{\pi^2}\int_0^\infty dp\, p^2\left(\frac{\sin\lambda_1}{\cos\lambda_1+\cosh(\varepsilon(p)/T)} + \frac{\sin\lambda_2}{\cos\lambda_2+\cosh(\varepsilon(p)/T)} + \frac{\sin\lambda_3}{\cos\lambda_3+\cosh(\varepsilon(p)/T)}\right)\end{aligned}$$

0 if Φ_8=0

odd function of Φ_8

Off-diagonal 2nd moment of the pressure

Mean Field:
comparison with lattice data



* C.R. Allton et al. PRD 71, 054508

** R.V. Gavai and S. Gupta PRD 78, 114503

χ_{11}

$$\frac{\partial^2}{\partial\mu_u\partial\mu_d}\ln\det[M(f;T,\mu_u,\mu_d)] = 0$$

π=0 in MF!

$$\frac{\partial}{\partial\mu_u}\ln\det[M(f;T,\mu_u,\mu_d)] = 0$$

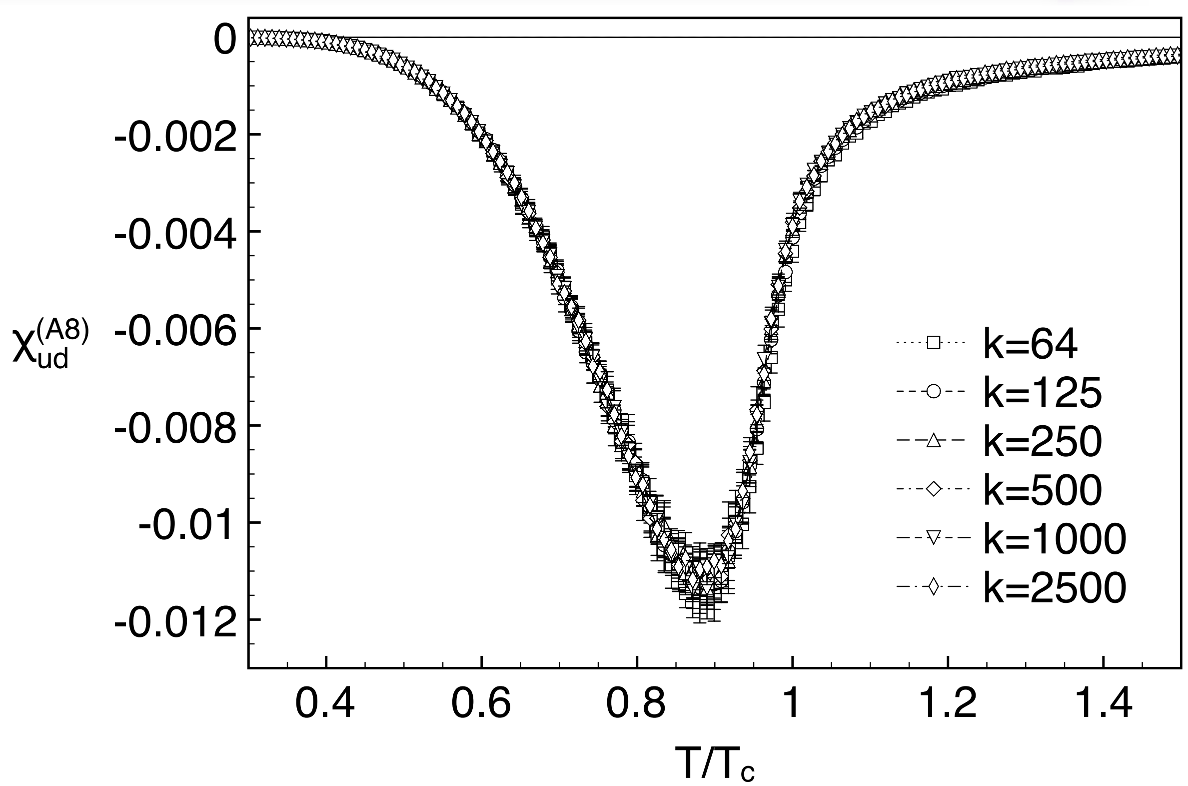
Φ_8=0 in MF!

