

CHARMONIUM-NUCLEON INTERACTION FROM LATTICE QCD

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Why $c\bar{c}$ -nucleon interaction ?

◆ Flavor singlet interaction

1) No quark interchange

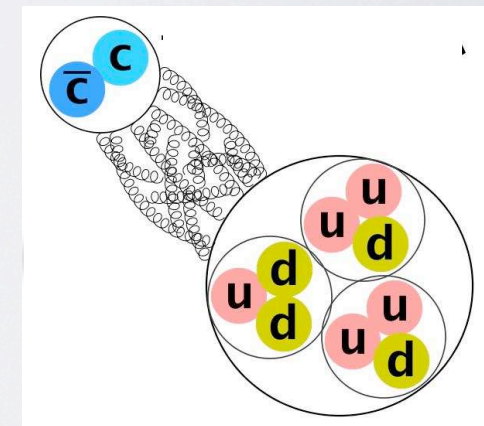
2) Multiple gluon exchange plays essential role

→ Interaction is described by **color van der Waals interaction**, which is weakly attractive in principle. (e.g. $-1/r^7$ behavior given by color dipoles)

H. Fujii and D. Kharzeev PRD60, 114039 (1999)



◆ If such an attraction is strong enough, charmonium may be bound to the nucleon or to the large nuclei.



Model study of nuclear-bound charmonium

- ◆ A semi-quantitative study of the charmonium-nucleus bound state was given by Brodsky et al.

Brodsky, Schmidt, de Teramond, PRL 64 (1990) 1011

1. A simple **Yukawa-type potential** is assumed for the cc^{bar} -N system.

$$V(r) = -\gamma \frac{\exp(-\alpha r)}{r} \quad \gamma=0.6 \quad \alpha=600 \text{ MeV}$$

2. The cc^{bar} -Nucleus potential

D. A. Wasson, PRL 67 (1991) 2237

$$V_{c\bar{c}-A}(r) = A \times V_{c\bar{c}-N}(r) \quad \text{or} \quad \int d^3\vec{r}' \rho(\vec{r}') V_{c\bar{c}-N}(\vec{r}')$$

They predicted a formation of nuclear-bound charmonium when $A \geq 3$.

- ◆ Precise information of the cc^{bar} -N potential $V_{cc^{\text{bar}}-N}(r)$ is indispensable for exploring nuclear-bound charmonium state.

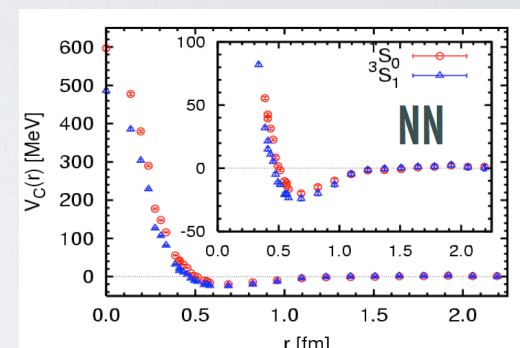
Our strategy

1. Tokyo-Tsukuba approach for hadron-hadron potential

To define the potential through the **Bethe-Salpeter wave function** measured on the lattice.

N. Ishii, S. Aoki and T. Hatsuda, *Phys. Rev. Lett.* 90, 022001 (2007)

S. Aoki, T. Hatsuda and N. Ishii, *Prog. Theor. Phys.* 123 (2010) 89.



2. Fermilab approach for heavy quark

To remove large discretization errors for heavy quarks.

