Confinement in G_2 Gauge Theories

Björn H. Wellegehausen

Theoretisch-Physikalisches Institut
Research Training Group (1523) 'Quantum and Gravitational Fields'
FSU Jena

with Christian Wozar and Andreas Wipf

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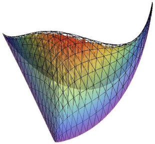

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Outline

Introduction

Why it is interesting to study G_2 gauge theories?

- In $SU(3)$ gauge theory confinement is related to the center of the gauge group
- The center of G_2 is trivial
- G_2 Yang Mills helps to clarify the relevance of center symmetry for confinement
- Possibility to distinguish between different confinement scenarios
- Similarity to QCD where center symmetry is explicitly broken by matter fields
- Test of Casimir scaling hypothesis and string breaking in different representations of the gauge group

- G_2 is the smallest Lie-group which is simply connected and has a trivial center
- The group has rank 2 and hence possesses two fundamental representations

$$\{7\} = [1,0], \quad \{14\} = [0,1].$$

- It is a subgroup of $SO(7)$, one can view the elements of the representation $\{7\}$ as matrices in the defining representation of $SO(7)$, subject to seven independent cubic constraints

$$T_{abc} = T_{def}\, g_{da}\, g_{eb}\, g_{fc}$$

where T is a total antisymmetric tensor given by

$$T_{127} = T_{154} = T_{163} = T_{235} = T_{264} = T_{374} = T_{576} = 1.$$

- The gauge group $SU(3)$ of QCD is a subgroup of G_2 and the corresponding coset space is a sphere

$$G_2/SU(3) \sim SO(7)/SO(6) \sim S^6.$$

Confinement in $SU(3)$ gluodynamics

- The center of $SU(3)$ is $\mathbb{Z}_3$
- Quarks and anti-quarks transform under the $\{3\}$ and $\{\bar{3}\}$ representation which have 3-ality (1) and (-1)
- Charges of quarks and anti-quarks can only be screened by particles with non-vanishing 3-ality
- The Polyakov loop expectation value serves as an order parameter for the $\mathbb{Z}_3$ center symmetry and for confinement / deconfinement

static inter-quark potential is linearly rising up to arbitrary long distances

Confinement in G_2 gluodynamics

- Quarks transform under the $\{7\}$, gluons under the $\{14\}$ representation
- Similarly as in $SU(3)$ two or three quarks can build a colour singlet

$$\{7\} \otimes \{7\} = \{1\} \oplus \cdots \quad , \quad \{7\} \otimes \{7\} \otimes \{7\} = \{1\} \oplus \cdots$$

- In contrast gluons can screen the colour charge of a single static quark

$$\{7\} \otimes \{14\} \otimes \{14\} \otimes \{14\} = \{1\} \oplus \cdots .$$

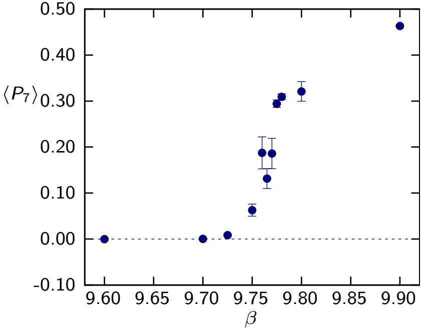
- The flux tube between two static quarks can break due to gluon production
- No linear rising potential up to arbitrary long distances

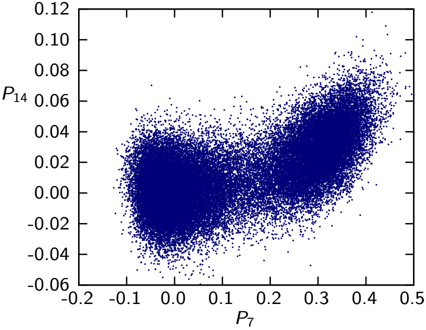
Confinement in G_2 gluodynamic really means — as in QCD

absence of free colour charges in the physical spectrum

linear rising potential only at intermediate scales

K. Holland, P. Minkowski, M. Pepe and U. J. Wiese, Nucl. Phys. **B668** (2003) 207





- The Polyakov loop is an approximate order parameter which changes rapidly at the phase transition and is small (but non-zero) in the confining phase
- First order confinement deconfinement phase transition [1]