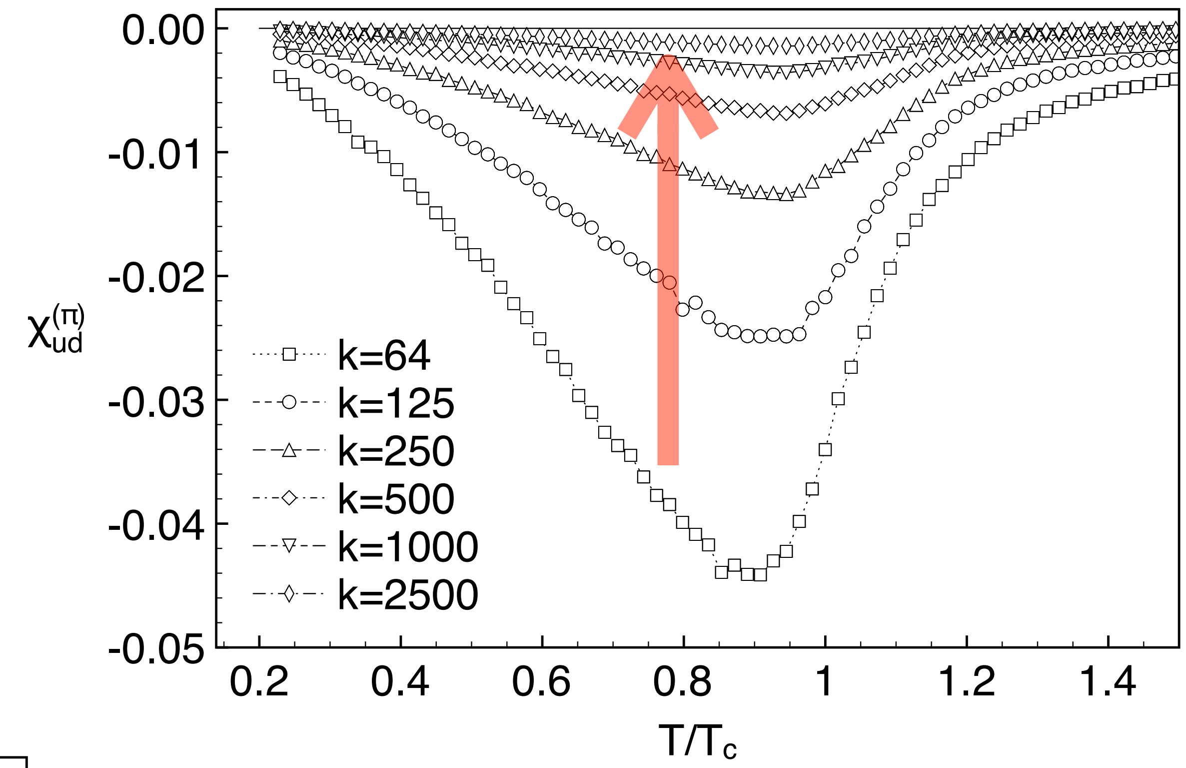
Remember MF result

Off-diagonal 2nd moment of the pressure

Separate contributions

δA8≠0 δπ=0



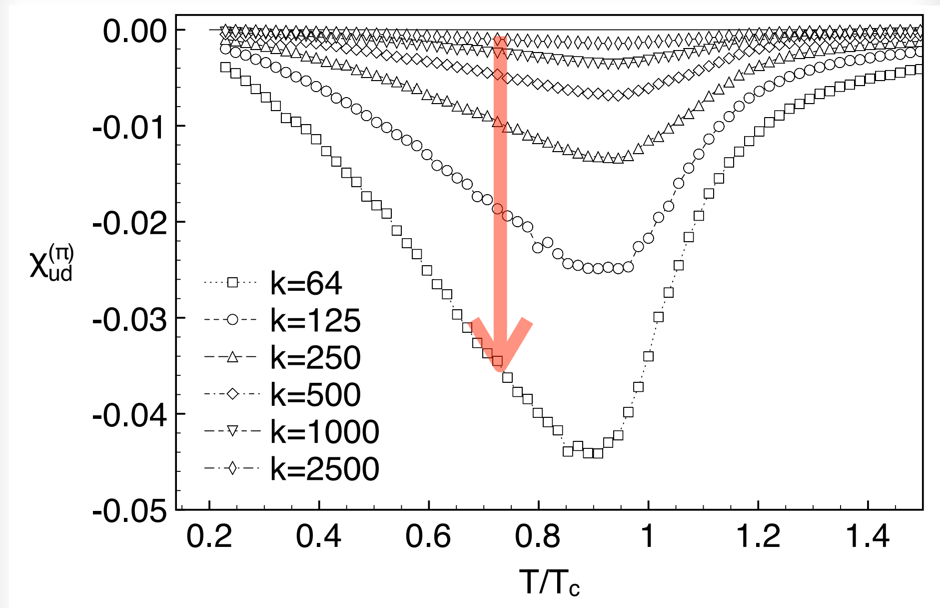


δA8=0 δπ≠0

- The contribution from the A8 field is volume independent
- The contribution from pionic zero modes is smaller for larger volumes

Off-diagonal 2nd moment of the pressure

Role of pions from ChPT



ChPT Lagrangian, 2nd order in the π fields

$$\mathcal{L} = \frac{1}{2}(\partial_\nu \pi^a)(\partial_\nu \pi^a) + i2\mu_I f_\pi(\partial_0\pi^3) + i2\mu_I\left[(\partial_0\pi^1)\pi^2 - (\partial_0\pi^2)\pi^1\right] + \frac{1}{2}m_\pi^2\pi^a\pi^a - 2\mu_I^2(\pi^1\pi^1+\pi^2\pi^2)$$

