Parton Distribution Amplitudes

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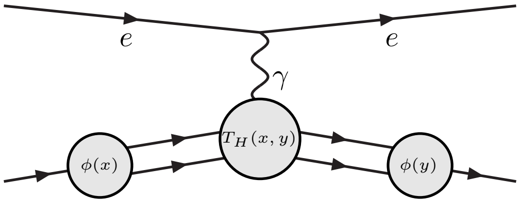
Introduction

Distribution Amplitudes:

- are introduced for the QCD description of hard exclusive processes - e.g. form factors with a large momentum transfer
- encode the non-perturbative QCD effects that occur from the factorisation of hard exclusive processes
- also appear in factorisation theorems relevant to measuring parameters of the SM:
  - $B \to \pi l \nu$ (CKM element $|V_{ub}|$)
  - $B \to D\pi$ (used in tagging)
  - $B \to \pi\pi,\ K\pi, ...$ (measuring CP violation)
- are calculable on the lattice through their moments

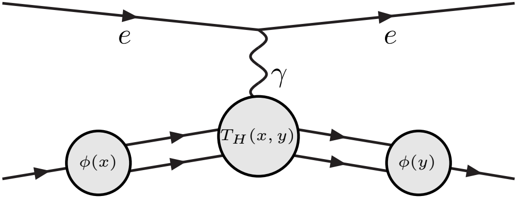
An Exclusive Process

- Consider the form factor, $F_\pi(Q^2)$ for elastic electron-pion scattering, at large $-q^2 = Q^2$
- The form factor is the amplitude for the composite pion to remain intact.



- The meson can be thought of as 2 valence quarks
- The quarks carry longitudinal momentum fractions x and $\bar{x} = 1 - x$ of the mesons momentum and move roughly parallel to the meson

An Exclusive Process



The form factor is a product of 3 probability amplitudes:

- The amplitude $\phi(x, Q^2)$ for finding the two quark valence state in the incoming meson
- The amplitude $T_H(x, y, Q^2)$ for this quark state to scatter with the photon producing two quarks in the final state
- The amplitude $\phi(y, Q^2)$ for this final state to reform into a meson

$$F_\pi(Q^2) = \int_0^1 \mathrm{dx} \int_0^1 \mathrm{dy} \phi_\pi(y, Q^2) T_H(x, y, Q^2) \phi_\pi(x, Q^2)$$

$\phi(x, Q^2)$ - The Quark Distribution Amplitude

In light cone gauge, the leading-twist pion distribution amplitude is defined through the matrix element :

$$\langle 0|\bar{q}(z)\gamma_\rho\gamma_5\mathcal{P}(z,-z)q(-z)|\pi(p)\rangle|_{z^2=0} \equiv f_\pi(ip_\rho)\int_0^1 \mathrm{d}x e^{i(x-\bar{x})p.z}\phi_\pi(x,\mu)$$

It is useful to parameterise the distribution amplitudes via their moments.

$$\langle\xi^n\rangle_\pi(\mu^2) = \int_{-1}^1 \mathrm{d}\xi\xi^n\phi_\pi(\xi,\mu^2)$$

where $\xi = x - \bar{x}$ is the difference between longitudinal momentum fractions

Moments

- The moments can be related to matrix elements of **local** operators $\longrightarrow$ Calculation of the DAs on the lattice

$$
\begin{aligned}
\langle 0|\bar{q}\gamma_{\{\rho}\gamma_5 \overleftrightarrow{D}_{\mu\}} q(0)|\pi\rangle &= \langle \xi^1 \rangle_\pi f_\pi p_\rho p_\mu \\
\langle 0|\bar{q}\gamma_{\{\rho}\gamma_5 \overleftrightarrow{D}_{\mu} \overleftrightarrow{D}_{\nu\}} q(0)|\pi\rangle &= -i\langle \xi^2 \rangle_\pi f_\pi p_\rho p_\mu p_\nu
\end{aligned}
$$

