

# Center Symmetry Restoration in Two Flavor Large N Yang Mills with Adjoint Fermions

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- 1 Physics Motivation
- 2 Eguchi-Kawai reduction
- 3 Numerical Results
- 4 Conclusions

# Technicolor Review

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- Higgs appears as a composite particle of "techniquarks" – new fermions with new gauge interaction.
- W,Z pick up masses by eating pseudo Goldstone bosons associated with spontaneous breaking of the new global chiral symmetries.

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- Technicolor dynamics require non-perturbative techniques to make predictions about chiral condensate, technihadron spectrum, etc.
- Dynamics cannot be exactly as scaled down QCD, due to possible conflicts with EW precision experiments.
- One possible scenario is for the theory to be near conformal or *walking*.
- Minimal key ingredients towards walking dynamics are using an  $SU(2)$  gauge group with  $N_f = 2$  (or 3?) (Dietrich, Sannino, et al).

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- Simulated by multiple groups by multiple means  
(Catterall-Giedt-Sannino-Schneible,  
Debbio-Luicini-Patella-Pica-Rago,  
Hietanen-Rummukainen-Tuominen, et al).

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- Also,  $N_f^*$  is independent of  $N_c$  at leading order in  $N$ . At infinite- $N$ , this leading behavior is independent of volume.
- Thus EK reduction may yield an alternative route to investigating near conformal behavior of ETC.

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- Polyakov (or Wilson) lines become relevant order parameters.

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- Heavy adjoint flavors with p.b.c. in circumstances which mimics  $R^3 \times S^1$  has been simulated (Cossu and D'elia).



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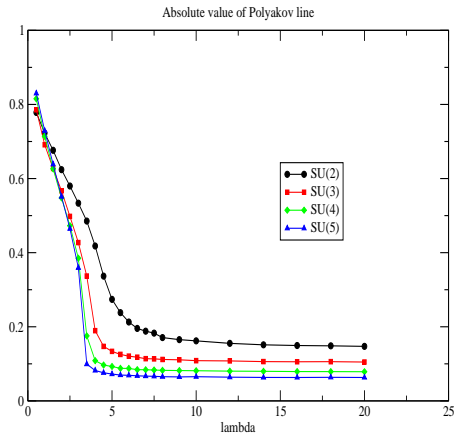
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- 6 Standard HMC algorithm used, order of 1000 measurements.

# Quenched Approximation

Pure gauge breaks center ( $m = \infty$ ), as expected.

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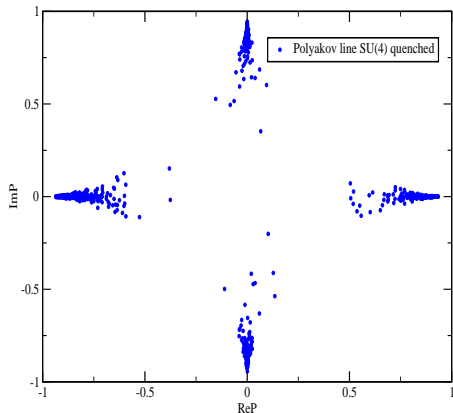
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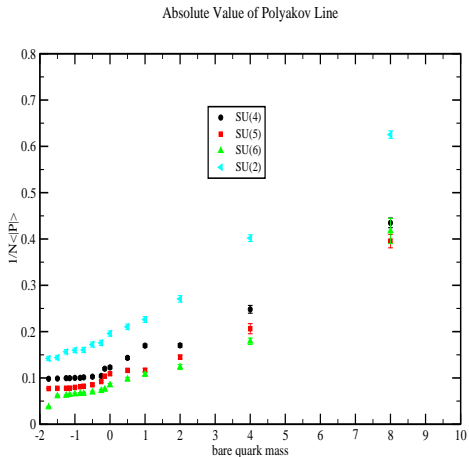


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Inclusion of adjoint fermions restores center