[1] K. Holland, P. Minkowski, M. Pepe and U. J. Wiese, Nucl. Phys. **B668** (2003) 207

- Wilson Action
- Local HMC algorithm for updating link variables
- Computation of the exponential map of G_2 via Coset space decomposition

$$\mathcal{U} = \mathcal{S} \cdot \mathcal{V}(\mathcal{W}) \quad \text{with} \quad \mathcal{S} \in G_2/SU(3)$$

- In terms of Lie algreba elements

$$\exp\{\delta\tau\,\mathfrak{u}\} = \exp\{\delta\tau\,\mathfrak{s}\}\cdot\exp\{\delta\tau\,\mathfrak{v}\} \quad \text{with generators} \quad \mathfrak{u} \in \mathfrak{g}_2, \;\; \mathfrak{v} \in \mathcal{V}_*(\mathfrak{su}(3))$$

- Baker-Campbell-Hausdorff relates $\mathfrak{u}$,$\mathfrak{s}$ and $\mathfrak{v}$

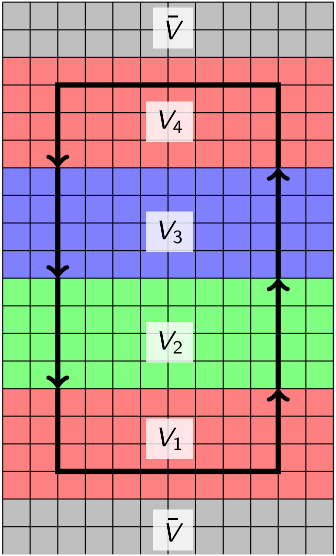
$$\delta\tau\,\mathfrak{u} = \delta\tau\,(\mathfrak{s} + \mathfrak{v}) + \frac{1}{2}\delta\tau^2\,[\,\mathfrak{s}, \mathfrak{v}\,] + \ldots$$

- In a confined phase Wilson loops obey an area law
- If we want to compute Wilson loops without any smearing, we need exponential error reduction M. Lüscher, P. Weisz, JHEP 0109:010,2001

- $\langle W(R,T)\rangle = \langle \mathrm{tr}[\mathcal{U}_1]_{V_1}[\mathcal{U}_2]_{V_2}[\mathcal{U}_3]_{V_3}\rangle$
- Sublattice expectation value $[\mathcal{U}]_{V_i}$ is computed with fixed boundary conditions
- Further refinement with multiple levels according to

$$[\mathcal{U}]_{V_i} = [[\mathcal{U}_1]_{V_{i1}}[\mathcal{U}_2]_{V_{i2}}]_V$$

- Splitting in temporal and spatial direction to avoid tensor products



Algorithmic considerations

Exponential error reduction for Wilson-lines

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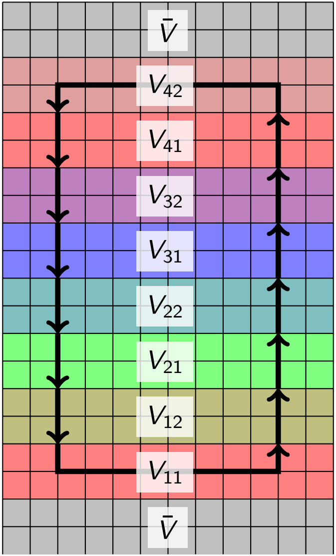

