Light meson form factors in $N_f = 2 + 1$ QCD with dynamical overlap quarks

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1. introduction

light meson form factors

- pion form factors : $F_V^\pi(q^2)$ and $F_S^\pi(q^2)$
  - good testing ground for consistency with ChPT
  - determination of LECs : $F_V^\pi(q^2) \to L_9, \quad F_S^\pi(q^2) \to 2L_4 + L_5$
- kaon EM form factors : $F_V^{K^+}(q^2)$ and $F_V^{K^0}(q^2)$
  - share (many) LECs with $F_V^{\pi^+}(q^2)$ (at NNLO) *(cf. Bijnens-Talavera, 2002)*
  - only old experimental data (~ 1980) are available $\Rightarrow$ calculate on lattice
- $K \to \pi$ form factor : $f_+(0)$
  - determination of CKM element $|V_{us}|$ $\Rightarrow$ test of SM

lattice calculation

- 3-pt. function : (much) noisier than 2-pt. functions
- calculation with various choices of initial / final mesons; meson momenta; ...
- disconnected diagrams for $F_S^\pi(q^2)$

1. introduction

this work

calculate light meson form factors in $N_f = 2+1$ QCD

- overlap quarks $\Rightarrow$ straightforward comparison w/ ChPT ($a = 0$)
- all-to-all quark propagators

  $\Rightarrow$ precise calculation of various 3-pt. functions

measurements are on-going

this talk : preliminary analysis of currently available data

outline

- simulation method
- EM form factors
- scalar form factors
- weak decay form factors

2.1 simulation method: configuration generation

configurations

- $N_f = 2+1$ QCD
- Iwasaki gauge + overlap quarks w/ std. Wilson kernel H_W
- determinant to suppress zero modes: $\det[H_W^2]/\det[H_W^2 + \mu^2] \quad (\mu = 0.2)$
- $\beta = 2.30$: $a =$ 0.1085(15) fm $\Leftarrow$ M_Ω as input *(talk by Noaki)*
- $16^3 \times 48$: $L \sim$ 1.74 fm

for form factors

- 4 m_{ud} : $M_\pi \simeq$ 310 – 560 MeV; $m_s =$ 0.080 : $m_{s,\rm phys} =$ 0.081
- in $Q = 0$ sector

  effects of fixed topology to $F_V^{\pi+}$: small for $N_f = 2$ *(JLQCD/TWQCD, 2009)*
- 50 conf $\times$ 50 HMC traj. for each (m_{ud},m_s)
- local and smeared operators : $\phi_{\rm l}(|\mathbf{r}|) = \delta_{\mathbf{r},\mathbf{0}}, \quad \phi_{\rm s}(|\mathbf{r}|) = \exp[-0.4|\mathbf{r}|]$
- periodic boundary condition $\Rightarrow$ 0.5 GeV$^2 \lesssim |q^2| \lesssim$ 2.0 GeV^2

on-going : $24^3 \times 48$, reweighting w.r.t. m_s, twisted boundary conditions (TBCs)

2.2 simulation method: measurements

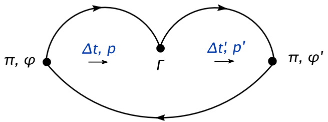
all-to-all quark propagator *(TrinLat, 2005)*

- *exact* low-mode contribution : $D\,u^{(k)} = \lambda^{(k)} u^{(k)} \quad (k \le N_e = 160)$
- noise method : $D\,x^{(d)} = \eta^{(d)}$ (single noise vector diluted w.r.t color/spinor/t)

$$D^{-1} = \sum_{k=1}^{N_e} \frac{u^{(k)}}{\lambda^{(k)}} u^{(k)\dagger} + (1 - P_{\rm low}) \sum_{d=1}^{N_d} x^{(d)} \eta^{(d)\dagger} = \sum_{k=1}^{N_v = N_e + N_d} v^{(k)}\, w^{(k)\dagger}$$