A. X. El-Khadra, A. S. Kronfeld and P. B. Mackenzie, (1997)

Hadron-hadron potential

◆ Equal-time Bethe-Salpeter amplitude

M. Lüscher, Nucl. Phys. B 354, 531 (1991)

$$\begin{aligned}
 F_{\eta_c-N}(\vec{x}, \vec{y}, t; t_0) &= \langle 0 | N(\vec{x}, t) \eta_c(\vec{y}, t) J_{\eta_c-N} | 0 \rangle \\
 &= \sum_n A_n \langle 0 | N(\vec{x}, t) \eta_c(\vec{y}, t) | n \rangle e^{-E_n(t-t_0)} \\
 &\longrightarrow A_0 \phi_0(\vec{r}) e^{-E_0(t-t_0)} \quad t \gg t_0, \quad \vec{r} = \vec{x} - \vec{y}
 \end{aligned}$$

Interpolating operators

$$N(\vec{x}) = \epsilon_{abc} (u_a^t C \gamma_5 d_b) d_c(\vec{x})$$

$$\eta_c(\vec{y}) = \bar{c} \gamma_5 c_a(\vec{y})$$

◆ Schrödinger type equation for general cases.

$$E\phi(\vec{r}) + \frac{1}{2m_{\text{red}}} \nabla^2 \phi(\vec{r}) = \int d^3 r' U(\vec{r}, \vec{r}') \phi(\vec{r}')$$

For $c\bar{c}$ -N scattering at low energy

$$U(\vec{r}, \vec{r}') = V_{\eta_c-N}(\vec{r}) \delta(\vec{r} - \vec{r}')$$

Reduced mass; $m_{\text{red}} = m_{\eta_c} m_N / (m_{\eta_c} + m_N)$

Relativistic heavy quark action

◆ Heavy quark mass introduces discretization errors of $O((ma)^n)$

✓ At charm quark mass, it becomes severe: $m_c \sim 1.5 \text{ GeV}$ and $1/a \sim 2 \text{ GeV}$, then $m_c a \sim O(1)$.

◆ The Fermilab group proposed **relativistic heavy quark action (RHQ)** approach where all $O((ma)^n)$ errors are removed by the appropriate choice of m_0, ξ, r_s, C_B, C_E . A. X. El-Khadra, A. S. Kronfeld and P. B. Mackenzie, (1997)

$$S_{\text{lat}} = \sum_{n,n'} \bar{\psi}_{n'} (\gamma^0 D^0 + \zeta \vec{\gamma} \cdot \vec{D} + m_0 a - \frac{r_t}{2} a (D^0)^2 - \frac{r_s}{2} a (\vec{D})^2 + \sum_{i,j} \frac{i}{4} c_B a \sigma_{ij} F_{ij} + \sum_i \frac{i}{2} c_E a \sigma_{0i} F_{0i})_{n',n} \psi_n$$

We take the Tsukuba procedure in our study.

S. Aoki, Y. Kuramashi, and S.-i. Tominaga, Prog. Theor. Phys. 109, 383 (2003)

Y. Kayaba et al. [CP-PACS Collaboration], JHEP 0702, 019 (2007).

Lattice set up

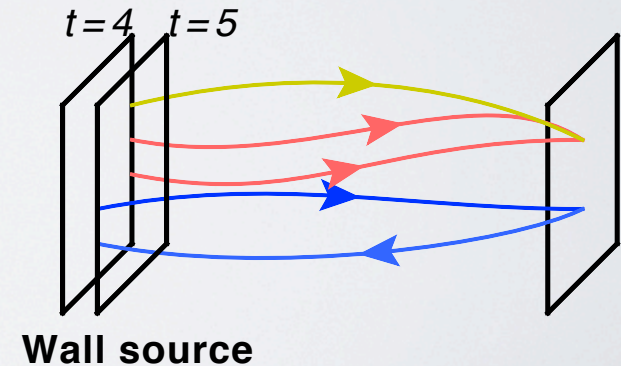
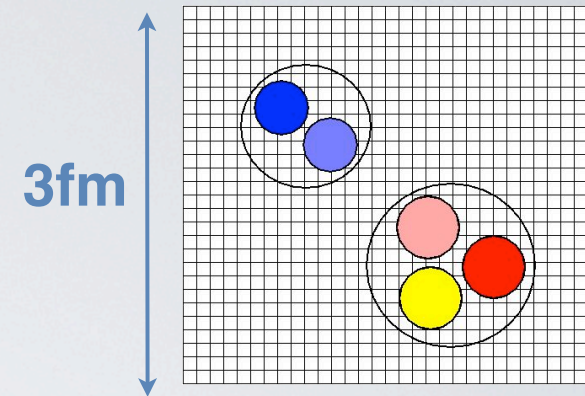
- ◆ low energy η_c -N interaction
- ◆ Quenched QCD simulation
- ◆ Lattice size:
 $L^3 \times T = 32^3 \times 48, 16^3 \times 48$ ($L a \approx 3.0, 1.5$ fm)
- ◆ plaquette action (gauge) $\beta=6.0$ ($a=0.093$ fm or $a^{-1}=2.1$ GeV)
 + non-perturbative $O(a)$ improvement action (up & down)
 + **RHQ** action with one-loop PT coefficients (charm)