δA8=0 δπ≠0

Partition function with only static part

$$Z_{\mathrm{static}} = \int\prod_{a=1}^{3} d\pi^a \exp -\frac{V}{T}\left[\frac{1}{2}m_\pi^2\pi^a\pi^a - 2\mu_I^2(\pi^1\pi^1+\pi^2\pi^2)\right] = \int\prod_{a=1}^{3} d\pi^a \exp -\frac{V}{T}\left[\frac{1}{2}m_\pi^2\pi^a\pi^a - 4\mu_I^2(\pi^+\pi^-)\right]$$

$$\chi_{ud} = -\frac{T}{V}\frac{2}{T^2}\frac{1}{m_\pi^2} = -\frac{2}{k}T^2\frac{1}{m_\pi^2}$$

Off-diagonal 2nd moment of the pressure

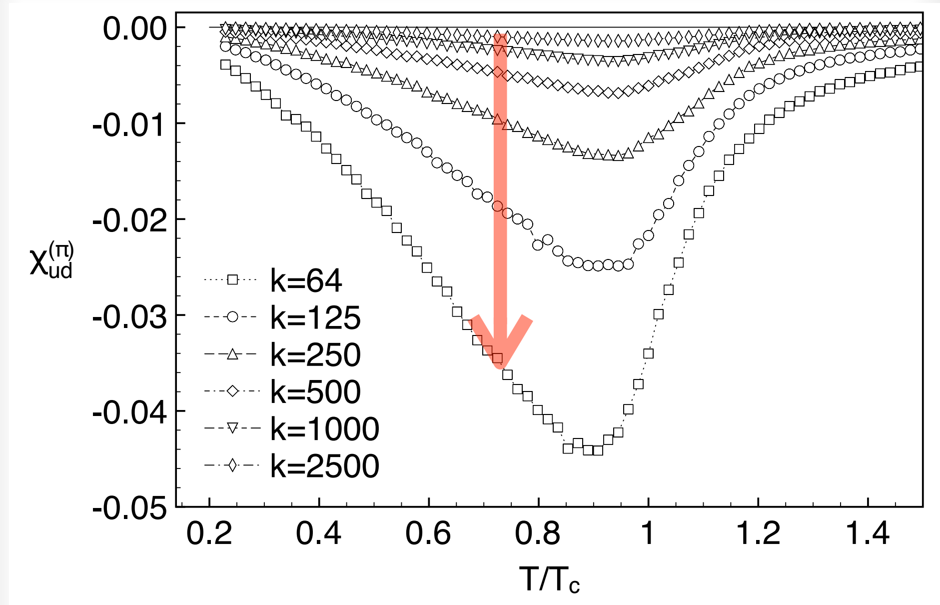
Role of pions from ChPT

$$\chi_{ud} = -\frac{T}{V}\frac{2}{T^2}\frac{1}{m_\pi^2} = -\frac{2}{k}T^2\frac{1}{m_\pi^2}$$