$$v^{(k)} = \{u^{(1)}/\lambda^{(1)}, ..., x^{(1)}/, ...\}, \quad w^{(k)} = \{u^{(1)}, ..., \eta^{(1)}, ...\}$$

connected 3-pt. functions



Δt : temporal separation src ⇔ opr

$\Delta t'$: temporal separation opr ⇔ snk

$\mathbf{p}$: initial meson momentum

$\mathbf{p}'$: final meson momentum

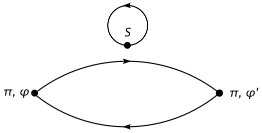
$$\mathcal{M}^{(k,l)}_{\Gamma,\phi}(t;\mathbf{p}) = \sum_{\mathbf{x},\mathbf{r}} \phi(\mathbf{r})\, w(\mathbf{x}+\mathbf{r},t)^{(k)\dagger}\, \Gamma\, v^{(l)}(\mathbf{x},t)\, \exp[-i\mathbf{p}\mathbf{x}]$$

$$C^{\pi\pi}_{\Gamma,\phi\phi',{\rm conn}}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}') = \frac{1}{N_t} \sum_{t=1}^{N_t} \sum_{k,l,m=1}^{N_v} \mathcal{M}^{(m,l)}_{\pi,\phi'}(t+\Delta t+\Delta t'; \mathbf{p}') \times \mathcal{M}^{(l,k)}_{\Gamma,\phi_1}(t+\Delta t; \mathbf{p}-\mathbf{p}')\, \mathcal{M}^{(k,m)}_{\pi,\phi}(t; -\mathbf{p})$$

2.2 simulation method: measurements

pros and cons

- low-mode contribution :
  dominates low-energy dynamics and is calculated exactly
- time-consuming steps : Lanczos and overlap solver
  - multi-shift solver for different $m_{q,\mathrm{val}}$
  - do not have to repeat to calculate 3-pt. functions with different $\mathbf{p}^{(\prime)}$, $\phi^{(\prime)}$, ...



$$C^{\pi\pi}_{S,\phi\phi',\mathrm{disc}}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}') = \frac{1}{N_t}\sum_{t=1}^{N_t}\sum_{k,l=1}^{N_v} \mathcal{M}^{(k,l)}_{\pi,\phi'}(t+\Delta t+\Delta t'; \mathbf{p}')\, \mathcal{M}^{(l,k)}_{\pi,\phi}(t; -\mathbf{p}) \times \sum_{m=1}^{N_v} \mathcal{M}^{(m,m)}_{S,\phi_1}(t+\Delta t; \mathbf{p}-\mathbf{p}')$$

but, have to repeat for different boundary conditions

- take average over source location $(\mathbf{x}, t)$ to (remarkably) improve statistical accuracy

3.1.1 pion EM form factor : determination of $F_V^{\pi^+}(q^2)$

$$\langle \pi^+(p')|j_\mu|\pi^+(p)\rangle = (p+p')_\mu F_V^{\pi^+}(q^2)$$

ratio method *(S. Hashimoto, et al., 2000)*

$$C^{\pi\pi}_{V_4,\phi\phi'}(\Delta t,\Delta t';\mathbf{p},\mathbf{p}') \rightarrow \frac{Z^*_{\pi,\phi}(|\mathbf{p}|)\, Z_{\pi,\phi'}(|\mathbf{p}'|)}{4E(p)E(p')\, Z_V}\, e^{-E(p)\Delta t}\, e^{-E(p')\Delta t'}\, \langle \pi(p')\,|V_4|\,\pi(p)\rangle$$

$$C^{\pi}_{\phi\phi'}(\Delta t;\mathbf{p}) \rightarrow \frac{Z^*_{\pi,\phi}(|\mathbf{p}|)\, Z_{\pi,\phi'}(|\mathbf{p}'|)}{2E(p)}\, e^{-E(p)\Delta t}, \qquad Z_{\pi,\phi}(|\mathbf{p}|) = \langle O_{\pi,\phi}(\mathbf{p})|\pi(p)\rangle$$