- Mesons with definite G-parity have a symmetry under $x \leftrightarrow \bar{x}$ $\therefore$ odd moments vanish $\langle \xi^1 \rangle_{\pi,}$ $\langle \xi^1 \rangle_\rho$
- $\langle \xi^1 \rangle_{K,}$ $\langle \xi^1 \rangle_{K^*}$ are important for $SU(3)$-breaking effects
- DAs are often expressed via Gegenbauer moments

$$
a_1 = \frac{5}{3}\langle \xi^1 \rangle, \qquad a_2 = \frac{7}{12}(5\langle \xi^2 \rangle - 1), \ldots
$$

Status

There have been 3 main approaches to calculating the DA's:

- **Extraction from experiment:** Shapes of leading-twist DA's can be determined from form factor data, such as $F_{\gamma\gamma^*\pi}$ at CLEO - Suffers from contamination of other hadronic uncertainties.
- **QCD Sum rules:** Has an irreducible error of $\sim 20\%$ as it is not possible to completely isolate the hadronic states. $a_1^K(1GeV) = 0.05(2), \quad 0.10(12), \quad 0.050(25) \quad \text{and} \quad 0.06(3)$
- **Lattice QCD:** Quite a few studies - including QCDSF and RBC/UKQCD

Simulation Details

- Calculations based on gauge field configurations drawn from joint UKQCD/RBC datasets
- Simulations use $N_f = 2 + 1$ flavours of domain wall fermions with an IWASAKI gauge action
- Results for lattices:
  - $16^3 \times 32(\times 16)$
  - $24^3 \times 64(\times 16)$
  - With a common lattice spacing $a^{-1} = 1.73(3)GeV$
  - Quark masses $am_s = 0.04, \quad am_l = 0.03,\ 0.02,\ 0.01,\ 0.005$
- Preliminary results:
  - for a finer lattice $32^3 \times 64(\times 16)$
  - $a^{-1} = 2.285(28)GeV$
  - Quark masses $am_s = 0.03, \quad am_l = 0.004,\ 0.006,\ 0.008$

Lattice Correlation Functions

Ratios of 2 point correlation functions $\longrightarrow$ Bare moments

- cancels f_π
- reduces statistical fluctuations

Pseudoscalar 1st Moment 2 pt functions

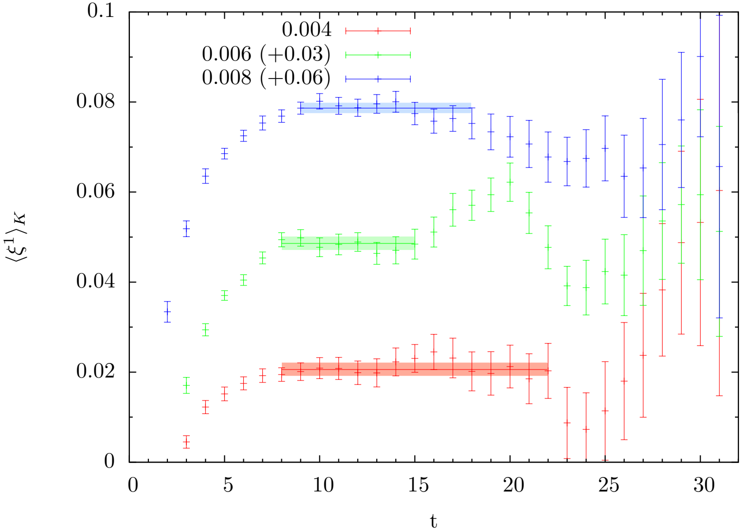
$$\begin{aligned} C_{\{\rho\mu\}P}(t,\mathbf{p}) &= \sum_{\mathbf{x}} e^{i\mathbf{p}.\mathbf{x}} \langle 0|O_{\{\rho\mu\}}(t,\mathbf{x})P^\dagger(0)|0\rangle \\ C_{A_\nu P}(t,\mathbf{p}) &= \sum_{\mathbf{x}} e^{i\mathbf{p}.\mathbf{x}} \langle 0|A_\nu(t,\mathbf{x})P^\dagger(0)|0\rangle \end{aligned}$$

Pseudoscalar ratio

$$\frac{C_{\{\rho\mu\}}(t,\mathbf{p})}{C_{A_\nu P}(t,\mathbf{p})} \stackrel{t\to\infty}{=} \frac{ip_\rho p_\mu}{p_\nu}\langle\xi^1\rangle$$