Y. Kayaba et al. [CP-PACS Collaboration], JHEP 0702, 019 (2007).

- ◆ Statistics : **O(600) configs**

- ◆ Quark mass
 - charm $\kappa_Q = 0.10190$ $m_{\eta_c} = 2.92$ GeV
 - Light

κ	0.1342	0.1339	0.1333
m_π [GeV]	0.64	0.73	0.87
m_N [GeV]	1.43	1.52	1.70



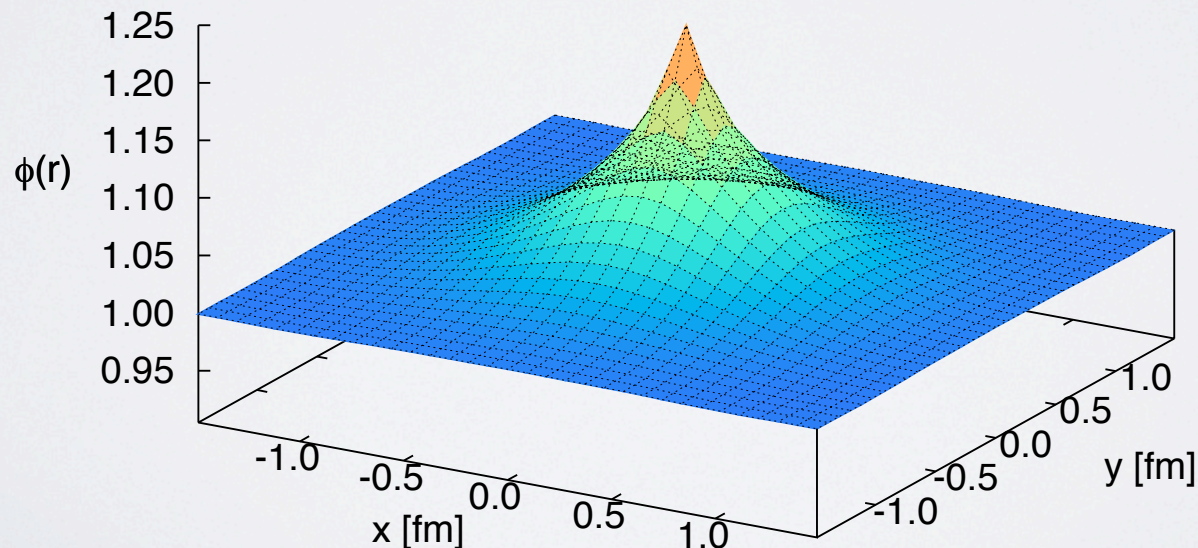
Result; η_c -N wave function

★ “S-wave” BS wave function can be projected out as

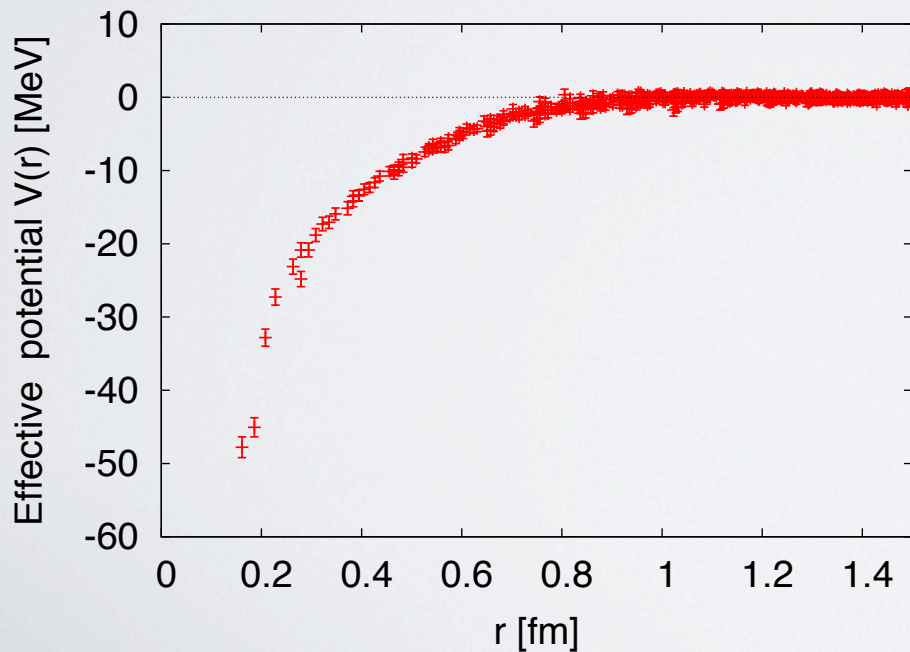