$$R_4(\Delta t,\Delta t';\mathbf{p},\mathbf{p}') = \frac{C^{\pi\pi}_{V_4,\phi_s\phi_s}(\Delta t,\Delta t';\mathbf{p},\mathbf{p}')}{C^{\pi}_{\phi_s\phi_1}(\Delta t;\mathbf{p})\, C^{\pi}_{\phi_1\phi_s}(\Delta t';\mathbf{p}')} = \frac{\langle \pi(p')\,|V_4|\,\pi(p)\rangle}{Z^*_{\pi,\mathrm{lcl}}\, Z_{\pi,\mathrm{lcl}}\, Z_V}$$

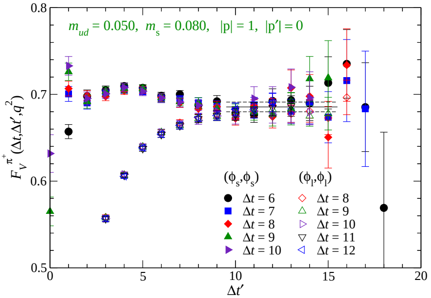
$$F_V(\Delta t,\Delta t';q^2) = \frac{2M_\pi}{E_\pi(p)+E_\pi(p')}\,\frac{R_4(\Delta t,\Delta t';\mathbf{p},\mathbf{p}')}{R_4(\Delta t,\Delta t';\mathbf{0},\mathbf{0})}$$

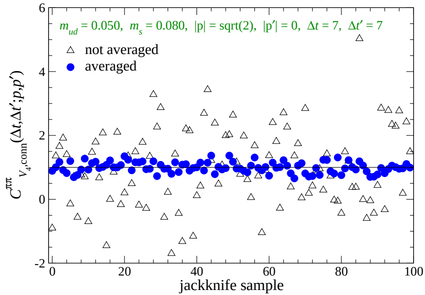
in this preliminary analysis

- use correlators w/ $(\phi,\phi')=(\phi_s,\phi_s)$
- use $E_\pi(p)$ calculated w/ dispersion relation

3.1.1 pion EM form factor : determination of $F_V^{\pi^+}(q^2)$

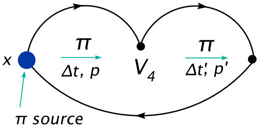
effective value $F_V^{\pi^+}(\Delta t, \Delta t'; q^2)$



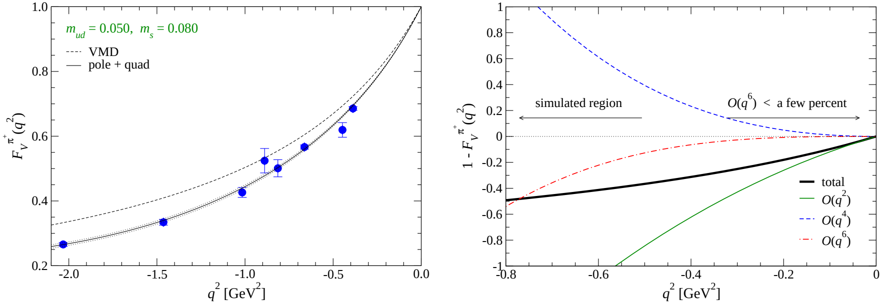


- data at arbitrary combinations of $(\Delta t, \Delta t')$

  $\Leftrightarrow$ conventional : $\Delta t + \Delta t'$ fixed
- can take average over source location x
- statistical accuracy $\sim 1-3\,\%$



3.1.2 pion EM form factor : q^2 dependence



- close to VMD near $q^2 = 0$ $\Rightarrow$ include ρ meson pole w/ measured mass