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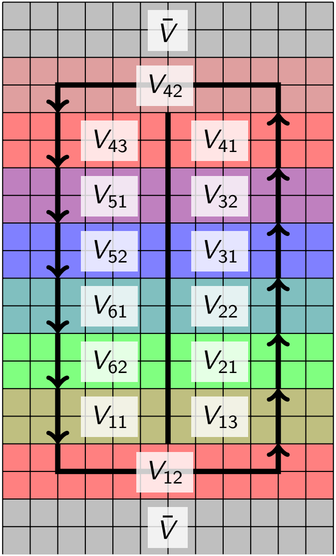


String tension and Casimir scaling
The quark anti-quark potential

- The quark anti-quark potential in representation $\mathcal{R}$ can be parametrised as

$$V_{\mathcal{R}}(R) = \gamma_{\mathcal{R}} - \frac{\alpha_{\mathcal{R}}}{R} + \sigma_{\mathcal{R}} R$$

  with string tension $\sigma_{\mathcal{R}}$ and Coulomb constant $\alpha_{\mathcal{R}}$
- Lattice derivative defines the local string tension $\sigma_{\mathcal{R}}(R)$

$$\sigma_{\mathcal{R}}(R+\rho/2) = \frac{V_{\mathcal{R}}(R+\rho) - V_{\mathcal{R}}(R)}{\rho} = \frac{\alpha_{\mathcal{R}}}{R(R+\rho)} + \sigma_{\mathcal{R}}$$

Casimir scaling

At intermediate scales we expect Casimir scaling $\sigma_{\mathcal{R}}/\sigma'_{\mathcal{R}} = c_{\mathcal{R}}/c'_{\mathcal{R}}$ with quadratic Casimir $c_{\mathcal{R}}$ of representation $\mathcal{R}$

$\mathcal{R}$	$[1,0]$	$[0,1]$	$[2,0]$	$[1,1]$	$[0,2]$	$[3,0]$	$[4,0]$	$[2,1]$
$d_{\mathcal{R}}$	7	14	27	64	77	77	182	189
$c_{\mathcal{R}}$	12	24	28	42	60	48	72	64
$C_{\mathcal{R}}$	1	2	7/3	3.5	5	4	6	16/3

Potential from Wilson loops

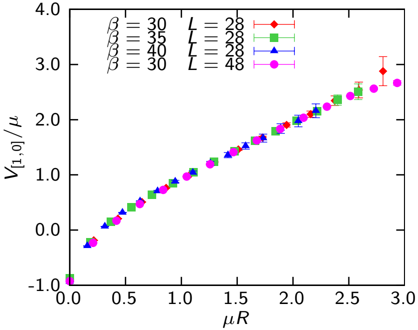
$$V_{\mathcal{R}}(R) = \frac{1}{\tau} \ln \frac{\langle W_{\mathcal{R}}(R,T)\rangle}{\langle W_{\mathcal{R}}(R,T+\tau)\rangle}$$

$$\sigma_{\mathcal{R}}(R+\rho/2) = -\frac{1}{\tau\rho} \ln \frac{\langle W_{\mathcal{R}}(R+\rho,T+\tau)\rangle \langle W_{\mathcal{R}}(R,T)\rangle}{\langle W_{\mathcal{R}}(R+\rho,T)\rangle \langle W_{\mathcal{R}}(R,T+\tau)\rangle}$$

Potential from Polyakov loops

$$V_{\mathcal{R}}(R) = -\frac{1}{\beta_T} \ln \langle P_{\mathcal{R}}(0) P_{\mathcal{R}}(R)\rangle$$

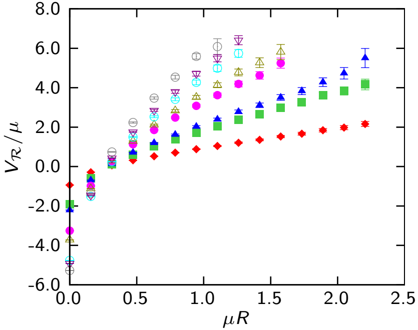
$$\sigma_{\mathcal{R}}(R+\rho/2) = -\frac{1}{\beta_T\rho} \ln \frac{\langle P_{\mathcal{R}}(0) P_{\mathcal{R}}(R+\rho)\rangle}{\langle P_{\mathcal{R}}(0) P_{\mathcal{R}}(R)\rangle}$$



