PERTURBATIVE IMPROVEMENT OF SU(N) GAUGE THEORY WITH WILSON FERMIONS IN HIGHER REPRESENTATIONS

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Motivation: Technicolor

- Electro-weak symmetry breaking
  - In the SM: "Unnatural" Higgs field (=Fund. scalar)
  - Can be replaced by new strong dynamics
  - New chiral symmetry, spontaneously broken
  - M_W, M_Z from this condensate

Lagrangian of EW theory with TC

$$\mathcal{L} = \mathcal{L}_{\mathrm{EW}}(\mathrm{H}=0) + \mathcal{L}_{\mathrm{TC}}$$

$$\mathcal{L}_{\mathrm{TC}} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \sum_f \bar{\psi}_f \not{D} \psi$$

Motivation: First studies

- In the first studies[1] the action was

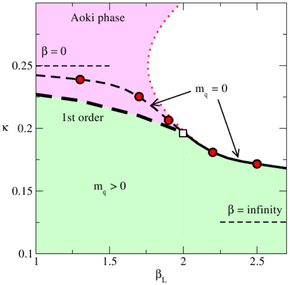
Unimproved wilson action for $SU(2)$ with 2 adjoint fermions

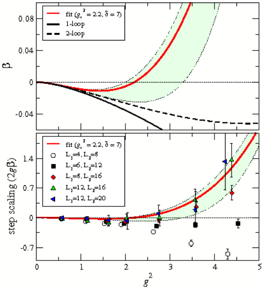
$$S_u = S_G + S_F$$

$$\begin{aligned} S_G &= \frac{1}{g_0^2}\sum_p \mathrm{Tr}[1 - U(p)], \\ S_F &= \sum_f \sum_x \bar{\psi}_f(x)(D_x + m_0)\psi_f(x) + \mathcal{O}(a), \end{aligned}$$

[1] S. Catterall, F. Sannino arXiv:0705.1664v1
L. Del Debbio, A. Patella, C. Pica arXiv:0805.2058v1
Catterall, J. Giedt, Sannino, J. Schneible arXiv:0807.0792v2
A. Hietanen, J. Rantaharju, K. Rummukainen, K. Tuominen arXiv:0812.1467
A. Hietanen, K. Rummukainen, K. Tuominen arXiv:0904.0864

Motivation: First studies





- Mass spectrum: Chiral symmetry unbroken when $\beta_L = \frac{4}{g_0^2} > 2$
- Chiral condensate can't appear in IR conformal theory: IRFP possible
- Behavior of coupling constant: IRFP at $g_{SF}^2 = 2.2$
- Step scaling function: Possibility of lattice artifacts

Introduction: Schrödinger functional

- Spacetime is a cylinder and its size L in all directions
- The boundary conditions $C(\eta)$, $C'(\eta)$ induce a constant color-electric background field

SF can be seen as an effective action

$$\Gamma = -\ln Z[C'(\eta), C(\eta)] = g_0^{-2}\Gamma_0 + \Gamma_1 + \mathcal{O}(g_0^2)$$

The running coupling can be defined as the system's response to the background field

$$\begin{aligned} \frac{1}{\bar{g}^2} &= \frac{\partial \Gamma}{\partial \eta} \\ \Rightarrow \bar{g}^2 &= g_0^2 - \frac{\partial \Gamma_1}{\partial \eta} / \frac{\partial \Gamma_0}{\partial \eta} g_0^4 + \mathcal{O}(g_0^6) \end{aligned}$$