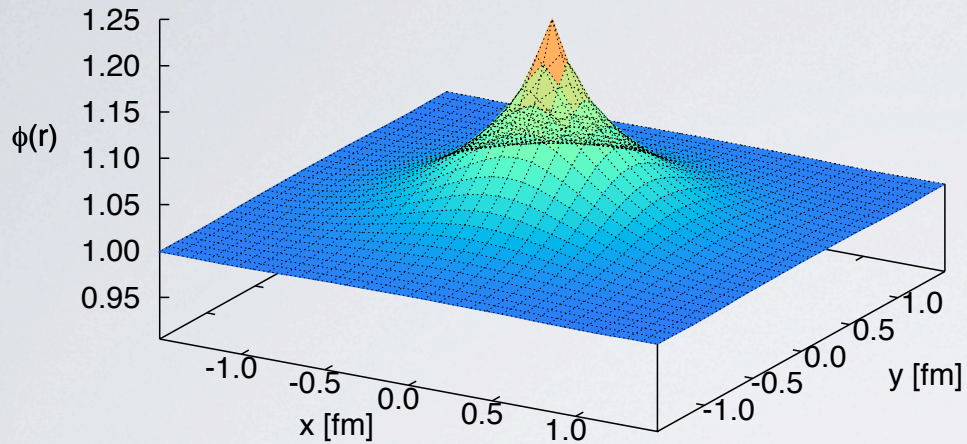
$$\phi(\vec{r}) = \frac{1}{24} \sum_{R \in O} \frac{1}{L^3} \sum_{\vec{x}} \langle 0 | N(R[\vec{r}] + \vec{x}) \eta_c(\vec{x}) | N \eta_c \rangle$$

R represents an element of cubic group. The summation over R and x projects out the A_1^+ sector of cubic group and zero total momentum.

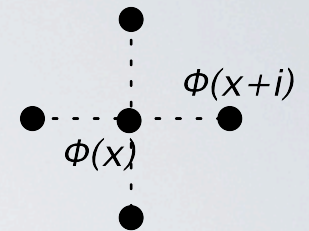
★ The “S-wave” η_c -N wave function.