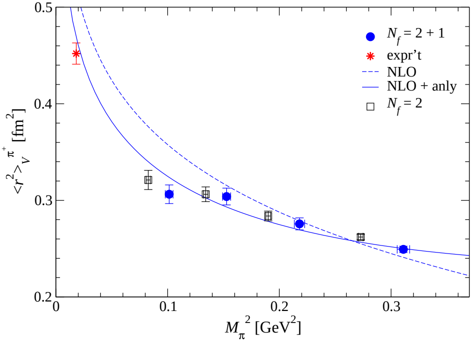
  $\Rightarrow$ approximate deviation (higher poles/cuts) by generic polynomial form

$$F_V(q^2) = \frac{1}{1 - q^2/M_\rho^2} + c_1\, q^2 + c_2\, (q^2)^2 + c_3\, (q^2)^3 = 1 + \frac{\langle r^2 \rangle_V}{6}\, q^2 + \ldots$$

- do not fit based on ChPT : $O(q^6)$ (NNNLO) contribu. is small at $|q^2| \lesssim$ (0.550 GeV)^2
- simulated pion masses : $M_\pi^2 \lesssim$ "(0.550 GeV)^2"

  m_q dependence of $\langle r^2 \rangle_V^{\pi^+}$ may be described by NNLO ChPT ?

3.1.3 pion EM form factor : chiral fit of radius



- in NLO ChPT *(Gasser-Leutwyler, 1985)*

$$\langle r^2\rangle_V^{\pi^+} = \frac{2}{NF_0^2}(-3+24NL_9^r) - 2\nu_\pi - \nu_K$$

$$\nu_X = (1/2NF_0^2)\ln[M_X^2/\mu^2]$$

  - $N=(4\pi)^2;\quad \mu=4\pi F_0$
  - use $F_0 = 52$ MeV from $M_{\pi,K}$, $F_{\pi,K}$ *(talk by Noaki)*

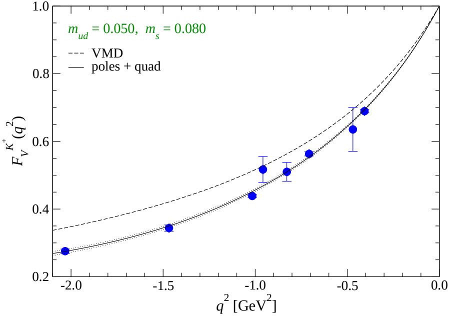
    $\Leftrightarrow\quad F_0=88$ MeV *(Bijnens, 2009)*

- small effect of sea strange quarks
- small F_0 $\Rightarrow$ enhanced NLO log $\Leftrightarrow$ $N_f=2$
- NLO fit : large $\chi^2/\mathrm{dof} \sim 11$
- $M_\pi^2/F^2 \Rightarrow \xi = M_\pi^2/F_\pi^2$: does not help ...
- NLO + anly : reduce χ^2/dof to ~ 2.8, extrap. $\sim$ expr't $\Rightarrow$ need NNLO analysis

3.2.1 kaon form factor : q^2 dependence

K^+

$$\langle K^+(p')|j_\mu|K^+(p)\rangle = (p+p')_\mu\, F_V^{K^+}(q^2)$$

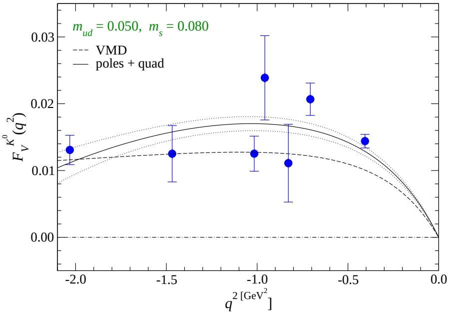


- vector poles + polynomial

$$\frac{2}{3}\frac{1}{1-q^2/M_\rho^2} + \frac{1}{3}\frac{1}{1-q^2/M_\phi^2} + \dots$$