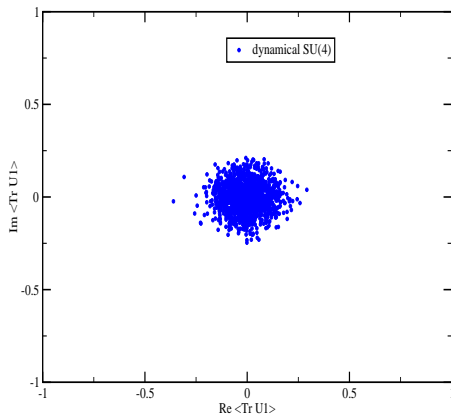
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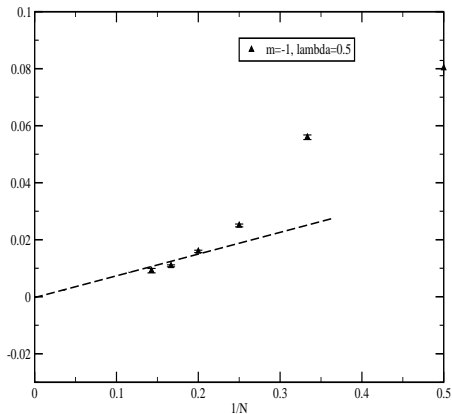
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$1/N$  behavior of absolute value of polyakov line



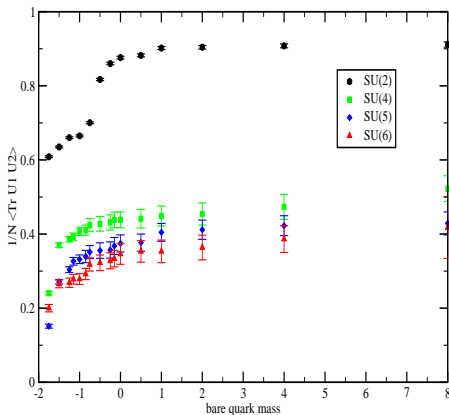
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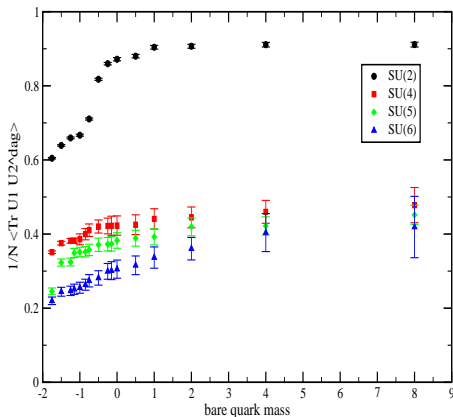
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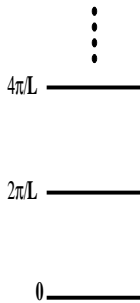




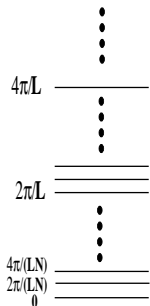
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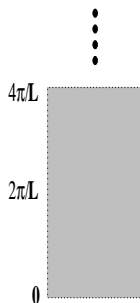
Kaluza Klein towers emerge from pbc and compactification..



a) Center-broken  
finite or large N



b1) Center-symmetric  
finite N



b2) Center-symmetric  
large N

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- More exotic order parameters like  $\langle P_1 P_2^\dagger \rangle \rightarrow 0$ .
- Our results indicate that center symmetry is also restored in the two flavor case.

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- We may now determine  $N_f$  critical by simulating the  $SU(N)$  theory at small volumes with large  $N$ .
- For more information: arXiv:1006.2469v1, "Realization of Center Symmetry in Two Adjoint Flavor Large- $N$  Yang-Mills."

## FIN

Thank you very much! Also, I thank A. Hietanen, R. Narayanan and M. Ünsal for useful discussions. I am also thankful for funding from Simon Catterall's DOE grant and the Syracuse STEM Graduate Fellowship.