Result; η_c -N potential



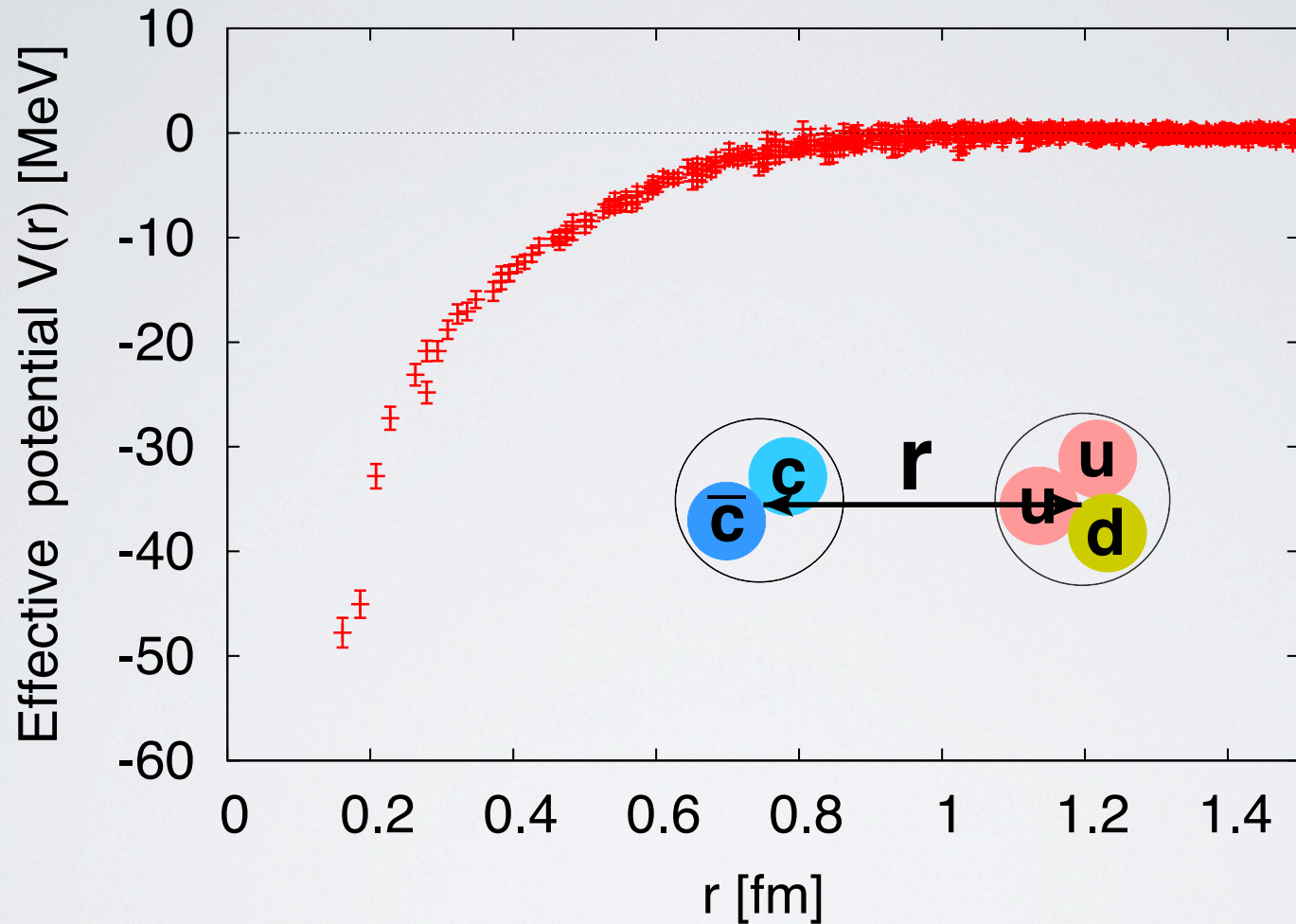
- ▶ The reduced mass is estimated from 2pt correlation functions
- ▶ ∇^2 denotes the discrete Laplacian