K^0

$$\langle K^0(p')|j_\mu|K^0(p)\rangle = (p+p')_\mu\, F_V^{K^0}(q^2)$$



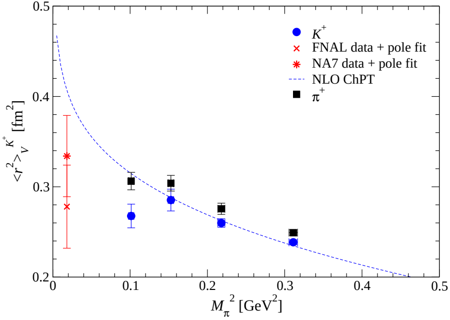
- small (“*d*”-”*s*”) but nonzero signal
- vector poles + polynomial

$$-\frac{1}{3}\frac{1}{1-q^2/M_\rho^2} + \frac{1}{3}\frac{1}{1-q^2/M_\phi^2} + \dots$$

3.2.2 kaon EM form factor : chiral fit of radius

K^+

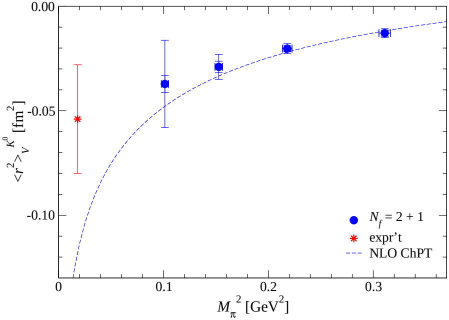
$$\langle r^2\rangle_V^{K^+} = (2/NF^2)(-3+24NL_9^r)-\nu_\pi - 2\nu_K$$



- slightly smaller than $\langle r^2\rangle_V^{\pi^+}$
- $\chi^2/\mathrm{dof} \sim 3.2$

K^0

$$\langle r^2\rangle_V^{K^0} = \nu_\pi - \nu_K$$



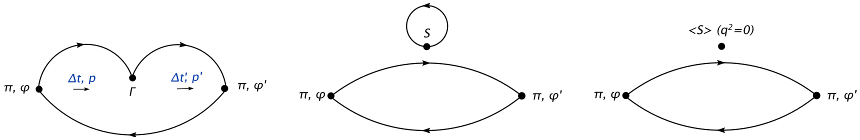
- no $O(p^4)$ coupling at NLO
- consistent w/ NLO ChPT within large sys. err. $\Rightarrow$ TBC could be helpful

analysis to be extended to NNLO : $\langle r^2\rangle_V^{\pi^+}$, $\langle r^2\rangle_V^{K^+}$, $\langle r^2\rangle_V^{K^0}$ share $O(p^6)$ LECs

4.1 scalar form factor: determination of $F_S^\pi(q^2)$

ratio method

$$\langle \pi(p') \,|S|\, \pi(p) \rangle \quad = \quad F_S^\pi(q^2)$$



$$\frac{F_S(\Delta t, \Delta t'; q^2)}{F_S(\Delta t, \Delta t'; 0)} \quad = \quad \frac{R_S(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}{R_S(\Delta t, \Delta t'; \mathbf{0}, \mathbf{0})} \; \Rightarrow \; \langle r^2 \rangle_S^\pi$$

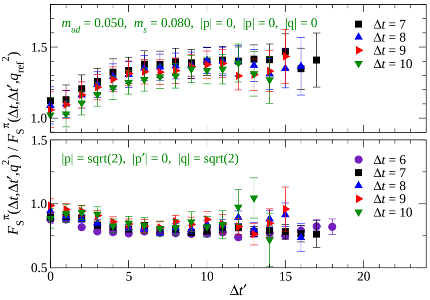
$$R_S(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}') \quad = \quad \frac{C^{\pi\pi}_{S,\phi_s\phi_s,\mathrm{sngl}}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}{C^{\pi}_{\phi_s\phi_1}(\Delta t; \mathbf{p})\, C^{\pi}_{\phi_1\phi_s}(\Delta t'; \mathbf{p}')} = \frac{\langle \pi(p') \,|S|\, \pi(p) \rangle}{Z^*_{\pi,\mathrm{lcl}}\, Z_{\pi,\mathrm{lcl}}\, Z_S}$$

$$F_S(\Delta t, \Delta t'; 0) \quad \Leftarrow \quad C^{\pi\pi}_{S,\mathrm{sngl}} = C^{\pi\pi}_{S,\mathrm{conn}} - \left(C^{\pi\pi}_{S,\mathrm{disc}} - C^{\pi\pi}_{S,\mathrm{vev}} \right) \quad \text{at } q^2 = 0$$