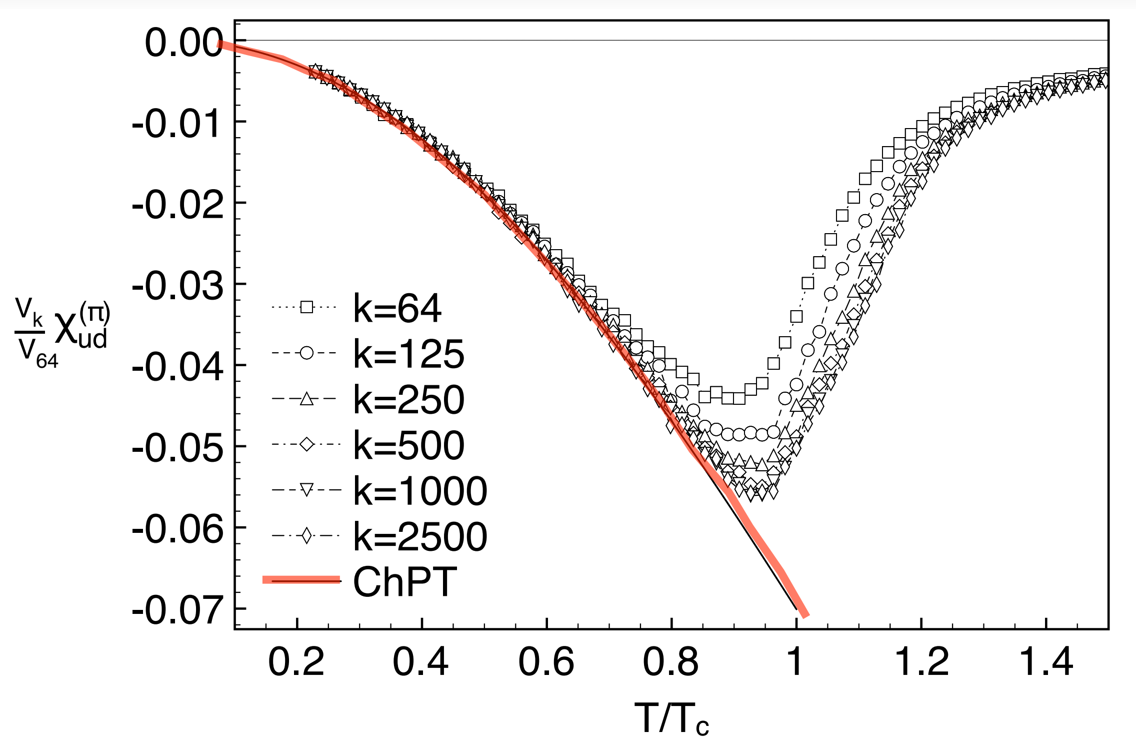
$$m_\pi(T) = m_\pi\left(1 + \frac{1}{2N_f}\frac{g_1(m_\pi^2, T, L)}{f_\pi^2} + \mathcal{O}(p^4)\right)$$

$$g_1(m_\pi^2, T, L) = \frac{1}{(4\pi)^2}\int_0^\infty \mathrm{d}\lambda^{r-3}\sum_{n\neq 0}\exp(-m_\pi^2\lambda - n^2/(4\lambda))$$