$$\frac{1}{2m_{\text{red}}} \frac{\nabla^2 \phi(r)}{\phi(r)} = V(r) - E$$

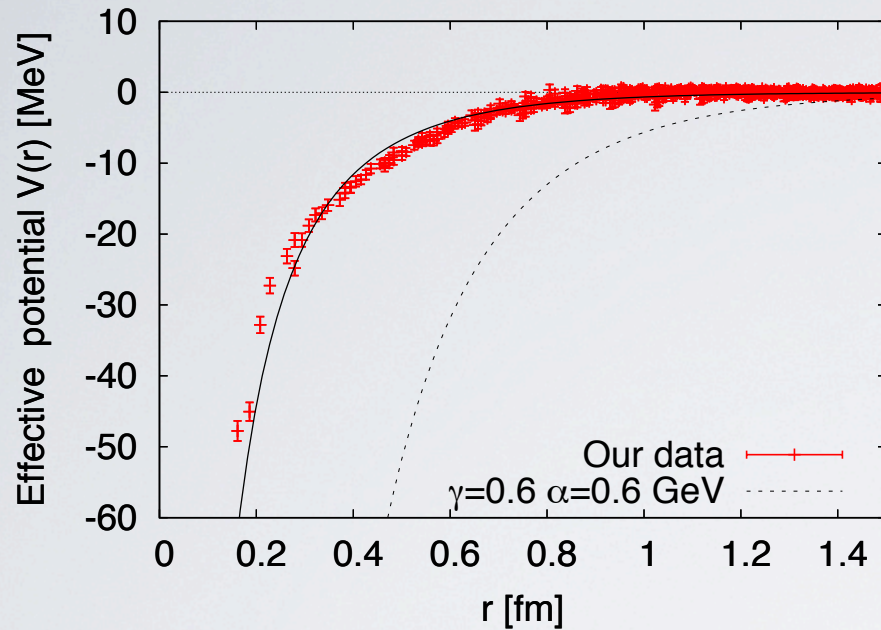
- ▶ $V(r)$ is adjusted as $V(r)=0$ for $r>1$ fm
→ Energy shift E

Result; η_c -N potential



- ▶ The η_c -N potential exhibits **entire attraction** without any repulsion.
- ▶ The interaction is **exponentially screened** in long distance region.

Result; η_c -N potential



Check of the long range screening.

-We have tried to fit data with two types of fitting function.

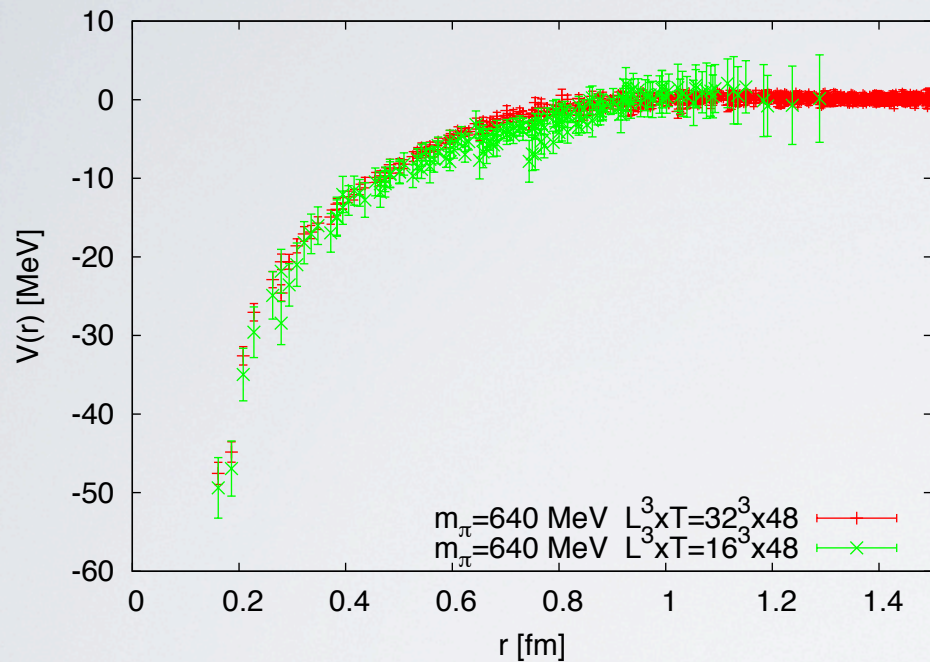
- 1) Exponential type function - $\exp(-r^n)/r^m$
→ gives a good fit with small χ^2/ndf
- 2) Inverse power law function $-1/r^n$
→ cannot give a reasonable fit.