Improvement

- Remove $\mathcal{O}(a)$ contributions

Schrödinger functional scheme action

$$S_i = S_u + \delta S_V + \delta S_{G,b} + \delta S_{F,b}$$

$$\begin{aligned}
\delta S_V &= \frac{ia^5}{4} c_{sw} \sum_{x_0=a}^{L-a} \sum_{\vec{x}} \bar{\psi}(x)\sigma_{\mu\nu}\hat{F}_{\mu\nu}(x)\psi(x) \\
\delta S_{G,b} &= \frac{1}{2g_0^2}(c_s - 1)\sum_{p_s} \mathrm{Tr}[1 - U(p_s)] \\
&\quad + \frac{1}{g_0^2}(c_t - 1)\sum_{p_t} \mathrm{Tr}[1 - U(p_t)] \\
\delta S_{F,b} &= a^4(\tilde{c}_s - 1)\sum_{\vec{x}}[\hat{O}_s(\vec{x}) + \hat{O}_s'(\vec{x})] \\
&\quad + a^4(\tilde{c}_t - 1)\sum_{\vec{x}}[\hat{O}_t(\vec{x}) - \hat{O}_t'(\vec{x})]
\end{aligned}$$

Improvement

- Coefficients known so far only in fundamental representation

Expansion of the perturbative improvement coefficients

$$c_x = 1 + c_x^{(1)} g_0^2 + \mathcal{O}(g_0^4)$$

Improvement: $\tilde{c}_s$ and c_s

- Coefficients $\tilde{c}_s$ and c_s are not needed

Fermion fields are set to zero on the boundaries $T=0$, $T=L$

$$\begin{aligned}
\hat{O}_s(\vec{x}) &= \frac{1}{2}\bar{\psi}(0,\vec{x})P_-\gamma_k(\nabla_k^* + \nabla_k)P_+\psi(0,\vec{x}) \\
\hat{O}'_s(\vec{x}) &= \frac{1}{2}\bar{\psi}(L,\vec{x})P_+\gamma_k(\nabla_k^* + \nabla_k)P_-\psi(L,\vec{x}) \\
\Rightarrow \delta S_{F_s,b} &= a^4(\tilde{c}_s - 1)\sum_{\vec{x}}[\hat{O}_s(\vec{x}) + \hat{O}'_s(\vec{x})] = 0
\end{aligned}$$

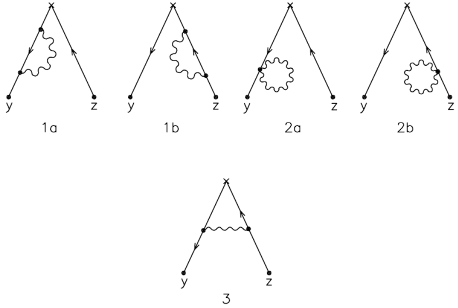
Abelian boundary conditions: $U(p_s) = 1$

$$\Rightarrow \delta S_{G_s,b} = \frac{1}{2g_0^2}(c_s - 1)\sum_{p_s}\mathrm{Tr}[1 - U(p_s)] = 0$$

Improvement: $\tilde{c}_t$

- $\tilde{c}_t^{(1)} = -0.0135(1) \times C_F$ in the fundamental rep.[2]

Figure: Diagrams contributing to calculation of $\tilde{c}_t$.[3]



- Diagrams proportional to $\sum_a (T^a)^2 = C_R$
- Higher representations of $SU(N)$: replace C_F with C_R

[2] M. Lüscher, P. Weisz hep-lat/9606016

[3] M. Lüscher, P. Weisz hep-lat/9606016

Improvement: c_t, fundamental representation

- The coefficient $c_t^{(1)}$ can be written as
  $c_t^{(1)} = c_t^{(1,0)} + N_F \times c_t^{(1,1)}$
- $c_t^{(1,0)}$ is due to the pure gauge part of the action
  Known for $SU(2)$[4] and $SU(3)$[5]
- $c_t^{(1,1)}$ is due to the fermionic part of the action
  Known for $SU(2)$ and $SU(3)$ for fund. rep. fermions[6]

N	$c_t^{(1,0)}$	$c_t^{(1,1)}$
2	$-0.0543(5)$	$0.0191410(1)$
3	$-0.08900(5)$	$0.0191410(1)$