$$n = (n_1 L, n_2 L, n_3 L)$$



δA8=0 δπ≠0



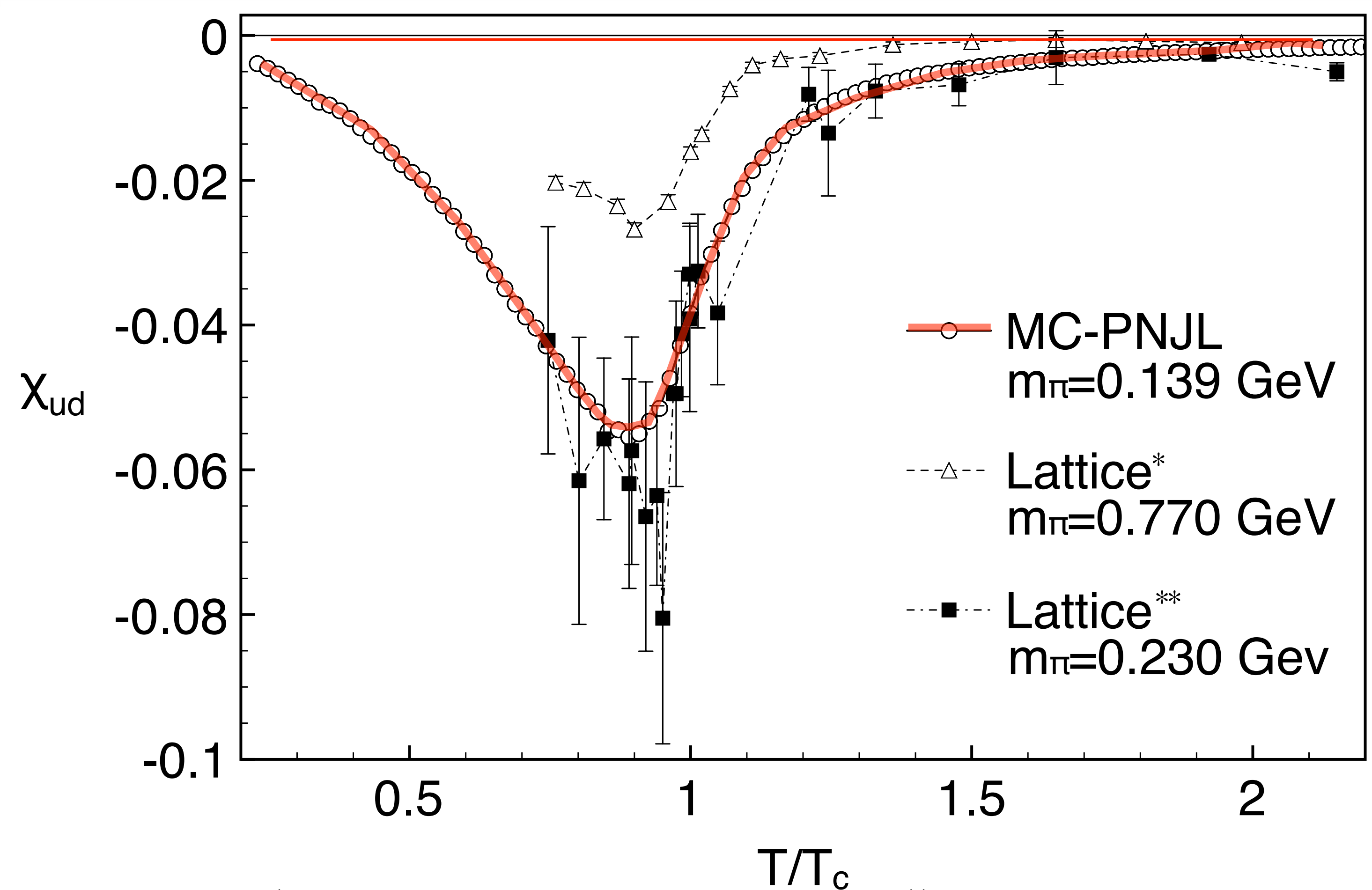
- MC calculations agree with ChPT in the range of validity of the theory

Off-diagonal 2nd moment of the pressure

MC-PNJL:

full result

$$\frac{\partial^2}{\partial\mu_u\partial\mu_d}\ln\det[M(f;T,\mu_u,\mu_d)]$$



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χ_{ud}

Conclusions

Conclusions and Outlook

Monte-Carlo PNJL

- Order parameters calculated using Monte-Carlo close to MF result
- Zero mode fluctuations are taken into account
- Off-diagonal moment of the pressure can be calculated
- In agreement with ChPT and lattice calculations

Outlook

- Move to non-local PNJL model in 2+1 flavor thank **you**