Physical units

- There is no physical length scale in G_2 gluodynamics
- $\sigma = \hat{\sigma} a^2, \quad R = \hat{R} a, \quad V = \hat{V} a^{-1}$
- $V \sigma^{-\frac{1}{2}} = \hat{V} \hat{\sigma}^{-\frac{1}{2}}$
- $R \sigma^{\frac{1}{2}} = \hat{R} \hat{\sigma}^{\frac{1}{2}}$
- We can fix a length scale except for a constant $\sigma_{[1,0]} = \mu$

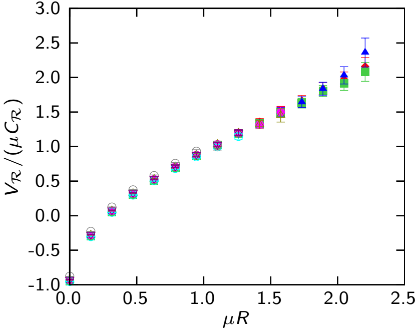
- Simulations on lattices $L \times L \times \beta_T = 28^3$ and $L \times L \times \beta_T = 48^3$ and different values of β (lattice spacing a)
- No difference between potentials from Wilson loops or Polyakov loops
- In physical units no difference between different lattice spacings and volumes $\rightarrow$ close to the continuum limit



- Lattice
  $L \times L \times \beta_T = 28^3, \quad \beta = 40$
- Wilson loops
- Two level algorithm
- Potential in 8 different representations

Potential unscaled

■ $= [1, 0]$ ■ $= [0, 1]$ ■ $= [2, 0]$ ■ $= [1, 1]$ ■ $= [0, 2]$ ■ $= [3, 0]$ ■ $= [4, 0]$ ■ $= [2, 1]$



- Lattice
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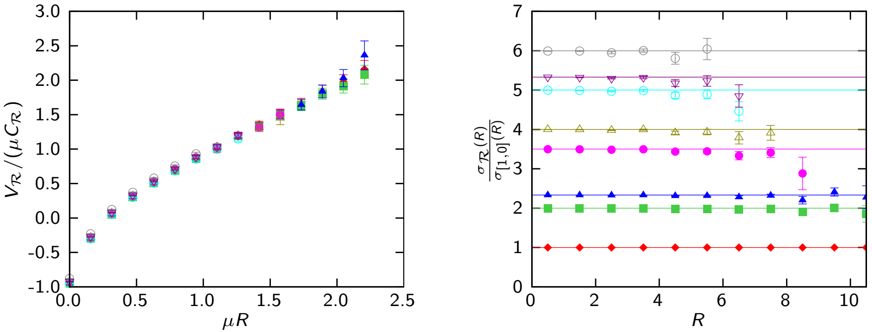
Potential scaled with Casimir values $C_{\mathcal{R}}$

$= [1, 0]$ $= [0, 1]$ $= [2, 0]$ $= [1, 1]$ $= [0, 2]$ $= [3, 0]$ $= [4, 0]$ $= [2, 1]$



Casimir scaling in 3 dimensions works!

= [1, 0] = [0, 1] = [2, 0] = [1, 1] = [0, 2] = [3, 0] = [4, 0] = [2, 1]