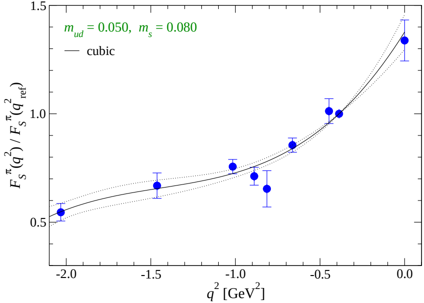
$$\frac{F_S(\Delta t, \Delta t'; q^2)}{F_S(\Delta t, \Delta t'; q^2_{\mathrm{ref}})} \quad = \quad \frac{R_S(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}{R_S(\Delta t, \Delta t'; \mathbf{1}, \mathbf{0})} \quad (\text{normalized @ } |\mathbf{p}| = 1, |\mathbf{p}'| = 0)$$

4.2 scalar form factor: effective plot ; q^2 dependence

effective value



q^2 dependence

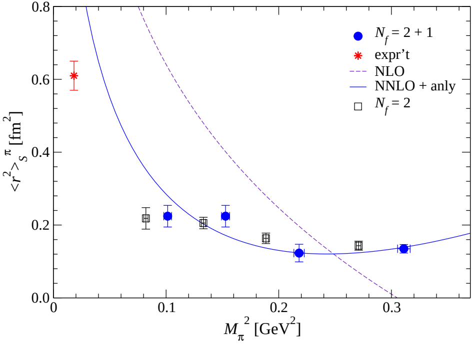


- statistical accuracy $\sim$ 5 – 10 % $\Rightarrow$ $C^{\pi\pi}_{S,\mathrm{disc}}, C^{\pi\pi}_{S,\mathrm{VEV}}$
- q^2 dependence : lack of knowledge on scalar resonances at simulated m_q

  $\Rightarrow$ use simple/generic polynomial form $\Rightarrow$ $\chi^2/\mathrm{dof} \sim 1$

$$F_S(q^2) = 1 + \frac{\langle r^2 \rangle_S}{6} q^2 + c_S (q^2)^2 \ \left[+d_S (q^2)^3 \right]$$

4.3 scalar form factor: chiral fit of radius



- in NLO ChPT *(Gasser-Leutwyler, 1985)*

$$\langle r^2 \rangle_S^\pi = \frac{1}{N F_0^2} \{-8 + 24N(2L_5^r + L_4^r)\}$$
$$-12\,\nu_\pi - 3\,\nu_K$$

  - $N = (4\pi)^2; \quad \mu = 4\pi F_0$
  - use $F_0 = 52$ MeV

- small effect of sea strange quarks
- $N_f = 2$ and $N_f = 2+1$: $\langle r^2 \rangle_S^\pi$ has 6 times larger NLO log than $\langle r^2 \rangle_V^\pi$
- $N_f = 2+1$: small F_0 further enhances chiral log
  $\Rightarrow$ fail to reproduce lattice data $(\chi^2/\mathrm{dof} \sim 100)$
- need NNLO corrections
  cf. much smaller $\chi^2/\mathrm{dof} \sim 7$ by including NNLO analytic