► If we adopt the Yukawa form $-\gamma \exp(-\alpha r)/r$ to fit our potential, we obtain $\gamma \sim 0.1$ $\alpha \sim 600 \text{ MeV}$.

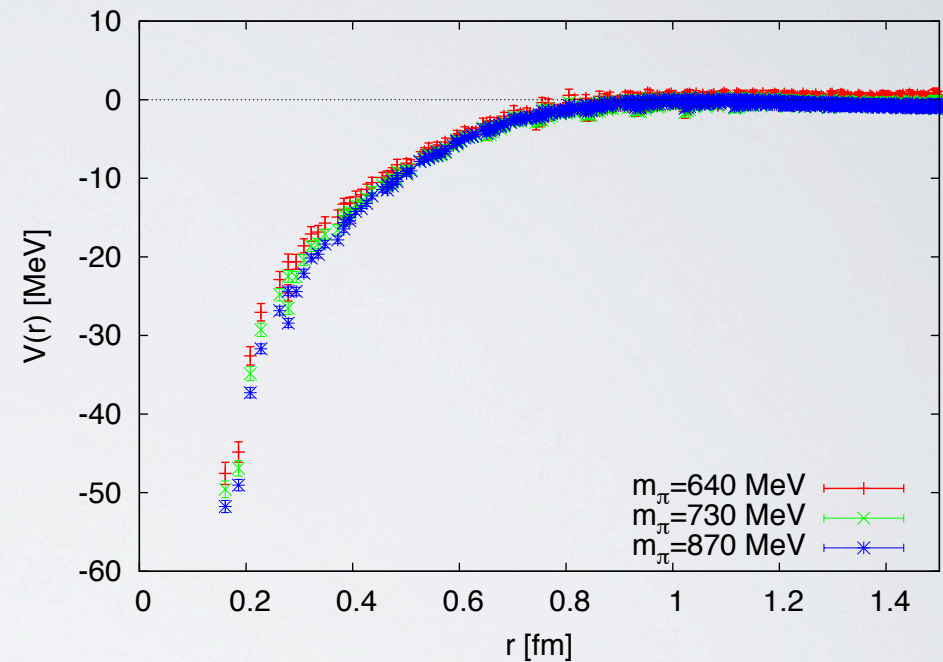
- cf. Phenomenological model $\gamma=0.6$ $\alpha = 600 \text{ MeV}$

→ The $c\bar{c}$ -N potential observed from lattice QCD is rather weak.