Results in 3 dimensions
Observing string breaking

- Three gluons can screen the color of a fundamental quark

$$(7) \otimes (14) \otimes (14) \otimes (14) = \color{red}{(1)} \oplus \cdots$$

- One gluon can screen the color of an adjoint quark

$$(14) \otimes (14) = \color{red}{(1)} \oplus \cdots$$

- Confining string can break if

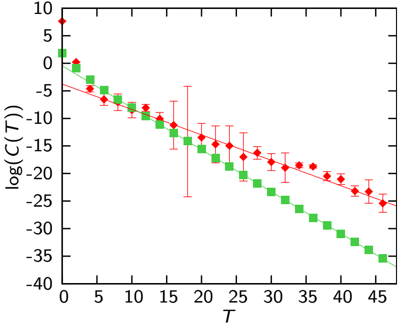
$$\color{red}{V_{\mathcal{R}}(R^c) = E \approx 2m_{qg}}$$

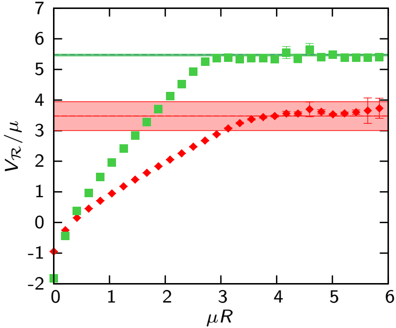
Mass of a quark-gluon bound state

Can be obtained from the correlation function

$$C(T) = \left\langle \left(\bigotimes_{n=1}^{N(\mathcal{R})} F_{\mu\nu}(x) \right) \Bigg|_{\mathcal{R},a} \mathcal{R}(\mathcal{U}_{xy})_{ab} \left(\bigotimes_{n=1}^{N(\mathcal{R})} F^{\dagger}_{\mu\nu}(y) \right) \Bigg|_{\mathcal{R},b} \right\rangle \propto \exp\left(-m_{\text{qg}} T\right)$$

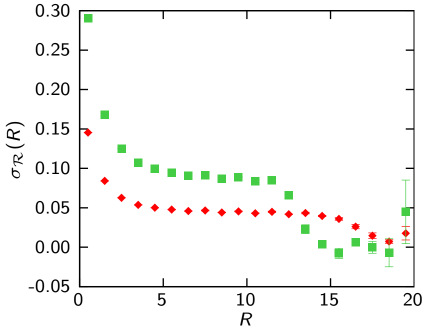
- Lattice $L \times L \times \beta_T = 48^3$, $\beta = 30$ and Polyakov loops (three level algorithm)

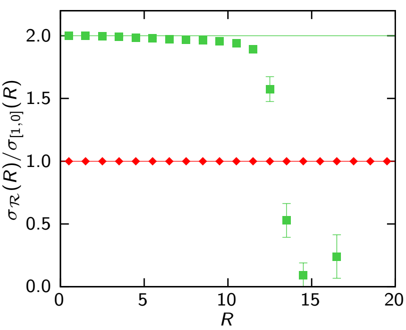




String breaking in the fundamental and adjoint representation

= [1, 0] = [0, 1] = [2, 0] = [1, 1] = [0, 2] = [3, 0] = [4, 0] = [2, 1]





Casimir scaling breaks down at large distances

$= [1,0]$ $= [0,1]$ $= [2,0]$ $= [1,1]$ $= [0,2]$ $= [3,0]$ $= [4,0]$ $= [2,1]$

Conclusions

- Casimir scaling at intermediate scales in 3 dimensions was confirmed for 8 different representations without smearing within 1 percent
- String breaking at larger distances was seen in both fundamental representations

B. Wellegehausen, A. Wipf, C. Wozar, Casimir Scaling and String Breaking in $\mathbf{G_2}$ Gluodynamics, arxiv:hep-lat 1006.2305 (2010)

Further results / Outlook

- Casimir scaling in 4 dimensions — also L. Liptak and S. Olejnik, Phys. Rev. **D78** (2008)
- Full phase diagram of the G_2 gauge higgs model — to be published soon
- Effective G_2 Polyakov Loop and spin models and their relation to G_2 Yang-Mills theorie

B. Wellegehausen, A. Wipf, C. Wozar, Effective Polyakov Loop Dynamics for Finite Temperature G(2) Gluodynamics, Phys. Rev. **D80** (2009)