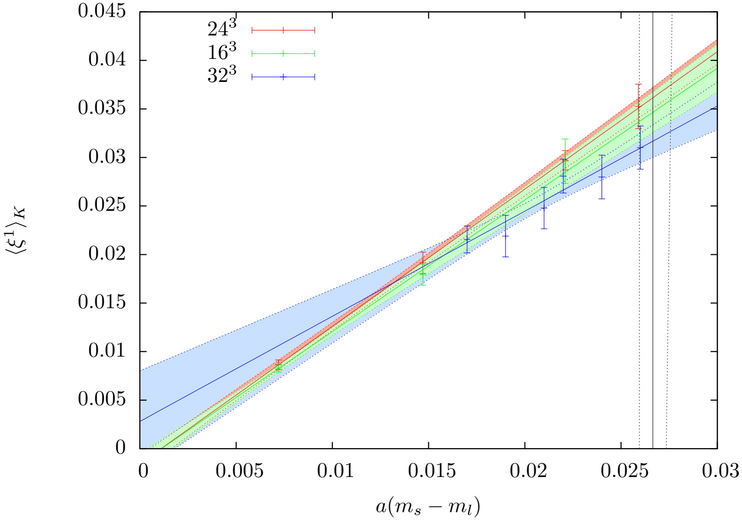
Results: 1st Moment Kaon 32

- $R^{P}_{\{4k\};4}(t, p_k = \pm 2\pi/L) = \pm i \frac{2\pi}{L} \langle \xi^1 \rangle, \quad k = 1, 2, 3$

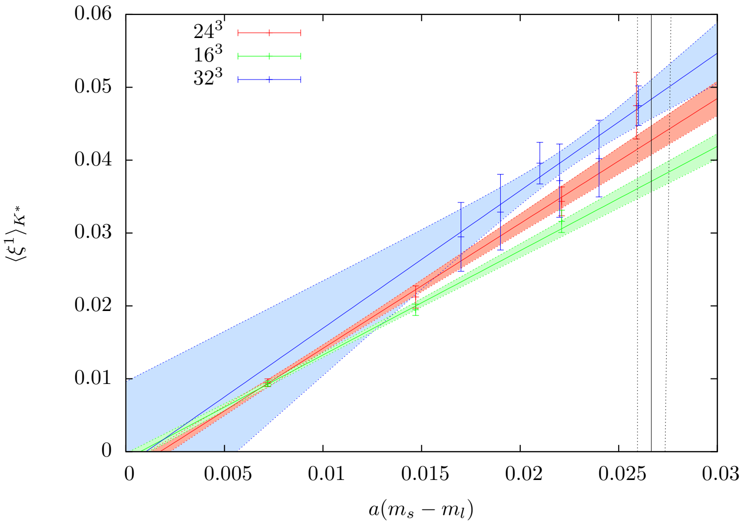


Results: 1st Moment Chiral extrapolation Kaon

- NLO χPT tells us: $\langle \xi^1 \rangle_K = \frac{8B_0}{f^2}(m_s - m_{u/d})b_{1,2}$