5.1 $K \to \pi$ form factor: determination of $f_{0,+}(q^2)$

$$\langle \pi^+(p')|V_\mu|K^0(p)\rangle = (p+p')_\mu f_+(q^2) + (p-p')_\mu f_-(q^2), \quad f_0(q^2) = f_+(q^2) + \frac{q^2}{M_K^2 - M_\pi^2} f_-(q^2)$$

ratio method

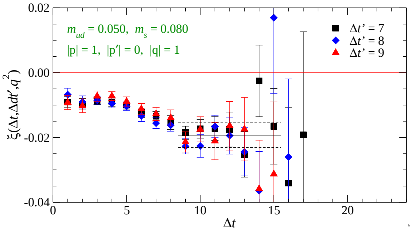
use ratios employed in previous studies *(Bećirević et al., 2005; JLQCD, 2006; RBC, 2006)*

$$R = \frac{C_{V_4}^{K\pi}(\Delta t, \Delta t'; \mathbf{0}, \mathbf{0}) C_{V_4}^{\pi K}(\Delta t, \Delta t'; \mathbf{0}, \mathbf{0})}{C_{V_4}^{KK}(\Delta t, \Delta t'; \mathbf{0}, \mathbf{0}) C_{V_4}^{\pi\pi}(\Delta t, \Delta t'; \mathbf{0}, \mathbf{0})} \to \frac{(M_K + M_\pi)^2}{4 M_K M_\pi} f_0(q^2_{\rm max})^2 \quad (q^2_{\rm max} = (M_K - M_\pi)^2)$$

$$\tilde{R} = \frac{C_{V_4}^{K\pi}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}') C^\pi(\Delta t, \mathbf{0})\, C^\pi(\Delta t', \mathbf{0})}{C_{V_4}^{K\pi}(\Delta t, \Delta t'; \mathbf{0}, \mathbf{0}) C^\pi(\Delta t, \mathbf{p})\, C^\pi(\Delta t', \mathbf{p}')} \to \left\{ 1 + \frac{E_K(\mathbf{p}) - E_\pi(\mathbf{p}')}{E_K(\mathbf{p}) + E_\pi(\mathbf{p}')} \xi(q^2) \right\} \frac{f_+(q^2)}{f_0(q^2_{\rm max})}$$

$$R_k = \frac{C_{V_k}^{K\pi}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}') C_{V_4}^{KK}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}{C_{V_4}^{K\pi}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}') C_{V_k}^{KK}(\Delta t, \Delta t'; \mathbf{p}, \mathbf{p}')}$$

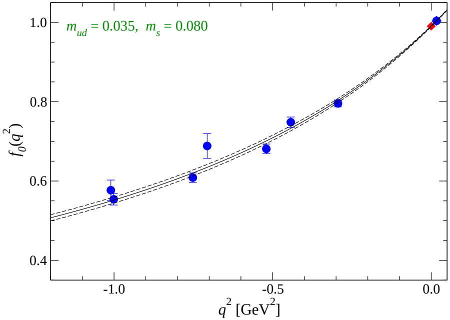
$\to$ a function of $\xi(q^2)$ $\quad (\xi(q^2) = f_-(q^2)/f_+(q^2))$



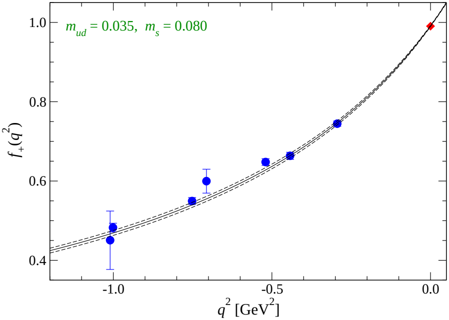
$\Rightarrow$ can construct $f_+(q^2)$ and $f_0(q^2)$

5.2 $K \to \pi$ form factor: q^2 dependence

$f_0(q^2)$ vs q^2



$f_+(q^2)$ vs q^2



- described reasonably well by polynomial, free-pole, free-pole + poly forms

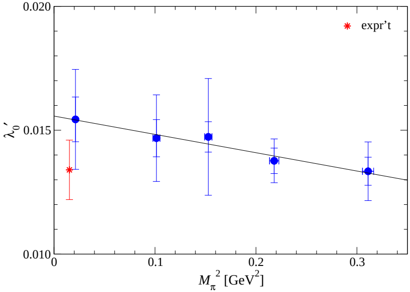
$$f_X(q^2) = f_X(0)\left\{1 + c_{X,1}q^2 + c_{X,2}(q^2)^2 + [c_{X,3}(q^2)^3]\right\}, \quad \frac{f_X(0)}{1 - q^2/M^2_{\mathrm{X,pole}}}, \ldots \quad (X = 0, +)$$