Result; volume & quark mass dependence



► No volume dependence

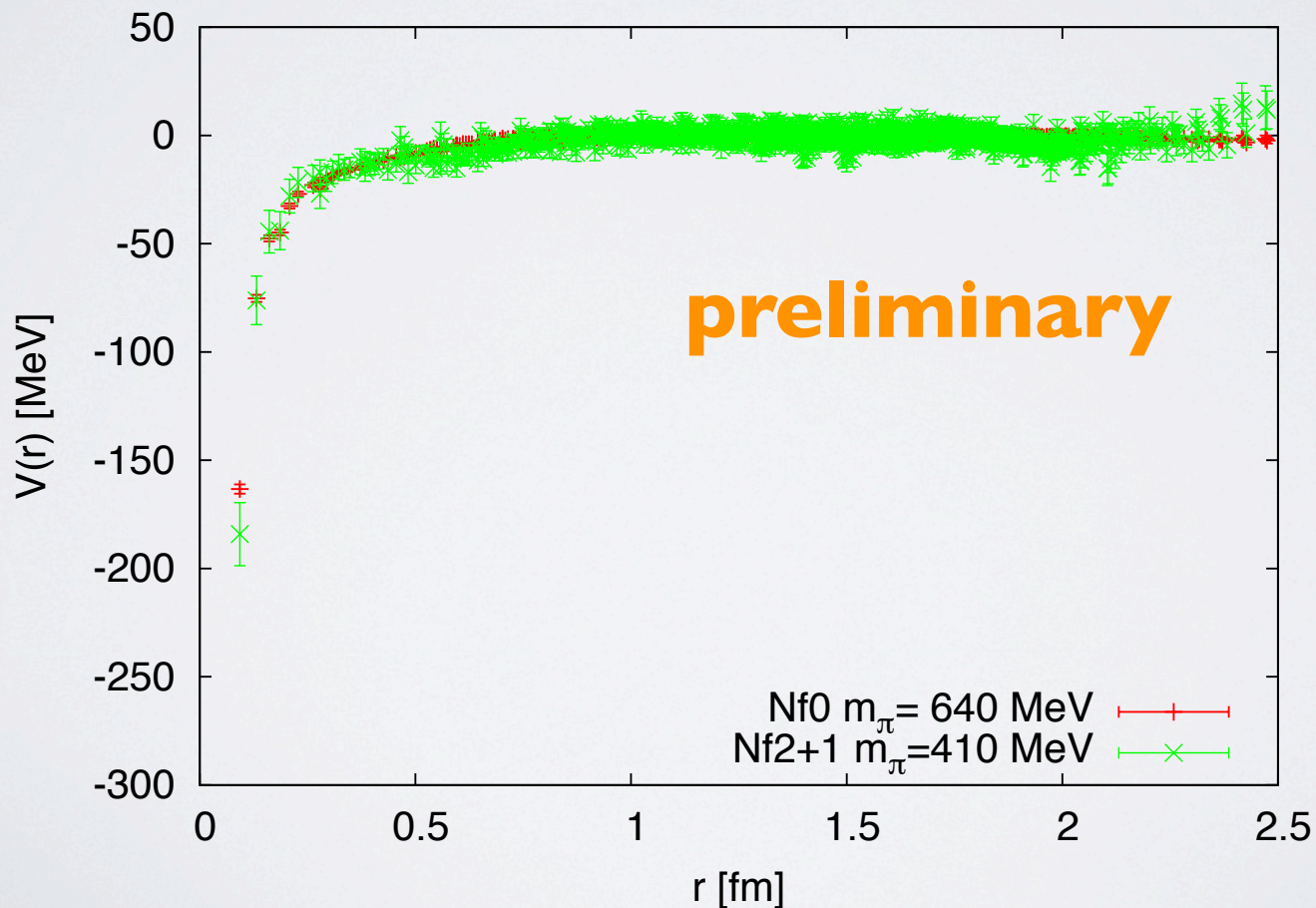


► No quark mass dependence

Result; η_c -N potential using PACS-CS 2+1 flavor dynamical configuration

S.Aoki et al., PRD 79, 034503 (2009)

- ▶ Quark mass; $\kappa_{ud}=0.13754$, $\kappa_s=0.13640$ ($m_\pi=0.41\text{GeV}$, $m_N=1.2\text{GeV}$)
 $\kappa_c = 0.106787$ ($m_{\eta_c} = 3\text{GeV}$)
- ▶ Lattice size; $L^3 \times T = 32^3 \times 64$ ($L_a \approx 3.0$ fm)
- ▶ Statistics ; 200 measurements



Summary

- ◆ We derived the $c\bar{c}$ -nucleon potential with **quenched QCD simulations** (pilot study) and **$N_f=2+1$ full QCD simulations** (preliminary)
 - ✓ The low energy $c\bar{c}$ -N interaction is **attractive in the whole range of r** .
 - ✓ The Long-range part is likely **suppressed exponentially**.
 - ┌ No quark mass dependence up to $m_\pi \sim 400\text{MeV}$.
 - └ No drastic difference between quenched QCD and Full QCD.
- ◆ Future perspective
 - ✓ We need to perform the simulation in **lighter quark mass region**.
 - ✓ Calculation of the spin dependent system; **J/ψ -N state**
 - ✓ **Exploring nuclear-bound charmonium state** with theoretical inputs.