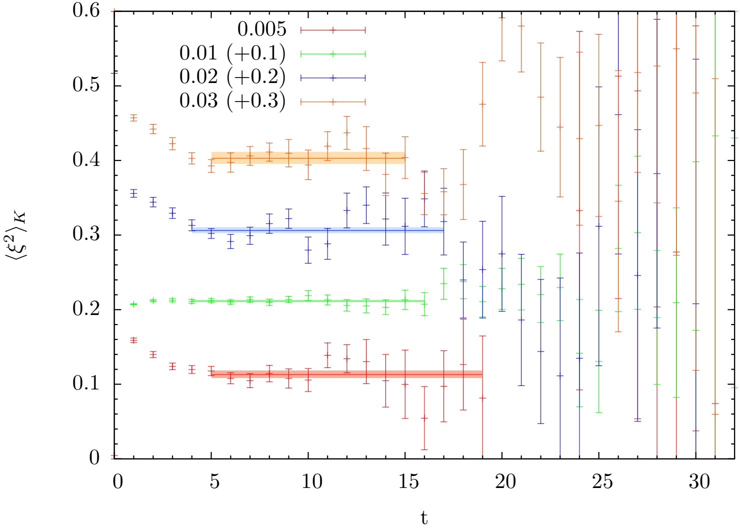


Results: 1st Moment Chiral extrapolation K^*



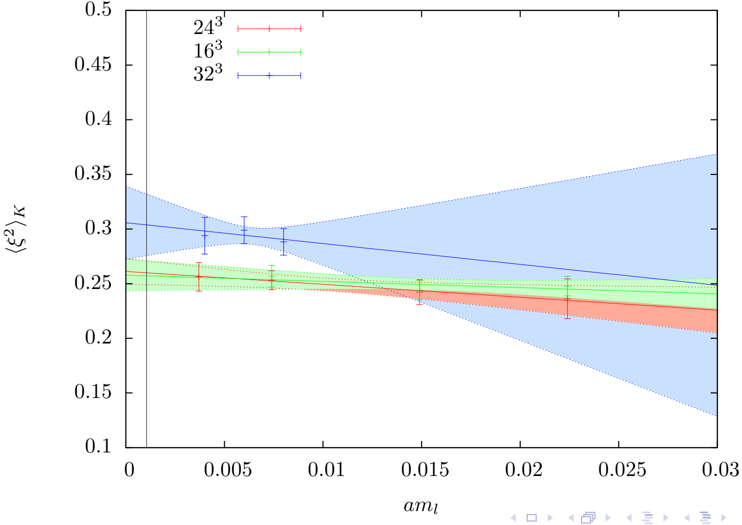
Results: 2nd Moment Kaon 24

- $R^P_{\{4jk\};4}(t, p_j = \pm 2\pi/L, p_k = \pm 2\pi/L) = -\left(\pm\frac{2\pi}{L}\right)\left(\pm\frac{2\pi}{L}\right)\langle\xi^2\rangle$

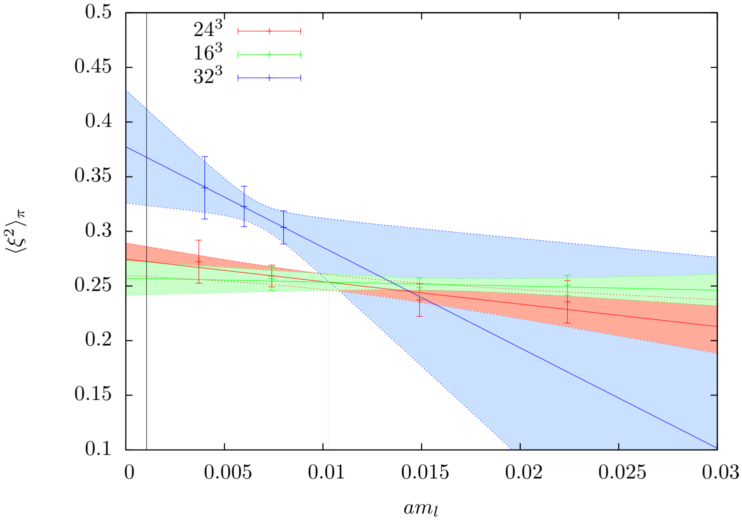


Results: 2nd Moment Chiral extrapolation K

- No strong nonlinear quark mass dependence - Linear Fit.
- χPT says no non-analytic dependence at 1 loop



Results: 2nd Moment Chiral extrapolation π

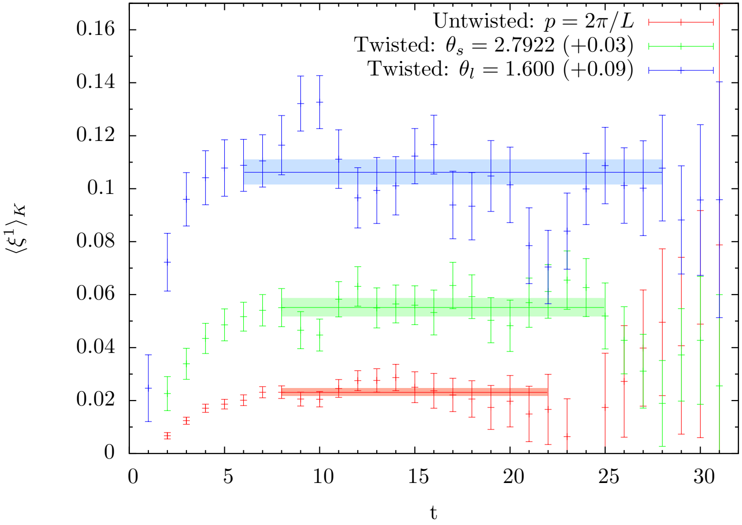


DAs Using Twisted Boundary Conditions

An initial look at using twisted boundary conditions to induce the momentum for DAs using existing 24^3 data.