- $f_+(0) = f_0(0)$
- $\lesssim 1\,\%$ deviation in $f_+(0)$ $\Rightarrow$ to be confirmed w/ TBC

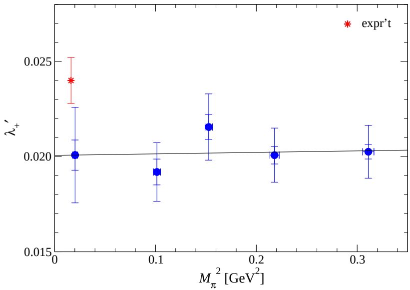
5.3 $K \to \pi$ form factor: q^2 dependence

$$f_X(q^2) = f_X(0) + c_{X,1} q^2 + c_{X,2} q^2 + \ldots, \quad \lambda'_X = M_\pi^2 c_{X,1} \quad (X = 0, +)$$

λ'_0



λ'_+

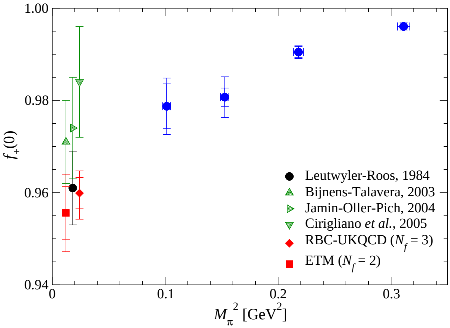


- mild quark mass dependence : $m_{s,\mathrm{sim}} - m_{s,\mathrm{phys}} \Rightarrow$ not large effect (?)
- reasonably consistent with experiment *(PDG,2008)*
- curvature

$$\lambda''_+ = 2c_{+,2} M_\pi^4 = 0.08\,(0.10) \times 10^{-2} \quad \Leftrightarrow \quad 0.20(0.05) \times 10^{-2} \text{ (expr't)}$$

5.4 $K \to \pi$ form factor: chiral behavior

- $f_+(0) : \Gamma \propto |V_{us} f_+(0)|^2$



- smaller M_π^2 $\Rightarrow$ $q^2_{\rm max}$ deviates from 0 $\Rightarrow$ larger uncertainty of $f_+(0)$
- to be improved by using TBC on larger volume

  (and reweighting of m_s)

6. summary

light meson form factors in $N_f = 2 + 1$ QCD with overlap quarks

- w/ all-to-all propagators
  - can re-use to calculate various observables : $F^{\pi}_{\{V,S\}}$, $F_V^{\{K^+,K^0\}}$, $f_{\{+,0\}}$
  - precise determination : exact low-mode + average over source location
- overlap action $\Rightarrow$ comparison w/ $a = 0$ ChPT
  - NLO ChPT fits : fail to reproduce $\langle r^2\rangle_V^\pi$ and $\langle r^2\rangle_S^\pi$
  - extension to NNLO (*cf.* $N_f = 2$: JLQCD/TWQCD, 2009)
    very complicated form w/ many $O(p^6)$ couplings
    $\Rightarrow$ simultaneous fit to different observables : *cf.* $\langle r^2\rangle_V^{\{\pi^+,K^+,K^0\}}$
- being extended to ...
  - larger volume $24^3 \times 48$: $M_\pi L \gtrsim 4$ at all m_{ud}'s
  - twisted boundary conditions : important for $f_+(0)$, $F_V^{K^0}$
  - non-trivial topological sectors
  - other observables : *cf.* pion strange form factor $\langle\pi|\bar{s}s|\pi\rangle$ $\Rightarrow$ L_4^r