[4] M. Lüscher, P. Weisz, U. Wolff, R. Narayanan hep-lat/9207009v1
[5] Lüscher, Weisz, Wolff, R. Sommer hep-lat/9309005
[6] S. Sint, R. Sommer hep-lat/9508012

Improvement: c_t, higher representations

- Set $c_{sw} = \tilde{c}_s = \tilde{c}_t = c_s = 1$
  - On-shell improvement
- $c_t^{1,1}$ is obtained by calculating running coupling to 1-loop

Match running ($\bar{g}$) and renormalized (g_{lat}) coupling

$$\bar{g}^2 = g_{lat}^2 + [p_{1,0} + N_F p_{1,1} + 2(b_{0,0} + N_F b_{0,1}) \ln L] g_{lat}^4 + \mathcal{O}(g_{lat}^6)$$

$$p_{1,1} = \frac{\frac{\partial}{\partial \eta} \ln \det \Delta_2}{2 N_F \frac{\partial}{\partial \eta} \Gamma_0}, \qquad \Delta_2 = [\gamma_5 (D + m_0)]^2$$

- $p_{1,1}$ can be calculated by solving the eigenvalues of Δ_2 for $L \in [2, 64]$

Improvement: c_t, higher representations

$p_{1,1}$ has a series expansion in large L limit

$$p_{1,1} \sim \sum_{n=0}^{\infty} (r_n + s_n \ln L)/L^n$$

$$r_1 = 2c_t^{1,1} \quad s_0 = 2b_{0,1} = -2 \times \frac{4}{3} T_R \quad s_1 = 0$$

- $c_t^{1,1}$ is obtained by making fit to $p_{1,1}$ data using Blocking transformation[7]

[7] M. Lüscher, P. Weisz Nucl.Phys.B266:309,1986

Improvement: c_t, higher representations

- Our perturbative results for $c_t^{(1,1)}$ are

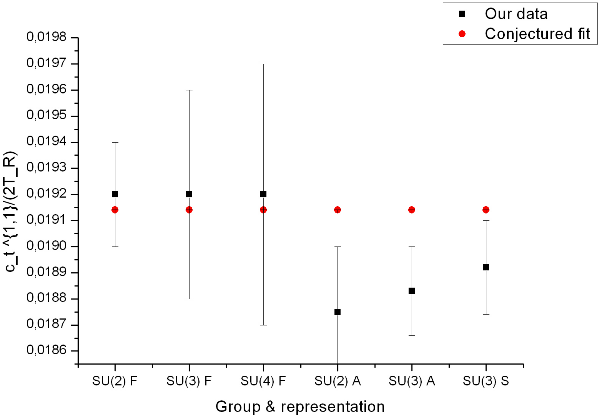
N	Fundamental	Adjoint	Sextet
2	0.0192(2)	0.075(1)	
3	0.0192(4)	0.113(1)	0.0946(9)
4	0.0192(5)		

All of these results are in agreement with

$$c_t^{(1,1)} = 0.019141 \times (2T_R)$$

Improvement: c_t, higher representations

Figure: Our results of $c_t^{1,1}$ scaled with $2T_R$ compared with conjectured value of $c_t^{1,1}/(2T_R)$



Improvement: c_{sw}

- Coefficient c_{sw} can be determined nonperturbatively
- Our results are represented in the poster session: "Non-perturbative improvement of SU(2) gauge theory with fundamental or adjoint representation fermions" by Anne-Mari Mykkänen and Jarno Rantaharju

Conclusion

- The groundwork for simulations on improved $SU(2)$ gauge theory with adjoint fermions has been done

  (T.Karavirta, J.Rantaharju, A-M. Mykkänen, K. Rummukainen, K.Tuominen, to appear)

- Numerical production for physical results can begin

  (T.Karavirta, J.Rantaharju, A-M. Mykkänen, K. Rummukainen, K.Tuominen, in progress)

Thank you!