- First preliminary results for Kaon 1st moment.
  - Using 2 different twist angles and hence momenta.
- Compared results to existing quantised momenta.
  - However the twisted and untwisted results have different smearings, also twisted has more gauge configurations but doesn't average over equivalent momenta.
  - A direct comparison is therefore difficult

Twisted DAs



Untwisted: $p = 2\pi/L$
Twisted: $\theta_s = 2.7922$ (+0.03)
Twisted: $\theta_l = 1.600$ (+0.09)
$\langle \xi^1 \rangle_K$
t
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
5
10
15
20
25
30

Summary

- Chiral limit results in $\overline{MS}$ at $2GeV$ using NPR, where errors are (stat)(syst)

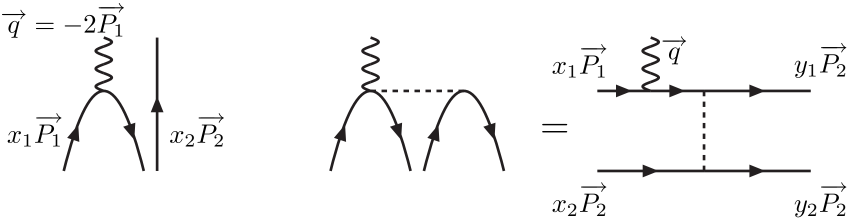
	$16^3 \times 32$	$24^3 \times 64$	$32^3 \times 64$ (Preliminary)
$\langle \xi^2 \rangle_\pi$	0.25(1)(1)	0.28(1)(1)	0.36(4)(1)
$\langle \xi^1 \rangle_K$	0.35(2)(2)	0.36(1)(2)	0.032(2)(2)
$\langle \xi^2 \rangle_K$	0.25(1)(1)	0.26(1)(1)	0.30(2)(1)
$\langle \xi^2 \rangle_\rho^{\vert\vert}$	0.25(1)(1)	0.27(1)(1)	0.32(7)(1)
$\langle \xi^1 \rangle_{K^*}^{\vert\vert}$	0.37(1)(2)	0.43(2)(3)	0.048(3)(4)

- Additional error of $\sim 2.5\%$ from discretisation effects
- $24/16$ cubed results agree with Sum-rules and QCDSF
- Work in progress: Finalisation of 32 cubed results, continuum extrapolation.

Additional Slides

T_H - The Hard Scattering Amplitude

- This can be calculated perturbatively.
- Photon couples to a single parton
- Partons only reform if they are collinear
- A Hard gluon exchange is required to turn spectators around
- Valence Fock state dominates



Scaling the 1st moment $24^3/16^3$ results to 32^3 results

- For a comparison it was necessary to scale the results
- $a_{32} \sim 3/4 a_{16/24}$
- $\langle \xi^1 \rangle_{32} \rightarrow \langle \xi^1 \rangle_{16/24}$ more tricky
- Renormalise at $\mu a = 1$, and run the Gegebauer moments

$$
\begin{aligned}
\langle \xi^1 \rangle^b_{24} Z(\mu a = 1) &= \langle \xi^1 \rangle (a_{24}^{-1}) \\
\langle \xi^1 \rangle^b_{32} Z(\mu a = 1) &= \langle \xi^1 \rangle (a_{32}^{-1}) \\
&= \langle \xi^1 \rangle (a_{24}^{-1}) L^{\gamma_1^0/(2\beta_0)}
\end{aligned}
$$

- Where $L = \alpha_s(\mu^2)/\alpha_s(\mu_0^2)$, $\beta_0 = 11 - 2N_f/3$ and $a_1 = 5/3 \langle \xi_1 \rangle$

$$
\langle \xi^1 \rangle^b_{32} = \langle \xi^1 \rangle^b_{24} L^{\gamma_1^0/(2\beta_0)}.
$$